The matrix representation of quasi centroids of Heisenberg superalgebra

Abstract

This article studies the matrix representation of the quasi-centroids of Heisenberg superalgebras. Based on the definition of Heisenberg superalgebras and quasi-centroids, a matrix representation of quasi-centroids of Heisenberg superalgebras with even center is studied by using the method of solving system of linear equations and supersymmetry operation of Lie superalgebras. Finally, a matrix representation of the quasi centroids of a Heisenberg superalgebra with an odd center is obtained through similar calculations.

Keywords: Heisenberg superalgebra; Quasi centroid; Matrix representation.
MR(2020) Subject Classification: 17B40; 17B05.

1. Introduction

Lie superalgebras are closely linked to Lie algebras, which are highly significant in physics and other areas of mathematics ([4]). In 1967, physicist Stawraki proposed the concept of Lie superalgebras as a natural extension of Lie algebras. These originated from supersymmetry in mathematical physics, and with the support of physical background, the study of Lie superalgebra theory received increasing attention from researchers. A Lie superalgebra is a specific type of $\mathbb{Z}_2$-graded algebra, with its even part being a Lie algebra and its odd part being an adjoint module of the even part ([7]). Similar to the study of Lie groups, Lie superalgebras are linear objects of Lie supergroups. As a generalization of Lie algebras, the theory of Lie superalgebras is an essential part of Lie theory, with numerous applications in other related branches of mathematics ([2]).

The field of Lie superalgebras has seen significant progress in terms of structure and representation. In reference [3], the derivation superalgebra and local derivation superalgebra definitions, along with the modular Lie superalgebra structure, are used to determine the specific forms of the derivation superalgebra and local derivation superalgebra of the model linear Lie superalgebra. Key elements are calculated to arrive at this conclusion. Additionally, some outer superderivations of the modular Lie superalgebras are presented, and it is established that these are all of their outer superderivations. It is also proven that the local superderivations of the linear Lie superalgebras are all superderivations.

Lie superalgebras are essential in theoretical Physics and Mathematics. Depending on the base field's unique characteristics, Lie superalgebras can be categorized into modular and non-modular Lie superalgebras. When it comes to modular Lie superalgebras, analyzing the related theories directly can be challenging. Hence, it becomes crucial to establish a constraint structure on the studied Lie superalgebras and employ the related theories of restricted Lie superalgebras for further research ([1]).

This article explores the relationship between linear transformations on Heisenberg superalgebras and matrices to find the quasi-centroid matrices of $(2n+3)$-dimensional Heisenberg superalgebras. Section 1 is the introduction. Section 2 contains important definitions and symbols needed for the calculations. The main part of the paper is in section 3, where we determine the block matrix representations of the quasi-centroids of $(2n+3)$-dimensional Heisenberg superalgebra using the method of solving systems of linear equations, as defined in the quasi-centroid of Heisenberg superalgebra.

2. Preliminaries

All Heisenberg Lie superalgebras are categorized based on the number of centers they have. They are divided into two types - even and odd Heisenberg Lie superalgebras.[5]

Definition 1. *([4]) Definition of Lie superalgebra: Let $L = L_0 \oplus L_1$ be a Z_2- order linear space, if any Z_2- homogeneous element satisfies super antisymmetry,*

$$[x, y] = -(-1)^{d(x)d(y)}[y, x],$$

and Jacobi superidentity

$$(-1)^{d(z)d(x)}[x, [y, z]] + (-1)^{d(x)d(y)}[y, [z, x]] + (-1)^{d(y)d(z)}[z, [x, y]] = 0,$$

It is called a Lie superalgebra. The x,y,z are homogeneous elements in Lie superalgebra, which are the degree of order.

Definition 2. *([4]) If L is a complex Lie superalgebra, then*

$$\Gamma_\theta(L) = \left\{ f \in End_\theta(L) |\, f[x, y] = [f(x), y] = (-1)^{d(x)d(f)}[x, f(y)], x, y \in L, \theta \in Z_2 \right\}$$

is the Z_2- order centroid on L;

$$Q\Gamma_\theta(L) = \left\{ f \in End_\theta(L) |\, f[x, y] = (-1)^{d(x)d(f)}[x, f(y)], x, y \in L, \theta \in Z_2 \right\}$$

is called the Z_2-order quasi centroid on L, where $End_\theta(L)$ represents the set of all Z_2--order linear transformations in L.

Note: For any $f \in End_\theta(L)$, if $\theta = \overline{0}$, then f is called an even transformation, that is $d(f) = \overline{0}$; If $\theta = \overline{1}$, f is called an odd transformation, that is $d(f) = \overline{1}$.

Lemma 2.1. *([6]) Heisenberg Lie superalgebras can be divided into Heisenberg Lie superalgebras with even centers $H_{m,n}$ and Heisenberg Lie superalgebras with odd centers H_n:*

1) $H_{m,n} = (H_{m,n})_{\bar{0}} \oplus (H_{m,n})_{\bar{1}}$. *The standard basis is:*

$$\{u_1, \cdots, u_m, u_{m+1}, \cdots, u_{2m}, z |\, w_1, \cdots, w_n\},$$

Nonzero Lie superoperation:

$$[u_i, u_{m+i}] = z = [w_j, w_j], \forall i = 1, \cdots, m, j = 1, \cdots, n.$$

2) $H_n = (H_n)_{\bar{0}} \oplus (H_n)_{\bar{1}}$ *The standard basis is:* $\{v_1, \cdots, v_n |\, z, w_1, \cdots, w_n\}$, *Nonzero Lie superoperation:* $[v_i, w_i] = z$, $\forall i = 1, \cdots, n$.

3. Main Conclusion and Proof

Theorem 3.1. *If the center c is element of $H_{\bar{0}}$, then the matrix representations of quasi-centroid of Heisenberg superalgebra are as follows.*

$$\begin{pmatrix} \Lambda & A & B & \alpha \\ C & D & E & \beta \\ F & G & H & \gamma \\ 0 & 0 & 0 & k_{2m+3,2m+3} \end{pmatrix}$$

where $\Lambda = diag(a, -a)$, $a \in \mathbb{C}$, A and B are $m \times 2$ matrices, C and F are $2 \times m$ matrices, and $C = B^T$, $F = -A^T$. D, E, G, H are $m \times m$ matrices, $H = D^T$, $U^T = -U$, $X^T = X$, α, β, γ are 2, m, m dimensional column vectors, respectively.

Proof. Let $\{s_1, t_1\}$ be a base of $H_{\bar{1}}$ and $\{x_1, x_2, \cdots x_m, y_1, y_2, \cdots y_m, c\}$ a base of $H_{\bar{0}}$ Then $\{s_1, t_1, x_1, x_2, \cdots x_m, y_1, y_2, \cdots y_m, c\}$ is a base of H.
There are linear equations as follows.

$$Q(s_1) = k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c,$$
$$Q(t_1) = k_{21}s_1 + k_{22}t_1 + \sum_{j=1}^{m}(k_{2,j+2}x_j + k_{2,j+2+m}y_j) + k_{2,2m+3}c,$$
$$Q(x_1) = k_{31}s_1 + k_{32}t_1 + \sum_{j=1}^{m}(k_{3,j+2}x_j + k_{3,j+2+m}y_j) + k_{3,2m+3}c,$$

$$\vdots$$

$$Q(x_m) = k_{m+2,1}s_1 + k_{m+2,2}t_1 + \sum_{j=1}^{m}(k_{m+2,j+2}x_j + k_{m+2,j+2+m}y_j) + k_{m+2,2m+3}c,$$
$$Q(y_1) = k_{m+3,1}s_1 + k_{m+3,2}t_1 + \sum_{j=1}^{m}(k_{m+3,j+2}x_j + k_{m+3,j+2+m}y_j) + k_{m+3,2m+3}c,$$

$$\vdots$$

$$Q(y_m) = k_{2m+2,1}s_1 + k_{2m+2,2}t_1 + \sum_{j=1}^{m}(k_{2m+2,j+2}x_j + k_{2m+2,j+2+m}y_j) + k_{2m+2,2m+3}c,$$
$$Q(c) = k_{2m+3,1}s_1 + k_{2m+3,2}t_1 + \sum_{j=1}^{m}(k_{2m+3,j+2}x_j + k_{2m+3,j+2+m}y_j) + k_{2m+3,2m+3}c.$$

According to algorithms on Heisenberg superalgebra:
i $[s_i, t_j] = [t_j, s_i] = \delta_{ij} c$.
ii $[x_i, y_j] = -[y_j, x_i] = \delta_{ij} c$.
iii $[s_i, s_j] = [t_i, t_j] = [x_i, x_j] = [y_i, y_j] = [s_i, c] = [t_i, c] = [x_i, c] = [y_i, c] = 0$
and $Q \in EndL = End_{\overline{0}}L \oplus End_{\overline{1}}L$, we have
1. When $Q \in End_{\overline{0}}L$, $dQ = \overline{0}$. At this point, the matrix representations of quasi-centroids of Heisenberg superalgebra are same as that of Heisenberg algebra.
2. When $Q \in End_{\overline{1}}L$, $dQ = \overline{1}$. Definition of the quasimatrix centroid is $[Qx, y] = (-1)^{dxdQ}[x, Qy]$. When replace x with s_1, it follows that

$$[Q(s_1), s_1] = -[s_1, Q(s_1)], \quad [Q(s_1), t_1] = -[s_1, Q(t_1)], \quad [Q(s_1), x_1] = -[s_1, Q(x_1)],$$

$$\vdots$$

$$[Q(s_1), x_m] = -[s_1, Q(x_m)], \quad [Q(s_1), y_1] = -[s_1, Q(y_1)],$$

$$\vdots$$

$$[Q(s_1), y_m] = -[s_1, Q(y_m)], \quad [Q(s_1), c] = -[s_1, Q(c)].$$

i.e.

$$\left[k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c, s_1\right]$$

$$= -\left[s_1, k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c\right],$$

$$\left[k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c, t_1\right]$$

$$= -\left[s_1, k_{21}s_1 + k_{22}t_1 + \sum_{j=1}^{m}(k_{2,j+2}x_j + k_{2,j+2+m}y_j) + k_{2,2m+3}c\right],$$

$$\left[k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c, x_1\right]$$

$$= -\left[s_1, k_{31}s_1 + k_{32}t_1 + \sum_{j=1}^{m}(k_{3,j+2}x_j + k_{3,j+2+m}y_j) + k_{3,2m+3}c\right],$$

$$\vdots$$

$$\left[k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c, x_m\right]$$

$$= -[s_1, k_{m+2,1}s_1 + k_{m+2,2}t_1 + \sum_{j=1}^{m}(k_{m+2,j+2}x_j + k_{m+2,j+2+m}y_j) + k_{m+2,2m+3}c],$$

$$\left[k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c, y_1\right]$$

$$= -[s_1, k_{m+3,1}s_1 + k_{m+3,2}t_1 + \sum_{j=1}^{m}(k_{m+3,j+2}x_j + k_{m+3,j+2+m}y_j) + k_{m+3,2m+3}c],$$

$$\vdots$$

$$\left[k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c, y_m\right]$$

$$= -[s_1, k_{2m+2,1}s_1 + k_{2m+2,2}t_1 + \sum_{j=1}^{m}(k_{2m+2,j+2}x_j + k_{2m+2,j+2+m}y_j) + k_{2m+2,2m+3}c],$$

$$\left[k_{11}s_1 + k_{12}t_1 + \sum_{j=1}^{m}(k_{1,j+2}x_j + k_{1,j+2+m}y_j) + k_{1,2m+3}c, c\right]$$

$$= -[s_1, k_{2m+3,1}s_1 + k_{2m+3,2}t_1 + \sum_{j=1}^{m}(k_{2m+3,j+2}x_j + k_{2m+3,j+2+m}y_j) + k_{2m+3,2m+3}c].$$

Solving the above system of linear equations yields

$$k_{12} = k_{2m+3,2} = 0, \quad k_{11} = -k_{22}, \quad k_{1,m+3} = k_{32}, \cdots, k_{1,2m+2} = k_{m+2,2},$$

$$k_{13} = -k_{m+3,2}, \cdots, k_{1,m+2} = -k_{2m+2,2}.$$

Similarly, when replace x with $t_1, x_1, x_2, \cdots x_m, y_1, y_2, \cdots y_m, c$, we obtain that

$$\begin{aligned} &k_{21} = k_{3,m+3} = k_{m+2,2m+2} = k_{m+3,3} = k_{2m+3,3} = k_{2m+2,m+2} = k_{2m+3,1} \\ &= \cdots = k_{2m+3,m+2} = k_{2m+3,m+3} = \cdots = k_{2m+3,2m+2} = 0. \end{aligned}$$

$$k_{23} = -k_{m+3,1}, \cdots, k_{2,m+2} = -k_{2m+2,1}, k_{2,m+3} = k_{31}, \cdots, k_{2,2m+2} = k_{m+2,1},$$

$$k_{33} = k_{m+3,m+3}, \cdots, k_{3,m+2} = k_{2m+2,m+3}, \quad k_{3,m+4} = -k_{4,m+3}, \cdots, k_{3,2m+2} = -k_{m+2,m+3},$$

$$k_{m+2,3} = k_{m+3,2m+2}, \cdots, k_{m+2,m+2} = k_{2m+2,2m+2}, \quad k_{m+3,4} = k_{m+4,3}, \cdots, k_{m+3,m+2} = k_{2m+2,3}.$$

It follows from above results that the matrix representations of quasi-centroid of the Heisenberg superalgebra are

$$\begin{pmatrix} \Lambda & A & B & \alpha \\ C & D & E & \beta \\ F & G & H & \gamma \\ 0 & 0 & 0 & k_{2m+3,2m+3} \end{pmatrix}$$

where $\Lambda = diag(a, -a)$, $a \in \mathbb{C}$, A and B are $m \times 2$ matrices, C and F are $2 \times m$ matrices, and $C = B^T$, $F = -A^T$. D, E, G, H are $m \times m$ matrices, $H = D^T$, $U^T = -U$, $X^T = X$, α, β, γ are 2, m, m dimensional column vectors, respectively. □

Theorem 3.2. *If the center c is an element of $H_{\bar{1}}$, then the matrix representations of quasi-centroid of the Heisenberg superalgebra are as follows.*

$$\begin{pmatrix} k_{11} & 0 & 0 & 0 \\ \alpha & \Lambda & A & B \\ \beta & C & D & E \\ \gamma & F & G & H \end{pmatrix}$$

where α, β, γ are 2, m, m dimensional column vectors, $\Lambda = diag(a, -a)$, $a \in \mathbb{C}$, A, B are $2 \times m$ matrices, C, F are $m \times 2$ matrices, $C = B^T$, $A = \begin{pmatrix} I \\ J \end{pmatrix}$, $F = \begin{pmatrix} -J^T & I^T \end{pmatrix}$, D, E, G, H are $m \times m$ matrices, $H = D^T$, $G^T = -G$, $E^T = -E$.

Proof. Let $\{c, s_1, t_1\}$ be a base of $H_{\bar{1}}$ and $\{x_1, x_2, \cdots x_m, y_1, y_2, \cdots y_m\}$ a base of $H_{\bar{0}}$, then $\{c, s_1, t_1, x_1, x_2, \cdots x_m, y_1, y_2, \cdots y_m\}$ is base of H.
According to linear equations

$$Q(c) = k_{11}c + k_{12}s_1 + k_{13}t_1 + \sum_{j=1}^{m}(k_{1,j+3}x_j + k_{1,j+m+3}y_j) + k_{1,2m+3}y_m,$$
$$Q(s_1) = k_{21}c + k_{22}s_1 + k_{23}t_1 + \sum_{j=1}^{m}(k_{2,j+3}x_j + k_{2,j+m+3}y_j) + k_{2,2m+3}y_m,$$
$$Q(t_1) = k_{31}c + k_{32}s_1 + k_{33}t_1 + \sum_{j=1}^{m}(k_{3,j+3}x_j + k_{3,j+m+3}y_j) + k_{3,2m+3}y_m,$$
$$Q(x_1) = k_{41}c + k_{42}s_1 + k_{43}t_1 + \sum_{j=1}^{m}(k_{4,j+3}x_j + k_{4,j+m+3}y_j) + k_{4,2m+3}y_m,$$
$$\vdots$$
$$Q(x_m) = k_{m+3,1}c + k_{m+3,2}s_1 + k_{m+3,3}t_1 + \sum_{j=1}^{m}(k_{m+3,j+3}x_j + k_{m+3,j+m+3}y_j) + k_{m+3,2m+3}y_m,$$
$$Q(y_1) = k_{m+4,1}c + k_{m+4,2}s_1 + k_{m+4,3}t_1 + \sum_{j=1}^{m}(k_{m+4,j+3}x_j + k_{m+4,j+m+3}y_j) + k_{m+4,2m+3}y_m,$$
$$\vdots$$
$$Q(y_m) = k_{2m+3,1}c + k_{2m+3,2}s_1 + k_{2m+3,3}t_1 + \sum_{j=1}^{m}(k_{2m+3,j+3}x_j + k_{2m+3,j+m+3}y_j) + k_{2m+3,2m+3}y_m.$$

and the operation rules on Heisenberg superalgebra are the same as Theorem 3.1, by replacing x with c, we obtain that

$$[Q(c), c] = -[c, Q(c)], \quad [Q(c), s_1] = -[c, Q(s_1)],$$
$$[Q(c), t_1] = -[c, Q(t_1)], \quad [Q(c), x_1] = -[c, Q(x_1)],$$
$$\vdots$$
$$[Q(c), x_m] = -[c, Q(x_m)], \quad [Q(c), y_1] = -[c, Q(y_1)],$$
$$\vdots$$
$$[Q(c), y_m] = -[c, Q(y_m)].$$

i.e.

$$\left[k_{11}c+k_{12}s_1+k_{13}t_1+\sum_{j=1}^{m}(k_{1,j+3}x_j+k_{1,j+m+3}y_j),c\right]$$
$$=-\left[c,k_{11}c+k_{12}s_1+k_{13}t_1+\sum_{j=1}^{m}(k_{1,j+3}x_j+k_{1,j+m+3}y_j)\right],\cdots(1)$$
$$\left[k_{11}c+k_{12}s_1+k_{13}t_1+\sum_{j=1}^{m}(k_{1,j+3}x_j+k_{1,j+m+3}y_j),s_1\right]$$
$$=-\left[c,k_{21}c+k_{22}s_1+k_{23}t_1+\sum_{j=1}^{m}(k_{2,j+3}x_j+k_{2,j+m+3}y_j)\right],$$
$$\left[k_{11}c+k_{12}s_1+k_{13}t_1+\sum_{j=1}^{m}(k_{1,j+3}x_j+k_{1,j+m+3}y_j),x_1\right]$$
$$=-\left[c,k_{31}c+k_{32}s_1+k_{33}t_1+\sum_{j=1}^{m}(k_{3,j+3}x_j+k_{3,j+m+3}y_j)\right],$$
$$\vdots$$
$$\left[k_{11}c+k_{12}s_1+k_{13}t_1+\sum_{j=1}^{m}(k_{1,j+3}x_j+k_{1,j+m+3}y_j),x_m\right]$$
$$=-\left[c,k_{m+3,1}c+k_{m+3,2}s_1+k_{m+3,3}t_1+\sum_{j=1}^{m}(k_{m+3,j+3}x_j+k_{m+3,j+m+3}y_j)\right],$$
$$\left[k_{11}c+k_{12}s_1+k_{13}t_1+\sum_{j=1}^{m}(k_{1,j+3}x_j+k_{1,j+m+3}y_j),y_1\right]$$
$$=-\left[c,k_{m+4,1}c+k_{m+4,2}s_1+k_{m+4,3}t_1+\sum_{j=1}^{m}(k_{m+4,j+3}x_j+k_{m+4,j+m+3}y_j)\right],$$
$$\vdots$$
$$\left[k_{11}c+k_{12}s_1+k_{13}t_1+\sum_{j=1}^{m}(k_{1,j+3}x_j+k_{1,j+m+3}y_j),y_m\right]$$
$$=-\left[c,k_{2m+3,1}c+k_{2m+3,2}s_1+k_{2m+3,3}t_1+\sum_{j=1}^{m}(k_{2m+3,j+3}x_j+k_{2m+3,j+m+3}y_j)\right].$$

By solving the system of linear equations, it follows that $k_{12}=k_{13}=\cdots=k_{1,2m+3}=0$.

Similarly, by replacing x with $s_1,t_1,x_1,x_2,\cdots x_m,y_1,y_2,\cdots y_m$, we get

$$k_{22}=-k_{33},\quad k_{23}=0,\quad k_{24}=k_{m+4,3},\cdots,k_{2,m+3}=k_{2m+3,3},\cdots,k_{2,2m+3}=k_{m+3,3}.$$
$$k_{32}=0,\quad k_{42}=k_{3,m+4},\cdots,k_{52}=k_{3,m+5},\cdots,k_{m+3,2}=k_{3,2m+3},$$
$$k_{m+4,2}=-k_{34},\cdots,k_{2m+3,2}=-k_{3,m+3}.$$
$$k_{4,m+4}=0,\quad k_{4,m+5}=-k_{5,m+4},\cdots,k_{4,2m+3}=-k_{m+3,m+4},$$
$$k_{44}=k_{m+4,m+4},\cdots,k_{4,m+3}=k_{2m+3,m+4}.$$
$$k_{m+3,2m+3}=0,\quad k_{m+3,4}=k_{m+4,2m+3},\cdots,k_{m+3,m+3}=k_{2m+3,2m+3}.$$
$$k_{m+4,4}=0,\quad k_{m+4,5}=-k_{m+5,4},\cdots,k_{m+4,m+3}=-k_{2m+3,4}.$$

$$\vdots$$

$$k_{2m+3,m+3} = 0.$$

Therefore, if the center c is an element of $H_{\bar{1}}$, then the matrix representations of quasi-centroid of the Heisenberg superalgebra are as follows.

$$\begin{pmatrix} k_{11} & 0 & 0 & 0 \\ \alpha & \Lambda & A & B \\ \beta & C & D & E \\ \gamma & F & G & H \end{pmatrix}$$

where α, β, γ are 2, m, m dimensional column vectors respectively, $\Lambda = diag(a, -a)$, $a \in \mathbb{C}$, A, B are $2 \times m$ matrices, C, F are $m \times 2$ matrices, $C = B^T$,$A = \begin{pmatrix} I \\ J \end{pmatrix}$, $F = \begin{pmatrix} -J^T & I^T \end{pmatrix}$, D, E, G, H are $m \times m$ matrices, $H = D^T$, $G^T = -G$, $E^T = -E$. □

4. Conclusions

The type of this article is Short Research Article. The matrix representations of quasi-centroid of the Heisenberg superalgebra are shown in Theorem 3.1 and Theorem 3.2.

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