

# A Review of Studies on the Seismic Vulnerability of Structures in China

## ABSTRACT

The performance-based seismic vulnerability analysis is the basis for evaluating the seismic reliability of structures, which is of great significance to the prediction of seismic damage and the establishment of seismic design codes based on the reliability theory. In this paper, a comprehensive review of the methods of seismic vulnerability analysis, the current research status, and the remaining problems in this field is presented. Firstly, the methods of structural seismic vulnerability analysis are categorized, and the advantages and disadvantages of each method are summarized, and the research progress in this field is discussed in detail; Finally, it is concluded that the research on the seismic vulnerability of structures in China has not yet produced research results of practical engineering significance. A set of more effective seismic vulnerability analysis theories should be established to directly guide the performance-based seismic design of structures by reasonably and accurately quantifying relevant factors such as ground shaking input parameters and performance level damage indicators.

*Keywords: structure ; seismic vulnerability analysis ; reliability.*

## 1. INTRODUCTION

Earthquakes are sudden-onset natural disasters with great randomness and uncertainty. In the past few decades, earthquakes have been occurring frequently around the world, and the damage caused by them has increased dramatically. Seismic vulnerability refers to the probability of various damage states occurring in the structure under the action of earthquakes of different intensities, which embodies the seismic performance of engineering structures in a probabilistic sense, describes the relationship between the intensity of ground shaking and the degree of damage to the structure from a macroscopic point of view, and has become a research hotspot in the field of earthquake engineering and structural engineering. The purpose of structural seismic vulnerability assessment is to calculate the damage probability of a structure by selecting a set of ground shaking records for a given structure type with selected damage criteria [1].

The analysis of seismic vulnerability of building structures is the basis for evaluating the seismic reliability of building structures, and the improvement of the theory of seismic vulnerability of building structures is of great significance to the seismic design of building structures, the prediction of seismic damages, the maintenance and reinforcement of building structures in the later stage and even the establishment of the seismic design code based on the theory of reliability. The paper summarizes various methods of analyzing the seismic vulnerability of structures, and discusses in detail the current research progress in this field at home and abroad.

## 2. BASIC THEORY OF SEISMIC VULNERABILITY ANALYSIS OF STRUCTURES

Seismic vulnerability refers to the probability of different damage states of structures and the specific damage state of structures under the action of different ground motion intensities (such as peak ground acceleration, spectral acceleration or spectral displacement). The conditional

probability of the structure can predict the probability of damage at all levels of the structure. Earthquake vulnerability analysis can be aimed at one or a type of building structure in a certain area, and can also be aimed at building groups in a certain area. The analysis results are usually expressed in terms of vulnerability probability matrix or vulnerability. It is represented by a damage function. It was originally used for seismic risk assessment of nuclear power plants, and later it was gradually developed and applied to earthquake damage prediction of various civil infrastructure and engineering structures. Vulnerability analysis is a probability-based structural seismic performance assessment method, the vulnerability analysis results are usually described by a vulnerability curve, and the vulnerability curve usually takes the ground motion intensity parameter as an independent variable, and on the basis of selecting the damage index and formulating the limit value of the damage state, the structural earthquake requirement reaches (Examples include: strain, curvature, interlayer displacement angle, displacement, and other indicators of damage) or The probability of exceeding a certain damage state limit is the dependent variable. The probability formula when the structure reaches a certain damage state under a certain earthquake intensity can be expressed as:

$$P_f = P[DI \geq C | IM] \quad (1-1)$$

In the formula, IM is the ground motion parameter; C is the structural capacity; DI is the damage index, corresponding to the structural requirements.

In seismic vulnerability analysis, the theoretical analysis method is a mature technology with a wide range of applications and is suitable for analyzing complex special structures. In this paper, the silo structure is a high-rise special structure, which is suitable to be analyzed by the theoretical seismic vulnerability analysis method and adopts the capacity to demand ratio model.

It has been mentioned that the expression for a structure to reach a certain damage state under a certain seismic intensity is shown in the above equation, and the relationship between the ground vibration parameters and the structural demand parameter (EDP) samples is satisfied as follows [2]:

$$EDP = \alpha(IM)^\beta \quad (1-2)$$

Assuming that the median value and the ground shaking parameters obey an exponential relationship then:

$$\hat{D} = \alpha(IM)^\beta \quad (1-3)$$

Taking the logarithm of both sides of the above equation:

$$\ln \hat{D} = a + b(IM) \quad (1-4)$$

The probability function D of the structural response is represented by a lognormal distribution function with the statistical parameter:

$$\lambda_d = \ln \hat{D} \quad (1-5)$$

$$\beta_d = \sqrt{\frac{1}{N-2} \sum_{i=1}^N (\ln D - \ln \hat{D})^2} \quad (1-6)$$

where  $\lambda_d$  is the logarithmic mean of D and  $\beta_d$  is the logarithmic standard deviation of D.

Define the probability function of the structural capacity parameter as C, assuming that it can also be represented by a lognormal distribution function and defined by  $\lambda_c$  and  $\beta_c$ .

In Eq. (1-4),  $a=\ln\alpha$  and  $b=\beta$ , where  $a$  and  $b$  are obtained by statistical regression of data from a large number of incremental dynamic analyses of the structure, then the values of  $\alpha$  and  $\beta$  are easily derived.

The probability that the structural response  $D$  reaches or exceeds a certain damage state limit  $C$  under different intensity of ground shaking forms a susceptibility curve. This probability can be expressed by the following equation:

$$P_f = P(C / D < 1) \rightarrow P_f = P(C - D < 0) \quad (1-7)$$

Let  $Z = C - D$ , since  $C, D$  are independent random variables, and they all obey the normal distribution, then  $Z = C - D$  also obey the normal distribution, and its average value  $\lambda_z = \lambda_c - \lambda_d$ , standard deviation  $\beta_z = (\beta_c^2 + \beta_d^2)^{1/2}$ .

The failure probability of a structure can be directly expressed by the probability that  $Z < 0$ , i.e.:

$$P_f = P(Z < 0) = \int_{-\infty}^0 f(Z) dZ = \int_{-\infty}^0 \frac{1}{\beta_z \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{Z - \lambda_z}{\beta_z}\right)^2\right] dZ \quad (1-8)$$

For ease of tabulation,  $N(\lambda_z, \beta_z)$  is reduced to the standard normal variable  $N(0, 1)$ . Let  $Z = \lambda_z + t\beta_z < 0$ , then  $t < -\lambda_z/\beta_z$ . The standard normalization transformation process is as follows:

$$P_f = P\left(t < -\frac{\lambda_z}{\beta_z}\right) = \int_{-\infty}^{-\lambda_z/\beta_z} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{t^2}{2}\right] dZ = \Phi\left(-\frac{\lambda_z}{\beta_z}\right) = \Phi\left(-\frac{\lambda_c - \lambda_d}{\sqrt{\beta_c^2 + \beta_d^2}}\right) \quad (1-9)$$

The probability of failure at a given stage is derived:

$$P_f = \Phi\left(-\frac{\ln(\hat{C}/\hat{D})}{\sqrt{\beta_c^2 + \beta_d^2}}\right) = \Phi\left(\frac{\ln(\hat{D}/\hat{C})}{\sqrt{\beta_c^2 + \beta_d^2}}\right) \quad (1-10)$$

### 3. STRUCTURAL SEISMIC VULNERABILITY ANALYSIS

#### METHODS

At present, the main methods applied to the seismic vulnerability analysis of structures are: historical seismic damage statistical method, combined empirical and theoretical method, fuzzy comparison method, structural calculation method and dynamic analysis method, etc [3].

#### 3.1 Empirical Statistical Analysis Method

The empirical statistical method is to analyze the main seismic damage states of the building structure based on the seismic design theory of the building structure according to the data of the past earthquake damage, and then summarize the commonalities and characteristics of each type of damage. Through the statistical regression of the weights and influencing parties of each influencing factor, the corresponding vulnerability research method is obtained [4]. This method mainly adopts mathematical and statistical methods to find out the main damage-causing factors and the corresponding influence weight coefficients from a large number of historical seismic data, so as to accurately judge and predict the weak links and damage probability of the structure under the action of earthquakes. Due to the seismic data of the structure and the empirical limitations of the engineering designer, the empirical and statistical analysis method has a large limitation [5].

### **3.2 Theoretical Analysis Method**

The seismic vulnerability analysis of the structure needs to obtain the relationship between the seismic response of the structure and the ground shaking intensity of the region, and then predict the degree of earthquake damage. This is usually done by two methods: one is statistical analysis based on earthquake damage reports, and the other is numerical simulation with finite elements. The theoretical analysis method requires the establishment of a reasonable mechanical model for the structural object under study, and the dynamic time course analysis of the numerical model based on the seismic design theory, so as to obtain the whole process of the response of the structure under the effect of earthquakes of different intensities [6].

## **4. STRUCTURAL SEISMIC SUSCEPTIBILITY CURVE**

The form of susceptibility curve to study the seismic susceptibility of structures originated in the early 1970s from the probabilistic seismic risk assessment of nuclear power plants, in the form of a probability distribution function to represent the seismic performance of mechanical and structural systems, assess the seismic reliability of the system and predict the probability of structural damage. The seismic vulnerability curve graphically represents the seismic hazard to predict the probability of damage to a certain type of building structure under earthquake-induced ground motion. The seismic susceptibility curve describes the continuity of the probability that the seismic response of a structure will exceed a certain limit state when subjected to different intensities of seismic excitation [7].

Three parameters are involved in obtaining the seismic susceptibility curve of a structure: the ground shaking demand parameter (EDP) of the structure, the ground shaking intensity parameter, and the ultimate state point of the structure. The damage state of the structure is closely related to the intensity of seismic action, and a reasonable selection of parameter indicators can take into account a series of stochastic factors such as the structure's own performance and ground shaking parameters, and predict the degree of damage by calculating the exceeding probability of the structure to reach a specific state [8].

The seismic susceptibility curves are mainly obtained by three methods, namely, the frequency statistics method of exceeding the damage state, the linear fitting method of the probabilistic demand model, and the curve fitting method of the capacity to demand ratio model. It is found that the frequency counting method beyond the damage state is prone to large systematic errors, while the linear fitting method of the direct regression probability demand model and the curve fitting method of the capacity-to-demand ratio model are more accurate in the calculation results. Since curve fitting can reduce the dispersion of regression, it is more reasonable to choose the curve fitting method of the capacity-to-demand ratio model to obtain the seismic vulnerability curves of structures [9].

## **5. SEISMIC VULNERABILITY ANALYSIS BASED ON THE INCREMENTAL DYNAMIC ANALYSIS (IDA) METHOD**

The incremental dynamic analysis (IDA) method, as an analytical method to evaluate the seismic

capacity of the structure when it collapses as a whole, can examine the seismic demand capacity and the overall collapse resistance of the structure under the action of earthquakes of different intensity levels, and realize the analysis of the entire response course of the structure from linear elasticity to elasto-plasticity and then to the collapse of the structure as a whole, which can then comprehensively evaluate the seismic performance of the structure, and is therefore in the nonlinear analysis of the structure under the action of earthquakes. Therefore, it is widely used in the nonlinear analysis of structures under earthquake [10].

The IDA method [11] is based on the input of one or more ground shaking records to the structural model, each of which is "amplified" to a different ground shaking intensity by a series of scaling coefficients, and then the structure is analyzed under the excitation of this set of "amplified" ground shaking records. Then the structure is analyzed under the excitation of this set of "amplified" ground shaking records, and a series of structural elastic-plastic seismic responses are obtained, and one or more relationship curves between the damage index DM and the ground shaking intensity index IM are generated, i.e., IDA curves. Finally, the overall seismic performance of the structure is evaluated by the disposition points on these IDA curves. The IDA method can select multiple seismic waves for analysis, which makes up for the single and discrete nature of the traditional dynamic time-course analysis. Meanwhile, the IDA method provides a data base for the seismic vulnerability analysis of structures.

## 6. FLOW OF SEISMIC VULNERABILITY ANALYSIS BASED ON IDA

- (1) Select multiple ground shaking records according to relevant principles and determine the ground shaking intensity parameters. The number of ground shocks is generally between 10-20.
- (2) Adjust each ground shock to peak ground acceleration (PGA).
- (3) Apply the amplitude-tuned ground shaking to the silo finite element model for IDA analysis respectively, and obtain the seismic demand response of the structure.
- (4) Select the appropriate damage index and classify the damage level, respectively, the ratio of seismic demand to each damage index, take the logarithm of these values with the corresponding seismic intensity, and then plot them in the coordinate system to obtain a series of scatter plots, and then fit the quadratic curve to these scatters to obtain the IDA curves of the capacity-demand ratio for each damage state [12].
- (5) Use the quadratic polynomial regression of the least squares method to obtain the regression coefficients a, b, c, according to the following equations (5-1, 5-2) to obtain the mean  $\lambda$  and the standard deviation  $\sigma$ , and use the equation (5-3) to calculate the probability of exceeding the damage in the limit state.

$$\lambda = a(\ln(PGA))^2 + b\ln(PGA) + c \quad (5-1)$$

$$\sigma = \sqrt{\frac{S_r}{n-2}} \quad (5-2)$$

$$P_f = P\left[\frac{S_d}{S_c} \geq 1\right] = 1 - \phi\left[\frac{\ln(1) - \lambda}{\sigma}\right] = \phi\left[\frac{\lambda}{\sigma}\right] \quad (5-3)$$

Where,  $a$ ,  $b$ ,  $c$  are the coefficients obtained from the regression statistical analysis;  $S_r$  is the residual variance sum of each discrete point for the regression curve;  $S_d$  and  $S_c$  are the structural demand parameter and the capacity parameter, respectively;  $n$  is the number of discrete points.

(6) According to the above calculation results, the exceeding probability of exceeding a specific damage state under different seismic intensities is obtained, and the seismic susceptibility curve of the silo structure is plotted and susceptibility analysis is carried out [11].

(7)

## 7. RESEARCH STATUS OF SEISMIC VULNERABILITY

### ANALYSIS

Compared with Europe, America, Japan and other countries, in the field of structural seismic vulnerability research in China started relatively late. In the early 1980s, Gao Xiaowang and others began the related research on the probability of failure and seismic damage prediction of the brick house structure, although at that time there was no clear definition of the concept of vulnerability, but it laid a certain theoretical foundation for the later scholars to learn about seismic vulnerability. Starting from 2010, related research on structural Beginning in 2010, relevant studies on seismic vulnerability of structures began to appear in large numbers in China [13]. Comprehensively discussing the research results on seismic vulnerability of structures in China in recent years, the following features can be summarized:

(1) The theory of structural seismic vulnerability analysis is applied to different types of monolithic structures [14]. Huang Dong et al. used the vulnerability analysis of high-rise frame structures to predict their damage probability; Li Zhian et al. carried out a study of structural seismic vulnerability of large-span tensile string girder structures beyond the probability and rare earthquakes [15]; Chen Cheng carried out a study of seismic vulnerability of space rigid frame structures on the basis of considering the ground shaking and structural uncertainty, and compared and analyzed the differences of vulnerability curves with different performance indicators [16]; steel-concrete composite structures have been studied for their structural seismic vulnerability, and the differences of vulnerability curves under different performance indicators have been analyzed. The steel-concrete composite structure is widely used in today's society due to its many advantages. Zhang and Hai et al. analyzed the seismic vulnerability curves of seven typical masonry structures using the IDA method by taking into account the site conditions, height-to-width ratios, and cross-wall area ratios of the masonry structures [17]. Tang Xu et al. obtained the damage probability of each limit state of the frame shear structure by establishing the incremental dynamic time course analysis under the frame shear wall base isolation structural model and analyzing the probability with the help of probabilistic statistical analysis, and the results showed that the base isolation structure of this type of base isolation structure showed good seismic performance [18].

(2) Considering the degradation of material properties under time and environment, the multi-age time-varying seismic vulnerability based on the full life cycle of the structure is investigated [19]. Zheng Shanlock et al. take a 15-story 3-span steel frame structure as an example, and establish the seismic susceptibility curves of the steel frame structure under different service times by establishing a multi-age principal structure based on the consideration of the existing steel corrosion pattern [20]; Shi Yibo et al. take a single-story Kevitt mesh shell as the research object,

and propose a damage model of the mesh shell structure under strong earthquakes based on the consideration of the accumulation of material damages and initial defects [13]; Xu Shanhua et al. Based on the accelerated corrosion test of steel, they proposed the change rule of steel strength, ductility and corrosion degree, and established the seismic susceptibility curve of steel frame structure under multi-age to evaluate the seismic performance of corroded steel frame [21]. The above research results not only provide a theoretical basis for the seismic risk assessment under the whole life cycle of the structure, but also play a positive role in the long-term operation and maintenance of the structure.

(3) According to the seismic vulnerability requirements of different types of structures, the traditional seismic vulnerability analysis theories applied to structures are gradually revised and improved. Yu et al. addressed the problem that only the uncertainty of ground shaking is considered in the traditional seismic vulnerability analysis of structures, while the structural uncertainty is ignored. Based on the reliability theory, a stochastic IDA method considering structural uncertainty is proposed, and a reinforced concrete frame structure is taken as an example to show the necessity of considering structural uncertainty in the analysis of seismic vulnerability of structures [22]; space mesh and shell structures often do not apply to the research results of the seismic vulnerability of ordinary structures due to the complexity of their own damage mechanism. For example, Nie Guibo et al. proposed a damage factor based on multiple characteristic responses of such structures under seismic action to more accurately quantify the graded performance level of the mesh shell structure, which greatly promoted the development of seismic susceptibility analysis from a single-parameter damage index to a multi-parameter damage index [23]; Du Yongfeng et al. compared the performance of foundation isolation bearing in the case of uneven settlement with that of non-uniform settlement for the special property of wetted loess commonly found in the western region, and studied the damage factor of the mesh shell structure in the case of uneven settlement and that of non-uniform settlement. Du Yongfeng et al. compared the seismic susceptibility of the foundation isolation bearing in the case of uneven settlement with that in the case of no settlement, which is of great practical significance for the future study of susceptibility of soft ground structures [24]; Zhou Changdong et al. studied the seismic susceptibility of a 240 m-high reinforced concrete chimney under multidimensional excitation by one-dimensional, two-dimensional, and three-dimensional ground shaking respectively [25]. The study shows that the damage probability of the structure is significantly increased after considering the multidimensional seismic effects, and that the seismic vulnerability of the structure under the IDA method should use 3D ground shaking inputs that can more comprehensively reflect the weak locations of the structure [26].

It can be seen that with the gradual development of seismic susceptibility studies in recent years, the importance of seismic susceptibility studies to the development of performance-based seismic design has been widely recognized by many experts and scholars [27]. At present, although some valuable research results have been achieved in the theory of seismic vulnerability, the research results on seismic vulnerability of structures in China lack systematicity, but they still play a positive role in promoting the application and development of vulnerability in the field of construction engineering in China.

## 8. CONCLUSION

With the gradual deepening of the seismic vulnerability of building structures, China has also made certain achievements in the theory of seismic vulnerability, but from the current point of view, the seismic vulnerability of building structures in China is relatively independent of each other, and has not yet been able to systematically form the practical significance of the engineering and the value of a wide range of application of the results of the research [28].

On the basis of the existing seismic vulnerability analysis theories, it is necessary to consider multiple uncertainties in vulnerability analysis and the transfer linkage effect between them, to quantify the input parameters of ground shaking, the damage index of performance level, and the probability distribution model of vulnerability, and to establish a set of more effective seismic vulnerability analysis theories, which can be used to directly guide the seismic design of building structures based on the performance, and it requires the unremitting efforts of experts and scholars. The establishment of a more effective seismic vulnerability analysis theory to directly guide the performance-based seismic design of building structures requires the unremitting efforts of experts and scholars [29].

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