Comparison of Trend Parameters and Seasonal Indices in the Presence and Absence of Missing Valuesofan Exponential Trend-Cycle in Time Series Analysis

Comment [nvg1]: The title is too long. It is suggested to change it to: COMPARISON OF TREND PARAMETERS AND SEASONAL INDICES IN TIME SERIES ANALYSIS OF LOCATION XXX.

Abstract: This study examines the comparison of trend cycle and seasonal componentsin the presence and absence of missing observations. The methodadopted in this studyis based on the row,column and overall means of the time series arranged in a Buys-Ballot table with m rows and s columns. The method assumes that (1) Only one data is missing at a time in the Buys-Ballot table (2) the trending curve isexponential (3) the modal structure is additive. The study indicates that, the estimation of the missing observations as they occur consecutively with the errors being normally distributed. Result indicates that, the difference between the trend parameters With Missing Data and Without Missing Data are insignificant because they are approximately the same. While that of seasonal components are significant at seasons 1 (one) and 7 (seven) of the Buys-Ballot table

Keywords: Missing Value, Trend Parameter, Seasonal Indices, ExponentialComponent, Additive Model, Buys-Ballot Table.

1 Introduction

Missing observations in time series analysis occurred as a result of issues that have to do with technical fault and sometime in human errors.It happens when an observation may not be made at particular time because of industrial action, inability to keep proper record, faculty equipment, mistake that cannot be ratified.This factors mentioned are very common in time series analysis.The missing values may be replaced with naïve forecast or with the average of the last two known observations that bound the missing value Almed [1]. Missing observation can lead to wrong conclusions about time series data. The process of substitution of missing observations may introduce inaccuracies. It can lead to inaccurate results, forecast and errors or data skews can proliferate across subsequent runs causing a large cumulative error effect.

Brockwell and Davis [2] discussed the option that missing data at the beginning or end of the series are simply ignored while intermediate missing data are seen as problems in the input time series. Therefore, they observed that, interpolates values using interpolation algorithm linear, polynomial, smoothing, spline and filering.

Cheema [3] used different time series methods to handle missing data. This methods are mean imputation, regression imputation, maximum likelihood imputation, multiple imputation and listwise deletion. Iwuezee*et.al* [4] proposed three different methods for estimation of missing values in time series. The methods are Column Mean Imputation (CMI) and Decomposing Without the Missing Value (DWMV). According to them, Decomposition Without the Missing Value (DWMV) yielded the best result when compared with the others.The reason for this study is to contribute to many existing solution of the problem of imputing missing values to a time series already in the literature.

Three time series models commonly used are additive, multiplicative and mixed models**.** If short period of time are involved, the trend component is superimposed into the cyclicalChatfield [5] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \tag{3}$$

As far as the descriptive method of decomposition is concerned, the first step will usually be to estimate and eliminate trend-cycle (M_t) for each time period from the actual data either by subtraction, for Equation (1) or division, for Equation (2). The de-trended series is obtained as $X_t - \hat{M}_t$ for Equation (1) or $X_t / \hat{M}_t$ for Equations (2) and (3). The seasonal effect is obtained by estimating the average of the de-trended series at each season. The de-trended, de-seasonalized series is obtained as $X_t - \hat{M}_t - \hat{S}_t$ for Equation (1) or $X_t / \left(\hat{M}_t \hat{S}_t\right)$ for Equations (2) and (3). This gives the residual or irregular component. Having fitted a time series model, one often wants to see if the residuals are purely random. For details of residual analysis, see Box, *et al*, [6] and Ljung and Box [7].It is always assumed that the seasonal effect, when it exists, has period *s*, that is, it repeats after *s* time periods.

$$S_{t+s} = S_t, \text{ for all } t \tag{4}$$

For Equation (1), it is assumed to make the further assumption that the sum of the seasonal components over a complete period is zero, ie ,

$$\sum_{j=1}^{s} S_{t+j} = 0. \tag{5}$$

Similarly, for Equations (2) and (3), it is also assumed to make further assumption is that the sum of the seasonal components over a complete period is *s*.

$$\sum_{j=1}^{s} S_{t+j} = s. \tag{6}$$

In this article, the use of Buys-Ballot table for estimation of trend parameters and seasonal indices in the presence of missing data using the methods of Iwueze and Nwogu [8] will be shown. The missing values will be estimated using the decomposition methods of Iwuezeet.al [4] and entire process of estimation will repeated in the absence of missing data. The effect of Buys-Ballot estimation of exponential trend parameters and seasonal indices in the presence and absences of missing data will be determined.

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The ultimate objective of this study is therefore, to compare the result from trend parameters with that of seasonal indices in the presence and absence of missing data. Specific objectives are (1) to estimate the trend parameters and seasonal indices of the monthly registered infant baptism over the period under investigation using Buys-Ballot table with missing values. (2) to estimate the missing values of the monthly number of registered infant baptism over the period under investigation. (3) to estimate trend parameters and seasonal indices without missing values.

This article is limited to time series with exponential trend that admits the additive model using registered number of reported infant baptism over the period January, 2012 to December, 2021. The observed series is transformed and the trend parameters and seasonal indices estimated using Decomposing Without the Missing Value. The missing observationswill be estimated using this decomposition method and the entire process

2. Materials and Methods

2.1Buys-Ballot Procedure

The method adopted for this article is the Buys-Ballot method. For details of this method, see Iwuezeet.al [4], Nwogu et.al [9], Dozie and Uwaezuoke[10], Dozie and Ibebuogu [11], Dozie et.al [12], Dozie and Ijomah [13], Dozie and Nwanya [14], Dozie [15], Dozie and Uwaezuoke [16], Dozie and Ihekuna [17], Dozie and Ibebuogu[18], Akpanta and Iwueze [19]. Dozie and Ihekuna [20], Dozie [21] and Dozie and Uwaezuoke [22]

2.2Column Mean Imputation (CMI)

The mean imputation is given.

$$CMI = \hat{X}_{(i-1)s+j} = \frac{1}{m-1}\left[\sum_{u-1}^{i=1} X_{(u-1)s+j} + \sum_{u-i+1}^{m} X_{(u-1)s+j}\right] \quad (7)$$

2.3Decomposing Without Missing Value (DWMV)

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Next, the variables to be used, the unit of measurement, the respective volume of data and the time scale are mentioned.

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The estimates of the missing value at $(i-1)s+j$ of the trend-cycle component of the regression imputation method for the exponential is given as

$$\hat{M}_{(i-1)s+j} = \hat{b}e^{\hat{c}[(i-1)s+j]} \qquad (8)$$

2.4Estimation of Trend Cycle and Seasonal Components for Exponential

The estimates of trend cycle and seasonal components provided by Iwueze and Nwogu [8]

$$\hat{C} = \frac{C'}{S} \qquad (9)$$

$$\hat{b} = b'\ell^{c\left(\frac{s-1}{2}\right)} \qquad (10)$$

$$\hat{S}_{.j} = \overline{X}_{.j} - \overline{X}_{..} \quad (11)$$

where $\bar{X}_{.j} = \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right)e^{cj} + s_j$ (12)

and $\bar{X}_{..} = \frac{be^{c}}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)$(13)

2.5 Estimates of the Missing Values of Transformed Series

The estimates of the missing values of transformedseries.

$$\hat{X}_{ij} = \hat{b}\ell^{\hat{c}[(i-1)s+j]} + \hat{S}_j + \hat{e}_{ij} \quad (14)$$

2.6Choice of Appropriate Transformation

Akpanta and Iwueze [19] presented the slope of the regression equation of log of group standard deviation on log of group mean as stated in equation (15) is what is needed for choice of appropriate transformation. Some of the values of slope β and their implied transformation are stated in Table 1

$$\log_e \hat{\sigma}_i = a + \beta \log_e \hat{X}_{i.} \quad (15)$$

Table 1: Bartlett's Transformation for Some Values of β

S/No	1	2	3	4	5	6	7
β	**0**	$\frac{1}{2}$	**1**	$\frac{3}{2}$	**2**	**3**	**-1**
Transformation	**No transformation**	$\sqrt{X_t}$	$\log_e X_i$	$\frac{1}{\sqrt{X_t}}$	$\frac{1}{X_t}$	$\frac{1}{X_t^2}$	X_t^2

The method of Akpanta and Iwueze [19] is applied in selecting the appropriate transformation, the natural logarithm of standard deviation will be used to regress against the natural logarithm of periodic means and the result of the β - value will determine the type of transformation.

3. Analysis:

This section presents empirical example to illustrate the application of the methods of estimating missing data discussed in chapter 2. One hundred and twenty (120) of reported cases of registered infant baptism at St Jude Church Amuzi, Mbaise in Imo State, Nigeria from January, 2012 to December, 2021 are considered in which these points have two (2) missing values. One hundred and eighteen (118) observed values are shown in Appendix A. The time plots of actual and transformed series with missing data are given in figure 1. The amplitude of figures 1and 2 appears small in the first two months and appears to have increased in the later year indicating that the variance is not constant, suggesting that the data requires transformation to stabilize the variance. The natural logarithm of the periodic and standard deviation are given in Table 4. The data is transformed by using the natural logarithm of the one hundred and eighteen (118) observations by the method of Akpanta and Iwueze [19]. The row and column totals, means and standard deviations in the presence of missing values are shown in Tables 3 and 4 respectively.

Table 2: Natural Logarithm of Periodic Averages and Standard DeviationsWith Missing Values

$\bar{X}_{i.}$	$Log_e \bar{X}_{i.}$	$\hat{\sigma}_{i.}$	$Log_e \hat{\sigma}_{i.}$
9.00	2.20	4.69	1.55
6.82	1.92	2.48	0.91
6.67	1.90	2.84	1.04
6.42	1.86	3.58	1.28
7.67	2.04	3.70	1.31
7.08	1.96	4.12	1.46
9.83	2.29	4.63	1.53
8.92	2.19	3.60	1.28
7.33	1.99	5.84	1.76
9.50	2.25	3.92	1.37

The estimates of missing values using Column Mean Imputation in equation (7)

The missing values are: First missing observation

$$\frac{1}{10-1}\left[4+(4+3+4+3+6+7+4+5)\right]=\frac{40}{9}=4.44\approx 4$$

Second missing observation

$$\frac{1}{10-1}\left[12+7+10+5+5+19+13+3+12\right]=\frac{86}{9}=9.56\approx 10$$

These are the missing values estimated using Column Mean Imputation (CMI) in equation (1), the entire process of estimation is repeated in the absence using the method of Decomposing Without the Missing Value (DWMV) in equation (8). The results from trend parameters and seasonal indices with and without missing values are therefore compared.

3.1 Estimates of Exponential Trend-Cycle and Seasonal Effect With Missing Data

Using equations (9), (10) and (11) we obtain b′ = 1.758 and c′ = 1.001

$$\hat{C}=\frac{1.001}{12}=0.0834$$

$\hat{b} = 1.758 \times \ell^{0.0834\left(\frac{12-1}{2}\right)} = 1.758 \times \ell^{0.4587} = 1.758 \times 1.5820 = 2.781$

$$\hat{S}_{.j} = \overline{X}_{.j} - \overline{X}_{..}$$

Table 3: Estimates of Seasonal Indices WithMissing Values

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$	$Adj\ \hat{S}_j$
1	1.575	-0.271	-0.269
2	1.821	-0.025	-0.024
3	1.943	0.097	0.098
4	1.865	0.019	0.020
5	1.835	-0.011	-0.010
6	1.730	-0.116	-0.114
7	1.955	0.109	0.111
8	1.814	-0.032	-0.031
9	1.823	-0.023	-0.021
10	1.858	0.012	0.013
11	2.059	0.213	0.214
12	1.858	0.012	0.013
$\sum_{j=1}^{12} \hat{S}_{.j}$		-0.016	0.000

The exponential trend-cycle and seasonal indices for the transformed data with missing data are estimated using Decomposition Without Missing Value (DWMV) method and the missing data are also estimated which are shown in Tables 3, 4 and 5 respectively

Table 4: Seasonal Effect With Missing Data

Parameters	With missing values
$\hat{b}$	1.758
$\hat{c}$	1.001
$\hat{S}_1$	-0269
$\hat{S}_2$	-0.024
$\hat{S}_3$	0.098
$\hat{S}_4$	0.020

$\hat{S}_5$	0.010
$\hat{S}_6$	-0.114
$\hat{S}_7$	0.111
$\hat{S}_8$	-0.031
$\hat{S}_9$	-0.021
$\hat{S}_{10}$	0.013
$\hat{S}_{11}$	0.214
$\hat{S}_{12}$	0.013

The estimates of missing data with the transformed data and missing position are displayed in Table 8.

Table 5: Estimates of the Transformed and True Missing Data

Missing Position	$X_{2,1}$	$X_{1,7}$
Transformed Missing Data	7.954	5.097

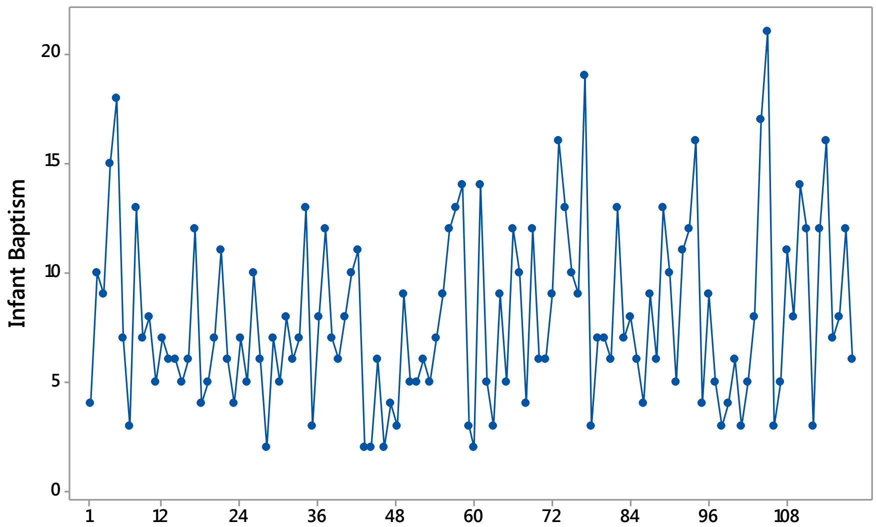

Fig 1: Original Time Series Data with Missing Data

3.2 Estimates of Exponential Trend-Cycle and Seasonal Effect Without Missing Data

Using equations (9), (10) and (11) we obtain b′ = 1.895 and c′ = 1.00

$$\hat{C} = \frac{1.00}{12} = 0.0833$$

$$\hat{b} = 1.895 \times \ell^{0.0833\left(\frac{12-1}{2}\right)} = 1.895 \times \ell^{0.4582} = 1.895 \times 1.5812 = 2.996$$

$$\hat{S}_{.j} = \overline{X}_{.j} - \overline{X}_{..}$$

Table 6: Estimates of Seasonal Indices Without Missing Values

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$
1	2.213	0.289
2	1.821	-0.103
3	1.943	0.019
4	1.865	-0.059
5	1.835	-0.089
6	1.730	-0.194
7	2.269	0.345
8	1.814	-0.110
9	1.823	-0.101
10	1.858	-0.066
11	2.059	0.135
12	1.858	-0.066
$\sum_{j=1}^{12} \hat{S}_{.j}$		0.000

Table 7: Estimates of Exponential Trend-Cycle and Seasonal Effect Without Missing Data

Parameters	Without missing values
$\hat{b}$	1.755
$\hat{c}$	1.00
$\hat{S}_1$	0.289
$\hat{S}_2$	-0.103
$\hat{S}_3$	0.019
$\hat{S}_4$	0.059

$\hat{S}_5$	-0.089
$\hat{S}_6$	-0.194
$\hat{S}_7$	0.345
$\hat{S}_8$	-0.110
$\hat{S}_9$	-0.101
$\hat{S}_{10}$	-0.066
$\hat{S}_{11}$	0.135
$\hat{S}_{12}$	-0.066

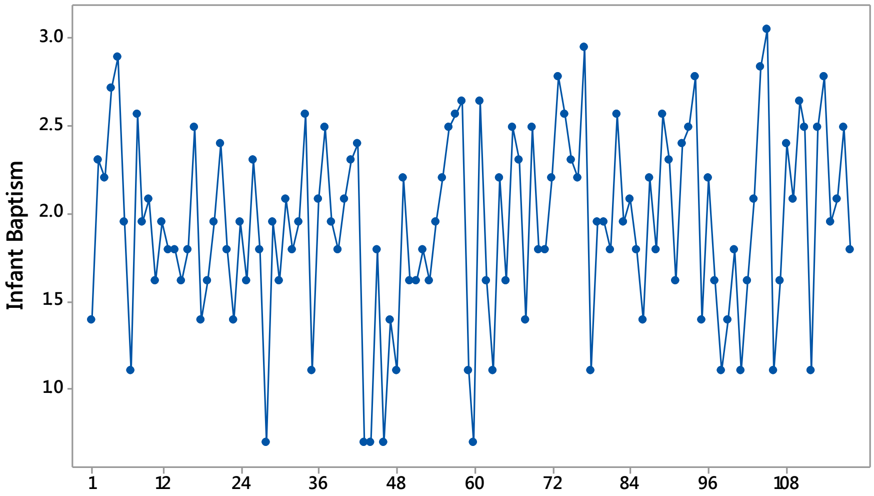

Fig 2: Transformed Time Series Data of Missing Data

The seasonal indices and exponential trend-cycle and seasonal indices for the transformed data without missing data are estimated using Decomposition Without Missing Value (DWMV) method are shown in Tables 6 and 7 respectively.

The row and column totals, means and standard deviations in the absence of missing values are listed in tables 8 and 9.

Table 8: Row totals, means and variances with missing observations.

Periods i	With missing observations			
	r_i	T_i	$\bar{X}_i$	$\hat{\sigma}_i^2$
1	9	21.149	1.923	0.967

2	9	19.721	1.793	1.788
3	10	21.202	1.767	0.245
4	10	20.554	1.713	0.346
5	10	22.048	1.837	0.280
6	10	21.230	1.769	0.344
7	10	23.824	1.965	0.303
8	10	23.261	1.938	0.249
9	10	21.186	1.765	0.387
10	10	23.647	1.971	0.281
Overall Total	98	217.819	1.846	0.304

$$n = \sum_{j=1}^{r} c_j = \sum_{i=1}^{c} r_i = total\ \ number\ \ of\ \ observation$$

Where,

r_i = Number of observation in the rth row

c_j = Number of observation in the jth column.

Table 9: Column totals, means and variances with missing observations

Seasons j		With missing observations		
	c_j	$T_{\cdot j}$	$\bar{X}_{.j}$	$\hat{\sigma}^2_{.j}$
1	12	14.177	1.575	0.142
2	11	18.209	1.821	0.279
3	12	19.426	1.943	0.255
4	12	18.645	1.865	0.324
5	12	18.350	1.835	0.318
6	12	17.302	1.730	0.238
7	11	17.596	1.955	0.345
8	12	18.137	1.814	0.337
9	12	18.233	1.823	0.267
10	12	18.579	1.858	0.326
11	12	20.588	2.059	0.257
12	12	18.578	1.858	0.387
Overall Total	142	217.819	1.846	0.304

Table 10: Row totals, means and variances without missing observations.

Periods i		Without missing observations		
	r_i	T_i	$\bar{X}_i$	$\hat{\sigma}^2_i$
1	10	26.245	2.187	0.967
2	10	27.675	2.306	1.788
3	10	21.202	1.767	0.246

4	10	20.554	1.713	0.346
5	10	22.048	1.837	0.280
6	10	21.230	1.769	0.344
7	10	23.824	1.985	0.303
8	10	23.261	1.938	0.249
9	10	21.186	1.765	0.387
10	10	23.647	1.971	0.281
Overall Total	98	230.870	1.924	0.698

Table 11: Column totals, means and variances without missing observations

Seasons		Withoutmissing observations		
j	c_j	$T_{\cdot j}$	$\bar{X}_{.j}$	$\hat{\sigma}^2_{.j}$
1	12	22.131	2.213	2.022
2	12	18.209	1.821	0.279
3	12	19.426	1.943	0.255
4	12	18.645	1.865	0.324
5	12	18.350	1.835	0.318
6	12	17.302	1.730	0.238
7	12	22.693	2.269	1.046
8	12	18.137	1.814	0.337
9	12	18.233	1.823	0.267
10	12	18.579	1.858	0.326
11	12	20.588	2.059	0.257
12	12	18.578	1.858	0.387
Overall Total	144	230.870	1.924	0.698

Table 12: Comparison of Exponential Trend-Cycle and Seasonal Indices With and Without Missing Data.

Parameters	With Missing Data	Without Missing Data	Difference
$\hat{b}$	1.758	1.755	0.003
$\hat{c}$	1.001	1.00	0.001
$\hat{S}_1$	-0.269	0.289	0.558
$\hat{S}_2$	-0.024	-0.103	0.079
$\hat{S}_3$	0.098	0.019	0.079
$\hat{S}_4$	0.020	0.059	0.039
$\hat{S}_5$	0.010	-0.089	0.099
$\hat{S}_6$	-0.114	-0.194	0.080
$\hat{S}_7$	0.111	0.345	0.234
$\hat{S}_8$	-0.031	-0.110	0.079

$\hat{S}_9$	-0.021	-0.101	0.080
$\hat{S}_{10}$	0.013	-0.066	0.079
$\hat{S}_{11}$	0.214	0.135	0.079
$\hat{S}_{12}$	0.013	-0.066	0.079

3.3Comparison of the estimation of trend parameters and seasonal indices in the presence/absence of missing values

As shown in table 12, the difference between the trend parameters in the presence and absence of the missing values are insignificant. Therefore, they are approximately the same. This indicates that the missing values have no effect on trend parameter. There are differences in the presence and absence of seasonal indices. The differences occurred at $j =$ 1 and 7. These are the points in the column of the Buys-Ballot table that have missing values. The estimates for the unobserved number of registered infant baptism are: (5.097) in July, 2012 and (7.954) in January, 2013.

Comment [nvg6]: This section does not provide a discussion that explains or indicates the meaning of the values obtained,
This is not related to references of other studies (as references) and there is no evidence of statistical evaluations (for example: RMSE, BIAS ...etc) that would indicate if the results obtained are of quality.

4. Summary, Conclusion and Recommendations

This Study has compared the results of Buys-Ballot estimates of exponential trend cycle component and that of seasonal indices in the presence and absence of missing observations in time series. The methodadopted is based on the row,Column and overall means of the time series arranged in a Buys-Ballot table with m rows and s columns. The study is limited to a series with only exponential trend and seasonal indices combined in the additive form. Only data missing at one point at a time is considered. This study provided a solution to the problem of missing observations not only one observation missing at a time but two or more observations missing consecutively.

Results indicate that, under the stated assumptions, (1) there is no significant difference in the trend parameters in the presence and absence shown in Table 7. Therefore, missing values have no effect in trend parameters. (2) there is significant effect in the seasonal indices of missing values. The differences occurred at $j =$ 1 and 7 as indicted in Table 12. These are the points in the column of the Buys-Ballot table that have missing values. The estimates for the unobserved number of registered infant baptism are: (5.097) in July, 2012 and (7.954) in January, 2013. The study has provided a basis for the comparison of exponential trend cycle and seasonality in the time in the presence and absence of missing observations when only data missing at one point at a time.No attempt has been made to discuss this method when the trend-cycle component is not exponential or when the cyclical component is separated from the trend. Therefore, further investigations in these directions are recommended

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Appendix A: Buys Ballot table of the actual data on the number of infant

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	4.0	10.0	9.0	15.0	18.0	7.0		3.0	13.0	7.0	8.0	5.0	99.0	9.00	4.69
2013		7.0	6.0	6.0	5.0	6.0	12.0	4.0	5.0	7.0	11.0	6.0	75.0	6.82	2.48
2014	4.0	7.0	5.0	10.0	6.0	2.0	7.0	5.0	8.0	6.0	7.0	13.0	80.0	6.67	2.84
2015	3.0	8.0	12.0	7.0	6.0	8.0	10.0	11.0	2.0	2.0	6.0	2.0	77.0	6.42	3.58
2016	4.0	3.0	9.0	5.0	5.0	6.0	5.0	7.0	9.0	12.0	13.0	14.0	92.0	7.67	3.70
2017	3.0	2.0	14.0	5.0	3.0	9.0	5.0	12.0	10.0	4.0	12.0	6.0	85.0	7.08	4.12
2018	6.0	9.0	16.0	13.0	10.0	9.0	19.0	3.0	7.0	7.0	6.0	13.0	118.0	9.83	4.63
2019	7.0	8.0	6.0	4.0	9.0	6.0	13.0	10.0	5.0	11.0	12.0	16.0	107.0	8.92	3.60
2020	4.0	9.0	5.0	3.0	4.0	6.0	3.0	5.0	8.0	17.0	21.0	3.0	88.0	7.33	5.84
2021	5.0	11.0	8.0	14.0	12.0	3.0	12.0	16.0	7.0	8.0	12.0	6.0	114.0	9.50	3.92
total	40.0	74.0	90.0	82.0	78.0	62.0	86.0	76.0	74.0	81.0	108.0	84.0	**935**		
$\bar{y}_{.j}$	4.44	7.40	9.00	8.20	7.80	6.20	9.56	7.60	7.40	8.10	10.80	8.40		7.92	
$\sigma_{.j}$	1.33	2.88	3.86	4.44	4.57	2.30	5.05	4.43	3.03	4.28	4.49	5.06			4.07

Appendix B: Buys-Ballot of transformed data without missing value

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	1.537	2.042	1.976	2.315	2.450	1.828		1.406	2.215	1.828	1.905	1.647	26.245	2.187	0.967
2013		1.828	1.743	1.743	1.647	1.743	2.160	1.537	1.647	1.828	2.103	1.743	27.675	2.306	1.788
2014	1.537	1.828	1.647	2.042	1.743	1.240	1.828	1.647	1.905	1.743	1.828	2.215	21.202	1.767	0.246
2015	1.406	1.905	2.160	1.828	1.743	1.905	2.042	2.103	1.240	1.240	1.743	1.240	20.554	1.713	0.346
2016	1.537	1.406	1.976	1.647	1.647	1.743	1.647	1.828	1.976	2.160	2.215	2.266	22.048	1.837	0.280
2017	1.406	1.240	2.266	1.647	1.406	1.976	1.647	2.160	2.042	1.537	2.160	1.743	21.230	1.769	0.344
2018	1.743	1.976	2.362	2.215	2.042	1.976	2.491	1.406	1.828	1.828	1.743	2.215	23.824	1.985	0.303
2019	1.828	1.905	1.743	1.537	1.976	1.743	2.215	2.042	1.647	2.103	2.160	2.362	23.261	1.938	0.249
2020	1.537	1.976	1.647	1.406	1.537	1.743	1.406	1.647	1.905	2.407	2.570	1.406	21.186	1.765	0.387
2021	1.647	2.103	1.905	2.266	2.160	1.406	2.160	2.362	1.828	1.905	2.160	1.743	23.647	1.971	0.281
total	**22.131**	**18.209**	**19.426**	**18.645**	**18.350**	**17.302**	**22.693**	**18.137**	**18.233**	**18.579**	**20.588**	**18.578**	**230.870**		
$\bar{y}_{.j}$	**2.213**	**1.821**	**1.943**	**1.865**	**1.835**	**1.730**	**2.269**	**1.814**	**1.823**	**1.858**	**2.059**	**1.858**		**1.924**	
$\sigma_{.j}$	**2.022**	**0.279**	**0.255**	**0.324**	**0.318**	**0.238**	**1.046**	**0.337**	**0.267**	**0.326**	**0.257**	**0.387**			0.698

Appendix C: Buys Ballot table of the actual data on the number of infant

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	4.0	10.0	9.0	15.0	18.0	7.0	10.0	3.0	13.0	7.0	8.0	5.0	99.0	9.00	4.69
2013	4.0	7.0	6.0	6.0	5.0	6.0	12.0	4.0	5.0	7.0	11.0	6.0	75.0	6.82	2.48
2014	4.0	7.0	5.0	10.0	6.0	2.0	7.0	5.0	8.0	6.0	7.0	13.0	80.0	6.67	2.84
2015	3.0	8.0	12.0	7.0	6.0	8.0	10.0	11.0	2.0	2.0	6.0	2.0	77.0	6.42	3.58
2016	4.0	3.0	9.0	5.0	5.0	6.0	5.0	7.0	9.0	12.0	13.0	14.0	92.0	7.67	3.70
2017	3.0	2.0	14.0	5.0	3.0	9.0	5.0	12.0	10.0	4.0	12.0	6.0	85.0	7.08	4.12
2018	6.0	9.0	16.0	13.0	10.0	9.0	19.0	3.0	7.0	7.0	6.0	13.0	118.0	9.83	4.63
2019	7.0	8.0	6.0	4.0	9.0	6.0	13.0	10.0	5.0	11.0	12.0	16.0	107.0	8.92	3.60
2020	4.0	9.0	5.0	3.0	4.0	6.0	3.0	5.0	8.0	17.0	21.0	3.0	88.0	7.33	5.84

2021	5.0	11.0	**8.0**	**14.0**	**12.0**	**3.0**	**12.0**	**16.0**	**7.0**	**8.0**	**12.0**	**6.0**	114.0	9.50	3.92
total	40.0	74.0	90.0	82.0	78.0	62.0	86.0	76.0	74.0	81.0	108.0	84.0	**935**		
$\bar{y}_{.j}$	4.44	7.40	9.00	8.20	7.80	6.20	9.56	7.60	7.40	8.10	10.80	8.40		7.92	
$\sigma_{.j}$	1.33	2.88	3.86	4.44	4.57	2.30	5.05	4.43	3.03	4.28	4.49	5.06			4.07

Appendix D: Buys-Ballot of transformed data without missing value

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	1.537	2.042	1.976	2.315	2.450	1.828	5.097	1.406	2.215	1.828	1.905	1.647	26.245	2.187	0.967
2013	7.954	1.828	1.743	1.743	1.647	1.743	2.160	1.537	1.647	1.828	2.103	1.743	27.675	2.306	1.788
2014	1.537	1.828	1.647	2.042	1.743	1.240	1.828	1.647	1.905	1.743	1.828	2.215	21.202	1.767	0.246
2015	1.406	1.905	2.160	1.828	1.743	1.905	2.042	2.103	1.240	1.240	1.743	1.240	20.554	1.713	0.346
2016	1.537	1.406	1.976	1.647	1.647	1.743	1.647	1.828	1.976	2.160	2.215	2.266	22.048	1.837	0.280
2017	1.406	1.240	2.266	1.647	1.406	1.976	1.647	2.160	2.042	1.537	2.160	1.743	21.230	1.769	0.344
2018	1.743	1.976	2.362	2.215	2.042	1.976	2.491	1.406	1.828	1.828	1.743	2.215	23.824	1.985	0.303
2019	1.828	1.905	1.743	1.537	1.976	1.743	2.215	2.042	1.647	2.103	2.160	2.362	23.261	1.938	0.249
2020	1.537	1.976	1.647	1.406	1.537	1.743	1.406	1.647	1.905	2.407	2.570	1.406	21.186	1.765	0.387
2021	1.647	2.103	1.905	2.266	2.160	1.406	2.160	2.362	1.828	1.905	2.160	1.743	23.647	1.971	0.281
total	**22.131**	**18.209**	**19.426**	**18.645**	**18.350**	**17.302**	**22.693**	**18.137**	**18.233**	**18.579**	**20.588**	**18.578**	**230.870**		
$\bar{y}_{.j}$	**2.213**	**1.821**	**1.943**	**1.865**	**1.835**	**1.730**	**2.269**	**1.814**	**1.823**	**1.858**	**2.059**	**1.858**		**1.924**	
$\sigma_{.j}$	**2.022**	**0.279**	**0.255**	**0.324**	**0.318**	**0.238**	**1.046**	**0.337**	**0.267**	**0.326**	**0.257**	**0.387**			0.698