Original Research Article

Computing Y-index of Different corona products of Graphs

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ABSTRACT

The Y-index of a graph is defined by the sum of four of degrees of the vertices of a graph . Among various operations, as corona product of two graphs is one of the most important . In this study, we obtain explicit expressions for the Y-index of different types of corona product.

Keywords: Topological index, Zagreb index, Y-index, Corona product, Graph operations.

1. INTRODUCTION

A topological is defined as a real valued function which maps each molecular graph to a real number and is necessarily invariant under automorphism of graphs. In Theoretical chemistry assigning a numerical value to molecular structure that will closely correlate with the physical quantities and activities. Molecular structure descriptors (also called topological indices) are used for modeling physicochemical pharmacologic, toxicologic, biological and other properties of chemical compounds.

In this paper, we consider only finite, connected and undirected graphs without any self-loops or multiple edges. Let A be a graph with vertex set V(A) and edge set E(A). So that the vertex set will be considered n and the edge set will be considered m. The vertices u and v is connected by edge and is denoted by u v. Let $d_G(v)$ denote the degree of the vertex v in A, which is the number of edges incident to v.

Zagreb indices are among the best applications for recognizing physical properties and chemical reactions in practical applications. The First Zagreb index $M_1(A)$ and Second Zagreb index $M_2(A)$ were firstly considered by I. Gutman and N. Trinajstic in 1972[12] defined as

$$M_1(A) = \sum_{v \in V(A)} d_A(u)^2 = \sum_{uv \in E(A)} [d_A(u) + d_A(v)]$$

$$M_2(A) = \sum_{u,v \in V(A)} d_A(u)\ d_A(v)$$

These indices have considerably studied with respect to both mathematical and chemical point of view. In 2005, Li and Zheng [7], introduce the First general Zagreb index as

$$M_1^{\alpha+1}(A) = \sum d_A{}^{\alpha+1} d_A(v)$$
$$= \sum_{uv \in E(A)} d_A^{\alpha}(u) + d_A^{\alpha}(v)$$

In 2018, Nilanjan De uses modern index to calculate the F-index and co index of some derived graphs[11]. Its special of First general Zagreb index where $\alpha = 3$

$$M_1^4(A) = \sum_{uv \in E(A)} d_A^3(u) + d_A^3(v)$$

So that the Y-index is defined as,

$$Y(A) = \sum_{u \in V(A)} d_A^4(u) = \sum_{uv \in E(A)} d_A^3(u) + d_A^3(v)$$

Let A_1 and A_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, for $i \in \{1,2\}$. The Corona product $A_1 \circ A_2$ of these two graphs is obtained by taking one copy of A_1 and n_1 copies of A_2: and by joining each vertex of the i-th copy of A_2 to the i-th vertex of A_1 where $1 \le i \le n$. The corona product of A_1 and A_2 has total number of $(n_1 n_2 + n_1)$ vertices and $(m_1 + n_1 m_2 + n_1 n_2)$ edges

The Subdivision graph S=S(A) is a graph obtained from A by replacing each of its edges by a path of length two , or equivalently by inserting an additional vertex into each edge of A

Among the most well-known graph products, the corona product of graphs is one of the most important graph operations by corona product of some general and particular graphs. In this paper, we derive some explicit expressions of different type of corona product of graph of graph such as Subdivision-vertex corona, Subdivision –edge corona, as Subdivision-vertex neighborhood corona, Subdivision –edge neighborhood corona, Vertex-edge corona of two graphs

2. MAIN RESULTS

Subdivision-vertex corona

Definition 2.1. [8] Let A_1 and A_2 be two vertex disjoint graphs . The Subdivision-vertex corona of A_1 and A_2 is denoted by $A_1 \boxdot A_2$ and obtained from S(A_1) and n_1 copies of A_2 , all vertex-disjoint, by joining the $i-th$ vertex of V(A_1) to every vertex in the $i-th$ copy of A_2

By definition it is clear that the Subdivision –vertex corona $A_1 \boxdot A_2$ has $n_1(1+n_2)+m_1$ vertices and $2m_1 + n_1(n_2+m_2)$ edges. Also the degree of the vertices of $A_1 \boxdot A_2$ are given by

$$d_{A_1 \boxdot A_2}(v_i) = d_{A_1}(v_i) + n_2 \qquad for \;\; i = 1,2,...n_1$$
$$d_{A_1 \boxdot A_2}(e_i) = 2 \qquad for \; i = 1,2,...m_1$$
$$d_{A_1 \boxdot A_2}(v_j^i) = d_{A_2}(u_j) + 1 \qquad for \; i = 1,2,.....n_1 and \quad j = 1,2,....n_2$$

We calculate the Y-index for the Subdivision-vertex corona $A_1 \boxdot A_2$
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Theorem.2.2 The Y-index of the Subdivision- vertex corona $A_1 \boxdot A_2$ is given by

$$Y(A_1 \boxdot A_2) = Y(A_1) + 4n_2 F(A_1) + 6{n_2}^2 M_1(A_1) + 8m_1 {n_2}^3 + n_1 {n_2}^4$$
$$+16m_1 + n_1\, Y(A_2) + 4n_1\, F(A_2) + 6n_1 M_1(A_2) + 8n_1 m_2 + n_1 n_2$$

Proof : From the definition of Subdivision-vertex corona $A_1 \boxdot A_2$, we get

$$Y(A_1 \boxdot A_2) = \sum_{i=1}^{n_1} (d_{A_1}(v_i) + n_2)^4 + \sum_{i=1}^{m_1} 2^4 + \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} (d_{A_2}(u_j) + 1)^4$$

$$= \sum_{i=1}^{n_1} d_{A_1}(v_i)^4 + 4n_2 d_{A_1}(v_i)^3 + 6d_{A_1}(v_i)^2 {n_2}^2 + 4d_{A_1}(v_i){n_2}^3 + n_2^4 +$$

$$16m_1 + n_1(\sum_{j=1}^{n_2} d_{a_2}(u_j)^4 + 4d_{A_2}(u_j)^3 + 6d_{A_2}(u_j) + 4d_{A_2}(u_j) + 1)$$

$$= \quad Y(A_1) + 4n_2F(A_1) + 6{n_2}^2 M_1(A_1) \; + 8m_1{n_2}^3 \; + n_1{n_2}^4$$

$$+16m_1 + n_1 \; Y(A_2) + 4n_1 \; \mathrm{F}(A_2) + 6n_1 M_1(A_2) + 8n_1 m_2 + n_1 n_2$$

Hence, we get the desired result.

Example. 2.3
$Y(P_n \boxdot \; P_m) = \; nm^4 + 8m^3(n-1) + 12m^2(2n-3) + 113nm - \; 56m - 98n - 46$
$Y(C_n \boxdot \; C_m) = \; nm^4 + 8nm^3 + 24nm^2 + 113nm + 32n$
$Y(C_n \boxdot \; P_m) = \; nm^4 + 8nm^3 + 24nm^2 + 113nm - 98n$

The Subdivision-vertex corona of $P_4, P_2 \; and \; C_3$ is given by

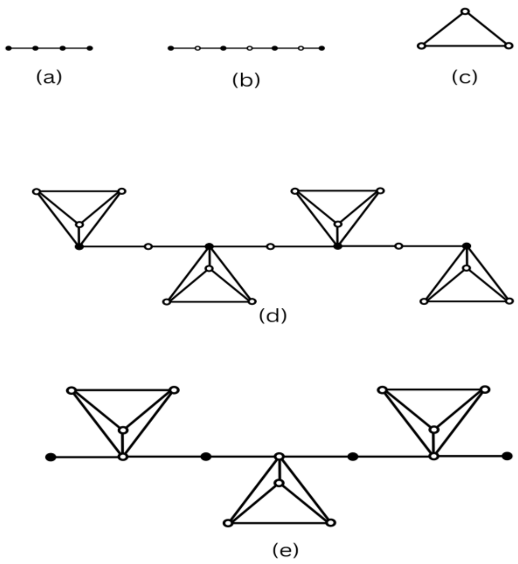

Figure 1: (a)-path p4, (b)-Subdivision S (p4), (c)-Cycle c3, (d)-Subdivision –vertex corona p4 c3, (e)- Subdivision-edge corona p4 c3

Subdivision-edge corona
Definition 2.4.[8] Again from the definition of subdivision graphs, The Subdivision-edge corona product $A_1 \oplus A_2$ of $A_1$and A_2 is obtained from the $S(A_1)$ and m_1 copies of A_2 such that for all disjoint vertices joining the i-th vertex of $S(A_1)$ to every vertex in the i-th copy of A_2. Clearly the product $A_1 \oplus A_2$ has $m_1(1+n_2)+n_1$ vertices and $m_1(n_2+m_2+2)$ edges. Also the degree of vertices of $A_1 \oplus A_2$ are given by

$$d_{A_1 \oplus A_2}(v_i) = d_{A_1}(v_i) \qquad for \;\; i = 1,2,\ldots n_1$$

$d_{A_1 \oplus A_2}(v_i) = 2 + n_2$ $\quad$ $for\ \ i = 1,2,...m_1$

$d_{A_1 \oplus A_2}(v_j^i) = d_{A_2}(u_j) + 1$ $\quad$ $for\ \ i = 1,2,.....n_1 and \quad j = 1,2,....n_2$

Theorem 2.5. The Y-index of Subdivision-edge corona $A_1 \oplus A_2$ is given by

$$Y(A_1 \oplus A_2) = Y(A_1) + [m_1\ (n_2 + 2)]^4 + m_1 Y(A_2) + 4m_1\ F(A_2)$$

$$+6m_1 M_1(A_2)\ \ +8m_1 m_2 + m_1 n_2$$

Proof: From the definition of Subdivision-edge corona $A_1 \oplus A_2$, we have

$$Y(A_1 \oplus A_2) = \sum_{i=1}^{n_1} (d_{A_1}(v_i))^4 + \sum_{i=1}^{m_1} (2 + n_2)^4 + \sum_{i=1}^{m_1} \sum_{j=1}^{n_2} (d_{A_2}(u_j) + 1)^4$$

$$= Y(A_1) + [m_1\ (n_2 + 2)]^4 + m_1 Y(A_2) + 4m_1\ F(A_2)$$

$$+6m_1 M_1(A_2)\ \ +8m_1 m_2 + m_1 n_2$$

Hence we get the result

Example.2.6

$Y(P_n \oplus P_m) = m^4(n-1) + 8m^3(n-1) + 2m^2(n-1) + 113nm - 113m - 98n + 84$

$Y(C_n \oplus C_m) = nm^4 + 8nm^3 + 2nm^2 + 113nm + 32n$

$Y(C_n \oplus P_m) = nm^4 + 8nm^3 + 2nm^2 + 113nm - 98n$

Subdivision- vertex neighborhood corona

Definition 2.7.[9] Subdivision –vertex neighborhood corona product $A_1 \odot A_2$ of A_1 and A_2 is obtained from the $S(A_1)$ and $n_1$copies of A_2 such that for all disjoint vertices joining the neighbors of the i-th vertex of $S(A_1)$ to every vertex in the i-th copy of A_2. Thus $A_1 \odot A_2$ has $m_1(1 + n_2) + n_1$ vertices $2m_1$ edges .

Let $V(A_1) = \{v_1, v_2, ... v_{n_1}\}, I(A_1) = \{e_1, e_2, ... e_{m_1}\}$ and $V(A_2) = \{v_1, v_2, ... v_{n_2}\}$. Also , let the vertex set of the i-th copy of A_2 is denoted by $V(A_2^i) = \{u_1^i, u_2^i, ... u_{n_2}^i\}$, for i=1,2,.., n_2. Then $(A_1 \odot A_2) = V(A_1) \cup I(A_1) \cup [V^1(A_2) \cup V^2(A_2) \cup ... \cup V^{n_1}(A_2)]$. The degree of the vertices of $A_1 \odot A_2$ are given by

$d_{A_1 \odot A_2}(v_i) = d_{A_1}(v_i)$ $\quad$ $for\ \ i = 1,2,...n_1$

$d_{A_1 \odot A_2}(e_i) = 2n_2 + 2$ $\quad$ $for\ \ i = 1,2,...m_1$

$d_{A_1 \odot A_2}(v_j^i) = d_{A_2}(u_j) + d_{A_1}(v_j)$ $\quad$ $for\ \ i = 1,2,.....n_1 and \quad j = 1,2,....n_2$

The Y-index of Subdivision-vertex neighborhood corona of two graphs is defined in the following theorem

Theorem 2.8. The Y-index of $A_1 \odot A_2$ is given by

$$Y(A_1 \odot A_2) = Y(A_1) + 16m_1(n_2 + 1)^4 + n_1 Y(A_2) + 8m_1\ F(A_2)$$

$$+6M_1(A_2)\ M_1(A_1) + 8m_2 F(A_1) + n_2 Y(A_1)$$

Proof: From the definition of $A_1 \odot A_2$, we have

$$Y(A_1 \odot A_2) = \sum_{i=1}^{n_1} (d_{A_1}(v_i))^4 + \sum_{i=1}^{m_1} (2n_2 + 2)^4 + \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} (d_{A_2}(u_j) + (d_{A_2}(v_j))^4$$

$$= Y(A_1) + 16m_1(n_2+1)^4 + n_1Y(A_2) + 8m_1\,F(A_2)$$

$$+6M_1(A_2)\,M_1(A_1) + 8m_2F(A_1) + n_2Y(A_1)$$

Hence we get the desired result

Example 2.9
$Y(P_n \odot P_m) = 16m^4(n-1) + 64m^3(n-1) + 96m^2(n-1) + 320nm - 414m - 318n + 394$
$Y(C_n \odot C_m) = 16nm^4 + 6nm^3 + 96nm^2 + 320nm + 32n$
$Y(C_n \odot P_m) = 16nm^4 + 64nm^3 + 96nm^2 + 320nm - 318n$

The Subdivision-vertex neighborhood corona of P_4, P_2 *and* C_3 is given in the figure.

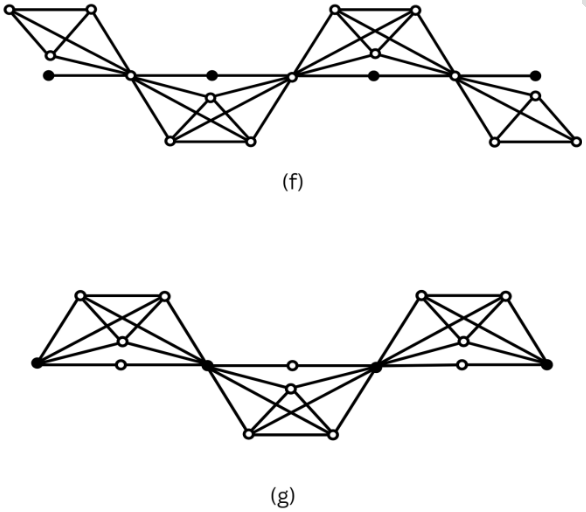

Figure.2: (f)- Subdivision-vertex neighborhood corona p4 c3, (g)- Subdivision-edge neighborhood corona p4 c3

Subdivision-edge neighborhood corona

Definition 2.10.[9]: For two vertex disjoint graphs $A_1$and A_2, the Subdivision –edge neighborhood corona $A_1$and A_2 is denoted by $A_1 \boxminus A_2$ and obtained from $S(A_1)$ and n_1 copies of A_2 all vertex disjoint, by joining the neighbors of the i-th vertex of $V(A_1)$ to every vertex in the i-th copy of A_2.

Let $V(A_1) = \{v_1, v_2, \dots v_{n_1}\}, I(A_1) = \{e_1, e_2, \dots e_{m_1}\}$ and $V(A_2) = \{v_1, v_2, \dots v_{n_2}\}$. Also , let the vertex set of the i-th copy of A_2 is denoted by $V(A_2^i) = \{u_1^i, u_2^i, \dots u_{n_2}^i\}$, for i=1,2,.., n_1. Then $A_1 \boxminus A_2 = V(A_1) \cup I(A_1) \cup [V^1(A_2) \cup V^2(A_2) \cup \dots \cup V^{n_1}(A_2)]$.

The degree of the vertices of $A_1 \boxminus A_2$ are given by

$d_{A_1 \boxminus A_2}(v_i) = (n_2+1)d_{A_1}(v_i)$ *for* $i = 1,2,\dots n_1$

$d_{A_1 \boxminus A_2}(e_i) = 2 \qquad for\ i = 1,2,...m_1$

$d_{A_1 \boxminus A_2}(v_j^i) = d_{A_2}(u_j) + 2 \qquad for\ i = 1,2,.....n_1\ \ and\ \ \ j = 1,2,....n_2$

The Subdivision-edge neighborhood corona of $P_4, P_2\ and\ C_3$ is given in the figure.2

So the result of this section is the following

Theorem.2.11

The Y-index of $A_1 \boxminus A_2$ is given by

$$Y(A_1 \boxminus A_2) = Y(A_1)(n_2+1)^4 + 16m_1 + n_1 Y(A_2) + 8n_1\, F(A_2)$$
$$+24n_1 M_1(A_2)\ \ +64n_1 m_2 + 16n_1 n_2$$

Proof: From the definition of Subdivision –edge neighborhood corona product of two graphs. We have

$$Y(A_1 \boxminus A_2) \ = \ \sum_{i=1}^{n_1}(n_2+1)^4\,(d_{A_1}(v_i)^4 + \sum_{i=1}^{m_1} 2^4 + \sum_{i=1}^{n_1}\ \sum_{j=1}^{n_2}\ (d_{A_2}(u_j)+2)^4$$

$$= \ (n_2+1)^4 Y(A_1) + 16m_1 + n_1(\sum_{i=1}^{n_2} d_{A_2}(u_j)^4 + 8d_{A_2}(u_j)^3$$
$$+ 24d_{A_2}(u_j)^2 + 4d_{A_2}(u_j).2^3 + 2^4)$$

$$= \ \ Y(A_1)(n_2+1)^4 + 16m_1 + n_1 Y(A_2) + 8n_1\, F(A_2)$$
$$+24n_1 M_1(A_2)\ \ +64n_1 m_2 + 16n_1 n_2$$

Hence we get the result

Example 2.12

$Y(P_n \boxminus P_m) \ = \ 16nm^4 - 30m^4 + 64nm^3 - 120m^3 + 96nm^2 - \ 180m^2 + 320nm - 318n - 120m - 46$

$Y(C_n \boxminus C_m) \ = \ 16nm^4 + 64nm^3 + 96nm^2 + 320nm + 32n$

$Y(C_n \boxminus P_m\ ) = \ 16nm^4 + 64nm^3 + 96nm^2 + 320nm\ - 318n$

The Vertex –edge corona.

Definition 2.13:[13] The vertex –edge corona of two graphs A_1 and A_2 is denoted by $A_1 \boxtimes A_2$ is the graph obtained by taking one copy of A_1, $n_1$copies of A_2 and also $m_1$copies of A_2, then joining the i-th vertex of $A_1$to every vertex in the i-th vertex copy of A_2 and also joining the end vertices of j-th edge of A_1 to every vertex in the j-th edge copy of A_2.

Let the vertex set of the j-th edge copy of A_2 is denoted by $V_{\ je}(A_2) = \{u_{ji}, u_{j2}, ... u_{jn_2}\}$ and the vertex set of the i-th vertex copy of A_2 is denoted by $V_{iv}(A_2) = \{w_{i1}, w_{i2}, ... w_{in}\}$. Also let us denote the edge set of the j-th edge and i-th vertex copy of A_2 by $E_{je}(A_2)$ and $E_{iv}(A_2)$ respectively. According to this definition we have the vertex –edge corona $A_1 \boxtimes A_2$ has $m_1 + m_1(m_2{+}2m_2) + n_1 n_2 + n_1 m_2$ edges and $n_1 + n_1 n_2 + mn_2$ vertices . Also the degree of the vertices of $A_1 \boxtimes A_2$ are given by

$d_{A_1 \boxtimes A_2}(v_i) = (n_2+1)d_{A_1}(v_i) + n_2 \qquad \forall\ v_i \in V(A_1)$

$d_{A_1 \boxtimes A_2}(e_i) = d_{A_2}(u_j) + 2 \qquad \forall\ u_{ij} \in V_{ie}(A_2)$

$d_{A_1 \boxtimes A_2}(v_j^i) = d_{A_2}(w_j) + 1 \qquad \forall\ \ w_{ij} \in V_{ie}(A_2)$

Theorem.2.14

The Y-index of $A_1 \boxtimes A_2$ is given by

$$Y(A_1 \boxtimes A_2)=Y(A_1)(n_2+1)^4+4(n_2+1)^3Y(A_1)n_2+\ 6{n_2}^2(n_2+1)^2M_1(A_1)+8m_1(n_2+1)$$

$$ {n_2}^3+n_1{n_2}^4+m_1Y(A_2)+8m_1F(A_2)+24M_1(A_2)m_1+8n_1m_2+n_1n_2$$

Proof: From the definition of vertex-edge corona of graphs, we have

$$Y(A_1 \boxtimes A_2) = \sum_{v_i \in V(A_1)} d_A(v_i)^4 + \sum_{e_i \in E(A_1)} \sum_{u_{ij} \in V(A_2)} d_A(u_{ij})^4 + \sum_{v_i \in V(A_1)} \sum_{w_{ij} \in V(A_2)} d_A(w_{ij})^4$$

$$= S_1 + S_2 + S_3 \text{ (say)}$$

We have now determined by calculating $S_1^{'}$s contribution

$$S_1 = \sum_{v_i \in V(A_1)} d_A(v_i)^4$$

$$= \sum_{v_i \in V(A_1)} [\ (n_2+1)d_{A_1}(v_i)+n_2]^4$$

$$= (n_2+1)^4Y(A_1)+4(n_2+1)^3F(A_1)n_2+6(n_2+1)^2M_1(A_1)n_2^2$$
$$+8(n_2+1)m_1n_2^3+n_1n_2^4$$

Also, we have the contribution of S_2 as

$$S_2 = \sum_{e_i \in E(A_1)} \sum_{u_{ij} \in V(A_2)} d_A(u_{ij})^4$$

$$= \sum_{e_i \in E(A_1)} \sum_{u_{ij} \in V(A_2)} [d_{A_2}(u_j)+2]^4$$

$$= = m_1Y(A_2)+m_18F(A_2)+24m_1M_1(A_2)+64m_1m_2+16m_1n_2$$

Similarly ,we get the $S_3^{'}$ s contribution

$$S_3 = \sum_{v_i \in V(A_1)} \sum_{w_{ij} \in V(A_2)} d_A(w_{ij})^4$$

$$= \sum_{v_i \in V(A_1)} \sum_{w_{ij} \in V(A_2)} [d_{A_2}(w_j)+1]^4$$

$$= n_1Y(A_2)+n_14F(A_2)+6n_1M_1(A_2)+8n_1m_2+n_1n_2$$

After adding S_1, S_2 *and* S_3 we obtain the equality in the statement of the theorem as required

Example 2.15

$Y(P_n \boxtimes P_m) = 81nm^4 - 130m^4 + 216nm^3 - 368m^3 + 216nm^2 - 384m^2 + 433nm - 432m - 480n + 336$

$Y(C_n \boxtimes C_m) = 81nm^4 + 216nm^3 + 216nm^2 + 465nm + 16n$

$Y(C_n \boxtimes P_m) = 81nm^4 + 216nm^3 + 216nm^2 + 465nm - 464n$

3. CONCLUSION

In this paper, we compute Y-index of different types of corona products of two graphs such as Subdivision-vertex corona, Subdivision-edge corona, Subdivision-vertex neighborhood corona, Subdivision- edge neighborhood corona, Vertex-edge corona. As an application we derive some explicit expressions of corona products of some particular graphs.

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