The Effect of Missing Data on Estimates of Exponential Trend-Cycle and Seasonal Components in Time Series: Additive Case

Abstract: This study discusses the effect of missing data on Buys-Ballot estimates of trend parameters and seasonal indices. The method adopted in this study is based on the row, column and overall means of the time series arranged in a Buys-Ballot table with m rows and s columns. The method assumes that (1) Only data missing at one point at a time in the Buys-Ballot table is considered. (2) the trending curve is either linear or exponential (3) the decomposition method is either additive or mixed. The article shows that, the estimation of the missing data as they occur consecutively with the errors being normally distributed. Result indicates that, under the stated assumptions, the differences between trend parameters in the presence and absence are insignificant, while that of seasonal indices are significant.

Keywords: Additive model, Missing data, Exponential trend, Seasonal indices, Buys-Ballot estimates

1 Introduction

Application of time series analysis has become very important in making crucial decision in several field. In trying to provide solutions to real life problems involving time series, most existing statistical models in solving these problems deals with observed values throughout the data. Time series data with missing data cannot be solved using these models.

A lot of works have previously been done on the use Buys-Ballot table in the estimation of trend parameters, seasonal indices and estimation of missing data. Rosen and Porat [1] proposed estimation of the covariance of stationary time series with missing data. The estimation provided general formulas for the asymptotic second- order moments of the sample covariance, for either random or deterministic pattern of missing data. Ferreiro [2] worked on different alternative for the estimation of missing data in stationary time series for autoregressive moving average models. He observed that, the occurrence of missing data is very common in time series and in many cases, it is important to estimate them. Ljung [3] obtained an expression for the likelihood function of the parameters in an autoregressive moving average model when there are missing data within the time series data. Luceno [4] extended Ljung's [3] method for estimating missing data and evaluating the corresponding likelihood function in scalar time series to the vector cases. Iwueze et.al [5] proposed new estimation methods for replacing missing data in time series analysis while comparing them with those of the existing methods. The new methods include Row Mean Imputation (RMI), Column Mean Imputation (CMI) and Decomposing Without the Missing Value (DWMV). This methods are based on the row, column and overall averages of time series data arranged in a Buys-Ballot table with m rows and s columns.

Three models commonly used in time series date are additive, multiplicative and mixed models**.** If short period of time are involved, the trend component is superimposed into the cyclical Chatfield [6] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \qquad (1)$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \qquad (2)$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \qquad (3)$$

As far as the descriptive method of decomposition is concerned, the first step will usually be to estimate and eliminate trend-cycle (M_t) for each time period from the actual data either by subtraction, for Equation (1) or division, for Equation (2). The de-trended series is obtained as $X_t - \hat{M}_t$ for Equation (1) or $X_t / \hat{M}_t$ for Equations (2) and (3). The seasonal effect is obtained by estimating the average of the de-trended series at each season. The de-trended, de-seasonalized series is obtained as $X_t - \hat{M}_t - \hat{S}_t$ for Equation (1) or $X_t / \left(\hat{M}_t \hat{S}_t\right)$ for Equations (2) and (3). This gives the residual or irregular component. Having fitted a time series model, one often wants to see if the residuals are purely random. For details of residual analysis, see Box, *et al*, [7] and Ljung and Box [8]. It is always assumed that the seasonal effect, when it exists, has period *s*, that is, it repeats after *s* time periods.

$$S_{t+s} = S_t, \text{ for all } t \tag{4}$$

For Equation (1), it is assumed to make the further assumption that the sum of the seasonal components over a complete period is zero, ie ,

$$\sum_{j=1}^{s} S_{t+j} = 0. \tag{5}$$

Similarly, for Equations (2) and (3), it is also assumed to make further assumption is that the sum of the seasonal components over a complete period is *s*.

$$\sum_{j=1}^{s} S_{t+j} = s. \tag{6}$$

In this study, the use of Buys-Ballot table for estimation of trend parameters and seasonal indices in the presence of missing data using the methods of Iwueze and Nwogu [9] will be shown. The missing values will be estimated using the decomposition methods of Iwueze et.al [5] and entire process of estimation will repeated in the absence of missing data. The effect of Buys-Ballot estimation of exponential trend parameters and seasonal indices in the presence and absences of missing data will be determined.

2 Methodology

The method adopted in this article is the Buys-Ballot procedure. For details of Buys-Ballot procedure, see the works of Iwueze *et.al* [5], Nwogu *et.al* [10], Dozie *et.al* [11], Dozie and Ihekuna [12], Dozie and Ijomah [13], Dozie and Nwanya [14], Dozie [15], Dozie and Ibebuogu [16], Dozie and Uwaezuoke [17], Dozie and Ihekuna [18], Dozie and Ibebuogu [19], Akpanta and Iwueze [18]. For a series with a complete observation, the appropriate Buys-Ballot table is given in Table 1 when observations are complete.

Table 1: Buys - Ballot Table when observations are complete

Rows/ Period (i)	Columns (season) j								
	1	2	$\cdots$	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	$\cdots$	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	$\cdots$	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	$\cdots$	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$\cdots$	$X_{(i-1)s+j}$	$\cdots$	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	$\cdots$	$X_{(m-1)s+j}$	$\cdots$	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	$\cdots$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\cdots$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	$\cdots$	$\hat{\sigma}_{.j}$	$\cdots$	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

In this arrangement each time period *t* is represented in terms of the period *i* (e.g. year) and season *j* (e.g. month of the year), as $t = (i-1)s + j$. Thus, the period (row), season (column) and overall totals, means and variances are defined as

$$T_{i.} = \sum_{j=1}^{s} X_{(i-1)s+j}, \qquad \bar{X}_{i.} = \frac{T_{i.}}{s}, \qquad \hat{\sigma}_{i.}^2 = \frac{1}{s-1}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{i.}\right)^2$$

$$T_{.j} = \sum_{i=1}^{m} X_{(i-1)s+j}, \qquad \bar{X}_{.j} = \frac{T_{.j}}{m}, \qquad \hat{\sigma}_{.j}^{2} = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij} - \bar{X}_{.j}\right)^{2}$$

$$T_{..} = \sum_{i=1}^{m}\sum_{j=1}^{s} X_{(i-1)+j}\ , \qquad \bar{X}_{..} = \frac{T_{..}}{n}, n = ms$$

For a series with exponential trend, the row, column and overall means obtained by Iwueze and Nwogu [9] when the observations are complele are given in Table 2

Table 2: Summary of row, column and overall means of a series when trend cycle is exponential: ($M_t = be^{ct}$)

Measure	Exponential trend-cycle component: $M_t = be^{ct}, \quad t = 1, 2, \ldots n = ms$
$\bar{X}_{i.}$	$\frac{b}{s}\left(\frac{e^{(1-s)} - e^{c}}{1-e^{c}}\right)e^{csi}$
$\bar{X}_{.j}$	$\frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right)e^{cj} + s_j$
$\bar{X}_{..}$	$\frac{be^{c}}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)$

Source: Iweze and Nwogu (2014)

where $\hat{X}_{ij} = \hat{b}\ell^{\hat{c}[(i-1)s+j]} + \hat{S}_j + \hat{e}_{ij}$ (7)

Using the row, column and overall means, they gave the following expressions for intercept, slop and seasonal indices

Table 3: Estimates of exponential of exponential trend-cycle and seasonal component

Parameter	Additive Model
b	$\hat{b}\,s\left(\frac{1-e^{-\hat{c}}}{1-e^{-\hat{c}s}}\right)$

c	$\dfrac{\hat{c}}{s}$
S_j	$\bar{X}_{.j}$ - $\dfrac{\hat{b}}{m}\left(\dfrac{1-e^{\hat{c}s}}{1-e^{\hat{c}}}\right)e^{\hat{c}j}$

Source: Iwueze and Nwogu (2014)

The corresponding estimates of parameters of exponential trend-cycle components and seasonal indices obtained by 1wueze and Nwogu [9] are given as

$$\hat{b} = \quad \hat{b}s\left(\frac{1-e^{-\hat{c}}}{1-e^{-\hat{c}s}}\right) \tag{8}$$

$$\hat{c} = \quad \frac{\hat{c}}{s} \tag{9}$$

$$\hat{s}_j = \quad \bar{X}_{.j} - \frac{\hat{b}}{m}\left(\frac{1-e^{\hat{c}s}}{1-e^{\hat{c}}}\right)e^{\hat{c}j} \tag{10}$$

Where b and c are the intercept and slope based on the row, column and means for a complete observation. In order to derive estimates of parameters of exponential trend-cycle components and seasonal indices based on complete observations, the decomposition method by Iwueze et.al [5] was applied to derive estimates of missing observations. According to them, the estimate of the missing observation in the i th row and j th column of the Buys-Ballot table is given by:

$$\hat{X}_{ij} = \hat{b}\ell^{\hat{c}[(i-1)s+j]} + \hat{S}_j \tag{11}$$

Where $\hat{b}$ and $\hat{c}$ are estimates of the intercept and slope of the exponential trend derived from the available data and $\hat{s}_j$ is the estimate of the corresponding seasonal index.

2.1 Choice of Appropriate Transformation

For a series arranged in Buys-Ballot table Akpanta and Iwueze [20] proposed that, the slope of the regression equation of log of group standard deviation on log of group mean as stated in equation (12) is what is needed for choice of appropriate transformation. Some of the values of slope β and their implied transformation are stated in Table 4

$$\log_e \hat{\sigma}_i = a + \beta \log_e \hat{X}_{i.} \tag{12}$$

Table 4: Bartlett's Transformation for Some Values of β

S/No	**1**	**2**	**3**	**4**	**5**	**6**	**7**
β	**0**	$\frac{1}{2}$	**1**	$\frac{3}{2}$	**2**	**3**	**-1**
Transformation	**No transformation**	$\sqrt{X_t}$	$\log_e X_i$	$\frac{1}{\sqrt{X_t}}$	$\frac{1}{X_t}$	$\frac{1}{X_t^2}$	X_t^2

The method of Akpanta and Iwueze [20] is applied in selecting the appropriate transformation, the natural logarithm of standard deviation will be used to regress against the natural logarithm of periodic means and the result of the β - value will determine the type of transformation.

3 Empirical Example

This section presents empirical example to illustrate the Buys-Ballot procedure discussed in chapter 2. One hundred and twenty (120) of reported cases of registered infant baptism at St Jude Church Amuzi, Mbaise in Imo State, Nigeria from January, 2012 to December, 2021 are considered in which these points have two (2) missing values. One hundred and eighteen (118) observed values are shown in Appendix A. The time plots of actual and transformed series with missing data are given in figure 1. The amplitude of figures 1 and 2 appear small in the first two months and appears to have increased in the later year indicating that the variance is not constant, suggesting that the data requires transformation to stabilize the variance. The natural logarithm of the periodic and standard deviation are given in Table 5. The data is transformed by using the natural logarithm of the one hundred and eighteen (118) observations by the method of Akpanta and Iwueze [20]

Table 5: Natural Logarithm of Periodic Averages and Standard Deviations With Missing Values

$\bar{X}_{i.}$	$Log_e \bar{X}_{i.}$	$\hat{\sigma}_{i.}$	$Log_e \hat{\sigma}_{i.}$
9.00	2.20	4.69	1.55
6.82	1.92	2.48	0.91
6.67	1.90	2.84	1.04
6.42	1.86	3.58	1.28
7.67	2.04	3.70	1.31
7.08	1.96	4.12	1.46
9.83	2.29	4.63	1.53
8.92	2.19	3.60	1.28
7.33	1.99	5.84	1.76
9.50	2.25	3.92	1.37

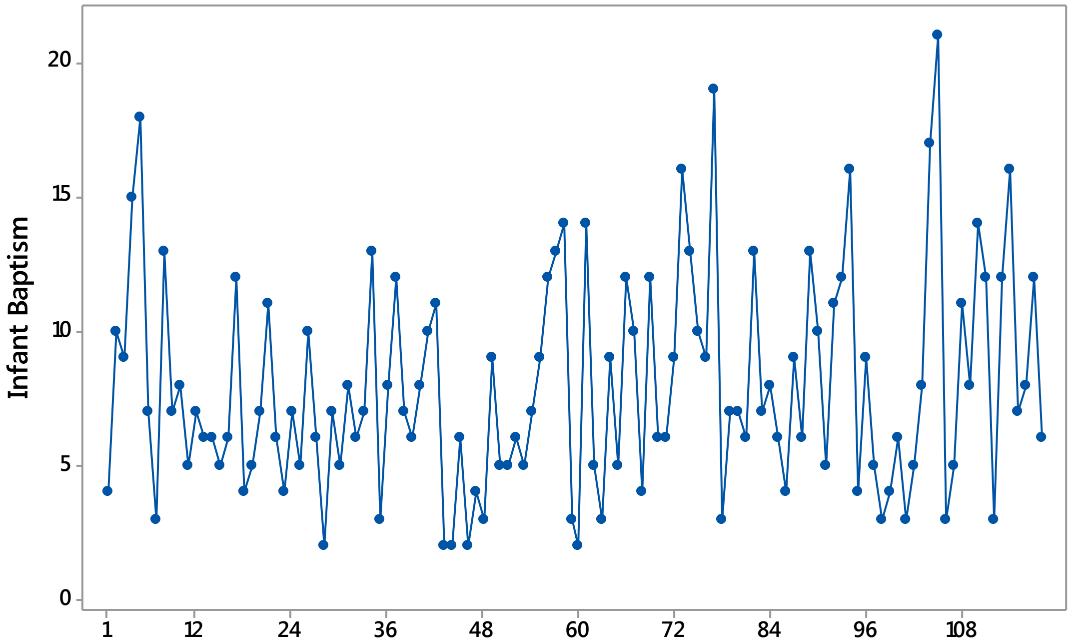

Fig 1: Original Time Series Data with Missing Data

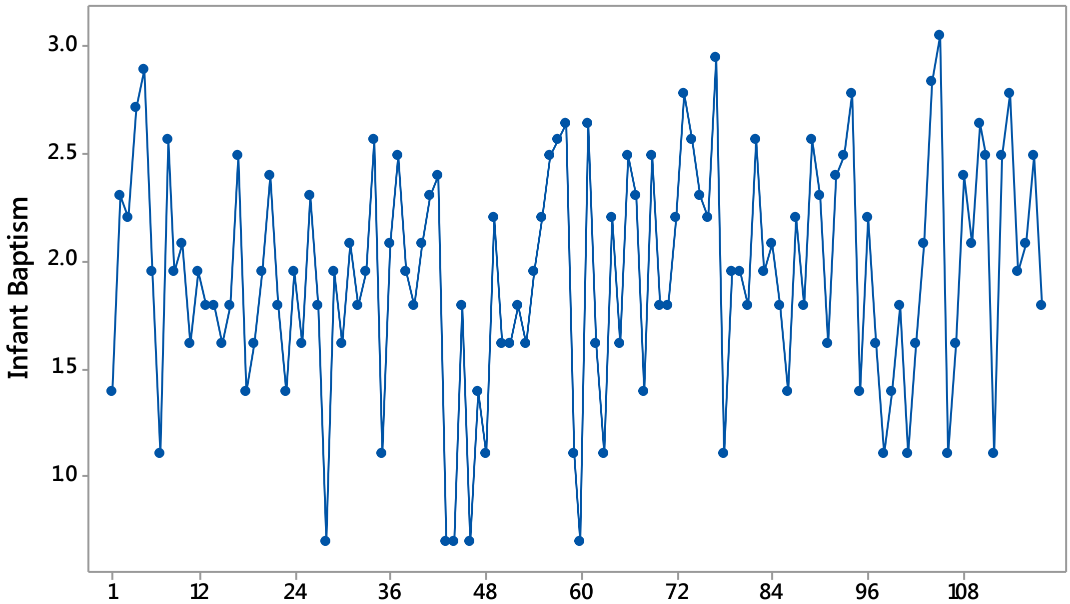

Fig 2: Transformed Time Series Data of Missing Data

The mean, standard deviation, natural logarithm of the mean and those of standard deviations are shown in Table 4.

3.1 Estimates of Exponential Trend-Cycle and Seasonal Effect With Missing Data

The exponential trend-cycle and seasonal indices for the transformed data with missing data are estimated using Decomposition Without the Missing Value (DWMV) method and the missing data are also estimated which are shown in Tables 6, 7 and 8 respectively.

Table 6: Estimates of Seasonal Indices With Missing Values

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$	$Adj\ \hat{S}_j$
1	1.575	-0.271	-0.269
2	1.821	-0.025	-0.024
3	1.943	0.097	0.098
4	1.865	0.019	0.020
5	1.835	-0.011	-0.010
6	1.730	-0.116	-0.114
7	1.955	0.109	0.111
8	1.814	-0.032	-0.031
9	1.823	-0.023	-0.021
10	1.858	0.012	0.013
11	2.059	0.213	0.214
12	1.858	0.012	0.013
$\sum_{j=1}^{12} \hat{S}_{.j}$		-0.016	0.000

Table 7: Estimates for Exponential Trend-Cycle and Seasonal Effect With Missing Data

Parameters	With missing values
$\hat{b}$	1.758
$\hat{c}$	1.001
$\hat{S}_1$	-0269
$\hat{S}_2$	-0.024
$\hat{S}_3$	0.098
$\hat{S}_4$	0.020
$\hat{S}_5$	0.010
$\hat{S}_6$	-0.114
$\hat{S}_7$	0.111

$\hat{S}_8$	-0.031
$\hat{S}_9$	-0.021
$\hat{S}_{10}$	0.013
$\hat{S}_{11}$	0.214
$\hat{S}_{12}$	0.013

The estimates of missing data with the transformed data and missing position are displayed in Table 8.

Table 8: Estimates of the Transformed and True Missing Data

Missing Position	$X_{2,1}$	$X_{1,7}$
Transformed Missing Data	7.954	5.097

3.2 Estimates of Exponential Trend-Cycle and Seasonal Effect Without Missing Data

The seasonal indices and exponential trend-cycle and seasonal indices for the transformed data without missing data are estimated using Decomposition Without Missing Value (DWMV) method are shown in Tables 9 and 10 respectively.

Table 9: Estimates of Seasonal Indices Without Missing Values

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$
1	2.213	0.289
2	1.821	-0.103
3	1.943	0.019
4	1.865	-0.059
5	1.835	-0.089
6	1.730	-0.194
7	2.269	0.345
8	1.814	-0.110
9	1.823	-0.101
10	1.858	-0.066
11	2.059	0.135

12	1.858	-0.066
$\sum_{j=1}^{12} \hat{S}_{.j}$		0.000

Table 10: Estimates of Exponential Trend-Cycle and Seasonal Effect Without Missing Data

Parameters	Without missing values
$\hat{b}$	1.755
$\hat{c}$	1.00
$\hat{S}_1$	0.289
$\hat{S}_2$	-0.103
$\hat{S}_3$	0.019
$\hat{S}_4$	0.059
$\hat{S}_5$	-0.089
$\hat{S}_6$	-0.194
$\hat{S}_7$	0.345
$\hat{S}_8$	-0.110
$\hat{S}_9$	-0.101
$\hat{S}_{10}$	-0.066
$\hat{S}_{11}$	0.135
$\hat{S}_{12}$	-0.066

Table 11: Difference in Exponential Trend-Cycle and Seasonal Indices With and Without Missing Data.

Parameters	With Missing Data	Without Missing Data	Difference
$\hat{b}$	1.758	1.755	0.003
$\hat{c}$	1.001	1.00	0.001
$\hat{S}_1$	-0.269	0.289	0.558
$\hat{S}_2$	-0.024	-0.103	0.079
$\hat{S}_3$	0.098	0.019	0.079
$\hat{S}_4$	0.020	0.059	0.039
$\hat{S}_5$	0.010	-0.089	0.099
$\hat{S}_6$	-0.114	-0.194	0.080

$\hat{S}_7$	0.111	0.345	0.234
$\hat{S}_8$	-0.031	-0.110	0.079
$\hat{S}_9$	-0.021	-0.101	0.080
$\hat{S}_{10}$	0.013	-0.066	0.079
$\hat{S}_{11}$	0.214	0.135	0.079
$\hat{S}_{12}$	0.013	-0.066	0.079

4 Summary, Conclusion and Recommendations

This article has discussed the effect on Buys-Ballot estimates of exponential trend-cycle and seasonal indices in time series decomposition. The method adopted is based on the row, column and overall means of the time series arranged in a Buys-Ballot table with m rows and s columns. The study is restricted to descriptive time series with only exponential trend and seasonal indices combined in the additive form. Only data missing at one point at a time is considered. This study provided a solution to the problem of missing observations not only one observation missing at a time but two or more observations missing consecutively.

The difference between the trend parameters With missing data and Without missing data are insignificant because they are approximately the same. While that of seasonal indices are significant at seasons 1 (one) and 7 (seven) of the Buys-Ballot table. Under acceptable assumption, the article shows that additive model satisfies $\left(\sum_{j=1}^{s} S_j = 0\right)$ as in (5)

The study has provided a basis for the estimation of missing values in descriptive time series with exponential trend and seasonality when only data missing at one point at a time. Other trending curves yet to be considered, are recommended for further investigation.

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Appendix A: Buys Ballot Table of the Actual Data on the number of Infant Baptism With Missing Data

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	4	10	9	15	18	7		3	13	7	8	5	99	9.00	4.69
2013		7	6	6	5	6	12	4	5	7	11	6	75	6.82	2.48
2014	4	7	5	10	6	2	7	5	8	6	7	13	80	6.67	2.84
2015	3	8	12	7	6	8	10	11	2	2	6	2	77	6.42	3.58
2016	4	3	9	5	5	6	5	7	9	12	13	14	92	7.67	3.70
2017	3	2	14	5	3	9	5	12	10	4	12	6	85	7.08	4.12
2018	6	9	16	13	10	9	19	3	7	7	6	13	118	9.83	4.63
2019	7	8	6	4	9	6	13	10	5	11	12	16	107	8.92	3.60
2020	4	9	5	3	4	6	3	5	8	17	21	3	88	7.33	5.84
2021	5	11	8	14	12	3	12	16	7	8	12	6	114	9.50	3.92
total	40	74	90	82	78	62	86	76	74	81	108	84	**935**		
$\bar{y}_{.j}$	4.44	7.40	9.00	8.20	7.80	6.20	9.56	7.60	7.40	8.10	10.80	8.40		7.92	
$\sigma_{.j}$	1.33	2.88	3.86	4.44	4.57	2.30	5.05	4.43	3.03	4.28	4.49	5.06			4.07

Appendix B: Buys-Ballot Table of Transformed Data on the number of Infant Baptism With Missing Data

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	1.537	2.042	1.976	2.315	2.450	1.828		1.406	2.215	1.828	1.905	1.647	26.245	2.187	0.967
2013		1.828	1.743	1.743	1.647	1.743	2.160	1.537	1.647	1.828	2.103	1.743	27.675	2.306	1.788
2014	1.537	1.828	1.647	2.042	1.743	1.240	1.828	1.647	1.905	1.743	1.828	2.215	21.202	1.767	0.246
2015	1.406	1.905	2.160	1.828	1.743	1.905	2.042	2.103	1.240	1.240	1.743	1.240	20.554	1.713	0.346
2016	1.537	1.406	1.976	1.647	1.647	1.743	1.647	1.828	1.976	2.160	2.215	2.266	22.048	1.837	0.280
2017	1.406	1.240	2.266	1.647	1.406	1.976	1.647	2.160	2.042	1.537	2.160	1.743	21.230	1.769	0.344
2018	1.743	1.976	2.362	2.215	2.042	1.976	2.491	1.406	1.828	1.828	1.743	2.215	23.824	1.985	0.303
2019	1.828	1.905	1.743	1.537	1.976	1.743	2.215	2.042	1.647	2.103	2.160	2.362	23.261	1.938	0.249
2020	1.537	1.976	1.647	1.406	1.537	1.743	1.406	1.647	1.905	2.407	2.570	1.406	21.186	1.765	0.387
2021	1.647	2.103	1.905	2.266	2.160	1.406	2.160	2.362	1.828	1.905	2.160	1.743	23.647	1.971	0.281
total	**22.131**	**18.209**	**19.426**	**18.645**	**18.350**	**17.302**	**22.693**	**18.137**	**18.233**	**18.579**	**20.588**	**18.578**	**230.870**		
$\bar{y}_{.j}$	**2.213**	**1.821**	**1.943**	**1.865**	**1.835**	**1.730**	**2.269**	**1.814**	**1.823**	**1.858**	**2.059**	**1.858**		**1.924**	
$\sigma_{.j}$	**2.022**	**0.279**	**0.255**	**0.324**	**0.318**	**0.238**	**1.046**	**0.337**	**0.267**	**0.326**	**0.257**	**0.387**			0.698

Appendix C: Buys Ballot Table of the Actual data on the number of Infant Baptism Without Missing Data

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	4	10	9	15	18	7	10	3	13	7	8	5	99	9.00	4.69
2013	4	7	6	6	5	6	12	4	5	7	11	6	75	6.82	2.48
2014	4	7	5	10	6	2	7	5	8	6	7	13	80	6.67	2.84
2015	3	8	12	7	6	8	10	11	2	2	6	2	77	6.42	3.58
2016	4	3	9	5	5	6	5	7	9	12	13	14	92	7.67	3.70
2017	3	2	14	5	3	9	5	12	10	4	12	6	85	7.08	4.12
2018	6	9	16	13	10	9	19	3	7	7	6	13	118	9.83	4.63
2019	7	8	6	4	9	6	13	10	5	11	12	16	107	8.92	3.60
2020	4	9	5	3	4	6	3	5	8	17	21	3	88	7.33	5.84
2021	5	11	8	14	12	3	12	16	7	8	12	6	114	9.50	3.92
total	40	74	90	82	78	62	86	76	74	81	108	84	**935**		
$\bar{y}_{.j}$	4.44	7.40	9.00	8.20	7.80	6.20	9.56	7.60	7.40	8.10	10.80	8.40		7.92	
$\sigma_{.j}$	1.33	2.88	3.86	4.44	4.57	2.30	5.05	4.43	3.03	4.28	4.49	5.06			4.07

Appendix D: Buys-Ballot Table of Transformed Data on the number of Infant Baptism Without Missing Data

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	1.537	2.042	1.976	2.315	2.450	1.828	5.097	1.406	2.215	1.828	1.905	1.647	26.245	2.187	0.967
2013	7.954	1.828	1.743	1.743	1.647	1.743	2.160	1.537	1.647	1.828	2.103	1.743	27.675	2.306	1.788
2014	1.537	1.828	1.647	2.042	1.743	1.240	1.828	1.647	1.905	1.743	1.828	2.215	21.202	1.767	0.246
2015	1.406	1.905	2.160	1.828	1.743	1.905	2.042	2.103	1.240	1.240	1.743	1.240	20.554	1.713	0.346
2016	1.537	1.406	1.976	1.647	1.647	1.743	1.647	1.828	1.976	2.160	2.215	2.266	22.048	1.837	0.280
2017	1.406	1.240	2.266	1.647	1.406	1.976	1.647	2.160	2.042	1.537	2.160	1.743	21.230	1.769	0.344
2018	1.743	1.976	2.362	2.215	2.042	1.976	2.491	1.406	1.828	1.828	1.743	2.215	23.824	1.985	0.303
2019	1.828	1.905	1.743	1.537	1.976	1.743	2.215	2.042	1.647	2.103	2.160	2.362	23.261	1.938	0.249
2020	1.537	1.976	1.647	1.406	1.537	1.743	1.406	1.647	1.905	2.407	2.570	1.406	21.186	1.765	0.387
2021	1.647	2.103	1.905	2.266	2.160	1.406	2.160	2.362	1.828	1.905	2.160	1.743	23.647	1.971	0.281

total	**22.131**	**18.209**	**19.426**	**18.645**	**18.350**	**17.302**	**22.693**	**18.137**	**18.233**	**18.579**	**20.588**	**18.578**	**230.870**		
$\bar{y}_{.j}$	**2.213**	**1.821**	**1.943**	**1.865**	**1.835**	**1.730**	**2.269**	**1.814**	**1.823**	**1.858**	**2.059**	**1.858**		**1.924**	
$\sigma_{.j}$	**2.022**	**0.279**	**0.255**	**0.324**	**0.318**	**0.238**	**1.046**	**0.337**	**0.267**	**0.326**	**0.257**	**0.387**			0.698