

## Original Research Article

# Entropy of Faze Space of Physical Systems, Free and Bond Energy of Closed Physical Systems and their Relativity Properties

### Abstract:

In the article[1] we introduced the concept of entropy for such topological spaces that admit pseudo-convex coverings[1], and it was shown here that the class of such topological spaces is quite wide.

The present article introduces the concepts of free and bond energies of a closed system, shows the relative nature this energies and entropy of the phase space of a closed system.

Key words: Entropy, faze space, Minkowski space, Lorentz Transformations.

### Introduction

Let's say there is a closed system with a phase space  $X$  whose entropy is  $S = \frac{N}{M}$  [1], where  $N$  represents the number of elements in the minimal pseudo-convex open covering [1] of the topological space and  $M$  the number of orbits acting on the covering elements of the homeomorphisms preserving this covering. The concept of entropy for phase spaces of dynamical systems was used by us to describe the evolution of closed physical systems. By means of the concept of entropy, a random process was constructed that described the evolution of such systems in time [1,6,7]. Since the concept of entropy is the base in such constructions the behavior of entropy in different reference systems during these dynamic processes is interesting.

### Free and bond energy of closed systems, Lorentz transformations of systems, Relative properties of free and bond energies

Let's energy of closed system is  $E$ , call the quantity  $E_M = \frac{\sqrt{E}}{S} M$  the bond energy of the system, and the quantity  $E_S = \frac{\sqrt{E}}{M} S$  the free energy of the system. Obviously, that is  $E = E_M E_S$ , there is an inversely proportional relationship between the bond and free energies of the closed system.

In closed systems, entropy increases continuously at all moments of time, and the free energy of the system also increases. In such a system, time is projected into the change in the free energy of the system. Since every change in a closed system is the result of an increase in entropy, therefore, we can assume that the same that  $E_S = Ct$ , where  $t$  is time and  $C$  is a speed of increase of free energy of systems which measured in J/sec .

Lorentz transformation  $L: R_{n-1}^n \rightarrow R_{n-1}^n$  of Minkowski  $R_{n-1}^n$  space [2,4,5] is transformation which saves scalar product:  $\langle L(a), L(b) \rangle = \langle a, b \rangle$ .

Consider the 2- dimensional vector space of events  $R_1^2$ , the elements of this space are pairs  $(Ct, E_M)$  of bond and free energies of closed systems. If we fix any isotropic base [3] in this space, and we will assume that the coordinates of event  $(Ct, E_M)$  is given then in this basis, then square of thi pseudo-scalar product of each this event (vector) will be the product  $E_M Ct = E$ .

For isotropic base Lorentz transformations [2,4,5] have the matrixes:

$$\begin{pmatrix} a & 0 \\ 0 & \frac{1}{a} \end{pmatrix}$$

or

$$\begin{pmatrix} 0 & a \\ \frac{1}{a} & 0 \end{pmatrix}.$$

Event  $(Ct, E_M)$  by this transformations in event  $(Ct', E'_M)$ , where

$E'_M = aE_M + 0t$ ,  $t' = E_M 0 + \frac{1}{a}t$ . Indeed,  $Ct' = \frac{1}{a}Ct$  therefore  $E'_M Ct' = E_M C \frac{1}{a}t = E_M Ct = E$  for second matrix will be some.

$$E'_M Ct' = E_M C \frac{1}{a}t = E_M Ct = E.$$

Let's say now we have events in the system  $(Ct_1, E_M^1)$ ,  $(Ct_2, E_M^2)$ . This pair of events will have the form interval between these events[2], it is

$$s_{12} = \sqrt{(E_M^2 - E_M^1)(Ct_2 - Ct_1)}.$$

The interval between the corresponding two events  $(Ct'_1, E_M^1)$ ,  $(Ct'_2, E_M^2)$  will be in the countdown system

$$s'_{12} = \sqrt{(E_M^2 - E_M^1)(Ct'_2 - Ct'_1)}.$$

If we make Lorentz transformation of one isotropic base to second, we will get

$$E_M'^2 - E_M'^1 = aE_M^2 - aE_M^1$$

and

$$C(t_2' - t_1') = C\left(\frac{1}{a}t_2 - \frac{1}{a}t_1\right)$$

It follows from these last two equations

$$s_{12}' = \sqrt{a(E_M^2 - E_M^1) \frac{C}{a}(t_2 - t_1)} = s_{12}$$

It means in two different isotropic reference system the energy and interval between event pairs are invariants. Therefore, the equality of the intervals between the events also took place in our case.

Consider the quantity  $(E_M'^2 - E_M'^1)$  using Lorentz transformations, we will have:

$$E_M'^2 - E_M'^1 = aE_M^2 - aE_M^1 = a(E_M^2 - E_M^1)$$

It follows, that the measure of difference between bond energies in two events first isotropic reference system will be more than measure of difference between bond energies between two the respective events of second isotropic reference system if  $a < 1$ . If  $a > 1$ , it will be vice versa.

Consider new equation

$$C(t_2' - t_1') = C\left(\frac{1}{a}t_2 - \frac{1}{a}t_1\right),$$

From this equation follows, that, the measure of difference between free energies in two events for observer of first isotropic reference system will be less than measure of difference between free energies between two the respective events of second isotropic reference system if  $a < 1$ . If  $a > 1$ , it also will be vice versa.

Let new, consider orthonormal  $e_1, e_2$  base in Minkowski vector space  $R_1^2$ . Let  $K$  The reference system associated with this base, in such system for event  $(Ct, E_M)$  product  $E_M Ct$  It does not represent a scalar square of this event. In case of a orthonormal base we can consider second  $K'$  reference system obtained from the system  $K$  by moving along the axis corresponding to component  $E_M$  at  $V$  joule/s a constant speed. In the isotropic case, we cannot consider such a reference system. Such transformations of coordinates in orthonormal case is called Lorentz transformations [2,4,5] and has a form:

$$E'_M = \frac{E_M - Vt}{\sqrt{1 - \left(\frac{V}{C}\right)^2}},$$

$$t' = \frac{t - \frac{V}{C^2}E_M}{\sqrt{1 - \left(\frac{V}{C}\right)^2}}.$$

As in the case of the isotropic basis, the energy of the system in the orthonormal basis is the square of the length of the vector representing the given event. It is clear, that energy of system is invariant for Lorentz transformations, indeed

$$E_M'^2 - C^2 t'^2 = \frac{(E_M - Vt)^2 - C^2 \left(\frac{V}{C^2}E_M - t\right)^2}{1 - \left(\frac{V}{C}\right)^2} = \frac{E_M^2 - 2E_M Vt + V^2 t^2 - C^2 \frac{V^2}{C^4} E_M^2 + 2C^2 E_M \frac{V}{C^2} t - C^2 t^2}{1 - \left(\frac{V}{C}\right)^2} =$$

$$\frac{E_M^2 \left(1 - \left(\frac{V^2}{C^2}\right)\right) - C^2 t^2 \left(1 - \frac{V^2}{C^2}\right)}{1 - \left(\frac{V}{C}\right)^2} = E_M^2 - C^2 t^2$$

In orthonormal reference system the interval between two pair of event  $(Ct_1, E_M^1), (Ct_2, E_M^2)$  in system  $K$  is

$$s_{12} = \sqrt{(Ct_2 - Ct_1)^2 - (E_M^2 - E_M^1)^2}.$$

Interval is invariant for such Lorentz transformation, Indeed, if  $(Ct'_1, E_M'^1), (Ct'_2, E_M'^2)$  is pair of events in system  $K'$  then interval between this events is

If we use the Lorentz transformation we will have:

$$s'_{12} = \sqrt{(Ct'_2 - Ct'_1)^2 - (E_M'^2 - E_M'^1)^2},$$

$$C^2 (t_2 - t_1)^2 = \frac{C^2 (t'_2 - t'_1)^2 + 2V(t_2 - t_1)(E_M'^2 - E_M'^1) + \frac{V^2}{C^2} (E_M'^2 - E_M'^1)^2}{1 - \frac{V^2}{C^2}}$$

and

$$(E_M^2 - E_M^1)^2 = \frac{(E_M'^2 - E_M'^1)^2 + 2V(t'_2 - t'_1)(E_M'^2 - E_M'^1) + V^2(t'_2 - t'_1)^2}{1 - \frac{V^2}{C^2}}.$$

From this equations, finally we will get:

$$s_{12} = \sqrt{(Ct_2 - Ct_1)^2 - (E_M^2 - E_M^1)^2} = s'_{12} = \sqrt{(Ct'_2 - Ct'_1)^2 - (E_M'^2 - E_M'^1)^2}.$$

Consider the magnitude  $\sqrt{(E_M'^2 - E_M'^1)^2}$ , If we use the Lorentz transformation we will have:

$$\sqrt{(E_M'^2 - E_M'^1)^2} = E_M'^2 - E_M'^1 = \frac{E_M^2 + Vt'}{\sqrt{1 - \left(\frac{V}{C}\right)^2}} - \frac{E_M^1 + Vt'}{\sqrt{1 - \left(\frac{V}{C}\right)^2}} = \frac{E_M^2 - E_M^1}{\sqrt{1 - \left(\frac{V}{C}\right)^2}},$$

from this follows

$$l = E_M'^2 - E_M'^1 = l_0 \sqrt{1 - \left(\frac{V}{C}\right)^2},$$

where

$$l_0 = E_M^2 - E_M^1.$$

This means that in stationary system, the difference in bond energies of two events in a moving system is smaller than for the stationary system.

The free energy in stationary reference system has form  $E_s = Ct$ . If we take into account Lorentz transformations system, and the fact that the free energy in this moving

system will have the form:  $Ct' = \frac{Ct - C \frac{V}{C^2} E_M}{\sqrt{1 - \left(\frac{V}{C}\right)^2}}$ .

If we take into account, since in zero moment of time the reference systems  $K, K'$  are match and  $E_M = 0$ , after time  $t$  will be  $E_M$  we can write  $E_M = Vt$  That's why we will have:

$$Ct' = \frac{Ct - C \frac{V}{C^2} E_M}{\sqrt{1 - \left(\frac{V}{C}\right)^2}} = \frac{Ct - \frac{V}{C^2} VCt}{\sqrt{1 - \left(\frac{V}{C}\right)^2}} = Ct \sqrt{1 - \left(\frac{V}{C}\right)^2}$$

For the magnitude we have  $\sqrt{1 - \left(\frac{V}{C}\right)^2} \leq 1$ , therefore we can make conclusion: in stationary system the free energy grows more slowly than in moving system, hence the time also flows more slowly and entropy grows more slowly.

### Conclusions:

1. Based on the concept of entropy of the phase space of a closed physical system, we defined the concepts of free energy and bond energy of this system.
2. We represented free energy and bond energy as components of the vector-two-dimensional Minkowski space.
3. We showed: If components of events in  $R_1^2$  Minkowski space is free energy and bond energy of closed physical space:  $(E_s = Ct, E_M)$  and  $K, K'$  two reference systems, for which  $K$  stationary and  $K'$  moving with speed  $V$  joule/second, along axis appropriate to  $E_M$ , then we have:

$$a) s_{12} = \sqrt{(Ct_2 - Ct_1)^2 - (E_M^2 - E_M^1)^2} = s'_{12} = \sqrt{(Ct'_2 - Ct'_1)^2 - (E_M'^2 - E_M'^1)^2},$$

$$b) E_M^2 - E_M^1 = (E_M'^2 - E_M'^1) \sqrt{1 - \left(\frac{V}{C}\right)^2},$$

$$c) E'_s = Ct' = (E_s = Ct) \sqrt{1 - \left(\frac{V}{C}\right)^2}$$

The meaning of these equations is as follows:

Interval between pairs of free energy and bond of closed system is invariant for Lorentz transformation.

The different of projections of events  $(Ct_1, E_M^1), (Ct_2, E_M^2)$  on the axis  $E_M$  is more than The different of projections of events  $(Ct'_1, E_M'^1), (Ct'_2, E_M'^2)$  on the axis  $E'_M$

The free energy of closed system in stationary reference system  $K$  is more than in moving reference system  $K'$ . This means that Free energy and entropy in system  $K'$  Free energy and entropy increase slowly than in system  $K$ , it is the same as that time flows more slowly in a moving system  $K'$  than in a stationary system  $K$

4. The relevant dependencies are shown for that case when  $K, K'$  reference systems are isotropic and  $K'$  is obtained from  $K$  by the above mentioned matrices

### References:

1. M. Mumladze Entropy of Topological Space and Evolution of Phase Space of Dynamical Systems // International Journal of Management and Fuzzy Systems. Pages: 8-13 Published Online: Jun. 15, 2020. DOI: 10.11648/j.ijmfs.2020060.
2. Ivo Terek Couto, Alexandre Lymberopoulos(2021)Introduction to Lorentz Geometry: Curves and Surfaces CRC press, Tailor & Francis group, Carman & Hall book,p.340.
3. M. Mumladze (2023) Basics of Geometry, Gori, p. 129, (in Georgian).
4. V. Pauli (1947) Теория Относительности, Moscow, p. 300,(in Russian).
- 5, F.I.Fiodorov, (1979) Lorentz groups, Moscow, p.384 с, (in Russian).
- 6.Stochastic model of evolution of homotopy type of phase space of dynamical system, East European scientific Journal,Volume1. 4(68) 2021, DOI: 10,32618/ESSA.2782-1994. 2021.1.68.17,
7. Entropy of State of Quantum System and Dynamic of This System, [Open Access Library Journal](#) > Vol.9 No.8, August 2022