Original Research Article

Non-Stationary Modeling of Annual Flood Peak Heights of Mahanadi River Basin with The q-Generalized Extreme Value Distribution

Abstract

In recent years, due to climate change, catastrophic events are increased largely in India. Hence researchers are forced to consider non-stationary flood frequency analysis as an improved method. In this paper, non-stationarity of annual daily maximum flood heights were studied at 12 sites of Mahanadi river basin by analyzing the flood frequency of a stationary model and 4 non-stationary models using time dependent q-GEV model by considering trend as a linear function of its location and scale parameters. The q-GEV distribution is utilized in this study because of its flexibility and accuracy than GEV distribution in modeling extreme flood heights. The results found that there is strong evidence of a linear trend existence for both the location and scale parameters at the Kesinga site; for the location parameter at Pathardi and Simga sites; for the scale parameter at Dharmajagarh, Kotni and Seorinarayan, and no linear trend exists for both location and scale parameters at Alipingal, Bomnidhi, Manendragarh, Mohana, Rajim and Sundargarh, there may be exists other form of trend at these sites. The findings also indicate that nonstationarity is present in the MRB due to climate change.

Keywords: Mahanadi River Basin; Non-stationarity; q-GEV Model; MLE.

1. Introduction

In the past few years, occurrence of extreme hydrological events such as floods, droughts are increasingly repeated due to impact of climate change, urbanization, deforestation and encroachment of river basin, particularly in India (Singh and Kumar, 2013; Black et al., 2018). In India, each year 1600 persons die as a result of the floods, Rs. 5600 Crores (73 Million USD) as a damage cost due to floods and 12% of geographic area of India is flood prone (Black et al., 2018). The assumption of stationary is that in a system, statistical characteristics such as mean and variance do not vary over time and show no trend (Sivakumar, 2016). The impact of climate change, global warming, deforestation and urbanization can invalidate the assumption of stationarity (Ray et al., 2019), Gumbel (1941) expressed invalidation of stationary model flood frequency analysis in the situations of climate change and other variations. It is therefore necessary to develop alternative methods that take nonstationarity into account for the effective design of hydraulic control structures. The floods are exacerbated by natural events such as reduced carrying capacity of the water course caused by silting of the river bed, landslide causing obstacles and change in river path and by manmade events such as unplanned urbanization and floodplain encroachment (Singh and Chinnasamy, 2021). The flood frequency analysis of one stationary and 12 non-stationary models were used to evaluate the nonstationarity of Periyar River flow, and their results indicate that climate change and anthropogenic activities are equally responsible for the nonstationarity of the Periyar River discharge (Singh and Chinnasamy, 2021). Seasonality and Trend in natural occurrences such as floods are common causes of nonstationarity (Maposa et al., 2016). In non-stationary settings, Hounkpe (2015) provided a statistical model for predicting flood probability, the model was used to five gauging stations in the Queme River Basin in Benin Republic, West Africa and it fits the annual maximum discharge using a time-dependent and covariate-dependent generalized extreme value (GEV) distribution.

In the above works, the GEV distribution was considered to be a good model for non-stationary modeling with different approaches, in our study, an extended GEV model q-GEV distribution was

considered for the first time as an alternate distribution for modeling the non-stationary situation in the Mahanadi River Basin (MRB).
In practice, the GEV model may be insufficient, but its generalizations should provide more modelling flexibility. Provost et al. (2018) introduced the extended model q-GEV distribution, which is a q analogue of the GEV, where q is an extra parameter that allows for greater flexibility in modeling extreme events than the GEV distribution. In previous work, the q-GEV distribution was shown that better flood frequency model than GEV distribution for twelve MRB sites using the block maxima technique with stationary assumption. For the same sites, our objective in this study is to model maximum flood height using time dependent q-GEV distribution under non-stationary assumption that is when covariates are present. This study involves a non-stationary q-GEV distribution as time dependent, expecting the variation linearly or nonlinearly over time in location and scale parameters while shape parameter is unchanged with the time. A statistical modeling approach is advocated by considering maximum likelihood estimation (MLE) in the presence of covariates like trends and cycles (Katz et al., 2002). Further related information can be found in the literature (Hesarkazzazi et al., 2021; Meis et al., 2021; Sen, 2021)
As far as we know no similar work has been done in earlier studies on extreme value theory in a changing climate for the Mahanadi River Basin.

2. Materials and methods

This section explains how the data was analyzed. The extended time homogeneous q-GEV distribution was used to investigate linear and quadratic trend models.

The data

The original form of twelve sites flood height data recorded thrice a day was provided by the Central Water Commission (CWC), Bhubaneshwar, which is the competent authority for water resources management in the river basin. Using successive steps in each hydrological year, the annual maximum series was obtained. Ferreira and De Haan (2015) provide detailed descriptions of the block maxima probability theory as well as practical considerations for choosing block maxima rather than peak over threshold, Dombry (2013) demonstrated that when utilizing the block maxima technique ML estimators are consistent. Block maxima and Peak over Threshold are two important methodologies used in extreme value theory for flood frequency analysis. When the sample size is large, the block maxima approach is utilized, in which each hydrological year is considered as its own block (Ferreira and De Haan, 2015).

Non-stationary extreme value models

The goal of the study was to model the behavior of annual maximum flood heights in the presence of covariates, in order to see if non-stationary models fit the data better than stationary (time independent) models. Trends, cycles, and physical factors are examples of covariates, according to Katz et al. (2002). Trends are considered as covariates in particular, and the method of time varying moments is the most commonly employed approach to non-stationary flood frequency analysis.

The models M_0, M_1, M_2, M_3, andM_4 were considered to compare the stationary (Time independent) and non-stationary (Time dependent) models, where

M_0 - q-GEV time independent (stationary) model,
M_1 - q-GEV model with linear trend in location and scale parameter,

M_2 - q-GEV model with linear trend in location parameter,
M_3 - q-GEV model with linear trend in scale parameter and
M_4 - q-GEV model with non-linear trend in location parameter.

In previous work, the best flood frequency model was shown to be the q-GEV (time independent) distribution at twelve MRB sites (Nagesh and Laxmi, 2022). This study looks at non-stationary (time-dependent) models for the same sites. M_0 is a stationary model that is not affected by time. Non-stationary models M_1, M_2, M_3, andM_4 have a linear or nonlinear trend in the location or scale parameter, or both.

The distribution function and density function of q-GEV distribution is given by

$$F(x; s, m, \xi, q) = \left[1 + q\left(1 + \xi(xs - m)\right)^{-\frac{1}{\xi}}\right]^{-\frac{1}{q}} \; ; \quad \xi \neq 0, q \neq 0 \qquad (1)$$

$$f(x; s, m, \xi, q) = s\left(1 + \xi(xs - m)\right)^{(-1/\xi)-1}\left[1 + q\left(1 + \xi(xs - m)\right)^{(-1/\xi)}\right]^{(-1/q)-1} \; ; \quad \xi \neq 0, q \neq 0 \qquad (2)$$

The log-likelihood function of q-GEV distribution is given by

$$l(s, m, \xi, q) = n * \log(s) + \left(-\frac{1}{q} - 1\right)\sum_{i=1}^{n} \log\left[q(\xi(x_i s - m) + 1)^{-1/\xi} + 1\right] + \left(-\frac{1}{\xi} - \right.$$
1i=1nlogξxis−m+1
(3)

where $m = \mu/\sigma$ and $s = 1/\sigma$, μ is location parameter, σ is scale parameter, ξ and q are shape parameters.

The distribution function, Probability density function and log-likelihood function of q-GEV distribution without re-parameterization are given by equations (4), (5) and (6) respectively.

$$F(x; \mu, \sigma, \xi, q) = \left\{1 + q\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}^{-1/q} \; ; \qquad \xi \neq 0, q \neq 0 \qquad (4)$$

$$f(x; \mu, \sigma, \xi, q) = \frac{1}{\sigma}\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{\left(-\frac{1}{\xi}\right)-1}\left\{1 + q\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}^{\left(-\frac{1}{q}\right)-1} \; ; \; \mu \in R, \sigma > 0, \; \xi \neq 0, q \neq 0$$
(5)

$$l(\mu, \sigma, \xi, q) = n * \log(\sigma) + \left(-\frac{1}{\xi} - 1\right)\sum_{i=1}^{n} log\left[1 + \xi\left(\frac{x_i-\mu}{\sigma}\right)\right] + \left(-\frac{1}{q} - 1\right)\sum_{i=1}^{n} log\left\{1 + \right.$$
qξxi−μσ−1/ξ
(6)

M_1 is non-stationary q-GEV model with linear trend in both location and scale parameter is

$\mu(t) = \mu_0 + \mu_1 t$, $\log\sigma(t) = \sigma_0 + \sigma_1 t$, $\quad \xi(t) = \xi \quad$ and $\quad q(t) = q$

The distribution function of M_1 is given by equation (7) while the log-likelihood function of M_1 is given by equation (8).

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + q\left[1 + \xi\left(\frac{x-(\mu_0+\mu_1 t)}{\exp(\sigma_0+\sigma_1 t)}\right)\right]^{-1/\xi}\right\}^{-1/q} \;;\; \xi \neq 0, q \neq 0 \quad (7)$$

$l\big(\mu(t),\sigma(t),\xi(t),q(t)\big) = \; n * log\left(-\frac{1}{\xi} - 1\right)\sum_{i=1}^{n} log\left[1 + \xi\left(\frac{x_i-(\mu_0+\mu_1 t)}{\exp(\sigma_0+\sigma_1 t)}\right)\right] + \;=\; \left(-\frac{1}{q} -\right.$
*1i=1nlog1+qξxi−(μ0+μ1t)*exp(*σ0+σ1t)−1/ξ*
(8)

where t is time (in years). The log-likelihood function of M_1 is given by equation (8), and the set of values is estimated by maximization of the log-likelihood function. The Newton Raphson method is used to solve the log-likelihood function equations.

M_2 is non-stationary (time dependent) q-GEV model with linear trend in the location parameter.

$\mu(t) = \mu_0 + \mu_1 t,\; \sigma(t) = \sigma,\; \xi(t) = \xi$ and $q(t) = q$

Cumulative distribution function and log-likelihood function of M_2 is given by equation (9) and equation (10) respectively

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + q\left[1 + \xi\left(\frac{x-(\mu_0+\mu_1 t)}{\sigma}\right)\right]^{-1/\xi}\right\}^{-1/q} \;;\; \xi \neq 0, q \neq 0 \quad (9)$$

$$l\big(\mu(t),\sigma(t),\xi(t),q(t)\big) = \; n * \log(\sigma) + \left(-\frac{1}{\xi} - 1\right)\sum_{i=1}^{n} log\left[1 + \xi\left(\frac{x_i-(\mu_0+\mu_1 t)}{\sigma}\right)\right] + \;=\; \left(-\frac{1}{q} - 1\right)\sum_{i=1}^{n} log\left\{1 + q\left[\xi\left(\frac{x_i-(\mu_0+\mu_1 t)}{\sigma}\right)\right]^{-1/\xi}\right\} \quad (10)$$

M_3 is non-stationary q-GEV model with linear trend in the scale parameter.

$log\sigma(t) = \sigma_0 + \sigma_1 t$, $\mu(t)=\mu$, $\xi(t) = \xi$ and $q(t) = q$

Distribution function and log-likelihood function of M_3 model are obtained by replacing above quantities in equations (4) and (6) respectively.

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + q\left[1 + \xi\left(\frac{x-\mu}{\exp(\sigma_0+\sigma_1 t)}\right)\right]^{-1/\xi}\right\}^{-1/q} \;;\; \xi \neq 0, q \neq 0 \quad (11)$$

$$l\big(\mu(t),\sigma(t),\xi(t),q(t)\big) = \; n * \ln(\sigma_0 + \sigma_1 t) + \left(-\frac{1}{\xi} - 1\right)\sum_{i=1}^{n} log\left[1 + \xi\left(\frac{x_i-\mu}{\exp(\sigma_0+\sigma_1 t)}\right)\right] + \;=\; \left(-\frac{1}{q} - 1\right)\sum_{i=1}^{n} log\left\{1 + q\left[\xi\left(\frac{x_i-\mu}{\exp(\sigma_0+\sigma_1 t)}\right)\right]^{-1/\xi}\right\} \quad (12)$$

M_4 is non-stationary q-GEV model with non-linear quadratic trend in the location parameter.

$\mu(t) = \mu_0 + \mu_1 t + \mu_2 t^2, \ \sigma(t) = \sigma, \ \xi(t) = \xi$ and $q(t) = q$

Similarly by replacing above quantities in equations (4) and (6) we can get distribution and log-likelihood function of M_4 q-GEV Model.

$$F\left(x;\mu(t),\sigma(t),\xi(t),q(t)\right) = \left\{1 + q\left[1 + \xi\left(\frac{x-(\mu_0+\mu_1 t+\mu_2 t^2)}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}^{-\frac{1}{q}}; \ \xi \neq 0, q \neq 0$$

(13)

$$l\left(\mu(t),\sigma(t),\xi(t),q(t)\right) = n * \log(\sigma) + \left(-\frac{1}{\xi} - 1\right)\sum_{i=1}^{n} log\left[1 + \xi\left(\frac{x_i-(\mu_0+\mu_1 t+\mu_2 t^2)}{\sigma}\right)\right] + \ = \left(-\frac{1}{q} - 1\right)\sum_{i=1}^{n} log\left\{1 + q\left[\xi\left(\frac{x_i-(\mu_0+\mu_1 t+\mu_2 t^2)}{\sigma}\right)\right]^{-1/\xi}\right\}$$

(14)

Using the maximum likelihood estimation technique, parameters of the models M_0, M_1, M_2, M_3, andM_4 are calculated using MLE technique. MLE is used to estimate the parameters of both stationary and non-stationary q-GEV models. Using the MLE methodology in the presence of variables is reliable in both the Block Maxima and Peaks over Threshold procedures (Yilmaz et al., 2014; Coles, 2001).

Model choice

To compare one model to the other, the MLE of nested models employs a simple procedure known as the Deviance (D) statistic (Yilmaz et al., 2014; Coles, 2001; Smith, 1987). The time-homogeneous GEV model, M_0, is a subset of the time-dependent models M_1, M_2, M_3, andM_4 in this research. To check significance of non-stationary models over stationary model D statistic is given by

$$D = 2\{l_i(M_i) - l_0(M_0)\}$$

(15)

where $l_i(M_i)$ is the maximized negative log-likelihood value of i^{th} model ($i = 1,2,3,4$), $l_0(M_0)$ is the maximized negative log-likelihood value of time independent(stationary) model. D follows chi-square distribution with k degrees of freedom, where k is difference in number of parameters. If $D > \chi^2_{k,\alpha}$, then we reject H_0. It suggests that M_i is more significant than M_0.

3. Results and discussion

The parameters of the models M_0, M_1, M_2, M_3, andM_4 were calculated using MLE. Parameter estimates of non-stationary q-GEV model and Deviance statistics calculated for pairs of stationary and non-stationary models for Bamnidhi site are given in Table-1. For Bamnidhi site, consider models M_0 and M_1, negative log-likelihood value for M_0 and M_1 is -52.9804 and -51.4985 respectively.

Using equation (15), D=2[(-51.4985) - (-52.9804)] = 2.9639, which is less than 5.991, i.e. $\chi^2_{2,\ 0.05} = 5.991$. So we can conclude that M_0 is better fit than M_1 for Bamnidhi site.

Table-1: Parameter estimates of annual maximum time heterogeneous q-GEV models for Bamnidhi site

Model	μ_0	μ_1	μ_2	σ_0	σ_1	ξ	q	ll	D
M_0	4.6943	0	0	0.8055	0	-0.5750	1.7019	-52.9804	
M_1	1.4083	1.408	0	0.9667	0.4028	-0.6900	1.5317	-51.4985	2.9639
M_2	4.2001	0.261	0	0.6427	0	-0.4263	1.2799	-51.5775	2.8058
M_3	4.8455	0	0	0.7180	-0.0287	-0.7785	1.0460	-51.9625	2.0359
M_4	4.6503	1.258	0.217	0.6620	0	-1.1672	2.6899	-52.6737	0.6135

In other words non-stationary model with linear trend in location parameter is insignificant. It clearly shows that the non-stationary model does not give any improvement in fit over the time-homogeneous q-GEV model. Here we can conclude that M_0 is better fit than M_1. Next consider M_0 andM_2.

Negative log-likelihood value for M_0 andM_2, is -52.9804 and -51.5775 respectively.
D = 2[(-51.5775) - (-52.9804)] = 2.8058, which is less than 3.8414. $\chi^2_{1,0.05} = 3.8414$.
Similarly the negative log-likelihood value and D statistics values for other pairs of models (M_0, M_3) and (M_0, M_4) are given in Table-1. The D statistic values are less than critical values (i.e. 2.0359 < 3.8414 and 0.6135 < 5.991) for the models respectively, p-values when μ=0 and σ=0 are not less than 0.05 for the above models. The models (M_1, M_2, M_3, andM_4)do not provide any improvement in fit over the time-homogeneous q-GEV model at Bamnidhi site. Therefore stationary model for Bamnidhi site is given by (using equation 4).

$$F(x;\mu,\sigma,\xi,q) = \left\{1 + 1.7019\left[1 - 0.575\left(\frac{x_i - 4.6943}{0.8055}\right)\right]^{1/0.575}\right\}^{-1/1.7019}$$

Table-2:Parameter estimates of annual maximum time heterogeneous q-GEV models for Dharamjaigarh site

Model	μ_0	μ_1	μ_2	σ_0	σ_1	ξ	q	ll	D
M_0	6.0787	0	0	0.7707	0	0.3404	0.6506	-44.7622	
M_1	6.0180	0.6079	0	0.6166	0.4624	0.3106	0.9425	-43.1313	3.2618
M_2	5.8273	0.0562	0	0.3776	0	0.2542	0.9349	-44.2648	0.9947
M_3	6.7685	0	0	0.6982	-0.049	0.2014	0.7433	-42.6482	4.2279
M_4	5.5320	0.2170	0.006	3.0090		0.3138	0.8911	-43.9958	1.5327

Consider the models (M_0, M_1), D statistic is 3.2618 which is less than $\chi^2_{2,0.05} = 5.991$

Linear trend in location parameter is not significant at 5% level of significance. D statistic value for the models (M_0, M_2) is 0.9947, which is less than 3.8414(from Table-2). D statistic value for the models (M_0, M_3) is 4.2279, which is greater than 3.8414.

The likelihood ratio test for $\sigma_1 = 0$ has p-value 0.0148 (indicates that M_3 is significant). It is worthwhile to consider time-heterogeneous (non-stationary) model M_3 that is linear trend in scale parameter at Dharamjaigarh. So we conclude that non-stationary model (M_3) outperform than stationary model.

Therefore non-stationary model with linear trend in scale parameter for Dharamjaigarh is given by (using equation 11)

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + 0.7433\left[1 + 0.2014\left(\frac{x_i - 6.7685}{\exp(0.6982 - 0.0489\,t)}\right)\right]^{-1/0.2014}\right\}^{-1/0.7433}$$

Parameter estimates of stationary and non-stationary q-GEV models for remaining ten sites are given in Table-3.

Table-3: Parameter estimates of time heterogeneous q-GEV models for ten sites

Site name	Model	μ_0	μ_1	μ_2	σ_0	σ_1	ξ	q
Kesinga	M_0	10.0670	0	0	1.8674	0	-0.6814	1.2930
	M_1	10.9203	1.0109	0	1.8639	0.1868	-0.6156	1.1996
	M_2	11.4792	1.0358	0	2.6923	0	-0.2112	2.4827
	M_3	10.2259	0	0	1.954	0.6547	-0.6952	2.3249
	M_4	10.0927	-4.2634	0.2132	1.9666	0	-0.3211	1.3031
Kotni	M_0	9.5410	0	0	1.7674	0	-0.5996	1.2471
	M_1	9.1822	0.9634	0	1.7580	0.1770	-0.6254	1.1993
	M_2	9.1917	1.0535	0	1.8565	0	-0.3397	1.5492
	M_3	8.5328	0	0	0.3842	-0.031	-0.7229	1.1002
	M_4	9.9001	-0.7310	0.0416	0.5767	0	-0.4306	2.0120
Manendragarh	M_0	4.3112	0	0	0.5755	0	0.1531	0.2173
	M_1	4.1216	0.0431	0	0.5180	0.4604	0.1791	0.1337
	M_2	4.3063	0.4574	0	0.5121	0	0.0772	0.2843
	M_3	4.8871	0	0	0.4896	-0.048	0.0627	0.1491
	M_4	4.5419	0.0149	0.0043	0.5528	0	0.1738	0.2498
Mohana	M_0	3.5996	0	0	0.6233	0	0.4391	1.2296
	M_1	3.4053	0.2746	0	0.9942	0.1221	0.3932	2.1456
	M_2	3.6147	0.1662	0	0.6176	0	0.2903	2.9925
	M_3	3.2420	0	0	0.3677	-0.005	0.3524	1.1606
	M_4	3.1959	-0.1583	0.0100	0.6223	0	0.4899	2.6569
Pathardhi	M_0	6.3879	0	0	1.4162	0	-0.4998	1.8411
	M_1	6.4684	0.6390	0	1.3623	0.1432	-0.5901	1.8992
	M_2	6.2766	0.7046	0	0.5444	0	-0.6028	1.9535
	M_3	6.4457	0	0	4.5827	-0.657	-0.5179	0.9781
	M_4	3.2121	0.8032	0.0623	0.2218	0	-0.7954	1.8510
Rajim	M_0	6.2691	0	0	1.1429	0	-0.4184	0.7030
	M_1	6.4530	0.6197	0	1.1333	0.1087	-0.7899	0.8978

	M_2	7.0967	0.8451	0	1.0812	0	-0.6399	0.9018
	M_3	6.9983	0	0	0.8316	0.8161	-0.5623	0.1563
	M_4	7.9598	-0.2092	0.0634	0.0966	0	-0.7796	0.9472
Seorinarayan	M_0	14.1171	0	0	2.0398	0	-1.2899	1.1590
	M_1	14.0027	1.4109	0	2.0608	0.2040	-1.1201	1.1992
	M_2	14.2159	1.458	0	1.9547	0	-1.654	1.2564
	M_3	13.9995	0	0	2.0956	0.2354	-1.441	1.9547
	M_4	14.2555	1.8647	0.0947	2.1587	0	-1.569	2.458
Simga	M_0	11.0891	0	0	1.9198	0	-0.6793	1.2716
	M_1	11.2147	1.1083	0	1.7283	0.1920	-0.7590	1.3015
	M_2	10.4293	0.1565	0	0.9750	0	-0.2570	1.8502
	M_3	12.2545	0	0	0.1885	-0.038	-0.2824	1.3522
	M_4	11.9654	-4.1557	0.0978	0.6826	0	-0.1515	1.1974
Sundargarh	M_0	7.3615	0	0	0.7668	0	-0.2991	0.5387
	M_1	7.1661	1.0138	0	0.7619	0.0768	-0.2665	0.4889
	M_2	7.7778	0.9919	0	0.8079	0	-0.2408	0.5498
	M_3	9.9346	0	0	1.0730	-0.267	-2.1195	0.5479
	M_4	7.2849	-1.4298	0.0965	0.3273	0	-0.2099	0.7350
Alipingal	M_0	11.4711	0	0	2.2865	0	-1.2679	3.7158
	M_1	11.3595	1.1470	0	2.0579	0.2286	-1.1936	3.5176
	M_2	11.6941	0.5456	0	0.8858	0	-0.0250	3.9536
	M_3	11.254	0	0	0.5648	0.198	-1.954	3.5578
	M_4	11.7532	3.6418	0.0317	0.5422	0	-1.1161	3.8538

Table-4 gives Negative log-likelihood (NLL), D statistic values and χ^2 critical value of stationary and non-stationary time series models for remaining ten sites.

Table-4:Annual maximum time heterogeneous q-GEV models for ten sites

Site name	**Model**	**NLL**	**D**	**χ^2 Critical Value**
Kesinga	M_0	-83.9958		
	M_1	-83.1	1.7916	5.991
	M_2	-83.3512	1.2892	3.8414
	M_3	-81.5821	4.8274	3.8414
	M_4	-83.2325	1.5266	5.991
Kotni	M_0	-82.4308		
	M_1	-82.1537	0.5542	5.991
	M_2	-82.3325	0.1966	3.8414
	M_3	-80.2243	4.413	3.8414
	M_4	-81.9637	0.9342	5.991
Manendragarh	M_0	-44.2274		
	M_1	-43.5480	1.35876	5.991
	M_2	-43.9629	0.52896	3.8414
	M_3	-43.649	1.15676	3.8414
	M_4	-43.3618	1.73116	5.991
Mohana	M_0	-39.6943		
	M_1	-39.547	0.2946	5.991

	M_2	-38.9243	1.54	3.8414
	M_3	-39.254	0.8806	3.8414
	M_4	-39.1657	1.0572	5.991
Pathardhi	M_0	-51.1474		
	M_1	-50.945	0.4047	5.991
	M_2	-48.3295	5.6357	3.8414
	M_3	-50.8495	0.5957	3.8414
	M_4	-50.5798	1.1351	5.991
Rajim	M_0	-79.2463		
	M_1	-78.5139	1.4648	5.991
	M_2	-79.1537	0.1852	3.8414
	M_3	-78.9381	0.6164	3.8414
	M_4	-78.349	1.7946	5.991
Seorinarayan	M_0	-66.772		
	M_1	-66.2973	0.9494	5.991
	M_2	-66.4967	0.5506	3.8414
	M_3	-63.9427	5.6586	3.8414
	M_4	-65.5468	2.4504	5.991
Simga	M_0	-100.255		
	M_1	-99.5826	1.345	5.991
	M_2	-98.2431	4.024	3.8414
	M_3	-99.5488	1.4126	3.8414
	M_4	-99.2849	1.9404	5.991
Sundaragarh	M_0	-54.8368		
	M_1	-53.9144	1.8448	5.991
	M_2	-54.015	1.6436	3.8414
	M_3	-54.3521	0.9694	3.8414
	M_4	-53.9986	1.6764	5.991
Alipingal	M_0	-66.6641		
	M_1	-66.315	0.6981	5.991
	M_2	-66.4983	0.3315	3.8414
	M_3	-66.3219	0.68424	3.8414
	M_4	-66.1999	0.92832	5.991

At Kesinga site, D statistic of models (M_0, M_3) is 4.8274 which is greater than tabulated value 3.8414, it shows that model M_3 is better than stationary model. The D statistic values for all other models except model (M_0, M_3) are less than tabulated value. Therefore at Kesinga non-stationary q-GEV model with linear trend in scale parameter M_3 gives improvisation over stationary model M_0.

So non-stationary model with linear trend in scale parameter for Kesinga site is given by

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + 0.7433\left[1 - 0.6952\left(\frac{x_i - 10.2259}{\exp(1.954 - 0.6547t)}\right)\right]^{1/0.6952}\right\}^{-1/2.3249}$$

Similarly from Table-4, D statistic value for the pair of model $M_0$and M_3 for Kotni and Seorinarayan is 4.413 and 5.6586 respectively, which are greater than tabulated value 3.8414 (i.e. D $>\chi^2_{1,0.05}$). Hence we can conclude that model M_3 is significant than M_0 for Kotni site and Seorinarayan site.

So non-stationary model with linear trend in scale parameter for Kotni site is given by

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + 1.1002\left[1 - 0.7229\left(\frac{x_i - 8.5328}{\exp(0.3842 - 0.031t)}\right)\right]^{1/0.7229}\right\}^{-1/1.1002}$$

Non-stationary model with linear trend in scale parameter for Seorinarayan site is given by

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + 1.9547\left[1 - 1.441\left(\frac{x_i - 13.9995}{\exp(2.0956 + 0.2354t)}\right)\right]^{1/1.441}\right\}^{-1/1.9547}$$

Next, the D statistic values for the pair of models (M_0, M_2)of Pathardhi and Simga sites are 5.6357 and 4.024 respectively. D statistic values are greater than 3.8414 (i.e. D $>\chi^2_{1,0.05}$). Hence we can conclude that non-stationary q-GEV model with linear trend in location parameter is better model than stationary model at these sites.

Non-stationary model with linear trend in location parameter for Pathardhi is given by (using equation 9)

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + 1.9535\left[1 - 0.6028\left(\frac{x - (6.2766 + 0.7046t)}{0.5444}\right)\right]^{1/0.6028}\right\}^{-1/1.9535}$$

Non-stationary model with linear trend in location parameter for Simga is given by (using equation 9)

$$F(x;\mu(t),\sigma(t),\xi(t),q(t)) = \left\{1 + 1.8502\left[1 - 0.257\left(\frac{x - (10.4293 + 0.1565t)}{0.9750}\right)\right]^{1/0.257}\right\}^{-1/1.8502}$$

D statistics value for all the pairs of models i.e. (M_0, M_1), (M_0, M_2) and (M_0, M_3), and (M_0, M_4) are less than tabulated values (5.991, 3.8414, 3.8414 and 5.991 respectively) for Manendragarh, Mohana, Rajim, Sundaragrh and Alipingal. Therefore we can conclude that at these sites stationary q-GEV model is better than non-stationary models.

Stationary q-GEV model for Manendragarh is given by

$$F(x;\mu,\sigma,\xi,q) = \left\{1 + 0.2173\left[1 + 0.1531\left(\frac{x - 4.3112}{0.5755}\right)\right]^{-1/0.1531}\right\}^{-1/0.2173}$$

Similarly Stationary q-GEV model for Mohana, Rajim, Sundaragrh and Alipingal are obtained by putting values of μ, σ, ξ and q in the equation (4) from Table-3.

D statistic value for M_0 and M_4 values of all the sites are less than 5.991, which indicates that M_4 is nowhere best model among twelve sites. It is better to ignore M_4 at all the sites. Fig. 1 gives return level plot for non-stationary model (i.e. model with linear trend in scale parameter) of Dharamjaigarh, which have higher return levels as compared to stationary model for a different return period. Same nature is found in the other non-stationary model return level plot.

Fig. 1 – Fig. 6 depicts Return level plots for the sites of MRB where non-stationary q-GEV model has improvisation over stationary model.

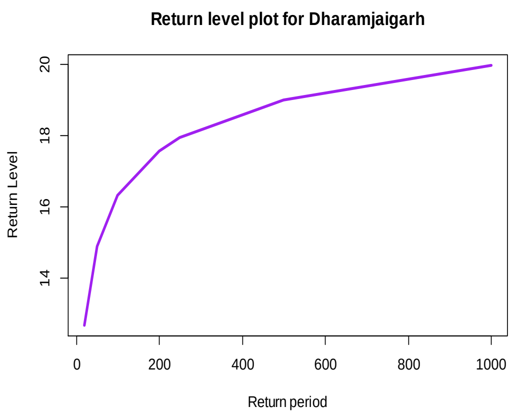


Fig. 1: Return level plot for Dharamjaigarh site

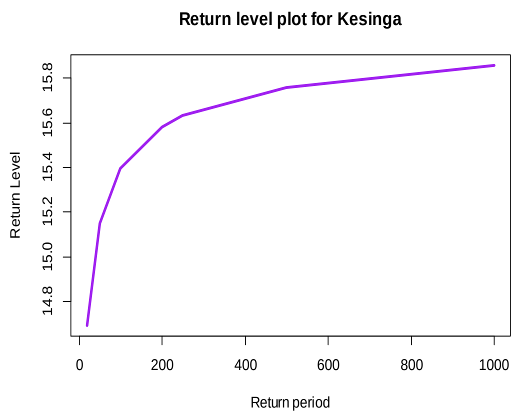


Fig. 2: Return level plot for Kesinga site

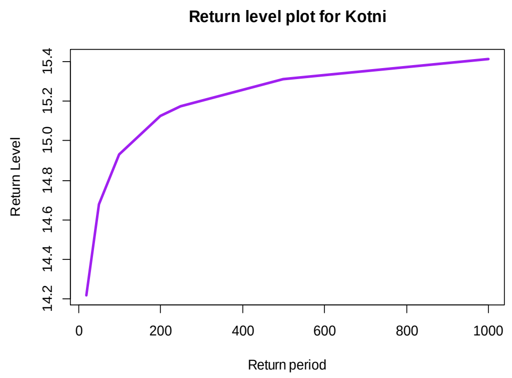


Fig. 3: Return level plot for Kotni site

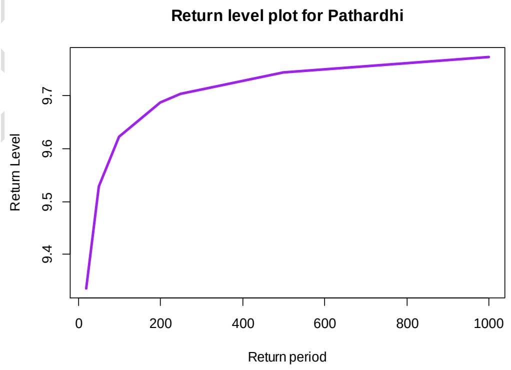
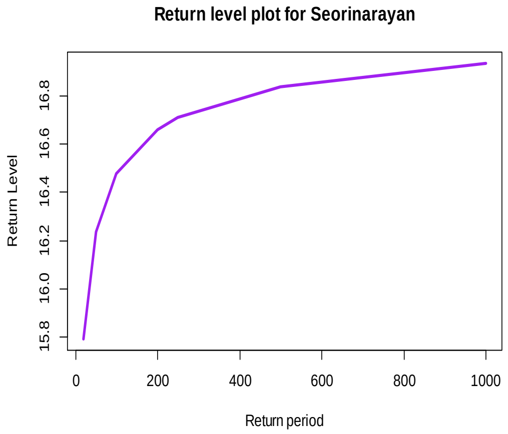


Fig. 4: Return level plot for Pathardhi site



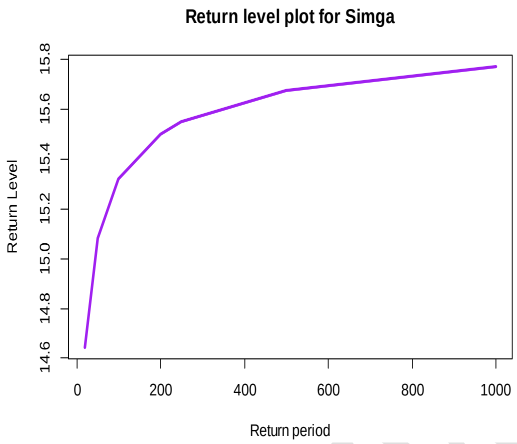


Fig. 5: Return level plot forSeorinarayan site

Fig. 6: Return level plot forSimga site

4. Conclusions

The study looked at the use of extreme value theory in a changing climate for the Mahanadi River Basin in India. The study looked at twelve hydrometric stations that represented twelve different sites along the MRB. The parameters of the q-GEV distribution were determined using the maximum likelihood estimation approach when a long-term trend covariate was present. The study revealed the importance of incorporating non-stationary linear and nonlinear trend models when using extreme value theory in a changing climate, as these models provide a significantly better fit than time-homogeneous models. This improvement in fitness is crucial for the government in planning and policy making.

The prevailing models at the twelve sites were successfully identified, six sites have a time-homogeneous GEV model, four sites have a prevailing time-heterogeneous GEV model with a dominant linear trend in the scale parameter, and two sites have a prevailing time-heterogeneous GEV model with a dominating linear trend in the scale parameter, according to this study.

The results of the study demonstrated that the time independent q-GEV model (M_0) is much better fit than time heterogeneous q-GEV models at six sites: Alipingal, Bamnidhi, Manendragarh, Mohana, Rajim, and Sundargarh.

At sites DharamjaigarhKesinga, Kotni and Seorinarayan, the time heterogeneous q-GEV model M_3 explains significant variability than M_0. Non-stationary model with linear trend in scale parameter outperformed than stationary models. At sites Pathardhi and Simga, time heterogeneous q-GEV model M_2 suits well as compare to M_0. Non-stationary model with linear trend in location parameter, M_2, is significant as compared to the Stationary model M_0.

Non-stationary model with Non-linear trend in location parameter does not fit well over stationary models at any site, which suggests that better to drop out this model consideration for application in the MRB.

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