Original Research Article

A Fuzzy-Based Control System for Customers' Admission in a Two-Station Queue Network

ABSTRACT

The study proposes a fuzzy-based control of admission of customers in a queue network with two stations in tandem. Each of the stations has individual arrival streams which may either be accepted or rejected. Class *i*arrivals occur in a Poissonstream with constant rate λ_i, $i= 1, 2$. Successive services in each station *j* areindependent and exponentially distributed, with mean $1/\mu_j$in station j, $j =1, 2$, irrespective of the customer's class. The objective of the study is to decide an optimal admission policy based on the state of the queue such that profit is maximized. The state of the system is described by (z_1, z_2), where z_i is the number of customers in station *i,* and$i$= 1, 2.The tool adopted is a fuzzy process which determines this policy using the fuzzy input values, *s* and λ giving a corresponding decision,*dec.* which is either a '1' or '0' representing 'Admit' or 'Reject' respectively.The membership functions of arrivals were defined and implemented using fuzzy rules to derive a fuzzy output of decision which either 'Admit' or 'Reject' an arrival. Numerical results show a considerable improvement in the control of customers' admission and it was concluded that the proposed method is efficient in the control of customers' admission in queue network.

Keywords: (Exponential Server, Reward, Holding Cost, State Transmission, Tandem Queues).

1. INTRODUCTION

Queuing theory is the mathematical study of waiting lines. Queue lengths and waiting times can be depicted via the use of a queuing model. Queuing theory plays an important role in our daily life. It is not possible to exactly determine the arrival and departure of customers when the number and types of facilities as well as the essential of the customers are not known. Queuing theory techniques, in particular, can help us to determine suitable number and type of service facilities to be provided to different types of customers [1].

There are many applications of queuing theory. This include traffic flow, programming patients in hospital, facility design in bank and other institutions,programming of service facilities in a repair and maintenance in workshop, programming of limited transport fleet to a large number of users,programming of reconstruction of used engines and assemblies of aircrafts, missile system, transport fleet, among others [2].

It is desirable to have customers' serviced within the shortest possible time in any service system. However, it is almost possible to meet up with all customers' service requirements at all times as these preferences for services changes with time. This is inevitable in a world of technology where service needs and customers' characteristics are dynamic in nature. In other to ensure that customers are serviced within the shortest possible time, it is imperative to have a system which can accommodateand service a sizeable number of customers.

The need for service providers to cope with dynamic and varyingcustomer needs with limited resources has become an issue of great concerns in recent times. This is because service providers have to find effective control mechanisms to manage their revenues as well as customer satisfaction in order to make the best use of their service capacities. In literature, these service systems are modeled as multi-class queuing systems with admission controls.

The optimality of trunk reservation policyfor a multi-class loss queuing system in which the rewards that customer classes pay for beingon their classes is significant as acceptance decisions on individual customer classes havethreshold structures, with respect to the number of customers in the system [3]. Consequently, if customerclass $i$is accepted when there are n customers in the system, then class $i$should also beaccepted when the system is less crowded. The admission controlstudies which consider class-dependent service rates have focused on providing heuristicpolicies [4]. The most common approaches of these studies include linear programmingtechniques [5] and asymptotic analyses [6].

The waiting times of customers awaiting service in any system is one of the importantservice quality indicators, which has significant consequential effects on revenuesor customer satisfaction levels. Consequently. waiting times can affect customer choicein selecting service providers. In essence, for a specific service provider, waitingtimes can be a determinant of the demand intensity of their system [6].

The effects of systemcongestion on customer behaviour in a queue system had been studied by [7] in which the objective of the study was to minimize a weighted difference between the average expected waiting time of those that enter, and the acceptance rate of customers.Similar studies to this include [8] and [9] which discussed the potentials of systems to obtain admission rewards when customers behave greedily based on congestion levels. These studies fail to consider heterogeneity among arrivals. Similarly, [10] studied congestion-related costs through the abandonmentof customers in a single-class multi-server model for controlling the admission decisionsof arriving customers.

Admission control in a single-server model with retrials where holding costs are used as means to incorporate congestion sensitivity of customers wasconsidered by [11].Similarly [12] investigated the callback option to mitigate congestion incall centres. The study was modeled such that arriving customers were routed to an offline queue to be called backlater when they accept the callback offer else customers were routed to the online queue inwhich they incur congestion-related waiting time costs. [13] considers a Make-to-Stockqueuing model with impatience when unsatisfied demands are backlogged. The control consists of both an admission decision tothe system and an admission in service decision. There are no ordering costs while threshold policies were optimal using the propagation of structural properties.

The congestion effect through class-dependent holding costs in the admission control problem of a multi-class queue model was considered by [14]. The model used a continuous-time Markov decision process formulation and relative bias functions intheir policy iteration algorithm for obtaining the optimal policy of an *M/M/c/N*queuewith class and congestion-dependent admission rewards. [15] considers a parametric admission in a retrial queue with impatience on retrials and introduces a Smoothed Rate Truncation method in order to work with models with bounded transition rates.

Stochastic controlled queuing models, have been largely studied in the literature as a result of its wide applications in networking, resources allocation, inventory control, etc. [16]. This approach had been used in solving a variety of optimization problems includingadmission control systems [17]; optimal scheduling [18] or optimal routing between queues [19] so as to minimizedeadline misses; scheduling in order to minimize long run costs [20]; inventory control in Make-to-Order systems [21];optimal control of the service rates [22]; admission in service involving slotted models [23].

The problem of dynamic control of admission of customers had been solely based on network metrics and this does not adequately meet the need and flexibility of a dynamic queue system. This is because the nature and service requirements of customers in contemporary business environments are changing on daily basis and this requires a large degree of flexibility on the admission control system. This is the reason the study is proposing a fuzzy-based admission control system which can adequately cater for these challenges.

Fuzzy set theory is a paradigm shift which helps to resolve classical and non-classical problems in a more convenient way than crisp systems by softening set boundaries. Fuzzy queues were first proposed by R. J. Lie and E. S. Lee in 1989 [24]. A Poisson arrival queueing system is a fairly reasonable appropriation where the arrival and service rates are really more realistic than probabilistic. However, in many practical situations the parameters of arrival rate (λ) and service rate $(\mu)$are frequently fuzzy and cannot be expressed in exact terms [24].

Optimality in queue control is an aspect of mathematical modeling which have been found to have multi-dimensional applications [25, 26]. Of focus, the present-day advancement in business requirements and expansion in the expectation of customers from essential service providers have made it necessary to have a system which regulates admission into a queue system with the sole aim of rendering efficient and timeless services.

The study is aimed at designing a system that dynamically manages customers' admission in a queue network. This is necessary to ensure that the buffer is not over-congested with customers awaiting service turns which could result in queue instability, customers' dropping and deficient customer management system. Particularly, the system adopted a two-station queue network structure with independent queue capacities.

2. MATERIAL AND METHODS

2.1. Problem Definition

We consider a continuous-time controlled queuing model in which customers arrive accordingto a Poisson process with a constant intensity. The model has two queues in tandem, each having independent input of arriving customers that may either be admitted or rejected. The service admission decision is made by a controller. Once admitted in service by the controller, the service begins instantly and it is not interrupted. The model is shown in figure 1.

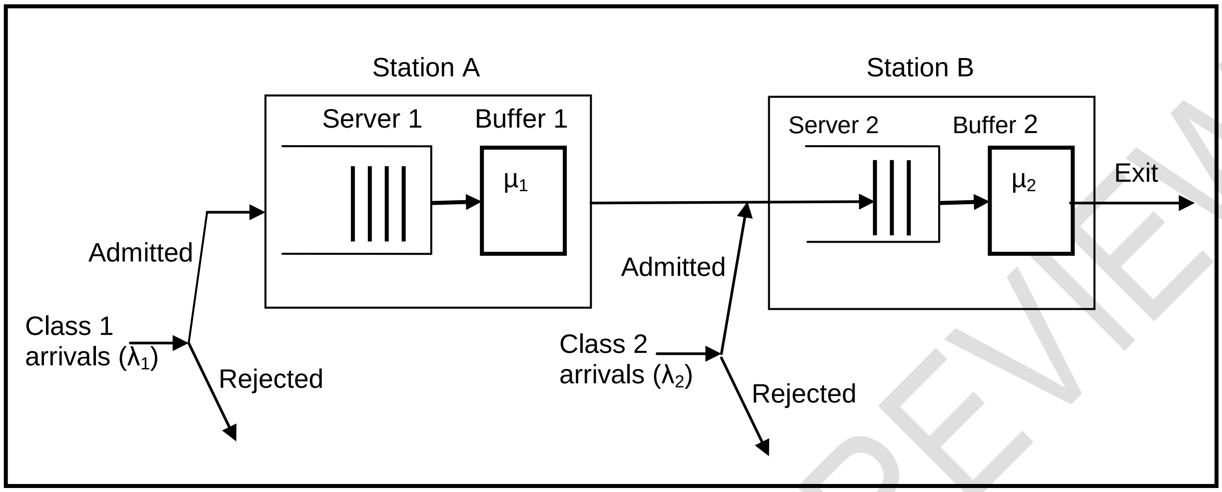

Figure 1: Proposed customers' admission system in a two-station queue network

The capacity of the server in each station is exponential while the buffer has an unlimited queueing capacity. Class1 customers seek admission to station A and later to station B after being servedin station A while Class 2 customers seek admission to station B only. However, what is common to both classes of customers is that they both exit the system after being served in station B. Class $i$arrivals occur in a Poissonstream with constant rate $\lambda_i$where i = 1, 2. Services in each station j areindependent and exponentially distributed, with mean $1/\mu_j$in station j, where j = 1, 2, irrespective of the customer's class.

If the system has a fixed reward $r_i$for each admitted customerof class $i$and pays a holding cost $h_j$for each customer per time unit in station j. In this case, it is possibleto decide the optimal admission policy, based on the state of the system in order to optimize profit. In this case, the state transitions depend on the current state while the times between two successive admissions areno longer exponential random variables because of the possibility of non-admittance of some arrivals.

2.2. ModelingCustomers' Admission and Rejection as a Dynamic Fuzzy Control System

Modeling a continuous-time dynamic queue system in the conventional approach is to describe the physical system by, for example, a non-linear ordinary differential equation of the form:

$$\begin{cases} \dot{x}(t) = f(\mathrm{x(t), u(t)}), & t_0 \le t < \infty \\ x(t_0) = \mathrm{x}_0 \end{cases} \quad (1)$$

where $x(t) \in \mathrm{R}^n$ is the state-vector, $u(t) \in \mathrm{R}^m (m \le n)$ is the control vector, f: $\mathrm{R}^n \times \mathrm{R}^m \rightarrow \mathrm{R}^n$is a non-linear integer function while x_0 is the initial state of the system. This mathematical model is well defined on $[t_0, \infty)$ because for each initial state $x_0$and each control input $u(t)$, there exists a unique solution $x(t)$. The mathematical model expressed in (1) represents a dynamic process via its explicit or implicit solution function: $x(t) = x(t; u(t), x0)$. At any instant $t = t^* \in [t_0, \infty)$, the system is described by the relationthat if the state vector x is equal to $x(t^*)$ and the control vector u is equal to $u(t^*)$, then the derivative of the state vector can be expressed as:

$$x = \frac{dx}{dy}\Big|_{t = t^*}$$

In this case, both the state and the control vectors, i.e. x and $u$respectively have "fuzzy values," instead of crisp values. This implies that these values at any instant are located within certain subsets with membership values. The derivative of the state, $\dot{x}$ has a fuzzy value at each instant which makes both the state and the control possible. By standard, a function $y = y(t)$ which assumes a fuzzy value at instant $t = \tilde{t}$ is described such that if I is an interval with μ_I being the membership function defined on it, then the fuzzy value $y(\tilde{t})$ is located in I with a corresponding membership value measured by μ_I which is a membership value equal to $\mu_I(y(\tilde{t}))$.

Consider a control system with state $x = x(t)$ and control input $u = u(t)$ defined on the time domain $[0, \infty)$ in which thesystem dynamics are described by the derivative of the state $\dot{x}$ which is a function of both x and u. If the relation between $\dot{x}$ and both (x, u) is exactly known, then mathematical modeling techniques and the systems control theory can be applied. The exact modeling $\dot{x} = ax + bu$simply implies that at any instant $t = t^* \in [0, \infty)$, if $x= x(t^*)$ and $u =u(t^*)$, then:

$$\dot{x} = \left.\frac{dx(t)}{dt}\right|_{t = t^*}$$

2.3. Design of the Fuzzy Controller

A fuzzy control system was designed to manage the admission and rejection of customers in the system.

2.3.1. Fuzzy Inputs and Rules

The system is described by (z_1, z_2), where z_1 is the number of customers in station i, where i= 1, 2. In order to prevent a situation whereby an arriving class 1 customer is rejected even when the system is empty or an arriving class 2 customer is rejected even when station 2 is empty, it is assumed that the reward at each station for every customer is greater than the corresponding expected holding cost. Consequently,

$$r_1 > \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} \qquad (2)$$

$$r_2 > \frac{h_2}{\mu_2} \qquad (3)$$

The system receives a reward for admitting customers while also incurring a cost for holdingcustomers. Considering (2) and (3) above, it is optimal for the systemto keep the server in station B busy provided there is a continuous arrival of customers and no customer is held in queue. It is important to observe that this condition is temporal as a result of the memoryless property of the exponential distribution whereby neitherthe inter-arrival nor the service times can be conditioned on the present observablestate. Similarly, if the system is highly rewarded for admitting customers and incurs a low cost forholding class 1 customers, then the system accepts class 1 customers easily.

The fuzzy inputs are s_i= 0, 1, …, where$i$= 1, 2 of customers in the bufferof station $i$and the customer arrival rates $\lambda_j \in [0, \infty)$, j = 1, 2 of class j. The fuzzy outputs are decisions,dec_j= 1, 0. When an arriving class j customer is admitted, it is a 1, while it is a 0 if it is rejected.The fuzzy inputs include “zero”, “fairly positive”, “positive” and “highly positive” represented as “NE”, “FP”, “PO” and “HP” respectively. A four fuzzy sets for each of the four inputs were chosen and the complete rule base consists of 256 (i.e. 4^4) rules combinations. However, only 15 combinations of the rules produce output decision ADMIT while the other 241 combinations produces the output decision REJECT. However all rule combinations that produce output decision ADMIT and only 10 rule combinations that produces output decision REJECT were recorded. The fuzzy rule baseis as shown in table 1.

S_A	S_B	λ_1	λ_1	***Decision (dec)***
NE	NE	NE	NE	ADMIT
NE	NE	NE	FP	ADMIT
NE	NE	NE	PO	ADMIT
NE	NE	FP	HP	ADMIT
NE	NE	FP	FP	ADMIT
NE	NE	PO	NE	ADMIT
NE	FP	NE	NE	ADMIT
NE	FP	NE	FP	ADMIT
NE	FP	FP	NE	ADMIT
NE	PO	NE	NE	ADMIT
FP	NE	NE	NE	ADMIT
FP	NE	NE	FP	ADMIT
FP	NE	FP	NE	ADMIT
FP	FP	NE	NE	ADMIT
PO	NE	NE	NE	ADMIT
NE	NE	NE	NE	REJECT
NE	FP	NE	FP	REJECT
NE	FP	NE	PO	REJECT
NE	FP	FP	FP	REJECT
NE	PO	PO	NE	REJECT
FP	NE	NE	FP	REJECT
FP	FP	NE	FP	REJECT
FP	NE	FP	HP	REJECT
FP	PO	NE	NE	REJECT
PO	NE	NE	HP	REJECT

Table 1: All rule combinations with ADMIT as output and only 10 rule combinations with REJECT decision

In table 1, ADMIT implies that an arriving customer of class *j* is admitted into station *j*. In this case, there are 15 combinations that produces ADMIT as output while 241 combinations producesREJECT as output.The membership functions for the fuzzy inputs s_i,*i*= 1, 2, and λ_j, *j* =1, 2, aredepicted shown in figure 2.

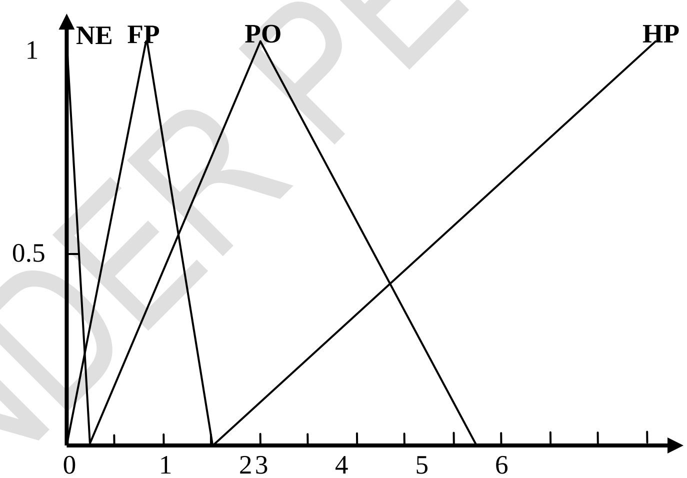

Figure 2: Membership functions for the fuzzy input,*s.*

Similarly, the membership for the fuzzy input, *λ*is given in figure 3 below.

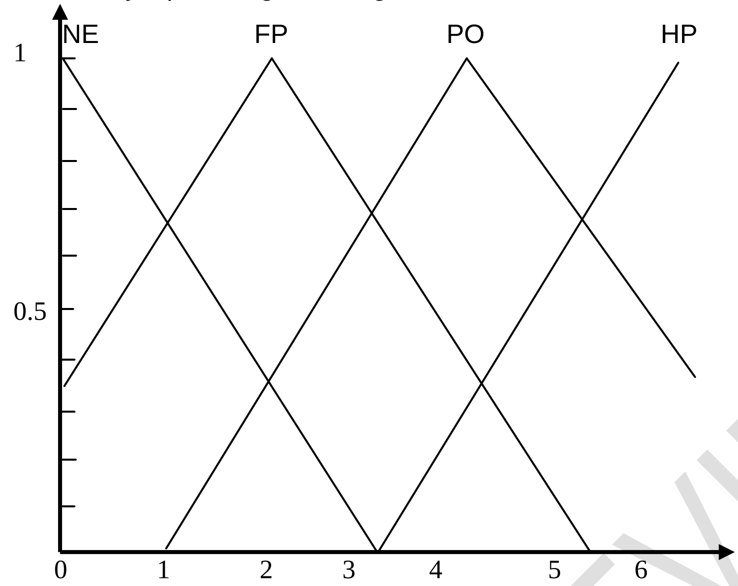

Figure 3: Membership for the fuzzy input *λ*

The fuzzy output, *dec.*is shown in figure 4.

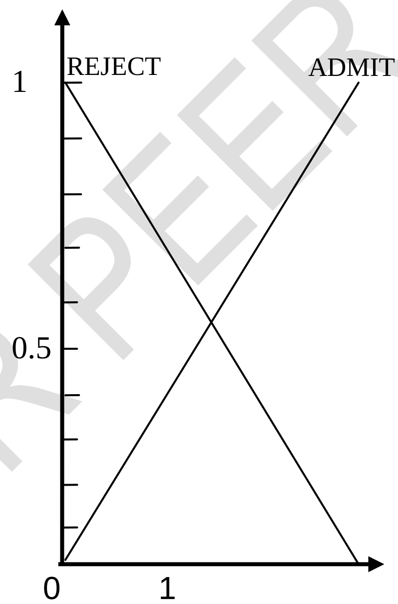

Figure 4: Fuzzy output,*dec*

The universes of discourse for the fuzzy inputs $s_i$and$\lambda_j$are [0, ∞] while for thefuzzy output*dec.is* [0, 1].

2.3.2. Membership functions applicable in the control of class 1 arrivals

Since it is necessary to determine the relationships between the fuzzy inputs $s_i$and λ_j, as well as fuzzy output *dec.*, it is assumed that $z_1 + z_2 > 0$. The scenario given by the rule:

If s_1 is HP while s_2 is zero (*NE)while each of* λ_1 *and* λ_2*is NE, then* dec_1 *is REJECT*.

In this case, it is necessary to decide whether to admitthelast class 1 customer when there are $s_1 \geq 0$ customers already in queue 1, onecustomer is in server 1 while all other input variables are zero, if$E_1$is the mean holding cost the last customer incurs in queue 1. Similarly, $E_2$is the mean holding cost from the time service starts at server 1 until exit from server2. In this case, the customer is admitted only if its reward compensates its expected holdingcost which is:

$$r_1 \geq E_1 + E_2. \qquad (4)$$

Since there are $s_1 + 1$ customers in station A, then $E_1 = h_1(s_1 + 1) / \mu_1$. Consequently, E_2 is computed as follows. Server 1 starts to service the customer just when server 2 starts to service the previous customer. In this case, the state of the systemis $(z_1, z_2) = (1, 1)$. The system continues to state (0, 2) if server 1is done with service before server 2. The system movesfrom state (0, 2) or (1, 0) to state (0, 1) and lastly to (0, 0). It is obvious from these transitions that the sojourn time in state (1, 1) is a random variable with exponential distribution and mean $1/(\mu_1 + \mu_2)$. As a result, the mean holding cost from server 1 to exit becomes:

$$E_2 = \frac{h_1}{\mu_1 + \mu_2}$$

$$E_2 = \frac{h_1}{\mu_1 + \mu_2} + \frac{\mu_1}{\mu_1 + \mu_2} x \text{ (mean cost from } (0,2) \textit{ to } \text{departure)}$$

$$E_2 = \frac{h_1}{\mu_1 + \mu_2} + \frac{\mu_1}{\mu_1 + \mu_2} x \,(\textit{mean} \text{ cost from } (1,0) \text{ to departure)}$$

$$E_2 = \frac{h_1}{\mu_1 + \mu_2} + \frac{\mu_1}{\mu_1 + \mu_2}\frac{2h_2}{\mu_2} + \frac{\mu_1}{\mu_1 + \mu_2}\left(\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}\right)$$

$$E_2 = \frac{h_1\left(1 + \frac{\mu_2}{\mu_1}\right) + h_2\left(1 + \frac{2\mu_1}{\mu_2}\right)}{\mu_1 + \mu_2}$$

Substituting $E_1 = h_1 (s_1 + 1)/\mu_1$ and the above into(3) and solving for s_1 gives:

$$s_1 \leq s_{1,dec_1} = \frac{\mu_1}{h_1}\left(r_1 - \frac{h_1}{\mu_1} - \frac{h_2}{\mu_2}\right) - \frac{\mu_1}{h_1}\frac{h_2}{\mu_2}\frac{\mu_1}{\mu_1 + \mu_2} - 1$$

However, if the number of customers in queue 1 is greater than the threshold s_1, then thedec_1 is REJECT.

2.3.3. Membership functions applicable in the control of class 2 arrivals:

It is necessary to specify the numerical settings of $s_i$and $\lambda_j$for the fuzzy output dec_2 with the assumption that $z_2 > 0$. Considering the rule scenario:

If s_1 is HP and$s_2$is NE while each ofλ_1*and* $\lambda_2$is 0, *then dec_2 is REJECT*.

It is necessary to determine a conditionfor $s_1$under which a class 2 customer is REJECTED when each of λ_1, λ_2, *and* s_2 is zero. If it is assumed $z_2 > 0$, then there is a customer in server 2. If this customer is accepted, the state of the system at time zero will be $(s_1 + 1, 2)$,else the system starts from state $(s_1 + 1, 1)$.

Let $F(i, j)$ be the total expected holding cost from state (i, j) to state (0, 0). If both $i$and j are greater than 0, then the system moves from state (i, j) to $(i - 1, j + 1)$ withprobability $\mu_1/(\mu_1 + \mu_2)$ or to state $(i, j - 1)$.When $i = 0$ ($j = 0$), the system will visit state $(0, j - 1)$ with probability 1. Consequently,$F(i, j)$ can be computed as follows:

$$F(i,j)\begin{cases} j\dfrac{h_2}{\mu_2}+F(0,j-1) \quad if \quad i=0 \\ i\dfrac{h_1}{\mu_1}+F(i-1,1)\, if \quad j=0 \\ \dfrac{ih_1+jh_2+\mu_1 F(i-1,j+1)+\mu_2 F(i,j-1)\, if\ i,j>0}{\mu_1+\mu_2} \end{cases}$$

Consequently, the only condition under which an arriving class 2 customer is rejected is given as:

$$r_2 - F(s_1+1,2) < - \quad F(s_1+1,1) \qquad (5)$$

If the condition depicted in (4) holds for some value s_1, dec_2thenit will also hold forevery $s_1 \geq s_{1,}, dec_2$. This implies that the fuzzy set HP for s_1 with membership grade 1.0 inthe fuzzy rule base for *dec.*$_2$is at:

$$s_1, \mathrm{dec}_2 = \min\left\{\ s_1 : F(s_1+1,2) - F(s_1+1,1) > r_2\ \right\} \qquad (6)$$

If a case described by the rule:*if $s_1$is NE and s_2 isHP while each of $\lambda_1$and λ_2 is NE, then dec.$_2$is REJECT*. It is important to determine a condition for $s_2$under which a class 2 arrival is rejected when $x_1=s_1=\lambda_1= \lambda_2=0$, while the number of existing customers in station B is$x_2=s_2+1$. This is the optimal decision provided the expected holding cost for the new customer is greater than the correspondingreward. This can be expressed as $r_2 < (s_2+2)h_2/\mu_2$. In this case, the fuzzy set HP for $s_2$with membership grade 1.0 in the fuzzy rule base for *dec.*$_2$ is fixed at:

$$s_2, dec_2 = \frac{\mu_2}{h_2} r_2 - 2$$

As for the fuzzy input λ_1, if we consider the rule: *if each of s_1 and $s_2$is NE while λ_1 is HP and λ_2 is NE, then dec.$_2$is REJECT*. Consequently, an arriving class 2 customer is REJECTED while$x_i = 1$, $s_i = 0$, i =1, 2. In this case, only class 1 arrivals are ADMITTED.

3. RESULTS AND DISCUSSION

Consideration is given to the proposed queue network structure in figure 1 with parameters $\mu_1= 1$, $\mu_2 = 1.5$, $\lambda_1= 0.5$, $\lambda_2 = 0.5$, $h_1= 1$, $h_2 = 1$, $r_1= 10$, $r_2 = 6$. The optimal policy for *dec.*$_1$ is determine using the fuzzy logic rule in table 1. The logic processes adopted are as follows:

a. The scaling were determined and the factors for the fuzzy inputs s_1, s_2, λ_1, and λ_2 in the rule base for *dec*$_1$*;*
b. The algorithm begin from an initial state $s_1 = s_2 = 0$;
c. With the current $s_1$and s_2 and the given $\lambda_1$and $\lambda_2$as inputs, the decision is made using fuzzification, fuzzy inference as well as de-fuzzification;
d. The decision *dec.*$_1$is plotted in the two-dimensional plane of $s_1$and s_2;
e. Proceed to step (f) if $dec_1 = 0$; else set $s_2= s_2 + 1$ and proceed to step (c); and
f. If $dec_1 = 0$, then stop; else set $s_2= s_2 + 1$ and proceed to step (c).

Step (c) in the logical processes can be illustrated with an example. Assume that $s_1= s_2 = 2$, then s_1 should be scaled down to 1.7 while s_2 is scaled down to 0.9. From figure 3, s_1 corresponds to NE with grade 0.43, FP with grade 0.90 and PO with grade 0.23, while s_1 corresponds to NE with grade 0.69 and FP with grade 0.64. Similarly, $\lambda_1 = 0.5$ is NE with grade 1.0 and $\lambda_2= 0.5$ is FP with grade 0.67 and PO with grade 0.67. The inputs s_1, s_2, λ_1 and λ_2 have 3, 2, 1, and 2 fuzzy sets respectively. Consequently, this gives 3 x 2 x 1 x 2 = 12 fuzzy decisions for fuzzy output *dec.*$_1$. Going by the fuzzy rule base, the 12 fuzzy decisions *dec.*$_1$are expressed as follows:

i. *If $s_1$is NE with grade 0.43,$s_2$is NE with grade 0.69, $\lambda_1$is NE with grade 1.00 and λ_2 is FP with grade 0.67, then dec.$_1$ is ADMIT with grade 0.43;*
ii. *If $s_1$is NE with grade 0.43, $s_2$is NE with grade 0.69, $\lambda_1$is NE with grade 1.00and $\lambda_2$is PO with grade 0.67, then dec.$_1$is ADMIT with grade 0.43;*

iii. *If s_1 is NE with grade 0.43, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00and λ_2 is FP with grade 0.67, then $dec._1$ is ADMIT with grade 0.43;*
iv. *If s_1 is NE with grade 0.43, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00and λ_2 is PO with grade 0.67, then $dec._1$ is REJECT with grade 0.43;*
v. *If s_1 is FP with grade 0.90, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00and λ_2 is FP with grade 0.67, then $dec._1$ is ADMIT with grade 0.67;*
vi. *If s_1 is FP with grade 0.90, s_2 is NE with grade 0.69, λ_1 is NE with grade1.00 and λ_2 is PO with grade 0.67, then $dec._1$ is REJECT with grade 0.67;*
vii. *If s_1 is FP with grade 0.90, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00and λ_2 is FP with grade 0.67, then $dec._1$ is REJECT with grade 0.64;*
viii. *If s_1 is FP with grade 0.90, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00and λ_2 is PO with grade 0.67, then $dec._1$ is REJECT with grade 0.64;*
ix. *If s_1 is PO with grade 0.23, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00and λ_2 is FP with grade 0.67, then $dec._1$ is REJECT with grade 0.23;*
x. *If s_1 is PO with grade 0.23, s_2 is NE with grade 0.69, λ_1 is NE with grade 1.00and λ_2 is PO with grade 0.67, then $dec._1$ is REJECT with grade 0.23;*
xi. *If s_1 is PO with grade 0.23, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00and λ_2 is FP with grade 0.67, then $dec._1$ is REJECT with grade 0.23; and*
xii. *If s_1 is PO with grade 0.23, s_2 is FP with grade 0.64, λ_1 is NE with grade 1.00and λ_2 is PO with grade 0.67, then $dec._1$ is REJECT with grade 0.23.*

The peak values and heights of the fuzzy decisions,*dec. from figure 3* are $e_1=1$, $e_2=1$, $e_3=1$, $e_4=0$, $e_5=1$, $e_6=0$, $e_7=0$, $e_8=0$, $e_9=0$ $e_{10}=0$ $e_{11}=0$ $e_{12}=0$. Similarly, $f_1 = 0.43$, $f_2 = 0.43$, $f_3 = 0.43$, $f_4 = 0.43$, $f_5 = 0.67$, $f_6 = 0.67$, $f_7 = 0.64$, $f_8 = 0.64$, $f_9 = 0.23$, $f_{10} = 0.23$, $f_{11} = 0.23$, $f_{12} = 0.23$. Using the height method of de-fuzzification, the crisp output *dec.** can be expressed as:

$$dec.^* = \frac{\sum_{i=1}^{12} e_i f_i}{\sum_{i=1}^{12} f_i} = 0.4$$

Since *dec.** < 0.5, the decision,$dec._1$ is REJECT which means that the server at station A will not admit an arriving class 1 customer. The outputsfor $dec._1$ is indicated in figure 5.

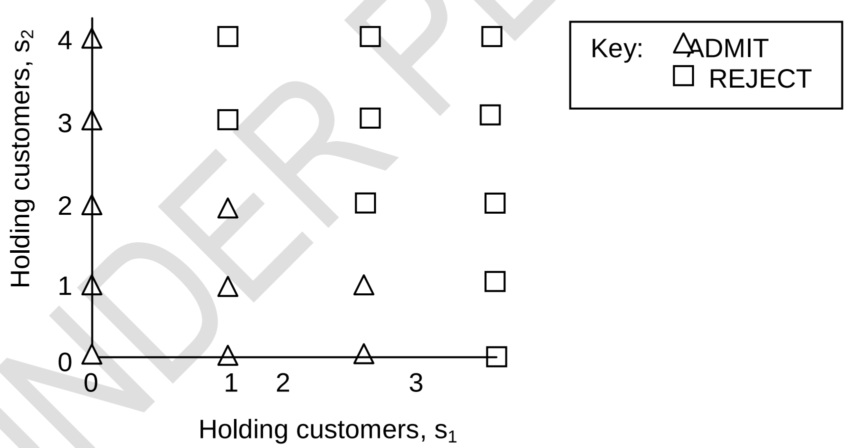

Figure 5: Outputs for $dec._1$

Similarly, the outputsfor$dec._2$ is indicated in figure 6.

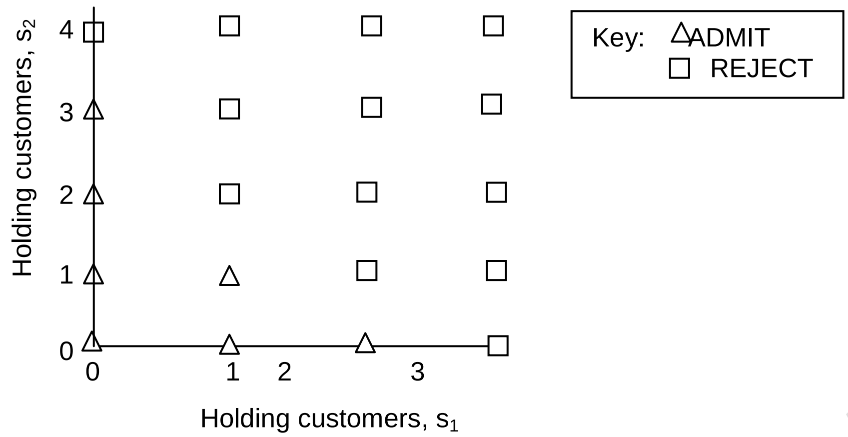

Figure 6: Outputs for *dec.*$_2$

4. CONCLUSION

The proposed model was able to manage the admission of arrivals in a two-server queue using a fuzzy-based policy such that the performance of the system is optimal. The tool adopted is a fuzzy process which determines this policy using the fuzzy input values, *s* and *λ* giving a corresponding decision, *dec*. Numerical results show a considerable improvement in the control of customers' admission and it was concluded that the proposed method is effective in the control of customers' admission in a queue network.

CONSENT

Not applicable

ETHICAL APPROVAL

Not applicable

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