Original Research Article

Combined Exponential-Type Estimators for Finite Population Mean in Two-Phase Sampling

Abstract

A family of exponential-type estimator for estimating population mean in two-phase sampling when the population proportion of the auxiliary character is available is proposed in this paper. Theoretically, the bias and minimum mean square error (MSE) for the proposed estimator are obtained. The expression for MSE of the proposed exponential-type of estimator is compared with the existing estimators in the literature.The optimum values of the parameters are determined. An empirical study was carried out by comparing the proposed estimators with some of the existing estimatorsreviewed in the literaturebased on the criteria of bias, mean square error (MSE) and relative efficiency using life datasets. The result of the comparisons showed that the proposed exponential-type estimators produce a better estimate of finite population mean than the existing estimators in the sense of having higher percentage relative efficiency which implies lesser mean square error and bias. Furthermore, the realistic conditions under which the proposed class of exponential-type estimators is more efficient were also presented. Thus, the proposed estimators can be considered as significant alternatives to estimating population characteristics of real life datasets.

Key words: Auxiliary variable, two-phase sampling, mean square error, bias, efficiency.

1. Introduction

The estimation of the population mean by ratio, regression, and other methods of estimation in single phase sampling calls for the auxiliary data in the form of a population parameter.A two-phase sampling strategy can be employed if this information is not accessible, where a large first-phase sample measured over the auxiliary variable x is utilized to get an accurate estimate of the population parameters. A second-phase sample can then be taken and the study variable *y* with an auxiliary variable *x* can be observed. Singh et al.[1] and Muhammad et al. [2] discussed that the major advantage of using two-phase sampling is the gain in high precision without a substantial increase in cost. Several authors improved ratio and regression estimators by adopting at least one auxiliary variable in two phase sampling scheme. Singh and Ruiz-Espejo[3], Muhammad et al.[4], Zakari et al. [5] suggested a class of ratio-product estimators in two phase sampling with its properties and identified asymptotically optimum estimators from proposed class of estimators. Zaman and Kadilar[6]proposed a new class of exponential type estimator in two phase sampling schemes. Rao [7], Audu et al. [8], Audu et al. [9]suggested some estimators in two-phase sampling to stratification, non-response problems and investigative comparisons. YadavandAdewara [10] worked on the estimation of population mean of the variable of study utilizing improved ratio-product type exponential estimator and qualitative auxiliary information and establish that the proposed estimator which under optimum conditions performs better than the usual sample mean estimator. Singh andUpadhyaya[11] suggested a generalized estimator to estimate the population mean using two auxiliary variables in the two-phase sampling. However, ratio-based estimators can only be applied when the correlation between the study and auxiliary variables is positively strong. Similarly, the regression type estimator can be applied, when the regression slope does not pass through the origin, and for the product-based estimators, when the estimators are negatively correlated.The reviewed existing estimators though efficient but possesses large values of bias and mean square error and as such they can be improve upon by obtaining most efficient estimators that possesses the least values of bias and mean square error than the reviewed existing estimators. Therefore, it is based on this background this paper under simple random sampling without replacement (SRSWOR) proposed an efficient combined exponential-type estimator in two phase sampling for finite population that handles all the situations.

Following the introduction is section two which contains usual notations and literature review while section three presents methodology of the study. Section four discusses the results while conclusion is presented in section five.

2.Materials and Method

Consider a sample of size n drawn by simple random sampling without replacement (SRSWOR) from a population of size N: Let y_i and ϕ_i denote the observations on variable y and ϕ espectively for ith unit $(i = 1, 2, \ldots, N)$
Let $\phi_i = 1$; if the ith unit of the population possesses attribute,$\phi = 0$; otherwise
Let $A = \sum_{i=1}^{N} \phi_i$ and $a = \sum_{i=1}^{n} \phi_i$, denote the total number of units in the population and sample respectively possessing attribute ϕ. Let $P = \frac{A}{N}$ and $p = \frac{a}{n}$denote the proportion of units in the population and sample respectively possessing attribute ϕ.
When P is not known, two-phase sampling is used to estimate the population mean of the study variable. Consider a finite population $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)$. Let y and p be the study and auxiliary variable, taking values y_i andp_i, respectively, for the ith unit ζ_i. Under the double sampling scheme, two cases are used for the selection of the required sample as follows:
Case-I. The first phase sample $S'(S' \subset \zeta)$ of a fixed size n' is drawn to measure only on the auxiliary attribute p in order to formulate a good estimate of a population proportion P.
Case-II. Given S' , the second phase sample $S(S \subset S')$ of a fixed size *n* is drawn to measure the study variable y.
Note that:
$\bar{y} = 1/n\sum_{i\epsilon S} y_i$, $p = 1/n\sum_{i\epsilon S} a_i$, $and\ p' = 1/n\sum_{i\epsilon S'} a_i$, $C_y = S_y/\bar{Y}, C_p = S_p/P$, $\rho_{pb} = S_{yp}/(S_y S_p)$, $S_y^2 = \frac{\sum_{i=1}^{N}(y_i-\bar{Y})^2}{N-1}$, $S_p^2 = \frac{\sum_{i=1}^{N}(p_i-P)^2}{N-1}$, and$S_{yp} = \frac{\sum_{i=1}^{N}(y_i-\bar{Y})(p_i-P)}{N-1}$.
where p' is the proportion of units possessing attribute ϕ in the first phase sample of size n'; p is the proportion of units possessing attribute ϕ in the second phase sample of size $n' > n$ and $\bar{y}$is the mean of the study variable y in the second phase sample Zaman and Kadilar[6].
Naik and Gupta [12], Zakari et al. [13] suggested the classical ratio type estimator of the population mean utilizing the auxiliary attribute under the simple random sampling as

$$t_{NG} = \bar{y}\frac{P}{p'} \qquad (1)$$

Kumar andBahl[14]suggested a ratio estimator of the population mean utilizing the auxiliary attribute under two-phase sampling as

$$t^{d}_{NG1} = \bar{y}\frac{p'}{P} \qquad (2)$$

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The mean square error (MSE)equations of$t^{d}{}_{NG1}$up to the first order of approximation, for Case-I and Case-II are given respectively as;

$$MSE(t^{d}{}_{NG1})_{I} = \overline{Y}^{2}\left[\lambda C_y^2 + (\lambda - \lambda')\left(C_p^2 - 2\rho_{pb}C_yC_p\right)\right] \quad (3)$$

$$MSE(t^{d}{}_{NG1})_{II} = \overline{Y}^{2}\left[\lambda C_y^2 + (\lambda + \lambda')C_p^2 - 2\lambda\rho_{pb}C_yC_p\right] \quad (4)$$

Where $\lambda = \frac{1-\frac{n}{N}}{n} = \frac{1}{n} - \frac{1}{N} = \frac{N-n}{Nn}$, $\lambda' = \frac{1-\frac{n'}{N}}{n'} = \frac{1}{n'} - \frac{1}{N} = \frac{N-n'}{Nn'}$, $\rho_{pb}$is the population coefficient of correlation between the auxiliary attribute and studyvariable.$C_p$is the population coefficient of variation for the form of attribute and C_y is the population coefficient of variation of the study variable.

Singh and Choudhury [15] product estimator of the population mean in the two-phase sampling using information about the population proportion is given by

$$t^{d}{}_{NG2} = \overline{y}\frac{p}{p'} \quad (5)$$

The mean square error (MSE) equations of$t^{d}{}_{NG2}$; up to the first order of approximation, for Case-I and Case-II are given respectively as;

$$MSE(t^{d}{}_{NG2})_{I} = \overline{Y}^{2}\left[\lambda C_y^2 + (\lambda - \lambda')\left(C_p^2 + 2\rho_{pb}C_yC_p\right)\right] \quad (6)$$

$$MSE(t^{d}{}_{NG2})_{II} = \overline{Y}^{2}\left[\lambda C_y^2 + (\lambda + \lambda')C_p^2 + 2\lambda\rho_{pb}C_yC_p\right] \quad (7)$$

Kumar andBahl[14] using the information about the population proportion, they suggested the dual to ratio estimator of the population mean under the two-phase sampling as

$$t^{*d}{}_{NG1} = \overline{y}\frac{p'^{*}}{P} \quad (8)$$

The mean square error (MSE) expressions of $t^{*d}{}_{NG1}$; up to the first order of approximation, for Case-I and Case-II respectively are

$$MSE(t^{*d}{}_{NG1})_{I} = \overline{Y}^{2}\left[\lambda C_y^2 + \frac{n}{n'-n}(\lambda + \lambda')\left(\frac{n}{n'-n}C_p^2 - 2\rho_{pb}C_yC_p\right)\right] \quad (9)$$

$$MSE(t^{*d}{}_{NG1})_{II} = \overline{Y}^{2}\left[\lambda C_y^2 + \frac{n}{n'-n}\left\{\frac{n}{n'-n}(\lambda + \lambda')C_p^2 - 2\lambda\rho_{pb}C_yC_p\right\}\right] \quad (10)$$

Singh and Choudhury [15]considering the information about the population proportion proposed the dual to product estimator of the population mean as

$$t^{*d}{}_{NG2} = \overline{y}\frac{p}{p'^{*}} \quad (11)$$

The mean square error (MSE) expressions of $t^{*d}{}_{NG2}$; up to the first order of approximation, for Case-I and Case-II respectively are

$$MSE(t^{*d}{}_{NG2})_{I} = \overline{Y}^{2}\left[\lambda C_y^2 + \frac{n}{n'-n}(\lambda + \lambda')\left(\frac{n}{n'-n}C_p^2 + 2\rho_{pb}C_yC_p\right)\right] \quad (12)$$

$$MSE(t^{*d}{}_{NG2})_{II} = \overline{Y}^{2}\left[\lambda C_y^2 + \frac{n}{n'-n}\left\{\frac{n}{n'-n}(\lambda + \lambda')C_p^2 + 2\lambda\rho_{pb}C_yC_p\right\}\right] \quad (13)$$

Singh et al. [1] suggested the two-phase ratio and product type exponential estimators when information about auxiliary attribute is available, respectively as:

$$t_{S1} = \overline{y}exp\left(\frac{p'-p}{p'+p}\right) \quad (14)$$

$$t_{S2} = \overline{y}exp\left(\frac{p-p'}{p+p'}\right) \quad (15)$$

The mean square error (MSE) expressions oft_{S1} and t_{S2}in equations (14) and (15) up to the first order of approximation, for Case-I and Case-II are given respectively by

$$MSE(t_{S1})_{I} = \overline{Y}^{2}\left[\lambda C_y^2 + (\lambda - \lambda')\left(\frac{C_p^2}{4} - \rho_{pb}C_yC_p\right)\right] \quad (16)$$

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$$MSE(t_{S1})_{II} = \overline{Y}^2\left[\lambda C_y^2 + \frac{1}{4}(\lambda + \lambda')C_p^2 - \lambda\rho_{pb}C_yC_p\right] \quad (17)$$

and

$$MSE(t_{S2})_I = \overline{Y}^2\left[\lambda C_y^2 + (\lambda - \lambda')\left(\frac{C_p^2}{4} + \rho_{pb}C_yC_p\right)\right] \quad (18)$$

$$MSE(t_{S2})_{II} = \overline{Y}^2\left[\lambda C_y^2 + \frac{1}{4}(\lambda + \lambda')C_p^2 + \lambda\rho_{pb}C_yC_p\right] \quad (19)$$

Kalita and Singh [16] proposed exponential dual to ratio and exponential dual to product estimator in the two-phase sampling respectively as

$$t_{S1}^* = \overline{y}exp\left(\frac{p'^*-p}{p'^*+p}\right) \quad (20)$$

$$t_{S2}^* = \overline{y}exp\left(\frac{p-p'^*}{p+p'^*}\right) \quad (21)$$

For Case-I and Case-II, the mean square error (MSE) equations of the ratio and product estimators in equations (20) and (21) are respectively given by

$$MSE(t_{S1}^*)_I = \overline{Y}^2\left[\lambda C_y^2 + \frac{n}{n'-n}(\lambda - \lambda')\left\{\frac{n}{4(n'-n)}C_p^2 - \rho_{pb}C_yC_p\right\}\right] \quad (22)$$

$$MSE(t_{S1}^*)_{II} = \overline{Y}^2\left[\lambda C_y^2 + \frac{n^2}{4(n'-n)^2}(\lambda + \lambda')C_p^2 - \lambda\frac{n}{n'-n}\rho_{pb}C_yC_p\right] \quad (23)$$

$$MSE(t_{S2}^*)_I = \overline{Y}^2\left[\vartheta\lambda + \frac{n}{n'-n}(\lambda - \lambda')\left\{\frac{n}{4(n'-n)}C_p^2 + \rho_{pb}C_yC_p\right\}\right] \quad (24)$$

$$MSE(t_{S2}^*)_{II} = \overline{Y}^2\left[\lambda C_y^2 + \frac{n^2}{4(n'-n)^2}(\lambda + \lambda')C_p^2 + \lambda\frac{n}{n'-n}\rho_{pb}C_yC_p\right] \quad (25)$$

3. Proposed Estimator

In this section, a new combined exponential-type estimator is proposed using information about the population proportion possessing certain attributes in two-phase sampling as:

$$\hat{t}_{CEi} = \overline{y}\left[\beta_1 + \beta_2\left(p' - p\right)\right]\exp\left\{\frac{\left(a_xp' + b_x\right) - \left(a_xp + b_x\right)}{\left(a_xp' + b_x\right) + \left(a_xp + b_x\right)}\right\} \quad (26)$$

where β_1 and β_2 are real parameters to be determined such that the mean square error of $\hat{t}_{CEi}$ is minimum, $a_x(\neq 0)$ and b_x are either real number or the functions of the known parameters of the attribute, C_p, $\beta_2(\phi)$ and the known parameter of the attribute with the study variable, ρ_{pb}. Some classes of the proposed estimator of the population mean are obtained using the suitable choices of constants a_x and b_x and shown in Table 1.

Table 1: Suggested Classes of Combined Exponential Estimators.

Estimators	**Values of**	
	$\boldsymbol{a_x}$	$\boldsymbol{b_x}$
$\hat{t}_{CE1} = \overline{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{p' - p}{(p' + p + 2\beta_2(\phi))}\right\}$	1	$\beta_2(\phi)$
$\hat{t}_{CE2} = \overline{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{p' - p}{(p' + p + 2C_p)}\right\}$	1	C_p
$\hat{t}_{CE3} = \overline{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{p' - p}{(p' + p + 2\rho_{pb})}\right\}$	1	ρ_{pb}
$\hat{t}_{CE4} = \overline{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{\beta_2(\phi)(p' - p)}{(\beta_2(\phi)(p' + p) + 2C_p)}\right\}$	$\beta_2(\phi)$	C_p

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$\hat{t}_{CE5} = \bar{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{\beta_2(\phi)(p' - p)}{\beta_2(\phi)(p' + p) + 2\rho_{pb})}\right\}$	$\beta_2(\phi)$	ρ_{pb}
$\hat{t}_{CE6} = \bar{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{C_p(p' - p)}{C_p(p' + p) + 2\beta_2(\phi))}\right\}$	C_p	$\beta_2(\phi)$
$\hat{t}_{CE7} = \bar{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{C_p(p' - p)}{C_p(p' + p) + 2\rho_{pb}}\right\}$	C_p	ρ_{pb}
$\hat{t}_{CE8} = \bar{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{\rho_{pb}(p' - p)}{\rho_{pb}(p' + p) + 2\beta_2(\phi)}\right\}$	ρ_{pb}	$\beta_2(\phi)$
$\hat{t}_{CE9} = \bar{y}[\beta_1 + \beta_2(p' - p)]exp\left\{\frac{\rho_{pb}(p' - p)}{\rho_{pb}(p' + p) + 2C_p}\right\}$	ρ_{pb}	C_p

$\theta_1 = \frac{P}{2(P + \beta_2(\phi))}$; $\theta_2 = \frac{P}{2(P + C_p)}$; $\theta_3 = \frac{P}{2(P + \rho_{pb})}$; $\theta_4 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P + C_p)}$;

$\theta_5 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P + \rho_{pb})}$; $\theta_6 = \frac{C_pP}{2(C_pP + \beta_2(\phi))}$; $\theta_7 = \frac{C_pP}{2(C_pP + \rho_{pb})}$; $\theta_8 = \frac{\rho_{pb}P}{2(\rho_{pb}P + \beta_2(\phi))}$;

$\theta_9 = \frac{\rho_{pb}P}{2(\rho_{pb}P + C_p)}$

Properties of the Proposed Estimator

Case-I. To obtain the properties of the estimator$\hat{t}_{CEi}$, let $\bar{y} = \bar{Y}(1 + e_0)$, $p = P(1 + e_1)$, *and* $p' = P(1 + e_1')$Such that

$$E(e_0) = E(e_1) = E(e_1') = 0$$
$$E(e_0^2) = \lambda C_y^2, \quad E(e_1^2) = \lambda C_p^2, \quad E(e_1'^2) = \lambda' C_p^2$$
$$E(e_0 e_1) = \lambda\rho C_y C_p, \quad E(e_0 e_1') = \lambda'\rho C_y C_p, \quad E(e_1 e_1') = \lambda' C_p^2$$

Expressing the estimator $\hat{t}_{CEi}$ in terms of e_i $(i = 0,\ 1)$ we can write (26) as

$$\hat{t}_{CEi} = \bar{Y}(1+e_0)\left[\beta_1 + \beta_2\left[P(1+e_1') - P(1+e_1)\right]\right]\exp\left\{\frac{\left[a_xP(1+e_1')+b_x\right] - \left[a_xP(1+e_1)+b_x\right]}{\left[a_xP(1+e_1')+b_x\right] + \left[a_xP(1+e_1)+b_x\right]}\right\} \quad (27)$$

By some appropriate simplifications, (27) becomes

$$\hat{t}_{CEi} = \bar{Y}(1+e_0)\left[\beta_1 + \beta_2\left(P + Pe_1' - P - Pe_1\right)\right]\exp\left\{\frac{a_xP(e_1' - e_1)}{2(a_xP+b_x)\left[1 + \frac{a_xP(e_1'+e_1)}{2(a_xP+b_x)}\right]}\right\} \quad (28)$$

By Letting $\theta = \frac{a_xP}{2(a_xP+b_x)}$ from (28), we get

$$\hat{t}_{CEi} = \bar{Y}(1+e_0)\left[\beta_1 + \beta_2(Pe_1' - pe_1)\right]\exp\left\{\theta(e_1' - e_1)\left[1+\theta(e_1'+e_1)\right]^{-1}\right\} \quad (29)$$

Thus it follows

$$\hat{t}_{CEi} = \bar{Y}\left[\beta_1 + \beta_1 e_0 - \beta_2 Pe_1 + \beta_2 Pe_1' + \beta_2 Pe_0e_1' - \beta_2 Pe_0e_1\right] * \left\{1 + \theta e_1' - \theta e_1 - \frac{\theta^2 e_1'^2}{2} + \frac{3\theta^2 e_1^2}{2} - \theta^2 e_1 e_1'\right\} \quad (30)$$

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Expanding the right hand side of (30) to the first order of approximation, multiplying out and neglecting the terms of e's greater than two, it gives

$$\hat{t}_{CEi}=\bar{Y}\left[\begin{array}{l}\beta_1+\beta_1 e_0-(\theta\beta_1+\beta_2 P)e_1+(\theta\beta_1+\beta_2 P)e_1^{'}+\dfrac{(3\theta^2\beta_1+2\theta P\beta_2)e_1^{2}}{2}-\dfrac{(\theta^2\beta_1-2\theta P\beta_2)e_1^{'2}}{2}\\ -(\theta\beta_1+\beta_2 P)e_0 e_1+(\theta\beta_1+\beta_2 P)e_0 e_1^{'}-(\theta^2\beta_1-2\theta P\beta_2)e_1 e_1^{'}\end{array}\right] \tag{31}$$

Subtracting $\overline{Y}$ and taking expectation to both sides of (31), the bias of the estimator $\hat{t}_{CEi}$ is derived as

$$Bias(\hat{t}_{CEi})=\bar{Y}\left[(\beta_1-1)+\left(\lambda-\lambda^{'}\right)\left\{\frac{\left(3\theta^2\beta_1+2\theta P\beta_2\right)C_p^2}{2}-(\theta\beta_1+P\beta_2)\rho C_y C_p\right\}\right] \tag{32}$$

Similarly, subtracting $\overline{Y}$, taking expectation, and squaring both sides of (31), the mean squared error is obtained as

$$E\left[\hat{t}_{CEi}-\bar{Y}\right]^2=\bar{Y}^2 E\left[\begin{array}{l}(\beta_1-1)+\beta_1 e_0-(\theta\beta_1+\beta_2 P)e_1+(\theta\beta_1+\beta_2 P)e_1^{'}+\dfrac{\left(3\theta^2\beta_1+2\theta P\beta_2\right)e_1^{2}}{2}\\ -\dfrac{\left(\theta^2\beta_1-2\theta P\beta_2\right)e_1^{'2}}{2}-(\theta\beta_1+\beta_2 P)e_0 e_1+(\theta\beta_1+\beta_2 P)e_0 e_1^{'}-\left(\theta^2\beta_1-2\theta P\beta_2\right)e_1 e_1^{'}\end{array}\right]^2 \tag{33}$$

$$MSE(\hat{t}_{CEi})_I=\bar{Y}^2 E\left[\begin{array}{l}\beta_1^2\left\{1+\lambda C_y^2+4\theta^2\left(\lambda-2\lambda^{'}\right)C_p^2-4\theta\left(\lambda-\lambda^{'}\right)\rho C_y C_p\right\}+\beta_2^2\left\{P^2\left(\lambda+\lambda^{'}\right)C_p^2\right\}\\ +\beta_1\beta_2\left\{4P(\lambda-\lambda^{'})\left[\theta C_p^2-\rho C_y C_p\right]\right\}-\beta_1\left\{2+3\theta^2(\lambda-3\lambda^{'})C_p^2-2\theta(\lambda-\lambda^{'})\rho C_y C_p\right\}\\ -\beta_2\left\{2P\left(\lambda-\lambda^{'}\right)\left[\theta C_p^2-\rho C_y C_p\right]\right\}+1\end{array}\right] \tag{34}$$

Denoting the known coefficients of β_1^2, β_2^2, $\beta_1\beta_2$, β_1, and β_2 by A_1, A_2, A_3, A_4, and A_5 respectively, the MSE of the estimator $\hat{t}_{REi}$ from (34) reduced to

$$MSE\left(\hat{t}_{CEi}\right)_I=\bar{Y}^2\left[\beta_1^2 A_1+\beta_2^2 A_2+\beta_1\beta_2 A_3-\beta_1 A_4-\beta_2 A_5+1\right] \tag{35}$$

Differentiating (35) partially with respect to β_1 and β_2, and equating it to be zero respectively, we get

$$\frac{\partial MSE\left(\hat{t}_{CEi}\right)}{\partial\beta_1}=0\Rightarrow\bar{Y}^2\left[2A_1\beta_1+A_3\beta_2-A_4\right]=0 \tag{36}$$

$$\frac{\partial MSE\left(\hat{t}_{CEi}\right)}{\partial\beta_2}=0\Rightarrow\bar{Y}^2\left[2A_2\beta_2+A_3\beta_1-A_5\right]=0 \tag{37}$$

Solving (36) and (37) simultaneously, we obtained the optimum values β_1 and β_2 respectively as

$$\beta_{1(opt)}\,\frac{2A_2A_4-A_3A_5}{4A_1A_2-A_3^2}$$

and

$$\beta_{1(opt)}\,\frac{2A_1A_5-A_3A_4}{4A_1A_2-A_3^2}$$

Substituting the optimum values of β_1 and β_2 into (35) to obtain the minimum mean square error, we have

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$$MSE_{\min}\left(\hat{t}_{CEi}\right)_I = \bar{Y}^2\left[1-\left\{\frac{A_3^2A_4\left(A_5-A_2A_4\right)-4A_1A_2A_4\left(A_3A_5-A_2A_4\right)-A_1A_5^2\left(A_3^2-4A_1A_2\right)}{\left(4A_1A_2-A_3^2\right)^2}\right\}\right]$$

(38)

Case-II. To obtain the properties of the estimator $\hat{t}_{CEi}$, let $\bar{y} = \bar{Y}(1+e_0)$, $p = P(1+e_1)$, and $p' = P(1+e_1')$ Such that

$$E(e_0) = E(e_1) = E(e_1') = 0$$
$$E(e_0{}^2) = \lambda C_y^2,\quad E(e_1{}^2) = \lambda C_p^2,\quad E(e_1'{}^2) = \lambda' C_p^2$$
$$E(e_0e_1) = \lambda\rho C_yC_p,\quad E(e_0e_1') = 0,\quad E(e_1e_1') = 0$$

Subtracting $\bar{Y}$ and taking expectation to both sides of (31), we obtained the bias of the estimator $\hat{t}_{CEi}$ for case II as

$$Bias\left(\hat{t}_{CEi}\right)_{II} = \bar{Y}\left[(\beta_1-1)+\frac{\left(3\theta^2\beta_1+2\theta P\beta_2\right)\lambda C_p^2}{2}-\frac{\left(\theta^2\beta_1-2\theta P\beta_2\right)\lambda' C_p^2}{2}-(\theta\beta_1+P\beta_2)\lambda\rho C_yC_p\right] \quad (39)$$

Which reduced to (40) after some appropriate simplification

$$Bias\left(\hat{t}_{CEi}\right)_{II} = \bar{Y}\left[(\beta_1-1)+\left\{\frac{\left(3\theta^2\beta_1+2\theta P\beta_2\right)\lambda}{2}-\frac{\left(\theta^2\beta_1-2\theta P\beta_2\right)\lambda'}{2}\right\}C_p^2-(\theta\beta_1+P\beta_2)\lambda\rho C_yC_p\right] \quad (40)$$

Similarly, subtracting $\bar{Y}$, taking expectation, and squaring both sides of (31), to the first order of approximation, we get the MSE of the estimator $\hat{t}_{CEi}$ for case II as

$$MSE\left(\hat{t}_{CEi}\right)_{II} = \bar{Y}^2E\begin{bmatrix}(\beta_1-1)^2+\beta_1^2\lambda C_y^2+(\theta\beta_1+P\beta_2)^2\lambda C_p^2+(\theta\beta_1+P\beta_2)^2\lambda' C_p^2\\+(\beta_1-1)(3\theta^2\beta_1+2\theta P\beta_2)\lambda C_p^2-(\beta_1-1)(\theta^2\beta_1-2\theta P\beta_2)\lambda' C_p^2\\-2(\beta_1-1)(\theta\beta_1+P\beta_2)\lambda\rho C_yC_p-2\beta_1(\theta\beta_1+P\beta_2)\lambda\rho C_yC_p\end{bmatrix} \quad (41)$$

Thus, it follows

$$MSE\left(\hat{t}_{CEi}\right)_{II} = \bar{Y}^2E\begin{bmatrix}\beta_1^2\left\{1+\lambda C_y^2+4\theta\lambda\left(\theta C_p^2-\rho C_yC_p\right)\right\}+\beta_2^2\left\{P^2\left(\lambda+\lambda'\right)C_p^2\right\}\\+\beta_1\beta_2\left\{4\theta P\left(\lambda-\lambda'\right)C_p^2-4P\lambda\rho C_yC_p\right\}-\beta_1\left\{2+\theta^2\left(3\lambda-\lambda'\right)C_p^2-2\theta\lambda\rho C_yC_p\right\}\\-\beta_2\left\{2\theta P\left(\lambda+\lambda'\right)C_p^2-2P\lambda\rho C_yC_p\right\}+1\end{bmatrix} \quad (42)$$

Denoting the known coefficients of β_1^2, β_2^2, $\beta_1\beta_2$, β_1, and β_2 by B_1, B_2, B_3, B_4, and B_5 respectively, the MSE of the estimator $\hat{t}_{CEi}$ from (43) reduced to

$$MSE\left(\hat{t}_{CEi}\right)_{II} = \bar{Y}^2\left[\beta_1^2B_1+\beta_2^2B_2+\beta_1\beta_2B_3-\beta_1B_4-\beta_2B_5+1\right] \quad (44)$$

Differentiating (44) partially with respect to β_1 and β_2, equating it to be zero respectively, we get

$$\frac{\partial MSE\left(\hat{t}_{CEi}\right)}{\partial\beta_1} = 0 \Rightarrow \bar{Y}^2\left[2B_1\beta_1+B_3\beta_2-B_4\right] = 0 \quad (45)$$

$$\frac{\partial MSE\left(\hat{t}_{CEi}\right)}{\partial\beta_2} = 0 \Rightarrow \bar{Y}^2\left[2B_2\beta_2+B_3\beta_1-B_5\right] = 0 \quad (46)$$

Solving (45) and (46) simultaneously, we obtained the optimum values β_1 and β_2 respectively as

$$\beta_{1(opt)}\ \frac{2B_2B_4-B_3B_5}{4B_1B_2-B_3^2}$$

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and

$$\beta_{1(opt)} \frac{2B_1B_5 - B_3B_4}{4B_1B_2 - B_3^2}$$

Substituting the optimum values of β_1 and β_2 into (44) to obtain the minimum mean square error for the case II, we get

$$MSE_{\min}\left(\hat{t}_{CEi}\right)_{II} = \bar{Y}^2\left[1 - \left\{\frac{B_3{}^2B_4\left(B_5 - B_2B_4\right) - 4B_1B_2B_4\left(B_3B_5 - B_2B_4\right) - B_1B_5{}^2\left(B_3{}^2 - 4B_1B_2\right)}{\left(4B_1B_2 - B_3{}^2\right)^2}\right\}\right] \tag{47}$$

3.1 Theoretical Efficiency Comparisons

Recall that the variance of the sample mean under simple random sampling without replacement (SRSWOR) is

$$V(\bar{y}) = \bar{Y}^2\lambda C_y^2 \tag{48}$$

Case-I and the Proposed Estimators

The proposed estimators in (38) are more efficient than sample mean in (48) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_I < V(\bar{y}); \qquad i = 1,2,\dots,9$$

$$\bar{Y}^2\left[1 - \frac{K}{V^2}\right] < \bar{Y}^2\lambda C_y^2$$

where, $K = A_3{}^2A_4(A_5 - A_2A_4) - 4A_1A_2A_4(A_3A_5 - A_2A_4) - A_1A_5{}^2\left(A_3{}^2 - 4A_1A_2\right)$ and $V = (4A_1A_2 - A_3{}^2)$

$$\left[1 - \frac{K}{V^2} - \lambda C_y^2\right] < 0 \tag{49}$$

The proposed estimators in (38) are more efficient than usual ratio estimator in (3) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_I < MSE(t^d{}_{NG1})_I; \qquad i = 1,2,\dots,9$$

$$\bar{Y}^2\left[1 - \frac{K}{V^2}\right] < \bar{Y}^2\left[\lambda C_y^2 + (\lambda + \lambda')\left(C_p^2 - 2\rho_{pb}C_yC_p\right)\right]$$

$$\left[1 - \frac{K}{V^2} - \left(\lambda C_y^2 + (\lambda + \lambda')\left(C_p^2 - 2\rho_{pb}C_yC_p\right)\right)\right] < 0 \tag{50}$$

The proposed estimators in (38) are more efficient than ratio estimator in (9) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_I < MSE(t^{*d}{}_{NG1})_I; \qquad i = 1,2,\dots,9$$

$$\bar{Y}^2\left[1 - \frac{K}{V^2}\right] < \bar{Y}^2\left[\lambda C_y^2 + \frac{n}{n' - n}(\lambda + \lambda')\left(\frac{n}{n' - n}C_p^2 - 2\rho_{pb}C_yC_p\right)\right]$$

$$\left[1 - \frac{K}{V^2} - \left(\lambda C_y^2 + \frac{n}{n'-n}(\lambda + \lambda')\left(\frac{n}{n'-n}C_p^2 - 2\rho_{pb}C_yC_p\right)\right)\right] < 0 \tag{51}$$

The proposed estimators in (38) are more efficient than ratio exponential estimator in (16) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_I < MSE(t_{S1})_I; \qquad i = 1,2,\dots,9$$

$$\bar{Y}^2\left[1 - \frac{K}{V^2}\right] < \bar{Y}^2\left[\lambda C_y^2 + (\lambda - \lambda')\left(\frac{C_p^2}{4} - \rho_{pb}C_yC_p\right)\right]$$

$$\left[1 - \frac{K}{V^2} - \left(\lambda C_y^2 + (\lambda - \lambda')\left(\frac{C_p^2}{4} - \rho_{pb}C_yC_p\right)\right)\right] < 0 \tag{52}$$

The proposed estimators in (38) are more efficient than ratio exponential estimator in (22) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_I < MSE(t_{S1}^*)_I; \qquad i = 1,2,\dots,9$$

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$$\overline{Y}^2\left[1-\frac{K}{V^2}\right] < \overline{Y}^2\left[\lambda C_y^2 + \frac{n}{n'-n}(\lambda-\lambda')\left\{\frac{n}{4(n'-n)}C_p^2 - \rho_{pb}C_yC_p\right\}\right]$$

$$\left[1-\frac{K}{V^2}-\left(\lambda C_y^2 + \frac{n}{n'-n}(\lambda-\lambda')\left\{\frac{n}{4(n'-n)}C_p^2 - \rho_{pb}C_yC_p\right\}\right)\right] < 0 \quad (53)$$

Case-IIand the Proposed Estimators

The proposed estimators in (47) are more efficient than sample mean in (48) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_{II} < V(\bar{y}); \qquad i = 1,2,\dots,9$$

$$\overline{Y}^2\left[1-\frac{T}{Z^2}\right] < \bar{Y}^2\lambda C_y^2$$

where,$T = {B_3}^2B_4(B_5 - B_2B_4) - 4B_1B_2B_4(B_3B_5 - B_2B_4) - B_1{B_5}^2({B_3}^2 - 4B_1B_2)$ and $Z = (4B_1B_2 - {B_3}^2)$

$$\left[1-\frac{T}{Z^2}-\lambda C_y^2\right] < 0 \quad (54)$$

The proposed estimators in (47) are more efficient than usual ratio estimator in (4) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_{II} < MSE({t^d}_{NG1})_{II}; \qquad i = 1,2,\dots,9$$

$$\overline{Y}^2\left[1-\frac{T}{Z^2}\right] < \overline{Y}^2\left[\lambda C_y^2 + (\lambda+\lambda')C_p^2 - 2\lambda\rho_{pb}C_yC_p\right]$$

$$\left[1-\frac{T}{Z^2}-\left(\lambda C_y^2 + (\lambda+\lambda')C_p^2 - 2\lambda\rho_{pb}C_yC_p\right)\right] < 0 \quad (55)$$

The proposed estimators in (47) are more efficient than ratio estimator in (10) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_{II} < MSE({t^{*d}}_{NG1})_{II}; \qquad i = 1,2,\dots,9$$

$$\overline{Y}^2\left[1-\frac{T}{Z^2}\right] < \overline{Y}^2\left[\lambda C_y^2 + \frac{n}{n'-n}\left\{\frac{n}{n'-n}(\lambda+\lambda')C_p^2 - 2\lambda\rho_{pb}C_yC_p\right\}\right]$$

$$\left[1-\frac{T}{Z^2}-\left(\lambda C_y^2 + \frac{n}{n'-n}\left\{\frac{n}{n'-n}(\lambda+\lambda')C_p^2 - 2\lambda\rho_{pb}C_yC_p\right\}\right)\right] < 0 \quad (56)$$

The proposed estimators in (47) are more efficient than ratio exponential estimator in (17) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_{II} < MSE(t_{S1})_{II}; \qquad i = 1,2,\dots,9$$

$$\overline{Y}^2\left[1-\frac{T}{Z^2}\right] < \overline{Y}^2\left[\lambda C_y^2 + \frac{1}{4}(\lambda+\lambda')C_p^2 - \lambda\rho_{pb}C_yC_p\right]$$

$$\left[1-\frac{T}{Z^2}-\left(\lambda C_y^2 + \frac{1}{4}(\lambda+\lambda')C_p^2 - \lambda\rho_{pb}C_yC_p\right)\right] < 0 \quad (57)$$

The proposed estimators in (47) are more efficient than ratio exponential estimator in (23) if the following condition hold:

$$MSE_{min}(\hat{t}_{CEi})_{II} < MSE(t^*_{S1})_{II}; \qquad i = 1,2,\dots,9$$

$$\overline{Y}^2\left[1-\frac{T}{Z^2}\right] < \overline{Y}^2\left[\lambda C_y^2 + \frac{n^2}{4(n'-n)^2}(\lambda+\lambda')C_p^2 - \lambda\frac{n}{n'-n}\rho_{pb}C_yC_p\right]$$

$$\left[1-\frac{T}{Z^2}-\left(\lambda C_y^2 + \frac{n^2}{4(n'-n)^2}(\lambda+\lambda')C_p^2 - \lambda\frac{n}{n'-n}\rho_{pb}C_yC_p\right)\right] < 0 \quad (58)$$

4.0 Results and Discussion

The three datasets in SukhatmeandSukhatme[17], Zaman et al. [18] and Mukhopadhyaya[19]mentioned as Population 1, Population 2 and Population 3, respectively, are used in order to examine the performances between the proposed combined exponential-type estimators and the existing estimators based on the criteria of means square error (MSE) and percentage relative efficiency (PRE) values.

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Table 2: Descriptive Statistics of Population 1

N = 89	$\overline{Y}$ = 3.3596	P = 0.1224	θ_4 =0.663639	θ_8 =0.168098
n = 20	n' = 45	θ_1 =0.221201	θ_5 =0.255046	θ_9 =0.129933
$\beta_2(\phi)$ = 3.492	C_y = 0.6008	θ_2 =0.171378	θ_6 =0.626013	
ρ_{pb} = 0.766	C_p = 2.6779	θ_3 = 0.054370	θ_7 =0.179256	

Table 3: Descriptive Statistics of Population 2

N = 111	$\overline{Y}$ = 29.279	P = 0.1162	θ_4 = 0.727196	θ_8 = 0.1847881
n = 30	n' = 55	θ_1 = 0.233225	θ_5 = 0.283080	θ_9 = 0.1319996
$\beta_2(\phi)$ = 3.898	C_y = 0.872	θ_2 = 0.166991	θ_6 = 0.675968	
ρ_{pb} = 0.797	C_p = 2.758	θ_3 = 0.053057	θ_7 = 0.179065	

Table 4: Descriptive Statistics of Population 3

N = 25	$\overline{Y}$ = 7.143	P = 0.294	θ_4 = 0.3792143	θ_8 = 0.0663664
n = 7	n' = 13	θ_1 = 0.2212013	θ_5 = 0.0242392	θ_9 = 0.0251466
$\beta_2(\phi)$ = 2.19	C_y = 0.36442	θ_2 = 0.0899278	θ_6 = 0.3014618	
ρ_{pb} = −0.314	C_p = 1.34701	θ_3 = 0.0117259	θ_7 = 0.0122935	

Table 5: Optimum value of the Parameters

		$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$
Case I	Population 1	5.470544e-06	-2.953245e-05
	Population 2	1.073719e-05	-1.609334e-07
	Population 3	-0.0003067824	-0.0003386136
Case II	Population 1	-0.0002031217	-0.0002532018
	Population 2	-2.337339e-05	-4.054335e-05
	Population 3	-0.001226135	-0.001275548

Table 5 showed the optimum values of the parameter of the proposed estimator under the three populations considered. The result revealed that $\beta_1 > \beta_2$ in both the case I and II under the three populations considered.

Table 6: BiasEstimates of the Existing Ratio Estimators and the Proposed Estimators.

	Case I		**Case II**	
Estimators	**PopI**	**Pop II**	**PopI**	**Pop II**
$\bar{y}$	*	*	*	*
t^{d}_{NG1}	0.115980	0.122411	0.106026	0.123461
t^{*d}_{NG1}	-0.103458	-0.062233	-0.02120	-0.04661
t_{S1}	0.005461	0.028671	0.005279	0.029386
t^{*}_{S1}	0.001259	0.04790	0.003337	0.048182
$\hat{t}_{CE1}$	0.000152	0.02685	0.000275	0.028344
$\hat{t}_{CE2}$	0.000208	0.01597	0.000742	0.028547
$\hat{t}_{CE3}$	0.000147	0.04541	0.000128	0.049968

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$\hat{t}_{CE4}$	0.009321	0.02287	0.004271	0.030069
$\hat{t}_{CE5}$	0.000123	0.09202	0.000551	0.099365
$\hat{t}_{CE6}$	0.000131	0.01788	0.000675	0.020029
$\hat{t}_{CE7}$	0.000204	0.03389	**0.000723**	0.041534
$\hat{t}_{CE8}$	0.000441	0.04565	0.000863	**0.013179**
$\hat{\mathbf{t}}_{\mathbf{CE9}}$	**0.000112**	**0.00108**	0.000101	0.063269

Table 7: MSE and Percent Relative Efficiency of the Existing Ratio and Proposed Estimators

	Case I				Case II			
Estimators	**Pop 1**		**Pop 2**		**Pop 1**		**Pop 2**	
	MSE	**PRE**	**MSE**	**PRE**	**MSE**	**PRE**	**MSE**	**PRE**
$\bar{y}$	0.157930	100	15.85573	100	0.157930	100	15.85573	100
${t^{d}}_{NG1}$	1.633285	9.67	64.86284	24.4	3.106026	5.085	154.3461	10.273
${t^{*d}}_{NG1}$	0.978480	16.1	98.37626	16.1	1.872120	8.436	234.4661	6.762
t_{S1}	0.333522	47.4	15.65928	101.3	0.625279	25.258	30.49386	51.996
t^{*}_{S1}	0.208471	75.8	21.54799	73.6	0.370737	42.599	46.52718	34.078
$\hat{t}_{CE1}$	0.176852	89.3	16.02685	98.9	0.468127	33.737	17.50083	90.599
$\hat{t}_{CE2}$	0.142909	110.5	13.65977	116.1	0.077874	202.80	8.505285	186.42
$\hat{t}_{CE3}$	0.147373	107.2	14.44541	109.7	0.152875	103.31	25.46468	62.266
$\hat{t}_{CE4}$	0.360097	43.85	18.88187	83.97	1.524271	10.361	34.4069	46.083
$\hat{t}_{CE5}$	0.199203	79.28	19.09202	83.05	0.745505	21.184	11.10365	142.80
$\hat{t}_{CE6}$	0.923413	17.10	21.11788	75.08	1.706715	9.2535	62.81029	25.244
$\hat{t}_{CE7}$	0.147804	106.9	13.89889	114.1	**0.072594**	**217.55**	8.467534	187.25
$\hat{t}_{CE8}$	0.140969	112.0	14.04565	112.8	0.079528	198.58	**8.463179**	**187.35**
$\hat{\mathbf{t}}_{\mathbf{CE9}}$	**0.124368**	126.9	**13.56808**	**116.9**	0.085061	185.67	9.487999	167.11

Table 6 and 7 shows respectively the bias, means square error (MSE) and percentage relative efficiency values of the sample mean; usual ratio; Singh et al. [1] exponential ratio-type estimator, Kalitaand Singh [16] exponential dual to ratio estimator,and the proposed combined exponential-type estimators in case I and case II. Evidence from the result signifies that the proposed $\hat{t}_{CEi}$ exponential-type estimators possessed minimum mean square error values than the existing ratio estimators considered,where $\hat{t}_{CE9}$ and $\hat{t}_{CE7}$ are the most efficient estimators for Population 1, while$\hat{t}_{CE8}$is the most efficient estimator for Population 2.

Table 8: Bias, MSE and PRE of the Proposed and Existing Product Estimators

	Case I			Case II		
Estimators	**Bias**	**MSE**	**PRE**	**Bias**	**MSE**	**PRE**
$\bar{y}$	*	0.6969476	100	*	0.6969476	100
${t^{d}}_{NG2}$	0.300845	7.837983	8.891925	0.295519	15.25519	4.5685930
${t^{*d}}_{NG2}$	5.441873	555.1851	0.1255343	4.992511	551.4325	0.1263886
t_{S2}	0.004883	1.704412	40.890794	0.033149	3.123148	22.315548
t^{*}_{S2}	1.068760	74.86570	0.9309305	7.996881	5706.482	0.0122132

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$\hat{t}_{CE1}$	**0.0000523**	**0.0096954**	**7188.4357**	0.009944	1.498544	46.508317
$\hat{t}_{CE2}$	0.0005252	0.1252534	556.43008	0.002073	0.1832079	380.41350
$\hat{t}_{CE3}$	0.0002996	0.1429093	487.68528	**0.000124**	**0.1294934**	**538.21090**
$\hat{t}_{CE4}$	0.0001413	0.1494134	466.45588	0.003483	1.344298	51.844725
$\hat{t}_{CE5}$	0.0003948	0.1394831	499.66454	0.000967	0.4139684	168.35768
$\hat{t}_{CE6}$	0.0002514	0.5648254	123.39168	0.001880	1.565188	44.528044
$\hat{t}_{CE7}$	0.0002958	0.1429585	487.51742	0.000964	0.1298864	536.58243
$\hat{t}_{CE8}$	0.0005067	0.1495002	466.18506	0.000415	0.1411975	493.59769
$\hat{t}_{CE9}$	0.0001723	0.1441146	483.60651	0.000613	0.1366131	510.16162

Table 8 shows respectively, the bias, means square error (MSE) and percentage relative efficiency (PRE) values of the sample mean; usual product; Singh et al. [1] exponential product-type estimator, Kalitaand Singh [16] exponential dual to product estimator,and the proposed exponential-type estimators in case I and case II. Evidence from the result signifies that the proposed $\hat{t}_{CE}$ exponential-type estimators possessed minimum mean square error values than the existing product estimators considered,where $\hat{t}_{CE1}$is the most efficient estimatorin case I, while$\hat{t}_{CE3}$is the most efficient estimator in case II.

Table 9: Region of Preference of ρ_{pb} under which the Proposed Estimator is better

ρ_{pb}	P	MSE	ρ_{pb}	P	MSE
	0.1	0.145967		0.1	0.1432010
	0.2	0.159834		0.2	0.1574037
	0.3	0.188921		0.3	0.1855774
	0.4	0.228896		0.4	0.2304614
0.300	0.5	0.279922	-0.300	0.5	0.2911532
	0.6	0.343510		0.6	0.3663207
	0.7	0.422206		0.7	0.4546116
	0.8	0.519637		0.8	0.5547210
	0.9	0.640659		0.9	0.6653893
	0.1	0.138967		0.1	0.1323412
	0.2	0.163723		0.2	0.1579494
	0.3	0.214654		0.3	0.2070158
	0.4	0.280803		0.4	0.2855278
0.533	0.5	0.362683	-0.533	0.5	0.3909368
	0.6	0.465046		0.6	0.5197609
	0.7	0.596479		0.7	0.6685015
	0.8	0.769938		0.8	0.8337651
	0.9	1.003538		0.9	1.0122420
	0.1	0.123329		0.1	0.1295358
	0.2	0.174576		0.2	0.1578198
	0.3	0.274009		0.3	0.2448164
	0.4	0.386202		0.4	0.3838459
0.725	0.5	0.515091	-0.725	0.5	0.5672152
	0.6	0.684773		0.6	0.7851993
	0.7	0.941247		0.7	1.0281800

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0.8	1.357366	0.8	1.2868940
0.9	2.039011	0.9	1.5524060

The regions of preferences proposed estimator is obtained by varying the degrees of correlation coefficient and presented in table 9. The results signifies that the proposed combined exponential-type estimator becomes more efficient under the assumption of when the correlation between the study variable and the auxiliary attribute is either strongly positive or negative, it could also be seen that the lower the value of proportion the more efficient the estimator becomes.

5.0 Conclusion

The properties of thecombined exponential estimators in two-phase sampling such as bias and means square error (MSE) equations are derived in two phases. The optimum value of the parameters along with the minimum mean square errors is obtained and tested using a real life datasets to examine their efficiencies vis-å-vis some other estimators in the literature at each phase and the proposed exponential estimators performed better to the datasets considered in this study. Thus, these estimators can be considered as alternatives to estimating real life datasets.

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