ON OPTIMAL PRODUCT-MIX FOR PROFIT OPTIMIZATION OF SHARKS BAKERY COMPANY

ABSTRACT

In this paper, linear programming model was applied to Sharks bread Production Company to determine the right product-mix and quantities that maximize the company's daily profit. The data collected on the products and resources were analyzed. The result showed that 30 loaves of coconut, 411 loaves of fruit, 50 loaves of almond and 270 loaves of wheat bread maximize the company's profit. Furthermore, the sensitivity analysis showed the lower and upper limits (of both per unit profits and resources) within which the optimal profit of the company is maintained.

Keywords: Product-mix, Optimal profit, Linear programming, Sensitivity analysis

1.1. Introduction

The greatest challenge of decision maker(s) in production planning is how best production resources can be combined so as to attain optimality in customers' satisfaction, cost minimization, profit maximization and business sustenance. This "best way" is called optimization. Every production Company needs to understand the concept behind the optimization problem of product-mix so as to be able to focus on her core business, meet her customers' needs and have a good inventory keeping and management.

The product-mix problem occurs in an industry where it is possible to manufacture a variety of products. The product-mix determination is very important as it helps a company to focus its production on relevant products that enhance business optimization: Preferred to See [13]. Nevertheless, a company may be tempted to add more product lines in their quest to have more customers, thereby entering into the risk of including such products that appeal only to a little fraction of its customer base: Preferred to See [6]. A quantitative decision making technique called linear programming can be used to effectively solve the product-mix problem of any production company.

Linear programming is therefore defined as a mathematical technique that can determine the right product-mix that will maximize the total profit, considering the fact that different products require different amounts of production resources, having different costs and revenues at different stages of production: See for example [1], [3] and [9]. [10] reported that, linear programming plays an important role in improving managerial decisions being that it is capable of solving problems such as production planning, allocation of resources, inventory control and advertisement. In solving linear programming problems, a well known mathematical technique called simplex algorithm which uses progressive iterative steps to arrive at optimal solution plays a very important role.

The simplex algorithm solution technique of linear programming has been used by several researchers in the determination of product-mix: See for example [3],[5],[16],[1], [14],[9],[13],[6] and [4]. [2] and [7] used M-method in solving their linear programming problems. [8] used big M and dual-simplex methods as alternative methods of finding solutions to linear programming that can reduce the number of iterations and also save valuable time. [12] applied linear programming for production planning and noted that it could reduce production cost to the barest minimum. [11] used LP in Software company to determine the number of projects of each type a company should take in one time period in order to maximize profit given the resources and the constraints. [15] used LP model to schedule drivers in a transport company so as to determine the minimum number of drivers needed for each shift in a day in order to reduce the amount of money spent for the reserved drivers.

In this study, we want to generate a production template for Sharks bakery production company that will bring to their knowledge the right product-mix with their profit limit specifications, quantities of the needed resources to be made available for their daily production and the very resources that could bring production to a halt due to their scarcity (so that there could be provision of extra units of such resources kept stand-by).

1.2. Sharks Bakery Production Company

Sharks bakery is a food and confectionary company that produces different kinds of bread and some other small chops like meat pie, doughnut and buns. They also have the food section where they prepare both local and continental dishes. This company is located in Kaduna and has been in existence for about nine months as at the time the data for this work was collected. This study focuses on the bread section of the company because of the customers' preference affection for their bread. This company has not been consistent in their production quantities and as a result, customers come sometimes only to be told that there is no bread available; and sometimes, they produce more than can be exhausted in a day. Their manager cannot even say for sure that this is what they make as profit per day. The resources are not being well managed. The scenario described above informed the research so as to provide a production template to be followed in order to optimize the business.

2.0. Materials and Methods

2.1. Materials.

The data used in this work was obtained from Sharks bakery company, Kaduna. The data collected include the raw materials used for the production of different types of bread and the profit made from each of the products. Table 1 shows the raw materials and their specified quantities.

Table 1: Raw materials and their available and required quantities

Ingredients	Quantity Required (Per Product)						Available
	x1	x2	x3	x4	x5	x6	Resources
All Purpose flour (Kg)	0.2	0.21	0.2	0.22	0	0	**150**
Sugar (g)	0.17	0.15	0.21	0.2	0.22	0.18	**500**
Yeast (Kg)	0.02	0.02	0.02	0.02	0.02	0.02	**20**
Salt (g)	0.0011	0.002	0.003	0.0027	0.0015	0.0013	**10**
Wheat gluten (g)	0.00015	0.0018	0.002	0.002	0.00016	0.0019	**15**
Olive oil (L)	0.0152	0.02	0.021	0.0167	0.0152	0	**10**
Butter (g)	0.03	0	0.046	0.054	0	0.05	**500**
Milk (L)	0.015	0	0	0.0175	0	0.0148	**4.0**
Coconut Milk (L)	0	0.05	0	0	0	0	**1.5**
Almond Flour (Kg)	0	0	0	0	0.21	0	**50**
Dried Fruit (Kg)	0.0026	0	0.0054	0	0	0	**3.5**
Wheat Flour (Kg)	0	0	0	0	0	0.23	**100**

Almond Milk (L)	0	0	0	0	0.02	0	**1.0**

The profits made from the bread types are as follows: A loaf each of special bread yields a profit of ₦150, coconut bread yields a profit of ₦130, fruit bread yields a profit of ₦85, buttered bread yields a profit of ₦110, almond bread yields a profit of ₦175 while wheat bread yields a profit of ₦200 on daily basis

2.2. Methodology

The fundamental aim of optimization is to arrive at the best possible decision in any given circumstance. LP is an operation research technique for solving optimization problems. It maximizes or minimizes a linear function having a number of decision variables and constraints or restrictions stated in the form of linear equations. The general LP problem with *n* decision variables and *m* constraints can be expressed as follows:

$$Max(Min)Z = p_1x_1 + p_2x_2 + \dots + p_nx_n = \sum_{i=1}^{n} p_ix_i$$

Subject to

$$r_{11}x_1 + r_{12}x_2 + \dots + r_{1n}x_n (\leq, =, \geq) b_1$$
$$r_{21}x_1 + r_{22}x_2 + \dots + r_{2n}x_n (\leq, =, \geq) b_2$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$
$$r_{m1}x_1 + r_{m2}x_2 + \dots + r_{mn}x_n (\leq, =, \geq) b_m$$
$$x_i \geq 0 \;\; for\; i = 1,2, \dots, n$$
$$b_j \geq 0 \; for\; j = 1,2, \dots, m$$

where

Z represents the measure of performance which is either to maximize profit or to minimize cost.

$p_1, p_2, \dots, p_n$ represent the per unit profits of the decision variables.

$x_1, x_2, \dots, x_n$ represent the quantities of the bread types to be produced.

$r_{11}, r_{12}, \dots, r_{2n}, r_{m1}, r_{m2}, \dots, r_{mn}$ represent the amounts of resources used for the production of a unit of the bread types.

$b_1, b_2, \dots, b_n$ represent the total available of each of the resources (raw materials or ingredients).

For *n* decision variables and *m* constraints, the standard form of the linear programming model is formulated as follows;

$$Max(Min)Z = p_1x_1 + p_2x_2 + \cdots + p_nx_n + 0s_1 + 0s_2 + \dots + 0s_m$$

subject to

$$r_{11}x_1 + rx_2 + \dots + r_{1n}x_n + s_1 = b_1$$
$$r_{21}x_1 + r_{22}x_2 + \dots + r_{2n}x_n + s_2 = b_2$$
$$r_{m1}x_1 + r_{m2}x_2 + \dots + r_{mn}x_n + s_m = b_m$$

$$x_1, x_2, \dots, x_n, s_1, s_2, \dots, s_n, b_1, b_2, \dots, b_m \geq 0 \qquad (Non-negativityconditions)$$

In matrix or vector notation, the standard linear programming problem can be expressed as:

$$Max(Min)Z = P'X$$

$S.t \qquad RX \leq B$

$\qquad\quad X \geq 0$

$where$

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} & 1 & \\ \vdots & \vdots & & \vdots & & \ddots 1 \\ r_{m1} & r_{m2} & \cdots & r_{mn} & & \end{bmatrix}$$

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \; P = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} \quad \text{and } X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

where

R, an ($m \times n$) matrix, is the amount of the production resources which serves as the coefficient of each decision variable

X is an n column vector which indicates the decision variables.

B is an m column vector which represents the available resource constraints.

P is an n column vector, is the (profit) objective functions coefficient of the decision variables.

2.3. Sensitivity Analysis

Sensitivity analysis also known as post optimality analysis helps to determine the limits within which the LP model parameters can be changed without affecting the current optimal solution. In this work, sensitivity analysis was carried out on the profit coefficients of the objective function and right hand side (RHS) resource constraint.

2.4. Mathematical Formulation of the Model for the Analyses

(i) Decision variables: The decision variables are defined as follows

x_1 represents the quantity of special bread to be produced

x_2 represents the quantity of coconut bread to be produced

x_3 represents the quantity of fruit bread to be produced

x_4 represents the quantity of buttered bread to be produced

x_5 represents the quantity of almond bread to be produced

x_6 represents the quantity of wheat bread to be produced.

(ii) Objective function:

$$Maximize\ Z = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4 + p_5x_5 + p_6x_6$$

Where

$p_1, p_2, p_3, p_4, p_5, p_6$ are the profit coefficients of special, coconut, fruit, buttered, almond and wheat breads respectively.

(iii) Constraints: The constraints are basically the raw materials (ingredients) used for the production of the six types of bread by the company. They are as shown in Table 1.

3.0. Result

3.1. The linear programming model for the collected data.

The linear programming model for this work is given as:

$Maximize\ Z = 150x_1 + 130x_2 + 85x_3 + 110x_4 + 175x_5 + 200x_6$

$S.t$

$0.2x_1 + 0.21x_2 + 0.20x_3 + 0.22x_4 \leq 150$ $(Flour)$

$0.17x_1 + 0.15x_2 + 0.21x_3 + 0.20x_4 + 0.22x_5 + 0.18x_6 \leq 500$ $(Sugar)$

$0.02x_1 + 0.02x_2 + 0.02x_3 + 0.02x_4 + 0.02x_5 + 0.02x_6 \leq 20$ $(Yeast)$

$0.0011x_1 + 0.002x_2 + 0.003x_3 + 0.0027x_4 + 0.0015x_5 + 0.0013x_6 \leq 10$ $(Salt)$

$0.00015x_1 + 0.0018x_2 + 0.002x_3 + 0.002x_4 + 0.00016x_5 + 0.0019x_6 \leq 15$ $(Wheat\ gluten)$

$0.0152x_1 + 0.02x_2 + 0.021x_3 + 0.0167x_4 + 0.0152x_5 \leq 10$ $(Olive\ oil)$

$0.03x_1 + 0.046x_3 + 0.054x_4 + 0.05x_6 \leq 500$ $(Butter)$

$0.015x_1 + 0.0175x_4 + 0.0148x_6 \leq 4.0$ $(Milk)$

$0.050x_2 \leq 1.5$ $(Coconut\ milk)$

$0.21\,x_5 \leq 50$ $(Almond\ flour)$

$0.0026\,x_1 + 0.0054x_3 \leq 3.5$ $(Dried\ fruits)$

$0.23x_6 \leq 100$ $(Wheat\ flour)$

$0.02x_5 \leq 1.0$ $(Almond\ Milk)$

$x_j \geq 0\ forj = 1,2,\dots,6$

To solve the above problem, Simplex method was applied using Micro Excel-Solver software.

3.1.1. Conversion of the linear programming model to a standard form

This is done by introducing the slack and surplus variables where necessary and conversion of inequality sign to equality sign. The standard model for the linear programming problem is given below;

$Maximize\ Z - 150x_1 - 130x_2 - 85x_3 - 110x_4 - 175x_5 - 200x_6 + 0S_1 + 0S_2 + 0S_3 + \cdots + 0S_{13} = 0$

$S.t.$

$0.2x_1 + 0.21x_2 + 0.20x_3 + 0.22x_4 + S_1 = 150$

$0.17x_1 + 0.15x_2 + 0.21x_3 + 0.20x_4 + 0.22x_5 + 0.18 + S_2 = 500$

$0.02x_1 + 0.02x_2 + 0.02x_3 + 0.02x_4 + 0.02x_5 + 0.02x_6 + S_3 = 20$

$0.0011x_1 + 0.002x_2 + 0.003x_3 + 0.0027x_4 + 0.0015x_5 + 0.0013x_6 + S_4 = 10$

$0.00015x_1 + 0.0018x_2 + 0.002x_3 + 0.002x_4 + 0.00016x_5 + 0.0019x_6 + S_5 = 15$

$0.0152x_1 + 0.02x_2 + 0.021x_3 + 0.0167x_4 + 0.0152x_5 + S_6 = 10$

$0.03\,x_1 + 0.046x_3 + 0.054x_4 + 0.05x_6 + S_7 = 500$

$0.015x_1 + 0.0175x_4 + 0.0148x_6 + S_8 = 4.0$

$0.050x_2 + S_9 = 1.5$

$0.21\, x_5 + S_{10} = 50$

$0.0026\, x_1 + 0.0054 x_3 + S_{11} = 3.5$

$0.23 x_6 + S_{12} = 100$

$0.02 x_5 + S_{13} = 1.0$

$x_{j^s} \geq 0\ for\ j = 1, 2, \dots, 6\ and\ S_{i^s} \geq 0\ for\ i = 1, 2, 3, \dots, 13$

Where;

$S_1, S_2, S_3, \dots, S_{13}$ are the added slack variables.

The solutions to the linear programming problem in section 3 are given in Tables 2, 3, 4 and 5.

Table 2: The summary of the optimum solution

S/N	Product (Variable)	Quantity (loaves) ()	Profit (Coefficient)	Profit Contribution in (₦)
1	Special Bread	0.00	150	0.00
2	Coconut Bread	30.00	130	3,900.00
3	Fruit Bread	411.43	85	34,971.55
4	Buttered Bread	0.00	110	0.00
5	Almond Bread	50.00	175	8,750.00
6	Wheat Bread	270.27	200	54,054.00

Table 3: The used and unused resource quantities

S/N	Constraint	R.H. S	Slack	Used
1	<=	150	61.41	88.59
2	<=	500	349.45	150.55
3	<=	20	4.77	15.23
4	<=	10	8.28	1.72
5	<=	15	13.60	1.40
6	<=	10	0	10.00
7	<=	500	467.56	32.44
8	<=	4.0	0	4.00
9	<=	1.5	0	1.50
10	<=	50	39.50	10.50
11	<=	3.5	1.28	2.22
12	<=	100	37.84	62.16
13	<=	1.0	0	1.00

Table 4: The summary of the sensitivity analysis on the objective function profit coefficients

S/N	Variable	Profit coefficient	Lower limit	Upper limit
1	Special Bread	150	-∞	114.2265
2	Coconut Bread	130	49.0476	∞

3	Fruit Bread	85	85	51.5
4	Buttered Bread	110	-∞	194.0817
5	Almond Bread	175	113.4762	∞
6	Wheat Bread	200	112.7035	∞

Table 5: Result of Sensitivity Analysis on the Resource Constraints (RHS)

S/N	Constraints	RHS Constraint	Lower limit	Upper limit	Dual Price
1	<=	150	61.4143	∞	0
2	<=	500	349.4514	∞	0
3	<=	20	4.7660	∞	0
4	<=	10	8.2794	∞	0
5	<=	15	13.6016	∞	0
6	<=	10	8.64	4.9711	4047.6191
7	<=	500	467.5608	∞	0
8	<=	4.0	4.0	2.4348	13513.51
9	<=	1.5	1.5	21.6	980.9524
10	<=	50	39.5	∞	0
11	<=	3.5	1.2783	∞	0
12	<=	100	37.8378	∞	0
13	<=	1.0	1.0	3.7619	5673.8095

4.0. Discussion

Microsoft Excel-Solver software was used to analyze the data in Table 1 and daily optimal profit of ₦101,675.48k was obtained. The respective contributions of the products to this optimal profit are as follows: 30 loaves of coconut bread (x_2) contributed ₦3,900, 411.43 loaves of fruit bread (x_3) contributed ₦34,971.55, 50 loaves of almond bread (x_5) contributed ₦8,750 and 270.27 loaves of wheat bread (x_6) contributed ₦54,054. The profit contribution of special bread (x_1) *and* buttered bread (x_4) is zero each meaning that their contribution to the profit of the company is so insignificant that the company can still succeed and survive without such products. The results in Table 3 showed that at the optimal solution, the available quantities of such ingredients (raw materials) as olive oil, milk, coconut milk and almond milk were exhausted or used up while the available quantities of all purpose flour, sugar, yeast, salt**,** wheat gluten, butter, almond flour, dried fruit and wheat flour were not all used up, meaning excess supply. The results of the sensitivity analysis in Table 4 showed the specified lower and upper limits of the profits made from the bread types within which the daily profit of the company remains optimal. Table 5 provides the allowable limits in the resources for the daily production with their dual or shadows prices (in case the company runs short of any). Finally, the study revealed olive oil, milk, coconut milk and almond milk (with zero dual prices) as scarce resources that can lead to a halt in production processes when lacked.

5.0. Conclusion

This study aims at using linear programming model to distribute raw materials to the production of different bread types in Sharks bakery company in order to know the right product-mix that actually maximizes the company's daily profit and in what capacity. The company produces six different types of bread as given below: the special, coconut, fruit, buttered, almond and wheat breads. The data collected from the company were analyzed using Microsoft Excel-Solver software and the results provided a template that will enable the company maintain optimality in their daily profit making. Based on the findings made from this study, it is concluded that LP model is a robust one that can be adopted by the production unit of any organization, company or firm since it makes known the right product-mix, their exact quantities and the amount of resources that can guarantee optimality in their profit making.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

[1] Akpan, N. P., and Iwok, I. A., "Application of linear programming for optimal use of raw materials in Bakery". *International Journal of Mathematics and Statistics Invention,* 4(8), 51-57, (2016).

[2] Anieting, A. E., Ezugwu, V. O and Ologun, S., "Application of linear programming technique in the determination of optimum production capacity". *IOSR Journal of Mathematics (IOSR-JM),* 5(6), 62-65, (2013).

[3] Balogun, O. S., Jolayemi, E. T., Akingbade, T. J. and Muazu, H. G., "Use of linear programming for optimal production in a production line in Coca-Cola Bottling Company", *International Journal of Engineering Research and Applications,* 2(5), 2004-2007, (2012).

[4] Egharevba, A. J. and Ojekudo N. A., "Optimal raw materials mix through linear programming in Tehinnah Cakes and Craft". *International Journal of Applied Science and Mathematical Theory.* 7(2) 36-43, (2021).

[5] Ezema, B. I. and Amakon, U., "Optimizing profit with linear programming model: A Focus on Golden Plastic Industry Limited Enugu Nigeria". *Interdisciplinary Journal of Research Business,* 2(2), 37-49, (2012).

[6] Ezra, P. N., Oladugba, A. V., Ohanuba, F. O., Igwe, N. O. and Okonta, C. A., "Application of linear programming on the profit maximization of Nigerian Bottling Company, *American Journal of Operational Research,* 10 (2),39-43, (2020).

[7] Jain, A. K., Saxena, H., Bhaardwaj, R., Jagannadha, G. V. and Nanda, C. S., "Application of linear programming for profit maximization of a Pharma Company", *Journal of Critical Reviews,* 7 (12). 1118-1123, (2020).

[8] Lokhande, K.., Khol, P. G and Khobragade, N. W., "Alternative approach to the optimum solution of linear programming problem", *International Journal of Latest Technology in Engineering, Management and Applied science,* 6 (6), 61-68, (2017).

[9] Mahesh, S. N., "Profit maximization in Bakery by optimal allocation of raw materials",http://aegaeum.com/ 8(5), 391-393, (2020).

[10] Maryam ,S. L., Samira M. B., Sharareh, K., Nastaran, M.,Fatemeh, Q. R., Marjan-ol-Sadat N. and Ojaghzadeh, M., "Linear programming and optimizing the resources". *Interdisciplinary Journal of ContemporaryResearch in business,* 4 (11), 701-705, (2013).

[11] Mustafa, A., Sayegh, M. and Rasheed, S., "Application of linear programming for optimal investments in Software Company". *Open Journal of Applied Sciences,* 11 (10), 1092-1101, (2021).

[12] Oladejo, L. G., Olusegun, K.. L. and Jibril, N. S. , "Application of linear programming for optimum production planning in Maidabino Investment Nigeria Limited, Katsina", *International Journal of Operational Research in Management, Social Sciences &Education,* 4(1), 166-175, (2018).

[13] Oluwaseyi, K.. O., Atsegameh, E. and Olaoluwa, O. E., "Profit maximization in a product-mix Bakery using linear programming technique". *Journal of Investment and Management,* 9(1), 27-30, (2020).

[14] Oyekan, E. A. and Temisan, G. O., "Application of linear programming to profit maximization of Johnsons Nigeria Limited., Bakery Division". *Journal of Advances in Mathematical & Computational Sciences,* 7(1), 11-20, (2019).

[15] Rama, S., Srividya, S. and Deepa, B., "A Linear programming approach for optimal scheduling of workers in a Transport Corporation". *International Journal of Engineering Trends and Technology (IJETT),* 45(10), 482-487, (2017).

[16] Zaman H., Rizwan, F., Muhammad, B.T., Syed, U., Naeem, U.and Sehrish K. (2016), "Application of Linear Programming for Profit Maximization: A Case of Paints Company, Pakistan". *International Journal of Management Sciences and Business Research,* 15(12), 144-151.