Thermal and dynamic study of convection-radiation coupling in a closed enclosure.

ABSTRAT

In this work, we present a two-dimensional study of natural convection in a square air-filled cavity whose two vertical sides are subjected to a temperature difference, while the other two horizontal ones are adiabatic. The numerical method used is that of finite elements used in the COMSOL calculation code. We first studied pure natural convection, by varying the Rayleigh number from 1 to 10^6, then its coupling with surface radiation, by varying the emissivity of the cavity surfaces from 0 to 0.6. The results are presented in the form of isotherms, pressure, current etc. It appears from this study that for a Rayleigh number less than 100, the fluid presents a vertical thermal stratification due to heat transfer only by conduction which does not generate any convective flow. . . The presence of surface radiation does not modify the equations governing fluid movements but only alternates the thermal boundary conditions. The coupling of natural convection and surface radiation occurs only through thermal boundary conditions. The variation in the emissivity of the walls does not influence the temperature isotherms.

Key words: Natural convection/square cavity/convection-radiation coupling/numerical simulation

I-INTRODUCTION

The study of the coupling of natural convection and surface radiation is the subject of several researches in different configurations. The researchers examined these studies experimentally and numerically using appropriate methods. This mode of transfer has a very wide field of application, namely heating a house (case of the radiator), energy savings, cooling of electronic components, thermal insulation of the home, etc. Among the most used methods, we can cite finite differences, finite volumes, finite elements. Given the importance of natural convection in many industrial applications, many researchers have been interested in the case of natural convection in regularly shaped enclosures. This is how the study of natural convection in a closed enclosure and its coupling with radiation were carried out by several researchers including: [1] developed a numerical code for surface convection-radiation coupling and studies are carried out for a cavity filled with air whose four walls have the same emissivity, the results show that compared to the case without radiation, the upper wall is cooled, the lower wall is heated. Horizontal flow is reinforced, and stratification is reduced. The aim of this work is to analyze the influence of surface radiation on the dynamic and thermal behavior of flows in natural convection. The differentially heated cavity is filled with air (Pr=0.71)and the emissivities of the walls vary from 0 to 0.6. The radiative flux imposed on the two horizontal sides are taken in accordance with the values of [1] with a temperature difference between the active walls of 10 k. [2] and [3] presented a numerical solution of natural convection in. A square cavity heated differentially. [4] studied a hybrid numerical scheme to simulate the interaction between natural convection and surface radiation in a rectangular and differentially heated cavity. [5] Studied a hybrid numerical scheme to simulate the interaction between natural convection and surface radiation in a rectangular and differentially heated cavity. The speeds are determined by the lattice Boltzmann method and the energy equation is discretized by the finite volume method. The algebraic systems obtained are solved by the conjugate gradient method. The radiation transfer equation is solved by the radiosity method. [6] Proposed a study of the interaction between natural convection and radiation for the low Mach approximation for transparent and semi-transparent media. The resolutions of the Navier-Stokes and energy equations written for an ideal gas use the finite volume method. The radiation transfer equation is solved by the discrete ordinate method. In the case of our work, the numerical method used is that of finite elements implemented in the COMSOL calculation code.The presence of surface radiation does not modify the equations governing the fluid movements but only alternates the thermal boundary conditions. The coupling of natural convection and surface radiation occurs only through thermal boundary conditions [7]. This study will make it possible to obtain isovalues of speed, temperature and current lines. The study would be carried out on the variation of the emissivity of the walls and the Rayleigh Number varying from 1 to 10^6. The flow is laminar in steady state. The cavity would be studied in two dimensions.

Materials and Method

I-Physical Model and Mathematical Formulations

***Physical model**

In the case of our study, we take the cavity as being a dwelling with a surface area of 1m^2. The figure represents the differentially heated cavity whose two vertical sides are at$T_c = 293{,}5K$ and$T_f = 288{,}5K$. The horizontal sides and the front and rear faces are adiabatic. The hypotheses adopted for this simulation are:the fluid is Newtonian and incompressible, the internal walls are considered gray, diffuse, opaque, have the same emissivity value Ɛ, the flux densities and the temperatures of the elementary surfaces are uniform. The cavity is filled with a transparent fluid (air), homogeneous and isotropic whose refractive index is equal to unity. The movement of air is governed by the Navier-Stokes equations under the Boussinesq hypothesis. For the resolution, we use the vorticity-current function formalism. Before the initial instant, we assume the air is at rest in the enclosure at an average temperature $(T_c + T_f)/2$. With T_c and T_f $(T_C > T_f)$ the temperatures of the hot and cold wall respectively. The figure below represents the cavity mesh intended for the two-dimensional study.

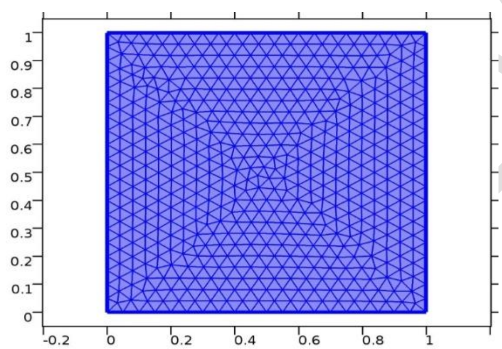


Figure 1: Mesh of the physical model of the cavity intended for the study.

II-2 Mathematical Formulation

- **dimensional governing equations**

*** Systems of equations**

The different fluid flows, with or without heat and mass transfer, are based on systems of nonlinear differential equations established from the conservation principles of physics. In these systems of non-linear equations, there are numerous parameters capable of being at the origin of the phenomena which cause the change in behavior of the flow of a fluid (bifurcation) in the cavity. In this cavity, the temperature gradient normal to the two adiabatic horizontal surfaces is zero. However, when radiation is present, the adiabatic condition is translated by the balance between the convective and radiative heat flow. For the numerical simulation, let us first present the equations which can model this natural convection. This expresses the laws of conservation of mass (continuity), momentum (Navier-stores) and energy. We apply the equations in the case of a planar cavity taking into account the discussion of the numerical method used for the resolution of these equations. When the vertical walls of a cavity filled with air are subjected to a constant temperature difference, the flow generated depends on several parameters, the main ones of which are:

- Rayleigh number

$$R_a = \frac{g.\beta.\Delta T.H^3}{\alpha \nu} \quad (1)$$

-The Prandtl number

$$P_r = \frac{\nu}{\alpha} \quad (2)$$

The thermo-physical properties of the air are taken at 20°C: Thermal conductivity λ; Stefan-Boltzmann constant σ; Kinematic viscosity $\mathcal{V}$; Thermal expansion coefficient β; heat capacity at pressure notes Cp; Density ρ;

- **Dimensionless equations**

Equation dimensionality consists of making equations dimensionless using reduced variables.

. Let's bring these equations into dimensionless form, to do this let's define the characteristic quantities by designating by: - H a characteristic linear dimension of the flow, - V0 the reference speed -t0 a reference time, Δt0 being the reference temperature difference - P0 a reference pressure:

$V_0 = \sqrt{g\beta H \Delta T_0}$; $t_0 = \frac{H}{V_0}$ $\quad p_o = \rho_o {V_o}^2$ By designating these characteristic quantities, we will obtain the following reduced variables:

$x = \frac{X}{H}$; $y = \frac{Y}{H}$; $u = \frac{U}{V_0}$; $v = \frac{V}{V_0}$; $\tau = \frac{t}{t_0}$; $\Theta = \frac{T - T_0}{T_C - T_f}$;$\acute{P} = \frac{P}{P_0}$The characteristic scales used for dimensionless equations analogous to those in references [9],[10]

- the continuityequation

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$(3)● momentum equations

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_0}\sqrt{\frac{P_r}{R_a}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial P}{\partial x}\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \Theta + \frac{1}{\rho_0}\sqrt{\frac{P_r}{R_a}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial P}{\partial y} \quad [5]$$

- the energy equation

$$\frac{\partial \Theta}{\partial \tau} + u\frac{\partial \Theta}{\partial x} + v\frac{\partial \Theta}{\partial y} = \frac{\Delta T}{\sqrt{R_a P_r}}\left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right) (4)$$

- Vorticity-current function formalism.

The advantage of using this formalism is to reduce the number of equations and to bring out dominant variables [11]. In the case of a two-dimensional Cartesian system defines: Vorticity:

$$\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial x} \quad (5)$$

La **fonction de courant Ψ** définie par : $\begin{cases} \frac{\partial \Psi}{\partial y} = u \\ \frac{\partial \Psi}{\partial x} = -v \end{cases}$ (6)

Alors $\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \Omega = 0$(7)

$$\frac{\partial \Omega}{\partial \tau} + u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y} = \frac{1}{\rho_0}\sqrt{\frac{P_r}{R_a}}\left(\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2}\right) - \frac{\partial \theta}{\partial x} \quad (8)$$

- dimensionless governing equations.

To dimensionless the conservation equations, we will use the same reference quantities on pure natural convection.

- Scaling of boundary conditions:

$\theta = \frac{T - T_0}{T_C - T_f}$, $Q_r = \frac{\varphi_r}{\sigma T_0^4}$, $y = \frac{Y}{H}$, $\Delta T = T_c - T_f$, $T_0 = \frac{T_C + T_f}{2}$, $\frac{\partial \theta}{\partial y} = N_r Q_r$,SO

$$\boldsymbol{N_r = \frac{H \sigma T_0^4}{\lambda \Delta T}}$$

The radiative condition:

$\lambda \frac{\partial T}{\partial y}/_{y=0} = N_r Q_r$ (9)

$\lambda \frac{\partial T}{\partial y}/_{y=1} = -N_r Q_r$ (10)

N_r Is the number of Radiation

($N_r = \frac{H \sigma T_0^4}{\lambda \Delta T}$), $Q_r = \frac{\varphi_r}{\sigma T_0^4}$, Q_r .Is the dimensionless radiative flux

$\theta = \frac{T - T_0}{\Delta T}$, $\theta_C = \frac{T - T_0}{\Delta T} = 0{,}5$,$\theta_f = \frac{T_f - T_0}{\Delta T} = -0{,}5$ Concerning the reference length H used in the work of [1],

$R_a = \frac{g.\beta.\Delta T.H^3}{\mathcal{V}\alpha}$, **$R_a$=1,10310^9 H^3**, Nr the number of radiation(($\boldsymbol{N_r} = \frac{\boldsymbol{\sigma.T_0{}^4 H}}{\boldsymbol{\lambda \Delta T}}$**)**

- The boundary conditions * U=V=0 on all

Surfaces :

$$\begin{cases} U(X=0,Y) = V(X=0,Y) = 0 \\ U(X=L,Y) = V(X=L,Y) = 0 \\ U(X,Y=0) = V(X,Y=0) = 0 \\ U(X,Y=H) = V(X,Y=H) = 0 \end{cases} \quad (11)$$

*The hot (TC) and cold (Tf) temperatures on the vertical walls, we set:

$$\begin{cases} T = T_C pour X = 0, \ 0 \leq Y \leq H \\ T = T_f pour X = L, \ 0 \leq Y \leq H \end{cases}$$

*In the presence of radiation and for a semi-transparent medium, the adiabatic condition is translated by the balance between the convective and radiative fluxes. The coupling of natural convection with surface radiation occurs only through thermal boundary conditions. We therefore have for the two horizontal walls of the cavity:

$$\lambda \frac{\partial T}{\partial y} /_{y=0} - \varphi_r /_{y=0} = 0 \quad (12)$$

$$\lambda \frac{\partial T}{\partial y} /_{y=H} + \varphi_r /_{y=H} = 0 \quad (13)$$

IV-Results and discussions

- **Validation**

In order to verify the accuracy of our numerical work, a validation of the numerical code is carried out by taking into account certain numerical and experimental studies which exist in the literature. Indeed, we will validate our work with the results of [1] in the case of a square cavity containing air whose two faces, ceiling and floor, are assumed to be adiabatic but at uniform temperatures. The left and right vertical faces are differentially heated to ΔT=10K. For this, we keep the same reference temperature To and the same length H used in the work of Hong and Al [1].

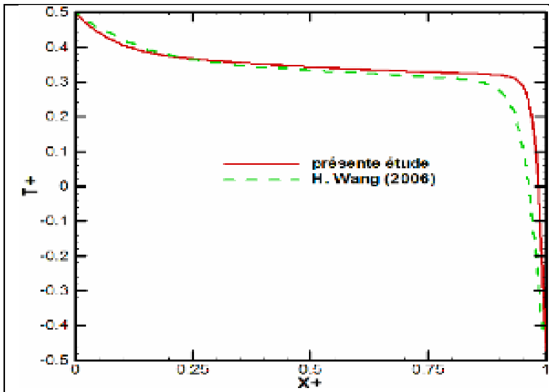


Figure 2a: comparisons of high wall temperature profiles at $R_a = 10^6$
Wang et al [1]

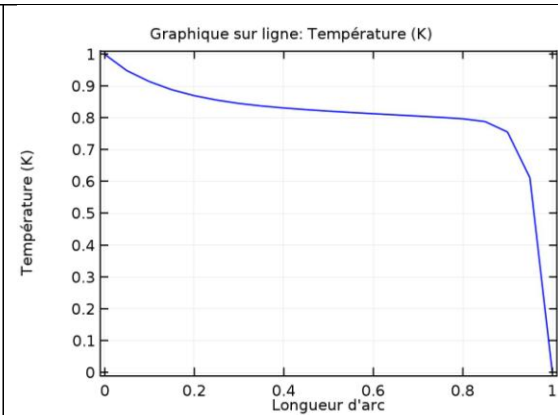


Figure 2b: upper wall temperature (the present work)

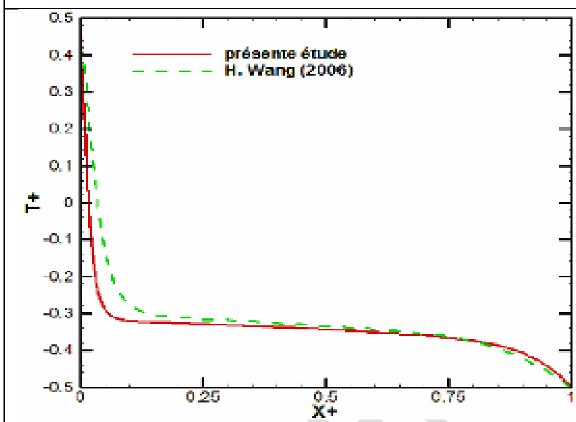


Figure 2c: comparisons of low wall temperature profiles at $R_a = 10^6$ Wang et al [1]

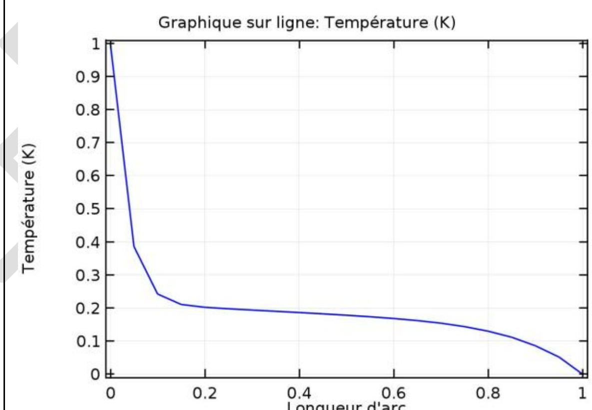


Figure 2d: temperature of the lower wall (the present work)

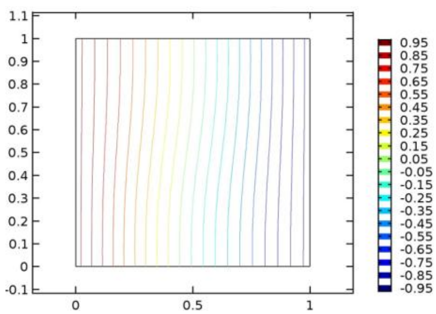
Figure 2: high and low temperature profile at $R_a = 10^6$.

Figures 2b and 2d represent the temperatures near the upper wall and the lower wall. As soon as the fluid first comes into contact with the hot wall, we notice an rise in the hot air.

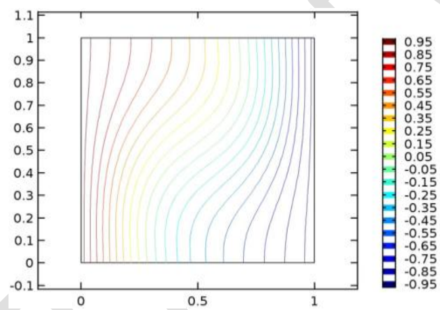
there is a strong heat exchange but as the fluid moves along the high adiabatic wall, it cools. Figure 2b shows that the air cools as the altitude descends towards the lower adiabatic wall.

- RADIATION EFFECT

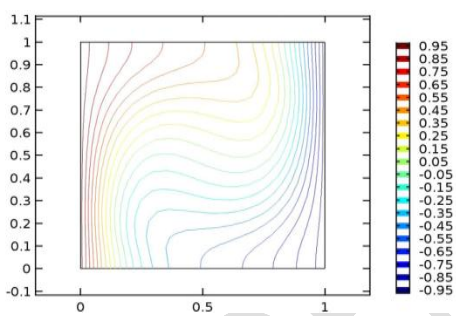
For the figure below, these are temperature isotherms for different values of Ra ranging from $R_a = 10^3 \; to R_a = 10^6$) The flow in pure convection and we will couple it with the Radiation of the walls.



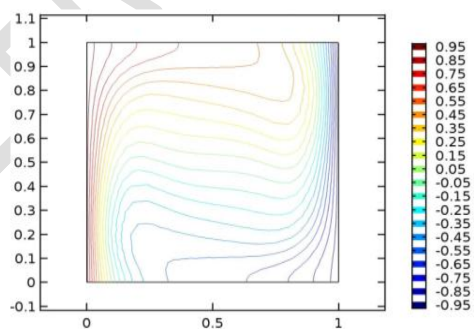
$R_a\!=\!10^3$ **(a)**



$R_a\!=\!10^4$ **(b**



$R_a\!=\!10^5$ **(c)**



$R_a\!=\!10^6$ **(d)**

Figure 3: Temperature isovalues for different numbers of .

Figure 3 represents the isovalues of the temperature in pure natural convection for different values of the Rayleigh number. This figure shows that for a low value of the Rayleigh number $R_a = 10^3$, the fluid presents a vertical thermal stratification due to heat transfer only by conduction which does not generate any convective flow. From the results presented in this figure, we can see that the energy transfer for $0 \prec X \prec 1$ is one-dimensional. This is explained by the fact that the isotherms are perpendicular to the main direction of heat transfer, direction ox. For $R_a \succ 10^3$ there is distortion of the isotherms and their contraction at the vertical interfaces is mainly due to the Rayleigh number which is relatively high $R_a = 10^6$ and to the significant temperature gradient in this interface.

Let's look for:

- the temperature profile and the influence of the emissivity on the isotherms at $R_a = 10^5$ and Ɛ=0; 0.1; 0.2; 0.4; 0.6

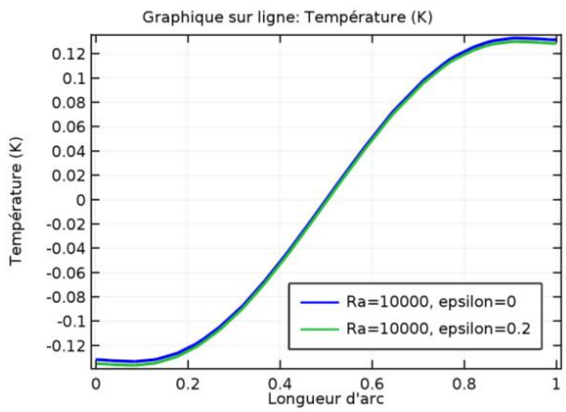


Figure 4: the temperature at . $R_a = 10^4$ and Ɛ=0, Ɛ=0.2

After analyzing this curve, when we increase the emissivity of the surfaces, this has no effect on the temperature variation.

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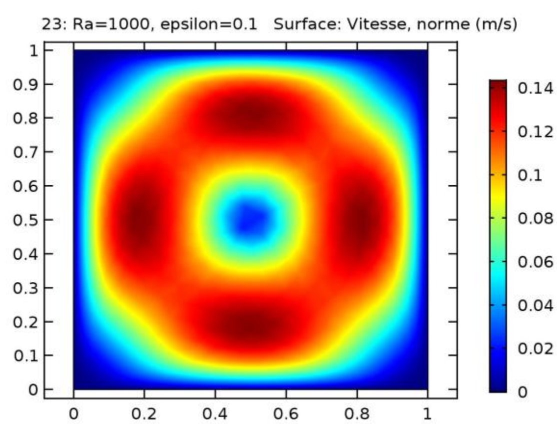
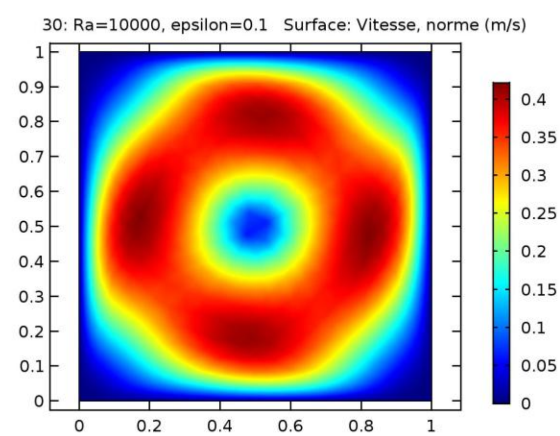
<tr><td>
</td><td>
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<tr><td>**Ra=10^3 et** Ɛ=0,1 (a)</td><td>**Ra=10^4 et** Ɛ=0,1 (b)</td></tr>
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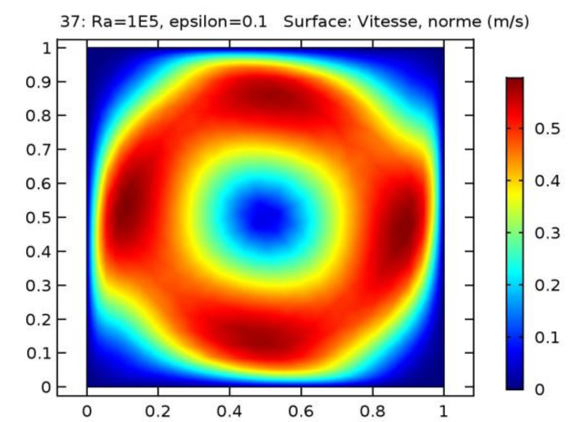
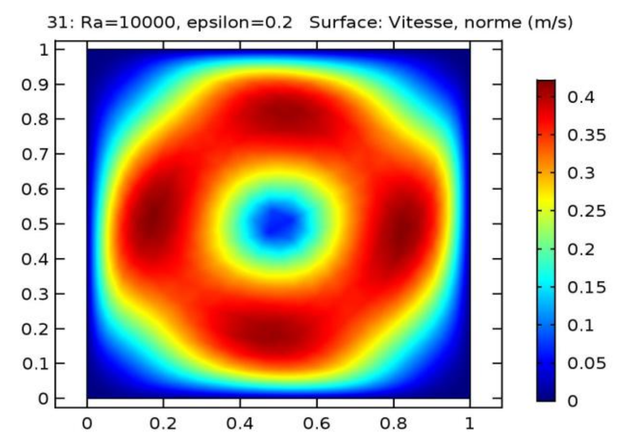
	
Ra=10^5 et Ɛ=0,1 (c)	**Ra=10^{4}et** Ɛ=0,2 (d)

Figure 5: velocity surface for different values of Ra and Ɛ=0.1; Ɛ=0.2

According to the analysis made for this figure of the velocity field, the variation of the emissivity does not modify the speed of the flow because for $R_a = 10^4$, when we vary the emissivity from 01 to 0.2 as in (b) and (d), the maximum speed (in red) is 0.4m/s. Close to the horizontal and vertical walls and in the center of the cavity (in blue), it is at 0.05m/s. the same figure shows that the flow speed increases close to the walls as well as in the middle of the enclosure for Ɛ=0.1 when Ra increases from $R_a = 10^3. R_a = 10^4 and R_a = 10^5$(respectively 0.14m/s, 0.4m/ s and 0.5m/s)The emissivity does not influence the dynamic behavior of the flow but the Rayleigh number which is a constant of natural convection modifies the speed of the wall flow.

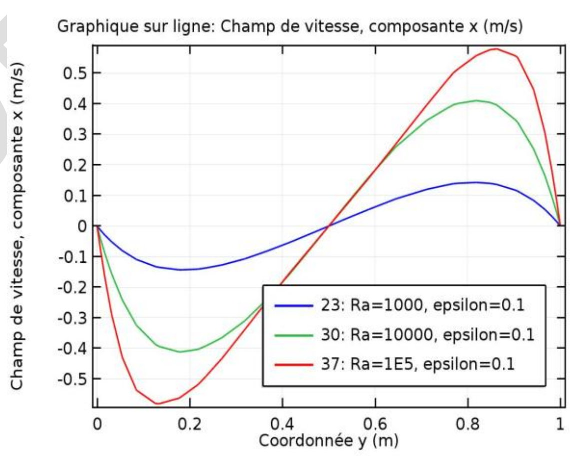
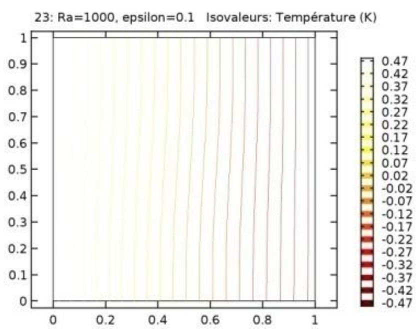
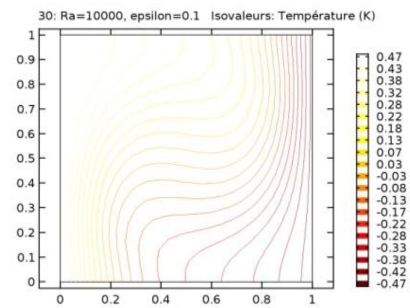


Figure 6: the speed lines following x at different values of Ra and at Ɛ=0.1

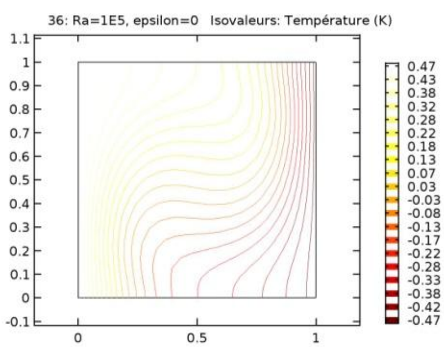
The figure represents the appearance of the horizontal velocities as a function of different values of Ra ($R_a = 10^3 . R_a = 10^4 . R_a = 10^5$.) and with an emissivity Ɛ=0.1. The velocity profiles indicate a strong acceleration near the horizontal walls, the negative velocities observed on half of the cavity (y<0) are characteristic of the recirculation zone. We see that the particle speed increases with the increase in the Rayleigh number.



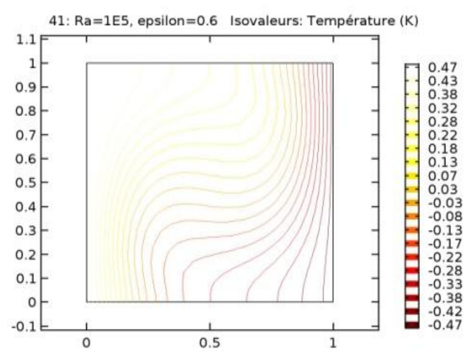
(7a) $R_a = 10^3$ **et** $\varepsilon = 0{,}1$



(7b) $R_a = 10^4$ **et** $\varepsilon = 0{,}1$



(7C) $R_a = 10^5$ **et** $\varepsilon = 0$



(7d) $R_a = 10^5$ **et** $\varepsilon = 0{,}6$

Figure 7: temperature isovalues a: $R_a = 10^3$, $R_a = 10^4$, $R_a = 10^5$ et $\varepsilon = 0{,}1$; $\varepsilon = 0{,}6$

Figure 7 shows that, the distortion of the isotherms inside the cavity, along the vertical walls and in the rest of the floor and roof is due to the two-dimensional energy transfer and the increase in the Rayleigh number. We see that the increase in surface emissivity (7c) and (7d) for $R_a = 10^3$ does not influence the isotherms. We move from the conductive state to the convective state when $R_a > 10^3$.

Conclusion

In order to verify the accuracy of our numerical work, a validation of the numerical code is carried out by taking into account certain numerical and experimental studies which exist in the literature. We studied pure natural convection, then the coupling of convection-radiation from radiative surfaces. In pure natural convection, the cavity is differentially heated on these two vertical sides and adiabatic on the other horizontal sides. In coupling, the two horizontal faces are subjected to a radiative flux. We presented a thermal study of pure natural convection for different values of the Rayleigh number. The numerical simulation was carried out for Rayleigh numbers ranging from 1 to 10^6. Surface radiation does not influence the temperature isotherms while increasing the Rayleigh number disrupts the isotherms.

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