Some Fixed Point Results on Boyd-Wong Type Generalized (α, ψ, F)-Geraghty Contraction Mappings in Partial Metric Spaces with Application

Abstract

Aims/ objectives: In this paper, we initiate Boyd-Wong type generalized (α, ψ, F)-Geraghty contraction mappings in the setting of partial metric spaces and investigate the existence and uniqueness of fixed points for the newly constructed contraction mappings. Our results are supported by an example. As an application of these well-established findings, we show that a class of nonlinear integral equations has a solution.

Keywords: F-contraction mapping, Boyd-Wong type generalized (α, ψ, F)-Geraghty contraction mapping, partial metric spaces.

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1 Introduction

Banach contraction principle [1] was a foundation for a development of metric fixed point theory which has been generalized by utilizing various contractive conditions in various contexts. Matthews [10] exhibited the idea of partial metric space (PMS) in which the self distance need not be equal to zero and proved the Banach fixed point theorem for such spaces. Geraghty [3] introduced a new class of contraction mappings and obtained a generalization of the Banach contraction principle in the setting of complete metric spaces by considering an auxiliary function. Further, Samet et al. [12] introduced another class of mappings, called the $\alpha - \psi$ contractive type mappings and obtained some fixed point results for this class of mappings. Karapinar and Samet [13] generalized the $\alpha - \psi$ contractive type mappings and established various fixed point theorems. On the other hand, Wardowski [4, 5, 6] introduced a new contraction called F-contraction and proved a fixed point theorem as a generalization of the Banach contraction principle. Many authors have formed different generalizations of Wardowski result (see [17, 18, 19, 20, 21]).

Recently, Singh et al. [15] introduced the Boyd-Wong type generalized F-ψ-contraction mappings in the setting of partial metric spaces and proved some fixed point results. On the other hand, Qawaqneh et al. [16] introduced (α, β, F)-contraction mappings by integrating the ideas of Wardowski [4] and Geraghty [3] and established some fixed point results.

Motivated by the results of Singh et al. [15] and Qawaqneh et al. [16] we established fixed point results for Boyd-Wong type generalized (α, ψ, F)-Geraghty contraction mappings in the setting of partial metric spaces.

In 1992, Matthews [10] presented generalization of metric space as follows:

Definition 1.1. ([10]) Let Φ_p be a non-empty set. A function $\rho : \Phi_p \times \Phi_p \to [0, \infty)$ is said to be a partial metric on Φ_p if the following conditions hold:

(PMS1) $\xi_p = \eta_p$ if and only if $\rho(\xi_p, \xi_p) = \rho(\eta_p, \eta_p) = \rho(\xi_p, \eta_p)$;

(PMS2) $\rho(\xi_p, \xi_p) \leq \rho(\xi_p, \eta_p)$;

(PMS3) $\rho(\xi_p, \eta_p) = \rho(\eta_p, \xi_p)$;

(PMS4) $\rho(\xi_p, \eta_p) \leq \rho(\xi_p, \delta_p) + \rho(\delta_p, \eta_p) - \rho(\delta_p, \delta_p)$. for all $\xi_p, \eta_p, \delta_p \in \Phi_p$.

The set Φ_p equipped with the metric ρ defined above is called a partial metric space and it is denoted by (Φ_p, ρ) (in short PMS). Each partial metric ρ on Φ_p generates a T_0 topology τ_ρ on Φ_p, which has a base of the family of open ρ-balls

$$\{B_\rho(\xi_p, \epsilon) : \xi_p \in \Phi_p, \epsilon > 0\}$$

Where

$$B_\rho(\xi_p, \epsilon) = \{\eta_p \in \Phi_p : \rho(\xi_p, \eta_p) < \rho(\xi_p, \xi_p) + \epsilon\}$$

for all $\xi_p \in \Phi_p$ and $\epsilon > 0$

Example 1.1. *([23]) Let $\Phi_p = \{[a, b] : a, b \in \mathbb{R}, a \leq b\}$ and define $\rho([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$. Then (Φ_p, ρ) is a partial metric space.*

Example 1.2. *([23]) Let $\Phi_p = [0, \infty)$ and define $\rho(\xi_p, \eta_p) = \max\{\xi_p, \eta_p\}$. Then Then (Φ_p, ρ) is a partial metric space.*

Lemma 1.3. *([10]) Let (Φ_p, ρ) be a partial metric space.*

(a) A sequence $\{\xi_{p_n}\}$ in (Φ_p, ρ) converges to a point $\xi_p \in \Phi_p$ if and only if

$$\rho(\xi_p, \xi_p) = \lim_{n\to\infty} \rho(\xi_{p_n}, \xi_p),$$

(b) A sequence $\{\xi_{p_n}\}$ in (Φ_p, ρ) is a Cauchy sequence if $\lim_{m,n\to\infty} \rho(\xi_{p_n}, \xi_{p_m})$ exists and finite ,

(c) (Φ_p, ρ) is complete if every Cauchy $\{\xi_{p_n}\}$ in Φ_p converges to a point $\xi_p \in \Phi_p$, such that

$$\rho(\xi_p, \xi_p) = \lim_{m,n\to\infty} \rho(\xi_{p_n}, \xi_{p_m}) = \lim_{n\to\infty} \rho(\xi_{p_n}, \xi_p) = \rho(\xi_p, \xi_p).$$

Lemma 1.4. *([24],[10],[11]) Let ρ be a partial metric on Φ_p, then the function $d^\rho : \Phi_p \times \Phi_p \to \mathbb{R}^+$ such that*

$$d^\rho(\xi_p, \eta_p) = 2\rho(\xi_p, \eta_p) - \rho(\xi_p, \xi_p) - \rho(\eta_p, \eta_p)$$

is metric on Φ_p. Let (Φ_p, ρ) be a partial metric space. Then

1. *A sequence $\{\xi_{p_n}\}$ in (Φ_p, ρ) is a Cauchy sequence if and only if it is a Cauchy sequence in the metric space (Φ_p, d^ρ),*
2. *(Φ_p, ρ) is complete if and only if the metric space (Φ_p, d^ρ) is complete. Moreover $\lim_{n\to\infty} d^\rho(\xi_{p_n}, \xi_p) = 0 \Leftrightarrow \rho(\xi_p, \xi_p) = \lim_{n\to\infty} \rho(\xi_{p_n}, \xi_p) = \lim_{n,m\to\infty} \rho(\xi_{p_n}, \xi_{p_m})$.*

Lemma 1.5. *([8]) Assume that $\xi_{p_n} \to \delta_p$ as $n \to \infty$ in a partial metric space (Φ_p, ρ) such that $\rho(\delta_p, \delta_p) = 0$ Then $\lim_{n\to\infty} \rho(\xi_{p_n}, \eta_p) = \rho(\delta_p, \eta_p)$ for every $\eta_p \in \Phi_p$.*

Lemma 1.6. *[8] If $\{\xi_{p_n}\}$ with $\lim_{n\to\infty} \rho(\xi_{p_n}, \xi_{p_{n+1}}) = 0$ is not a Cauchy sequence in (Φ_p, ρ), and two sequences $\{r(j)\}$ and $\{s(j)\}$ of positive integers such that $r(j) > s(j) > j$, then following four sequences*

$$\rho(\xi_{p_{s(j)}}, \xi_{p_{r(j)+1}}), \rho(\xi_{p_{s(j)}}, \xi_{p_{r(j)}}),$$
$$\rho(\xi_{p_{s(j)-1}}, \xi_{p_{r(j)+1}}), \rho(\xi_{p_{s(j)-1}}, \xi_{p_{r(j)}})$$

tend to $\mu_p > 0$ when $j \to \infty$

Lemma 1.7. *[9] Let (Φ_p, ρ) be a partial metric space.*

1. *if $\rho(\xi_p, \eta_p) = 0$ then $\xi_p = \eta_p$,*
2. *If $\xi_p \neq \eta_p$ then $\rho(\xi_p, \eta_p) > 0$.*

In 2012, Samet et al. [12] introduced α-admissible mapping as follows:

Definition 1.2. [12] Let $T : \Phi_p \to \Phi_p$ and $\alpha : \Phi_p \times \Phi_p \to [0, \infty)$. T is said to α-admissible if

$$\alpha(\xi_p, \eta_p) \geq 1 \Rightarrow \alpha(T\xi_p, T\eta_p) \geq 1$$

for all $\xi_p, \eta_p \in \Phi_p$.

Definition 1.3. [14] Let $T : \Phi_p \to \Phi_p$ and $\alpha : \Phi_p \times \Phi_p \to [0, \infty)$ be functions. Then T is said to triangular α-admissible if T is α-admissible and for $\xi_p, \eta_p, \delta_p \in \Phi_p$, $\alpha(\xi_p, \delta_p) \geq 1$ and $\alpha(\delta_p, \eta_p) \geq 1 \Rightarrow \alpha(\xi_p, \eta_p) \geq 1$.

Lemma 1.8. *[14] Let $T : \Phi_p \to \Phi_p$ be triangular α-admissible mapping. Assume that there exists $\xi_{p_0} \in \Phi_p$ such that $\alpha(\xi_{p_0}, T\xi_{p_0}) \geq 1$. Define a sequence $\{\xi_{p_n}\}$ by $\xi_{p_{n+1}} = T\xi_{p_n}$ for each $n \in \mathbb{N}_0$. Then we have $\alpha(\xi_{p_m}, \xi_{p_n}) \geq 1$ for all $m, n \in \mathbb{N}$ with $m > n$.*

Boyd and Wong [2] introduced a class of mappings called the φ-contraction mapping and obtained following result:

Definition 1.4. [2] Let (Φ_p, d) be a metric space and $\varphi : [0, \infty) \to [0, \infty)$ be a function such that $\varphi(t) < t$ for $t > 0$. A self map $T : \Phi_p \to \Phi_p$ is called φ-contraction if

$$d(T\xi_p, T\eta_p)) \leq \varphi(d(\xi_p, \eta_p))$$

for all $\xi_p, \eta_p \in \Phi_p$.

Theorem 1.9. *[2] Let (Φ_p, d) be a complete metric space and $T : \Phi_p \to \Phi_p$ a φ-contraction such that φ is upper semicontinuous from the right on $[0, \infty)$ and satisfies $\varphi(t) < t$ for all $t > 0$, Then T has a unique fixed point.*

Definition 1.5. [3] Let $\mathcal{S}$ denotes the class of the functions $\tilde{\beta} : [0, \infty) \to [0, 1)$ which satisfy the condition $\tilde{\beta}(t_n) \to 1$ implies $t_n \to 0$

Theorem 1.10. *[3] Let (Φ_p, d) be a complete metric space and $T : \Phi_p \to \Phi_p$ be a self-mapping. Assume that there exists $\tilde{\beta} \in \mathcal{S}$ such that for all $\xi_p, \eta_p \in \Phi_p$*

$$d(T\xi_p, T\eta_p) \leq \tilde{\beta}(d(\xi_p, \eta_p))d(\xi_p, \eta_p)$$

Then T has a unique fixed point in Φ_p.

Wardowski [4] introduced new class of contraction mappings as follows

Definition 1.6. Let $\mathfrak{F}$ be family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying:

(F1) F is strictly increasing, i.e. for all $a, b \in \mathbb{R}^+$ if $a < b$ then $F(a) < F(b)$;

(F2) for each sequence $\{a_n\}$ of positive numbers, $\lim_{n\to\infty} a_n = 0$ if and ony if $\lim_{n\to\infty} F(a_n) = -\infty$;

(F3) there exists $k \in (0, 1)$ such that $\lim_{a\to 0^+} a^k F(a) = 0$

Wardowski [4] defined F-contraction as follows:

Let (Φ_p, d) be a metric space, then the mapping $T : \Phi_p \to \Phi_p$ is said to be an F-contraction, if there exist $F \in \mathfrak{F}$ and $\tau > 0$ such that for all $\xi_p, \eta_p \in \Phi_p$ with $d(T\xi_p.T\eta_p) > 0$ we have

$$\tau + F(d(T\xi_p, T\eta_p)) \le F(d(\xi_p, \eta_p))$$

Recently, Piri and Kumam [7] extended the result of Wardowski [4] by changing the condition (F3) in Definition 1.6 as follows:

Definition 1.7. [7] Let $\Delta_{\mathfrak{F}}$ be family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying:

(i) F is strictly increasing, i.e. for all $a, b \in \mathbb{R}^+$ if $a < b$ then $F(a) < F(b)$;

(ii) for each sequence $\{a_n\}$ of positive numbers, $\lim_{n\to\infty} a_n = 0$ if and ony if $\lim_{n\to\infty} F(a_n) = -\infty$;

(iii) F is continuous on $(0, \infty)$.

Definition 1.8. [22] Let Φ be family of all functions $\varphi : [0, \infty) \to [0, \infty)$ satisfying:

(i) $\varphi(t) < t$ for $t > 0$;

(ii) φ is continuous function.

2 Main Results

Definition 2.1. Let Ψ be the family of all functions $\psi : [0, \infty) \to (0, \infty)$ satisfying following conditions:

1. ψ is a decreasing function ;
2. ψ is a continuous function.

Definition 2.2. Let (Φ_p, ρ) be partial metric space, $\alpha : \Phi_p \times \Phi_p \to [0, \infty)$ be a function. A self map $T : \Phi_p \to \Phi_p$ is said to be Boyd-Wong type generalized (α, ψ, F)-Geraghty contraction if for all $\xi_p, \eta_p \in \Phi_p$, $\varphi \in \Phi$, $\psi \in \Psi$, $F \in \Delta_{\mathfrak{F}}$, $\beta \in \mathcal{S}$ with $\alpha(\xi_p, \eta_p) \ge 1$ and $\rho(T\xi_p, T\eta_p) > 0$, we have

$$\alpha(\xi_p, \eta_p)(\psi(\rho(\xi_p, T\xi_p)) + F(\rho(T\xi_p, T\eta_p))) \le \beta(M(\xi_p, \eta_p))F(\varphi(M(\xi_p, \eta_p))) \tag{2.1}$$

where

$$M(\xi_p, \eta_p) = \max\left\{\rho(\xi_p, \eta_p), \frac{(1 + \rho(\xi_p, T\xi_p))\rho(\xi_p, T\xi_p)}{1 + \rho(\xi_p, \eta_p)}, \frac{(1 + \rho(\xi_p, T\xi_p))\rho(\eta_p, T\eta_p)}{1 + \rho(\xi_p, \eta_p)}, \frac{\rho(\xi_p, T\eta_p) + \rho(\eta_p, T\xi_p)}{2}\right\}$$

Remark 2.1. Since the functions belonging to $\mathcal{S}$ are strictly less than 1, the expression $\beta(M(\xi_p, \eta_p))$ in (2.1) can be estimated as follows

$$\beta(M(\xi_p, \eta_p)) < 1$$

for all $\xi_p, \eta_p \in \Phi_p$ with $\rho(T\xi_p, T\eta_p) > 0$

Theorem 2.1. *Let (Φ_p, ρ) be a complete partial metric space and $T : \Phi_p \to \Phi_p$ be self mapping. Suppose $\alpha : \Phi_p \times \Phi_p \to [0, \infty)$ be the mapping satisfying the conditions:*

(i) T is triangular α-admissible mapping;

(ii) T is Boyd-Wong type generalized (α, ψ, F)-Geraghty contraction mapping;

(iii) There exists $\xi_{p_0} \in \Phi_p$ such that $\alpha(\xi_{p_0}, T\xi_{p_0}) \geq 1$;

(iv) T is continuous;

Then T has a fixed point in Φ_p.

Proof. Let ξ_{p_0} be an arbitrary point such that $\alpha(\xi_{p_0}, T\xi_{p_0}) \geq 1$. Suppose we have a sequence $\{\xi_{p_n}\}$ in X such that $\xi_{p_{n+1}} = T\xi_{p_n}$ for all $n \in \mathbb{N}_0$.
If $\xi_{p_n} = \xi_{p_{n+1}}$ for some $n \in \mathbb{N}_0$, then ξ_{p_n} is a fixed point of T and the existence part of the proof is finished. Suppose $\xi_{p_n} \neq \xi_{p_{n+1}}$ for every $n \in \mathbb{N}_0$, Then $\rho(\xi_{p_n}, \xi_{p_{n+1}}) = \rho(T\xi_{p_{n-1}}, T\xi_{p_n}) > 0$ by lemma 1.7. Now, since T is α-admissible, so

$$\alpha(T\xi_{p_0}, T\xi_{p_1}) = \alpha(\xi_{p_1}, \xi_{p_2}) \geq 1$$
$$\alpha(T\xi_{p_1}, T\xi_{p_2}) = \alpha(\xi_{p_2}, \xi_{p_3}) \geq 1$$

and using induction we have $\alpha(\xi_{p_n}, \xi_{p_{n+1}}) \geq 1$ for all $n \in \mathbb{N}$.

By (2.1) we get

$$\begin{aligned}\psi(\rho(\xi_{p_{n-1}}, \xi_{p_n})) + F(\rho(\xi_{p_n}, \xi_{p_{n+1}})) &= \psi(\rho(\xi_{p_{n-1}}, T\xi_{p_{n-1}})) + F(\rho(T\xi_{p_{n-1}}, T\xi_{p_n})) \\ &\leq \alpha(\xi_{p_{n-1}}, \xi_{p_n})(\psi(\rho(\xi_{p_{n-1}}, T\xi_{p_{n-1}})) + F(\rho(T\xi_{p_{n-1}}, T\xi_{p_n}))) \\ &\leq \beta(M(\xi_{p_{n-1}}, \xi_{p_n}))F(\varphi(M(\xi_{p_{n-1}}, \xi_{p_n}))) \qquad (2.2)\end{aligned}$$

where

$$\begin{aligned}M(\xi_{p_{n-1}}, \xi_{p_n}) =& \max\Big\{\rho(\xi_{p_{n-1}}, \xi_{p_n}), \frac{(1+\rho(\xi_{p_{n-1}}, T\xi_{p_{n-1}}))\rho(\xi_{p_{n-1}}, T\xi_{p_{n-1}})}{1+\rho(\xi_{p_{n-1}}, \xi_{p_n})}, \\ & \frac{(1+\rho(\xi_{p_{n-1}}, T\xi_{p_{n-1}}))\rho(\xi_{p_n}, T\xi_{p_n})}{1+\rho(\xi_{p_{n-1}}, \xi_{p_n})}, \frac{\rho(\xi_{p_{n-1}}, T\xi_{p_n}) + \rho(\xi_{p_n}, T\xi_{p_{n-1}})}{2}\Big\} \\ =& \max\Big\{\rho(\xi_{p_{n-1}}, \xi_{p_n}), \frac{(1+\rho(\xi_{p_{n-1}}, \xi_{p_n}))\rho(\xi_{p_{n-1}}, \xi_{p_n})}{1+\rho(\xi_{p_{n-1}}, \xi_{p_n})}, \\ & \frac{(1+\rho(\xi_{p_{n-1}}, \xi_{p_n}))\rho(\xi_{p_n}, \xi_{p_{n+1}})}{1+\rho(\xi_{p_{n-1}}, \xi_{p_n})}, \frac{\rho(\xi_{p_{n-1}}, \xi_{p_{n+1}}) + \rho(\xi_{p_n}, \xi_{p_n})}{2}\Big\}\end{aligned} \qquad (2.3)$$

On the other side by (PMS4)

$$\rho(\xi_{p_{n-1}}, \xi_{p_{n+1}}) \leq \rho(\xi_{p_{n-1}}, \xi_{p_n}) + \rho(\xi_{p_n}, \xi_{p_{n+1}}) - \rho(\xi_{p_n}, \xi_{p_n}) \qquad (2.4)$$

replacing (2.4) in (2.3) we get that

$$M(\xi_{p_{n-1}}, \xi_{p_n}) \leq \max\{\rho(\xi_{p_{n-1}}, \xi_{p_n}), \rho(\xi_{p_n}, \xi_{p_{n+1}})\} \qquad (2.5)$$

Now, using (2.5) in (2.2) we get that

$$\begin{aligned}\psi(\rho(\xi_{p_{n-1}}, \xi_{p_n})) + F(\rho(\xi_{p_n}, \xi_{p_{n+1}}))) \leq& \beta(\max\{\rho(\xi_{p_{n-1}}, \xi_{p_n}), \rho(\xi_{p_n}, \xi_{p_{n+1}})\})F(\varphi(\max\{\rho(\xi_{p_{n-1}}, \xi_{p_n}), \\ & \rho(\xi_{p_n}, \xi_{p_{n+1}})\}) \qquad (2.6)\end{aligned}$$

Now, if $\rho(\xi_{p_n}, \xi_{p_{n+1}}) > \rho(\xi_{p_{n-1}}, \xi_{p_n})$, then we get

$$\begin{aligned}\psi(\rho(\xi_{p_{n-1}}, \xi_{p_n})) + F(\rho(\xi_{p_n}, \xi_{p_{n+1}}))) &\leq \beta(\rho(\xi_{p_n}, \xi_{p_{n+1}}))F(\varphi(\ \rho(\xi_{p_n}, \xi_{p_{n+1}})) \\ &\leq F(\varphi(\ \rho(\xi_{p_n}, \xi_{p_{n+1}})) \\ &< F(\rho(\xi_{p_n}, \xi_{p_{n+1}}))\end{aligned}$$

which is a contradiction since $\rho(\xi_{p_n}, \xi_{p_{n+1}}) > 0$. Therefore

$$M(\xi_{p_{n-1}}, \xi_{p_n}) = \rho(\xi_{p_{n-1}}, \xi_{p_n}) \tag{2.7}$$

Again, Using (2.7) in (2.6) and by definitions of β, F and φ we get

$$\begin{aligned}F(\rho(\xi_{p_n}, \xi_{p_{n+1}})) &\leq F(\varphi(\rho(\xi_{p_{n-1}}, \xi_{p_n})) - \psi(\rho(\xi_{p_{n-1}}, \xi_{p_n})) \\ &< F(\rho(\xi_{p_{n-1}}, \xi_{p_n}) - \psi(\rho(\xi_{p_{n-1}}, \xi_{p_n})) \\ &< F(\rho(\xi_{p_{n-1}}, \xi_{p_n}))\end{aligned} \tag{2.8}$$

As F is strictly increasing, we get that

$$\rho(\xi_{p_n}, \xi_{p_{n+1}}) < \rho(\xi_{p_{n-1}}, \xi_{p_n})$$

Thus, $p(\xi_{p_n}, \xi_{p_{n+1}})$ is a decreasing sequence of positive real numbers. Consequently from (2.8), we have

$$\begin{aligned}F(\rho(\xi_{p_n}, \xi_{p_{n+1}}))) &< F(\rho(\xi_{p_{n-1}}, \xi_{p_n})) - \psi(\rho(\xi_{p_{n-1}}, \xi_{p_n})) \\ &< F(\rho(\xi_{p_{n-2}}, \xi_{p_{n-1}})) - \psi(\rho(\xi_{p_{n-2}}, \xi_{p_{n-1}})) - \psi(\rho(\xi_{p_{n-1}}, \xi_{p_n}))\end{aligned}$$

As ψ is decreasing function, we get

$$F(\rho(\xi_{p_n}, \xi_{p_{n+1}}))) < F(\rho(\xi_{p_{n-2}}, \xi_{p_{n-1}})) - 2\psi(\rho(\xi_{p_{n-2}}, \xi_{p_{n-1}}))$$

On generalizing

$$F(\rho(\xi_{p_n}, \xi_{p_{n+1}}))) < F(\rho(\xi_{p_0}, \xi_{p_1})) - n\psi(\rho(\xi_{p_0}, \xi_{p_1})) \tag{2.9}$$

Letting the limit $n \to \infty$ in (2.9) we get

$$\lim_{n\to\infty} F(\rho(\xi_{p_n}, \xi_{p_{n+1}})) = -\infty \Leftrightarrow \lim_{n\to\infty} \rho(\xi_{p_n}, \xi_{p_{n+1}}) = 0 \tag{2.10}$$

consequently, we get

$$\lim_{n\to\infty} \rho(\xi_{p_n}, \xi_{p_{n+1}}) = 0 \tag{2.11}$$

Now, we show that $\{\xi_{p_n}\}$ is a Cauchy sequence in X i.e. We prove that $\lim_{n,m\to\infty} \rho(\xi_{p_n}, \xi_{p_m}) = 0$.
We prove it by contradiction.
Let

$$\lim_{n,m\to\infty} \rho(\xi_{p_n}, \xi_{p_m}) \neq 0$$

.

Then sequences in lemma 1.6 tends to $\mu_p > 0$, when $j \to \infty$.
So we can see that

$$\lim_{j\to\infty} \rho(\xi_{p_{s(j)}}, \xi_{p_{r(j)}}) = \mu_p \tag{2.12}$$

Further corresponding to $s(j)$, we can choose $r(j)$ in such a way that it is smallest integer with $r(j) > s(j) > j$. Then

$$\lim_{j\to\infty} \rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)}}) = \mu_p \tag{2.13}$$

Again,

$$\rho(\xi_{p_{s(j)-1}}, \xi_{p_{r(j)-1}}) \leq \rho(\xi_{p_{s(j)-1}}, \xi_{p_{r(j)}}) + \rho(\xi_{p_{r(j)}}, \xi_{p_{r(j)-1}}) - \rho(\xi_{p_{r(j)}}, \xi_{p_{r(j)}})$$

Letting $j \to \infty$ and using lemma 1.6 we get

$$\lim_{j\to\infty} \rho(\xi_{p_{s(j)-1}}, \xi_{p_{r(j)-1}}) = \mu_p \tag{2.14}$$

In (2.1) replacing ξ_p by $\xi_{p_{r(j)-1}}$ and η_p by $\xi_{p_{s(j)-1}}$ respectively and using Lemma 1.8, we get

$$\begin{aligned}
\psi(\rho(\xi_{p_{r(j)-1}}, \xi_{p_{r(j)}})) + F(\rho(\xi_{p_{r(j)}}, \xi_{p_{s(j)}})) &= \psi(\rho(\xi_{p_{r(j)-1}}, T\xi_{p_{r(j)-1}})) + F(\rho(T\xi_{p_{r(j)-1}}, T\xi_{p_{s(j)-1}})) \\
&\leq \alpha(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}})[\psi(\rho(\xi_{p_{r(j)-1}}, T\xi_{p_{r(j)-1}})) \\
&\quad + F(\rho(T\xi_{p_{r(j)-1}}, T\xi_{p_{s(j)-1}}))] \\
&\leq \beta(M(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}}))F(\varphi(M(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}})))
\end{aligned} \tag{2.15}$$

where

$$\begin{aligned}
M(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}}) = &\max\Bigg\{\rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}}), \frac{(1+\rho(\xi_{p_{r(j)-1}}, T\xi_{p_{r(j)-1}}))\rho(\xi_{p_{r(j)-1}}, T\xi_{p_{r(j)-1}})}{1+\rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}})}, \\
&\frac{(1+\rho(\xi_{p_{r(j)-1}}, T\xi_{p_{r(j)-1}}))\rho(\xi_{p_{s(j)-1}}, T\xi_{p_{s(j)-1}})}{1+\rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}})}, \\
&\frac{\rho(\xi_{p_{r(j)-1}}, T\xi_{p_{s(j)-1}}) + \rho(\xi_{p_{s(j)-1}}, T\xi_{p_{r(j)-1}})}{2}\Bigg\} \\
= &\max\Bigg\{\rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}}), \frac{(1+\rho(\xi_{p_{r(j)-1}}, \xi_{p_{r(j)}}))\rho(\xi_{p_{r(j)-1}}, \xi_{p_{r(j)}})}{1+\rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}})}, \\
&\frac{(1+\rho(\xi_{p_{r(j)-1}}, \xi_{p_{r(j)}}))\rho(\xi_{p_{s(j)-1}}, \xi_{p_{s(j)}})}{1+\rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}})}, \\
&\frac{\rho(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)}}) + \rho(\xi_{p_{s(j)-1}}, \xi_{p_{r(j)}})}{2}\Bigg\}
\end{aligned} \tag{2.16}$$

Letting $j \to \infty$ in (2.16) and using (2.11), (2.12), (2.13), (2.14) and lemma 1.6 we get

$$\lim_{j\to\infty} M(\xi_{p_{r(j)-1}}, \xi_{p_{s(j)-1}}) = \mu_p \tag{2.17}$$

Now Letting $k \to \infty$ in (2.15) and using definitions of ψ, F, β, φ and (2.12),(2.17) we get

$$\psi(\mu_p) + F(\mu_p) \leq \beta(\mu_p)F(\varphi(\mu_p)) \leq F(\varphi(\mu_p)) < F(\mu_p) \tag{2.18}$$

This is a contradiction, Therefore

$$\lim_{n,m\to\infty} \rho(\xi_{p_n}, \xi_{p_m}) = 0 \tag{2.19}$$

This implies that $\{\xi_{p_n}\}$ is a Cauchy sequence in (Φ_p, ρ) which is complete. Therefore the sequence $\{\xi_{p_n}\}$ is convergent in the space (Φ_p, ρ), say $\lim_{n\to\infty} \rho(\xi_{p_n}, \delta_p) = 0$. Again from Lemma 1.3, we get

$$\rho(\delta_p, \delta_p) = \lim_{n\to\infty} \rho(\xi_{p_n}, \delta_p) = \lim_{m,n\to\infty} \rho(\xi_{p_n}, \xi_{p_m}) = 0 \tag{2.20}$$

Moreover, As T is continuous, we have

$$\delta_p = \lim_{n\to\infty} \xi_{p_{n+1}} = \lim_{n\to\infty} T\xi_{p_n} = T\delta_p$$

□

In the following, we omit the continuity assumption of T in Theorem 2.1.

Theorem 2.2. *Let (Φ_p, ρ) be a complete partial metric space and $T : \Phi_p \to \Phi_p$ be self mapping. Suppose $\alpha : \Phi_p \times \Phi_p \to [0, \infty)$ be the mapping satisfying the conditions:*

(i) T is triangular α-admissible mapping;

(ii) T is Boyd-Wong type generalized (α, ψ, F)-Geraghty contraction mapping;

(iii) There exists $\xi_{p_0} \in \Phi_p$ such that $\alpha(\xi_{p_0}, T\xi_{p_0}) \geq 1$;

(iv) If for any sequence $\{\xi_{p_n}\}$ such that $\xi_{p_n} \to \delta_p$ as $n \to \infty$ and $\alpha(\xi_{p_n}, \xi_{p_{n+1}}) \geq 1$, then $\alpha(\xi_{p_n}, \delta_p) \geq 1$.

Then T has a fixed point in Φ_p. Further if δ_p, δ_q are fixed points of T with $\alpha(\delta_p, \delta_q) \geq 1$, then T has a unique fixed point in Φ_p.

Proof. From the proof of the Theorem 2.1, the sequence $\{\xi_{p_n}\}$ defined by $\xi_{p_{n+1}} = T\xi_{p_n}$ is Cauchy in Φ_p. Now, suppose that *(iv)* holds. We have to show that $T\delta_p = \delta_p$. Assume that $\rho(T\delta_p, \delta_p) > 0$. Since $\alpha(\xi_{p_n}, \xi_{p_{n+1}}) \geq 1$, from (2.1) we have

$$\begin{aligned}\psi(\rho(\xi_{p_n}, T\xi_{p_n})) + F(\rho(\xi_{p_{n+1}}, T\delta_p)) &= \psi(\xi_{p_n}, T\xi_{p_n}) + F(\rho(T\xi_{p_n}, T\delta_p)) \\ &\leq \alpha(\xi_{p_n}, \delta_p)(\psi(\xi_{p_n}, T\xi_{p_n}) + F(\rho(T\xi_{p_n}, T\delta_p))) \\ &\leq \beta(M(\xi_{p_n}, \delta_p))F(\varphi(M(\xi_{p_n}, \delta_p)))\end{aligned} \tag{2.21}$$

Where

$$\begin{aligned}M(\xi_{p_n}, \delta_p) = \max\Big\{&\rho(\xi_{p_n}, \delta_p), \frac{(1 + \rho(\xi_{p_n}, T\xi_{p_n}))\rho(\xi_{p_n}, T\xi_{p_n})}{1 + \rho(\xi_{p_n}, \delta_p)}, \frac{(1 + \rho(\xi_{p_n}, T\xi_{p_n}))\rho(\delta_p, T\delta_p)}{1 + \rho(\xi_{p_n}, \delta_p)}, \\ &\frac{\rho(\xi_{p_n}, T\delta_p) + \rho(\delta_p, T\xi_{p_n})}{2}\Big\}\end{aligned} \tag{2.22}$$

Now, taking $n \to \infty$ in (2.22) and using (2.11), (2.20) and Lemma 1.5 we get

$$\lim_{n\to\infty} M(\xi_{p_n}, \delta_p) = \rho(\delta_p, T\delta_p) \tag{2.23}$$

Again, taking $n \to \infty$ in (2.21) and using (2.23) we get

$$\psi(\rho(\delta_p, T\delta_p)) + F(\rho(\delta_p, T\delta_p)) \leq \beta(\rho(\delta_p, T\delta_p))F(\varphi(\rho(\delta_p, T\delta_p))) \leq F(\varphi(\rho(\delta_p, T\delta_p))) < F(\rho(\delta_p, T\delta_p))$$

which implies

$$F(\rho(\delta_p, T\delta_p)) < F(\rho(\delta_p, T\delta_p)) - \psi(\rho(\delta_p, T\delta_p)) < F(\rho(\delta_p, T\delta_p))$$

which is a contradiction. Therefore $T\delta_p = \delta_p$ i.e. δ_p is a fixed point.
Further, suppose δ_p and δ_q be two fixed point of T such that $\rho(\delta_p, \delta_q) > 0$ and $\alpha(\delta_p, \delta_q) \geq 1$. From (2.1) we have

$$\begin{aligned}\psi(\rho(\delta_p, T\delta_p)) + F(\rho(\delta_p, T\delta_q)) &\leq \alpha(\delta_p, \delta_q)(\psi(\rho(\delta_p, T\delta_p)) + F(\rho(\delta_p, T\delta_q))) \\ &\leq \beta(M(\delta_p, \delta_q))F(\varphi(M(\delta_p, \delta_q)))\end{aligned} \tag{2.24}$$

Where

$$\begin{aligned}M(\delta_p, \delta_q) &= \max\left\{\rho(\delta_p, \delta_q), \frac{(1+\rho(\delta_p, T\delta_p))\rho(\delta_p, T\delta_p)}{1+\rho(\delta_p, \delta_q)}, \frac{(1+\rho(\delta_p, T\delta_p))\rho(\delta_q, T\delta_q)}{1+\rho(\delta_p, \delta_q)}, \frac{\rho(\delta_p, T\delta_q)+\rho(\delta_q, T\delta_p)}{2}\right\} \\ &= \max\left\{\rho(\delta_p, \delta_q), \frac{(1+\rho(\delta_p, \delta_p))\rho(\delta_p, \delta_p)}{1+\rho(\delta_p, \delta_q)}, \frac{(1+\rho(\delta_p, \delta_p))\rho(\delta_q, \delta_q)}{1+\rho(\delta_p, \delta_q)}, \frac{\rho(\delta_p, \delta_q)+\rho(\delta_q, \delta_p)}{2}\right\} \\ &= \rho(\delta_p, \delta_q)\end{aligned} \tag{2.25}$$

putting (2.25) in (2.24) we get

$$\begin{aligned}\psi(\rho(\delta_p, \delta_p)) + F(\rho(\delta_p, \delta_q)) &\leq \alpha(\delta_p, \delta_q)(\psi(\rho(\delta_p, \delta_p)) + F(\rho(\delta_p, T\delta_q))) \\ &\leq \beta(\rho(\delta_p, \delta_q))F(\varphi(\rho(\delta_p, \delta_q))) \leq F(\varphi(\rho(\delta_p, \delta_q))) < F(\rho(\delta_p, \delta_q))\end{aligned} \tag{2.26}$$

which implies

$$F(\rho(\delta_p, \delta_q)) < F(\rho(\delta_p, \delta_q)) - \psi(\rho(\delta_p, \delta_p)) < F(\rho(\delta_p, \delta_q))$$

which is a contradiction. Hence T has a unique fixed point. This completes the proof. □

Following are consequences of the theorems.

Corollary 2.3. *Let (Φ_p, ρ) be partial metric space, $T : \Phi_p \to \Phi_p$ be a self map. Let $F \in \Delta_{\mathfrak{F}}$, $\beta \in \mathcal{S}$ and $\varphi \in \Phi$, If for all $\xi_p, \eta_p \in \Phi_p$, with $\rho(T\xi_p, T\eta_p) > 0$ we have*

$$\tau + F(\rho(T\xi_p, T\eta_p)) \leq \beta(M(\xi_p, \eta_p))F(\varphi(M(\xi_p, \eta_p))) \tag{2.27}$$

for some $\tau > 0$ and

$$M(\xi_p, \eta_p) = \max\left\{\rho(\xi_p, \eta_p), \frac{(1+\rho(\xi_p, T\xi_p))\rho(\xi_p, T\xi_p)}{1+\rho(\xi_p, \eta_p)}, \frac{(1+\rho(\xi_p, T\xi_p))\rho(\eta_p, T\eta_p)}{1+\rho(\xi_p, \eta_p)}, \frac{\rho(\xi_p, T\eta_p)+\rho(\eta_p, T\xi_p)}{2}\right\}$$

Then T has a unique fixed point in Φ_p.

Corollary 2.4. *Let (Φ_p, ρ) be partial metric space, $T : \Phi_p \to \Phi_p$ be a self map. Let $F \in \Delta_{\mathfrak{F}}$, $\varphi \in \Phi$, If for all $\xi_p, \eta_p \in \Phi_p$ with $\rho(T\xi_p, T\eta_p) > 0$ we have*

$$\tau + F(\rho(T\xi_p, T\eta_p)) \leq F(\varphi(M(\xi_p, \eta_p))) \tag{2.28}$$

for some $\tau > 0$ and

$$M(\xi_p, \eta_p) = \max\left\{\rho(\xi_p, \eta_p), \frac{(1+\rho(\xi_p, T\xi_p))\rho(\xi_p, T\xi_p)}{1+\rho(\xi_p, \eta_p)}, \frac{(1+\rho(\xi_p, T\xi_p))\rho(\eta_p, T\eta_p)}{1+\rho(\xi_p, \eta_p)}, \frac{\rho(\xi_p, T\eta_p)+\rho(\eta_p, T\xi_p)}{2}\right\}$$

Then T has a unique fixed point in Φ_p.

Example 2.5. *Let $\Phi_p = [0, 1]$ and define $\rho : \Phi_p \times \Phi_p \to \mathbb{R}^+$ by $\rho(\xi_p, \eta_p) = \max\{\xi_p, \eta_p\}$. Then (Φ_p, ρ) is a complete partial metric space. Consider the mapping $T : \Phi_p \to \Phi_p$ defined by $T(\delta_p) = \frac{\delta_p}{100}$ and* *let $\varphi : [0,\infty) \to [0,\infty)$ be such that $\varphi(t) = \frac{t}{10}$, $\psi : [0,\infty) \to (0,\infty)$ be such that $\psi(t) = \frac{1}{1+t}$. If we define the functions $\alpha : \Phi_p \times \Phi_p \to [0,\infty)$ as*

$$\alpha(\xi_p,\eta_p) = \begin{cases} \frac{3}{2} & \xi_p \geq \eta_p \\ o & \text{otherwise} \end{cases} \tag{2.29}$$

Define the function $F : \mathbb{R}^+ \to \mathbb{R}$ by $F(\alpha) = \log \alpha$ for all $\alpha \in \mathbb{R}^+ > 0$.
We show that contractive condition of Theorem 2.1 is satisfied. Let $\xi_p, \eta_p \in \Phi_p$, without loss of generality we assume that $\xi_p \geq \eta_p$. Suppose that $\rho(T\xi_p, T\eta_p) > 0$, then

$$\begin{aligned}\alpha(\xi_p,\eta_p)(\psi(\rho(\xi_p,T\xi_p)) + F(\rho(T\xi_p,T\eta_p))) &= \frac{3}{2}[\psi(\rho(\xi_p,\frac{\xi_p}{100})) + F(\rho(\frac{\xi_p}{100},\frac{\eta_p}{100}))] \\ &= \frac{3}{2}[\psi(\xi_p) + F(\frac{\xi_p}{100})] \\ &\frac{3}{2}[\frac{1}{1+\xi_p} + \log(\frac{\xi_p}{100})]\end{aligned} \tag{2.30}$$

On the other side

$$\begin{aligned}M(\xi_p,\eta_p) &= \max\left\{\rho(\xi_p,\eta_p), \frac{(1+\rho(\xi_p,T\xi_p))\rho(\xi_p,T\xi_p)}{1+\rho(\xi_p,\eta_p)}, \frac{(1+\rho(\xi_p,T\xi_p))\rho(\eta_p,T\eta_p)}{1+\rho(\xi_p,\eta_p)}, \frac{\rho(\xi_p,T\eta_p)+\rho(\eta_p,T\xi_p)}{2}\right\} \\ &= \max\left\{\rho(\xi_p,\eta_p), \frac{(1+\rho(\xi_p,\frac{\xi_p}{100}))\rho(\xi_p,\frac{\xi_p}{100})}{1+\rho(\xi_p,\eta_p)}, \frac{(1+\rho(\xi_p,\frac{\xi_p}{100}))\rho(\eta_p,\frac{\eta_p}{100})}{1+\rho(\xi_p,\eta_p)}, \frac{\rho(\xi_p,\frac{\eta_p}{100})+\rho(\eta_p,\frac{\xi_p}{100})}{2}\right\} \\ &= \xi_p\end{aligned} \tag{2.31}$$

$$\begin{aligned}\beta(M(\xi_p,\eta_p))F(\varphi(M(\xi_p,\eta_p))) &= \beta(\xi_p)F(\varphi(\xi_p)) \\ &= \frac{1}{1+\xi_p}\log(\varphi(\xi_p)) \\ &= \frac{1}{1+\xi_p}\log(\frac{\xi_p}{10})\end{aligned} \tag{2.32}$$

Therefore, it satisfies the condition of Theorem 2.1. Hence T has a fixed point, which in this case is 0.

3 Application

we will apply Corollary 2.3 to solving the existence and uniqueness of the solution of the following integral equations:

$$\xi_p(t) = f(t) + \lambda\int_0^1 G(t,s)F_n(s,\xi_p(s))\,ds \tag{3.1}$$

$$\eta_p(t) = f(t) + \lambda\int_0^1 G(t,s)G_n(s,\eta_p(s))\,ds \tag{3.2}$$

for all $t \in [0,1]$ and λ is a real number.

Let $\Phi_p = C([0,1],\mathbb{R})$ be set of all real valued continuous function on $[0,1]$. Let Φ_p be endowed with the partial metric $\rho : \Phi_p \times \Phi_p \to [0,\infty)$ defined by

$$\rho(\xi_p,\eta_p) = d(\xi_p,\eta_p) + c_n = \sup_{t\in[0,1]}|\xi_p(t) - \eta_p(t)| + c_n$$

for all $\xi_p, \eta_p \in \Phi_p$ and $\{c_n\}$ is a sequence of positive real numbers such $\lim_{n\to\infty} c_n = 0$. Now, we prove the following theorem to ensure the existence of solution of system of integral equations.

Theorem 3.1. *Assume the following conditions are satisfied:*

(i) $F_n, G_n : [0,1] \times \mathbb{R} \to \mathbb{R}$, $G : [0,1] \times [0,1] \to \mathbb{R}^+$ *and* $f : [0,1] \to \mathbb{R}$ *are continuous.*

(ii) Define

$$T\xi_p(t) = f(t) + \lambda \int_0^1 G(t,s) F_n(s, \xi_p(s))\, ds \tag{3.3}$$

$$T\eta_p(t) = f(t) + \lambda \int_0^1 G(t,s) G_n(s, \eta_p(s))\, ds \tag{3.4}$$

and when $n \to \infty$ *there exists* $\tau \geq 1$ *such that*

$$|F_n(t, \xi_p(t)) - G_n(t, \eta_p(t))| \leq e^{-\tau} \Big(\varphi(|\xi_p(t) - \eta_p(t)|) \Big)^k$$

for all $t \in [0,1]$, *where* $k = \frac{1}{1+\rho(\xi_p, \eta_p)}$ *and* $\varphi \in \Phi$

(iii)

$$\sup_{t\in[0,1]} \int_0^1 |G(t,s)|\, ds \leq R < +\infty$$

(iv) $\lambda R \leq 1$

Then the system of integral equations given in (3.1)and (3.2) has a solution.

Proof. Following the assumptions of Theorem 3.1, we have

$$\begin{aligned}
\rho(T\xi_p, T\eta_p) &= d(T\xi_p, T\eta_p) + c_n \\
&= \sup_{t\in[0,1]} |T\xi_p(t) - T\eta_p(t)| + c_n \\
&= \sup_{t\in[0,1]} \int_0^1 \Big| \lambda G(t,s)[F_n(s, \xi_p(s)) - G_n(s, \eta_p(s)] \Big| \, ds + c_n
\end{aligned}$$

Letting $n \to \infty$, we get

$$\begin{aligned}
\rho(T\xi_p, T\eta_p) &\leq |\lambda| e^{-\tau} \sup_{t\in[0,1]} \left[\Big(\varphi(|\xi_p(t) - \eta_p(t)|) \Big)^k \int_0^1 |G(t,s)|\, ds \right] \\
&\leq |\lambda| e^{-\tau} \Big(\varphi(d(\xi_p, \eta_p)) \Big)^k \sup_{t\in[0,1]} \int_0^t |G(t,s)|\, ds \\
&\leq |\lambda| R e^{-\tau} \Big(\varphi(d(\xi_p, \eta_p)) \Big)^k \\
&\leq e^{-\tau} \Big(\varphi(d(\xi_p, \eta_p)) \Big)^k \\
&\leq e^{-\tau} \Big(\varphi(\rho(\xi_p, \eta_p)) \Big)^k
\end{aligned}$$

By passing logarithm we obtain that

$$\ln(\rho(T\xi_p, T\eta_p)) \leq -\tau + k \ln(\varphi(\rho(\xi_p, \eta_p)) = -\tau + \frac{1}{1 + \rho(\xi_p, \eta_p)} \ln(\varphi(\rho(\xi_p, \eta_p))$$

where $k = \frac{1}{1+\rho(\xi_p,\eta_p)}$, By taking $F(r) = \ln(r)$, $\beta(r) = \frac{1}{1+r}$ we get

$$\tau + F(\rho(T\xi_p, T\eta_p)) \leq \beta(\rho(\xi_p, \eta_p))F(\varphi(\rho(\xi_p, \eta_p)) \leq \beta(M(\xi_p, \eta_p))F(\varphi(M(\xi_p, \eta_p)))$$

Clearly, all the conditions of Corollary 2.3 are satisfied and so T has a unique fixed point. Thus the system of integral equations (3.1)and (3.2) has a unique solution. □

4 CONCLUSIONS

In this paper, we generalized Wardowski F-contraction mappings by introducing Boyd-Wong type generalized (α, ψ, F)-Geraghty contraction mappings and established the corresponding fixed point theorems in partial metric spaces. We have applied our results to get the existence of a solution for a class of nonlinear integral equations.

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