An optimal model and Production Planning in upholstery by linear integer programming

ABSTRACT

This research effort used an upholstery factory as a case study to apply the idea of integer programming for an ideal model. In order to allocate limited and readily available material resources to competing products (Bed (6x6ft), Bed (6x4ft), Wardrobe (6x6ft), Wardrobe (6x4ft), Side Drawer, and Shoe Rack) in the furniture factory with a view to maximizing profit, the Revised Simplex method of solving linear programming and the branch and bound method of solving integer programming models were used in this study. The analysis was carried out with the help of the LINDO software version 6.1 using data containing the availability of materials, sales volume for each of the product types, machine and time it takes to complete the manufacturing process of each product type, as well as profit per unit of the product as collected from the factory. The outcome indicated that 38 units of the Bed (6x6ft), 10 units of the Wardrobe (6x4ft), and 0 units of the other products needed to be manufactured to make a monthly profit of no more than ₦1711700.. From the result, it was observed that the Wardrobe (6x4ft) and Bed (6x6ft) contributed majorly to the profit, hence the recommendation that more of the Wardrobe (6x4ft) should be produced and sold for optimum profit.

Keywords : linear programming, integer programming, optimal model, Revised Simplex method, profit maximization,

1. INTRODUCTION

Every organization, business, or enterprise strives to be profitable since doing so ensures its ongoing existence and productivity. Concerns regarding the dissolution or liquidation of organizations have grown over time; this may be due to a lack of profitable growth that is both effective and sustainable.

Profit is the main driver of people entering the business world, thus businesses can only develop and expand when they are making money. In the labor market, a lot has been done to secure profit maximization in Nigeria, ranging from rigorous adherence to the mathematical or economic concepts of profit maximization to stringent enforcement of worker layoffs, which is an anti-people attitude. Even while it sounds wonderful, the latter has proven difficult for businesses and organizations to achieve, leading many to downsize their workforces or retrench employees as a quick fix.

Growing rates of organizational liquidation have slowed the nation's economic growth by increasing unemployment or underemployment. Our youth are experiencing high unemployment rates, social vices have risen, and young people are turning to illicit means of making a living. All of these are actually a result of businesses and organizations closing down that would have given these young people productive employment and contributed to the country's gross domestic product (GDP). This is a result of businesses making bad decisions.

The linear programming method is one of the ways to guarantee profit maximization that has been mathematically demonstrated. A mathematical method for choosing how to allocate a company's limited resources to best achieve a goal is known as linear programming (LP). Additionally, it is a mathematical strategy applied in Operation Research (OR) or Management Sciences to resolve certain issues like allocation, transportation, and assignment issues that allow for a choice or options amongst potential solutions. Yahya (2004).

In accordance with a predetermined standard for optimality, linear programming is "a mathematical technique useful for allocating scarce or limited resources to several competing activities." Sharma (2008).

There are differing views on whether linear programming techniques should be used in various management decision-making processes as a result of continual advancement in the application of these approaches to real-world business challenges over a lengthy period of time. Since it is simpler to find the best use of scarce resources in developing economies, linear programming is a crucial tool for resource allocation. A Soviet mathematician named Leonid Kantorovich, who introduced the concept of linear programming to the Soviet Union in 1939, developed strategies for dealing with complex linear programming issues, such as the method of using the dual problem's optimal solution to solve the primal problem. He created the now-famous mathematical method of linear programming in 1939, a few years before Dantzig did.(1947). He used LP to organize the varied activities of the U.S. Air Force related with the issue of supplies to the Force and to solve practical challenges. Because LP modeling works to achieve a single target of either maximizing (profit or contribution) or minimization, Charles et al. (1963) referred to it as a single objective optimization strategy. (cost or time). According to Gupta and Hira (2009), LP modeling can be used to optimize a linear function known as the objective function, which is dependent on an assembly of constraint functions. According to Dowing (1992), linear programming is far better to alternative optimization approaches like the Lagrangian method and Graphic method. Lagrangian approach. In agreement with Turban and Meredith (1991), Dwivedi (2008) claimed that one of the most well-known management science tools is linear programming. The majority of scholars Wagner (2007) and Lucey (2002) studied the idea that linear programming is a method used in operations research. They believe that it is one of the most commercially successful operations research applications, with Wagner (2007) stating that there is strong evidence to suggest that it has the greatest economic impact. Kareem and Aderoba (2008) used data from a cocoa processing plant in Akure, Ondo State, Nigeria, to demonstrate the efficiency of the linear programming model in maintenance and personnel planning. In order to determine the highest profit from the manufacture of soft drink at the Nigeria Bottling Company, Ilorin facility, Balogun et al. (2012) applied the linear programming method. By using the software linear interactive optimizer (LINDO) to maximize the profit of the Khadi and Village Industries Commission (KVIC) connected to servodaya Sangham, Murugan and Manivel (2009) attempted to illustrate the usage of linear programming technique. The concept of the revised simplex technique with sensitivity analysis was used to Kingmos Paints Nig. Ltd. by Ikpang et al. in 2021. In order to calculate the best profit for the well-known neighborhood bakery Shukura Bakery in Zaria, Kaduna State, Nigeria, Zakariyya et al. (2022) used the linear programming technique.

However, there is no assurance that an integer valued solution will be obtained while solving an LP model. A non-integer valued solution, for instance, will have no importance when determining how many beds and wardrobe would be needed to produce in order to maximize profit. It is not possible to arrive at the best solution by rounding off to the nearest integer. In these situations, integer programming is employed to guarantee that the decision variables have integer values.

Thus, the revised simplex method is a commonly used algorithm for solving linear programming problems, including profit maximization problems.

This work demonstrates the pragmatic use of linear programming methods in maximization of profit at the Mudiame Business Concept Enterprise.

The purpose of this study is to apply integer linear programming to the Mudiame commercial enterprise in order to maximize profits, as well as to determine the optimal linear model for the company's products and the product mix that will maximize profits for the firm.

2. Brief Background on The Case Study

Mudiame Business Concept Enterprise is a business-based organization with a number of production facilities that is housed within the Mudiame University neighborhood in Irrua, Esan Central, Edo State, Nigeria. It was created to conduct business operations involving the production of items, the processing of raw materials, and the delivery of sales and services within her many sections. The study focused only on the furniture factory out of the several industrial sectors that exist, including water production, bakery and pastry making, soap making, fashion and design, and furniture manufacturing. The sector was created in an effort to fill the gap left by the high, medium, and low classes' lack of access to basic necessities. Its manufacturing is primarily focused on providing the common people with the necessities. The furniture factory produces six different items: Side Drawer, Shoe Rack, Wardrobe (6x6ft x 4ft), Bed (6x6ft x 6ft), and Wardrobe (6x6ft x 4ft). As a result, choosing the number combinations of the products produced is a crucial and significant management choice.

3. Methodology/ Method Of Data Analysis

The financial statement that includes the total production for one month was the tool used for this study project. The Mudiame Business Concept Enterprise, Irrua, Edo State, total production and sales volumes for one month were used in the study.

The study's methodology is the conventional maximizing problem's revised simplex method. The regular simplex method, which is a modified version of the simplex method, may not be practical for really big issues because it requires a lot of memory space to store the simplex when using a computer. The procedure needs only;

- The net evaluation row Δ_j to determine the non-basic variable that enters the basis.
- The pivot column

- The current basis variables and their values (X_B column) to determine the minimum positive ratio and then identify the basis variable to leave the basis.

Revised Simplex Method is an efficient computational procedure for solving an LPP with less labour. In this method, we need not to compute all the value of X_B, $/Y_j and\ Z_j - C_j$ at each iteration. The core computation is rooted in the basis B and its inverse B^{-1} The inverse of the next basis is computed directly from the current basis without actually inverting the next basis. The current basis inverse and the original data of the problem are used to determine the entering and departing vectors.

Revised Simplex Method In Standard Forms

Standard form I: In this form, an identity matrix is assumed to be obtained after introduction of

In order to solve LP problem using revised simplex method, the objective function is also considered as one of the constraints equation in which value of Z can be made as large as possible and unrestricted in sign. Thus, the set of constraints can be written as:

$$Z - c_1x_1 - c_2x_2 - \cdots - c_nx_n - 0.x_{n+1} - 0.x_{n+2} - \cdots - 0.x_{n+m} = 0$$

$$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n + x_{n+1} = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n + x_{n+2} = b_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n + x_{n+m} = b_m \qquad . \quad . \quad . \qquad (1)$$

$$x_1, x_2 \cdots x_{n+m} \geq 0$$

In matrix notations, the system of equation (3) can be expressed as $Z - Cx = 0$

$$Ax = b$$

$$x \geq 0$$

And in the system of equation (1), there are $(m + 1)$ simultaneous linear equations in $(n + m + 1)$ variables $(Z, x_1, x_2, \dots, x_{n+m})$. The aim now is to solve equation (1) such that Z is as large as possible and unrestricted in sign, subject to the condition $x_1, x_2, \cdots, x_{n+m} \geq 0$. By rewriting equation (1) in more symmetric notations as follows, we get:

$$1.x_0 + a_{01}x_1 + a_{02}x_2 + \cdots + a_{0n}x_n + a_{0,n+1}x_{n+1} - a_{0,n+2}x_{n+2} - \cdots - a_{0,n+m}x_{n+m} = 0$$

$$0.x_0 + a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n + x_{n+1} = b_1$$

$$0.x_0 + a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n + x_{n+2} = b_2$$

$$0.x_0 + a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n + x_{n+m} = b_m$$

Where $Z = x_0$ and $-c_j = a_{0j}$ $(j = 1, 2, \dots, n + m)$. In matrix notations it may be written as

$$\begin{bmatrix} 1 & a_{01} & a_{02} & \cdots & a_{0n} & a_{0,n+1} & \cdots & a_{0,n+m} \\ 0 & a_{11} & a_{12} & \cdots & a_{1n} & 1 & \cdots & 0 \\ 0 & a_{21} & a_{22} & \cdots & a_{2n} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{n+m} \end{bmatrix} = \begin{bmatrix} 0 \\ b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Or $$\begin{bmatrix} 1 & a_0 \\ 0 & A \end{bmatrix} \begin{bmatrix} x_0 \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix} \quad . \quad . \quad . \qquad (2)$$

Where, $a_0 = (a_{01}, a_{02}, \dots, a_{0,n+m})$

3.1 Computational Procedure For Standard Form I

For the initial basis matrix in revised simplex method, the column $a_j^{(1)}$ which form the initial identity matrix I are used. Since simplex method always start with an initial basis (identity) matrix B of order m, therefore, for the revised simplex method, the inverse of the initial basis matrix can be written as:

$$B = \begin{bmatrix} 1 & c_B B^{-1} \\ 0 & B^{-1} \end{bmatrix} = \begin{bmatrix} 1 & c_B \\ 0 & I_m \end{bmatrix}; \qquad B = I_m = B^{-1} \quad . \quad . \quad . \qquad (3)$$

Furthermore, if columns of matrix A form an initial basis matrix of order m that corresponds to the slack or surplus variables, then $c_{Bi} = 0$ $(i = 1, 2, \dots, m)$. Thus equation (5) reduces to the form:

$$B_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & I_m \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} = I_{m+1}$$

This implies that the inverse of initial basis matrix B_1 will be I_{m+1} to start the revised procedure. The initial basis solution is given by:

$$B_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & I_m \end{bmatrix} \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

This solution is feasible because the last m *components* are non-negative, and the first component Z can have any sign.

After obtaining a feasible solution and the inverse $(= I_{m+1})$ of the initial basis matrix, B_1^{-1}, we need to improve the solution by using the revised simplex method. For this, we first calculate

$c_j - z_j$ for each column $a_1^{(1)}$ not in the basis B_1, by taking scalar product of the first row of B_1^{-1} with each $a_1^{(1)}$. The vector $a_k^{(1)}$ to enter the basis is determined by the criterion

$c_k - z_k = Max\ \{c_j - z_j; c_j - z_j > 0\}$, for all j

Since x_0 (= Z) is always desired in the basis, the first column B_1^{-1} (= $e^{(1)}$) of the initial basis matrix inverse $B_1^{-1} = I_{m+1}$ will never be removed from the basis at any iteration. The vector to be removed from the basis is determined by the criterion:

$\frac{x_{Br}}{y_{rk}} = mi \left\{\frac{x_{Bi}}{y_{ik}}, y_{ik} > 0\right\}$, for all i

Where y_{ik} (i = 1, 2, … , m) are the components of vector $y_k^{(1)}$, and $y_k^{(1)} = B_1^{-1} a_1^{(1)} = \begin{bmatrix} z_k - c_k \\ y_k \end{bmatrix}$

Since we start with an identity matrix B_1, the new inverse denoted by B_0^{-1} shall be obtained by multiplying the basis matrix inverse B_0^{-1} at the previous iteration by an elementary matrix E, where E is the inverse of an identity matrix with r_{th} column replaced y_k.

If there is a tie in the selection of the key column, then choose the column from left to right that is the smallest index j. A tie in selecting the outgoing vector can be broken arbitrarily.

Step 1: Define/Identify the original column vector (P_n) and the coefficient matrix C_n of the objective row.

Step 2: Calculate the new value of column vector (P_n') for the non-basic variables and the X_B which is the RHS column vector. $P_n' = B^{-1} * P_n$ where B^{-1} is the inverse of original column of basic variables and P_n is the original value of the column vector.

Step 3: Identify the new basic variable (if exists) by calculating the (C_n') value for non-basic variables. $C_n' = C_n - C_B * P_n'$ where Cn′ is the new value of coefficient of objective row; C_n is the original value of coefficient of objective function; C_B is the value of coefficient of Basic variable in the objective row. Any C_n' that has the smallest value is the New Entering Variable.

Step 4: Finding the exiting row using the (X_B') and the new value of the (P_n') by calculating the ratio of the corresponding elements. The one with the smallest positive value is the leaving variable.

Step 5: Repeat Step 2, 3, 4 until no other new basic variables can be identified.

Data Presentation, Analysis And Result Interpretation

The source of the information for this study was Mudiame Business Concept Enterprise in Irrua, Edo State. The information includes the maximum amount of raw materials that can be used for manufacturing, the profit contribution per piece of furniture produced, and the amount of each resource used to make one piece of furniture.

The table1 below contains the data as it was acquired.

Table 1; Data on raw material mix per item

Resources	Product						Total available resources
	x_1	x_2	x_3	x_4	x_5	x_6	
Labour (hr) (LB)	4.0	4.0	7.0	4.0	1.5	1.0	192
Machinery/Diesel (hr) (MC)	1.25	1.25	3.50	1.33	0.58	0.58	96
Wood (pcs) (WD)	2.50	2.00	6.00	2.50	0.25	0.25	240
Edge Tape (roll) (ET)	0.25	0.20	0.27	0.15	0.05	0.05	25
3x30 Screw Nails (pkt) (SN 1)	0.25	0.25	0	0	0.15	0.15	30
5x30 Screw Nails (pkt) (SN 2)	0.25	0.25	0	0	0	0	20
Bed Hook (pair) (BH)	1	1	0	0	0	0	74
3x16 Screw Nails (pkt) (SN 3)	0	0	0.3	0.2	0	0	18
Bracket Iron (pkt) (BI)	0	0	0.25	0.25	0.17	0.17	15
Wardrobe Handles (pcs) (WHD)	0	0	8	2	1	0	386
Wardrobe Hinges (pair) (WHG)	0	0	6	3	0	0	220
Drawer Runner (pair) (DR)	0	0	2	0	1	0	145
Back Cover (pcs) (BC)	0	0	2	1	0.25	0.25	52
Unit Profit	33400	28700	72500	44250	8000	8500	

Where x_1 : Bed (6x6ft),

x_2 : Bed (6x4ft),

x_3 : Wardrobe (6x6ft),

x_4 : Wardrobe (6x4ft),

x_5 : Side Drawer,

x_6 : Shoe Rack.

By taking into account the objective function as one of the constraints and, if necessary, adding the slack and excess variables to the inequalities in order to transform them into equalities, we state the provided issue in the revised simplex form. LINDO software will be used for the data analysis. (Version 6.1).

4.0 MODEL FORMULATION

The model is given as:

$$Maximize\ Z = \sum_{j=1}^{6} p_j\, x_j$$

$$Subject\ to\ = \sum_{j=1}^{6} a_{ij}\, x_j \leq b_i$$

$x_j \geq 0$, j =1,….,6, i = 1,2,…. 13

where Z is the objective function.

x_j represents the types of furniture to be produced.

p_j represents the profit contribution per furniture produced.

b_j represents the maximum values for the production constraints

The Linear Programming Model formulated is thus:

$$Maximize\ Z = 33400x_1 + 28700x_2 + 72500x_3 + 44250x_4 + 8000x_5 + 8500x_6$$

Subject to

$4.0x_1 + 4.0x_2 + 7.0x_3 + 4.0x_4 + 1.5x_5 + 1.0x_6 \leq 192$ (Labour Constraints)

$1.25x_1 + 1.25x_2 + 3.50x_3 + 1.33x_4 + 0.58x_5 + 0.58x_6 \leq 96$ (Machinery/Diesel Constraints)

$2.50x_1 + 2.00x_2 + 6.00x_3 + 2.50x_4 + 0.25x_5 + 0.25x_6 \leq 240$ (Wood Constraints)

$0.25x_1 + 0.20x_2 + 0.27x_3 + 0.15x_4 + 0.50x_5 + 0.50x_6 \leq 25$ (Edge Tape Constraints)

$0.25x_1 + 0.25x_2 + 0.15x_5 + 0.15x_6 \leq 30$ (3x30 Screw nail Constraints)

$0.25x_1 + 0.25x_2 \leq 20$ (5x30 Screw nail Constraints)

$x_1 + x_2 \leq 74$ (Bed Hook Constraints)

$0.3x_3 + 0.2x_4 \leq 18$ (3x16 Screw Nail Constraints)

$0.25x_3 + 0.25x_4 + 0.17x_5 + 0.17x_6 \leq 15$ (Bracket Iron Constraints)

$8x_3 + 2x_4 + x_5 \leq 386$ (Wardrobe Handles Constraints)

$6x_3 + 3x_4 \leq 220$ (Wardrobe Hinges Constraints)

$2x_3 + x_5 \leq 145$ (Drawer Runner Constraints)

$2x_3 + x_4 + 0.25x_5 + 0.25x_6 \leq 52$ (Back Cover Constraints)

$\forall\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ for all non-negative condition

4.1 Solution Of Lpp Using Revised Simplex Method

By introducing the slack variables in the objective functions above we have and putting the problem clearly in standard form , we have that:

$$Z - 33400x_1 - 28700x_2 - 72500x_3 - 44250x_4 - 8000x_5 - 8500x_6 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6 + 0s_7 + 0s_8 + 0s_9 + 0s_{10} + 0s_{11} + 0s_{12} + 0s_{13} = 0$$

$4.0x_1 + 4.0x_2 + 7.0x_3 + 4.0x_4 + 1.5x_5 + 1.0x_6 + s_1 = 192$

$1.25x_1 + 1.25x_2 + 3.50x_3 + 1.33x_4 + 0.58x_5 + 0.58x_6 + s_2 = 96$

$2.50x_1 + 2.00x_2 + 6.00x_3 + 2.50x_4 + 0.25x_5 + 0.25x_6 + s_3 = 240$

$0.25x_1 + 0.20x_2 + 0.27x_3 + 0.15x_4 + 0.50x_5 + 0.50x_6 + s_4 = 25$

$0.25x_1 + 0.25x_2 + 0.15x_5 + 0.15x_6 + s_5 = 30$

$0.25x_1 + 0.25x_2 + s_6 = 20$

$x_1 + x_2 + s_7 = 74$

$0.3x_3 + 0.2x_4 + s_8 = 18$

$0.25x_3 + 0.25x_4 + 0.17x_5 + 0.17x_6 + s_9 = 15$

$8x_3 + 2x_4 + x_5 + s_{10} = 386$

$6x_3 + 3x_4 + s_{11} = 220$

$2x_3 + x_5 + s_{12} = 145$

$2x_3 + x_4 + 0.25x_5 + 0.25x_6 + s_{13} = 52$

And $x_1, x_2, x_3, x_4, x_5, x_6, s_1, s_2, .., s_{13} \geq 0$

Step 1: Define/Identify the original column vector (P_n) and the coefficient matrix C_n of the objective row.

$$P_n =$$

$$\begin{bmatrix} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 & P_{10} & P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} & P_{17} & P_{18} & P_{19} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & s_1 & s_2 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} & s_{12} \\ 4.0 & 4.0 & 7.0 & 4.0 & 1.5 & 1.0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.25 & 1.25 & 3.50 & 1.33 & 0.58 & 0.58 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.50 & 2.00 & 6.00 & 2.50 & 0.25 & 0.25 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.20 & 0.27 & 0.15 & 0.50 & 0.50 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0 & 0.15 & 0.15 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0.25 & 0.17 & 0.17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 6 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0.25 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

C_n = [33400 28700 72500 44250 8000 8500]

Z = [33400 28700 72500 44250 8000 8500 0 0 0 0 0 0 0 0 0 0 0 0 0]

Step 2: Calculate the new value of column vector (P_n') for the non-basic variables and the X_B which is the RHS column vector. $P_n' = B^{-1} * P_n$ where B^{-1} is the inverse of original column of basic variables and P_n is the original value of the column vector.

$$B^{-1} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \qquad X_B = \begin{bmatrix} 192 \\ 96 \\ 240 \\ 25 \\ 30 \\ 20 \\ 74 \\ 18 \\ 15 \\ 386 \\ 220 \\ 145 \\ 52 \end{bmatrix}$$

Step 3: Identify the new basic variable (if exists) by calculating the (C_n') value for non-basic variables. $C_n' = C_n - C_B * P_n'$ where Cn′ is the new value of coefficient of objective row; C_n is the original value of coefficient of objective function; C_B is the value of coefficient of Basic variable in the objective row. Any C_n' that has the smallest value is the New Entering Variable.

C_n = [33400 28700 72500 44250 8000 8500]

$C_B\ for\ s_1, s_2, s_3, s_4, \ldots, s_{13}$ = [0 0 0 0 0 0 0 0 0 0 0 0 0]

That is, $C_1' = C_1 - C_B * P_1'$, $C_2' = C_2 - C_B * P_2'$, $C_3' = C_3 - C_B * P_3'$ and so on.

Step 4: Finding the exiting row using the (X_B') and the new value of the (P_n') by calculating the ratio of the corresponding elements. The one with the smallest positive value is the leaving variable.

$$X_B = \begin{bmatrix} 192 \\ 96 \\ 240 \\ 25 \\ 30 \\ 20 \\ 74 \\ 18 \\ 15 \\ 386 \\ 220 \\ 145 \\ 52 \end{bmatrix} = X_B' = B^{-1} * X_B$$

Step 5: Repeat Step 2, 3, 4 until no other new basic variables can be identified.

The LINDO software is employed in solving the problem and the underlying algorithm in LINDO is Revised Simplex Method.

4.2 RESULT FROM LINDO

Fig. 1: The report window in LINDO showing optimum solution

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

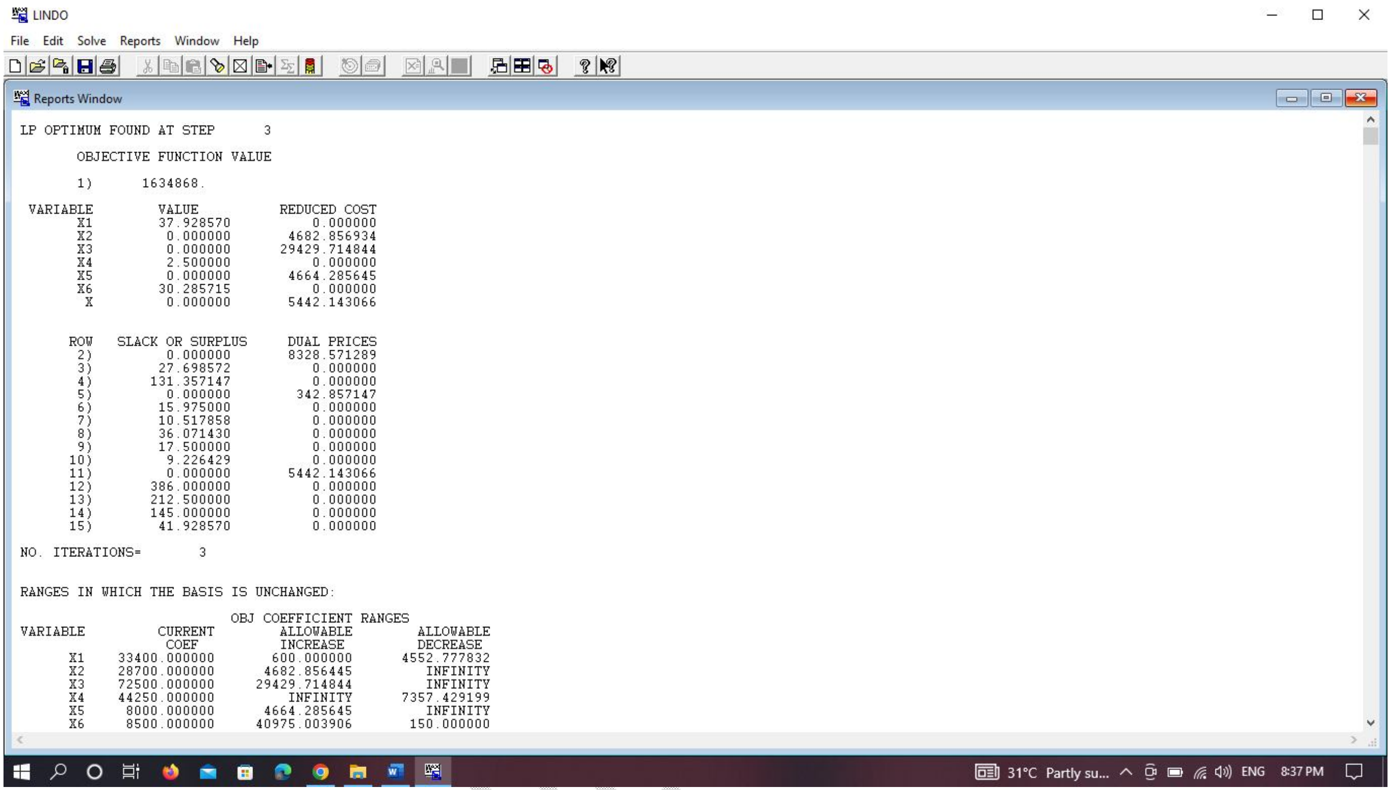
1) 1634868.

VARIABLE	VALUE	REDUCED COST
X1	37.928570	0.000000
X2	0.000000	4682.856934
X3	0.000000	29429.714844
X4	2.500000	0.000000
X5	0.000000	4664.285645
X6	30.285715	0.000000
X	0.000000	5442.143066

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	8328.571289
3)	27.698572	0.000000
4)	131.357147	0.000000
5)	0.000000	342.857147
6)	15.975000	0.000000
7)	10.517858	0.000000
8)	36.071430	0.000000
9)	17.500000	0.000000
10)	9.226429	0.000000
11)	0.000000	5442.143066

12)	386.000000	0.000000
13)	212.500000	0.000000
14)	145.000000	0.000000
15)	41.928570	0.000000

Fig. 1: The report window in LINDO showing optimum solution



```
LP OPTIMUM FOUND AT STEP      3

       OBJECTIVE FUNCTION VALUE

       1)      1634868.

 VARIABLE        VALUE          REDUCED COST
       X1        37.928570          0.000000
       X2         0.000000       4682.856934
       X3         0.000000      29429.714844
       X4         2.500000          0.000000
       X5         0.000000       4664.285645
       X6        30.285715          0.000000
        X         0.000000       5442.143066

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000       8328.571289
       3)        27.698572          0.000000
       4)       131.357147          0.000000
       5)         0.000000        342.857147
       6)        15.975000          0.000000
       7)        10.517858          0.000000
       8)        36.071430          0.000000
       9)        17.500000          0.000000
      10)         9.226429          0.000000
      11)         0.000000       5442.143066
      12)       386.000000          0.000000
      13)       212.500000          0.000000
      14)       145.000000          0.000000
      15)        41.928570          0.000000

NO. ITERATIONS=       3

RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
VARIABLE        CURRENT        ALLOWABLE        ALLOWABLE
                 COEF          INCREASE         DECREASE
      X1    33400.000000      600.000000      4552.777832
      X2    28700.000000     4682.856445         INFINITY
      X3    72500.000000    29429.714844         INFINITY
      X4    44250.000000        INFINITY      7357.429199
      X5     8000.000000     4664.285645         INFINITY
      X6     8500.000000    40975.003906       150.000000
```

Fig. 2: The report window in LINDO showing optimum solution continued

The solution shows that X1 = 37.928570 X4 = 2.500000 X6 = 30.285715
Since we cannot introduce or produce fraction of bed, integer programming will be introduced by applying branch and bound technique to have a new problem of

Maximize Z=33400x1+28700x2+72500x3+44250x4+8000x5+8500x6

Subject to

4.0x1+4.0x2+7.0x3+4.0x4+ 1.5x5+1.0x6 <192 (Labour Constraints)

1.25x1+1.25x2+3.50x3+1.33x4+ 0.58x5+0.58x6 <96 (Machinery/Diesel Constraints)

2.50x1+2.00x2+6.00x3+2.50x4+ 0.25x5+0.25x6 <240 (Wood Constraints)

0.25x1+0.20x2+0.27x3+0.15x4+ 0.50x5+0.50x6 <25 (Edge Tape Constraints)

0.25x1+0.25x2+ 0.15x5+0.15x6 <30 (3x30 Screw nail Constraints)

0.25x1+0.25x2 <20 (5x30 Screw nail Constraints)

x1+x2 <74 (Bed Hook Constraints)

0.3x3+0.2x4 <18 (3x16 Screw Nail Constraints)

0.25x3+0.25x4+ 0.17x5+0.17x6 <15 (Bracket Iron Constraints)

8x3+2x4+ x5 <386 (Wardrobe Handles Constraints)

6x3+3x4 <220 (Wardrobe Hinges Constraints)

2x3+x5 <145 (Drawer Runner Constraints)

2x3+x4+ 0.25x5+0.25x6 <52 (Back Cover Constraints)
X1 < = 37

Solving gives

LP OPTIMUM FOUND AT STEP 1

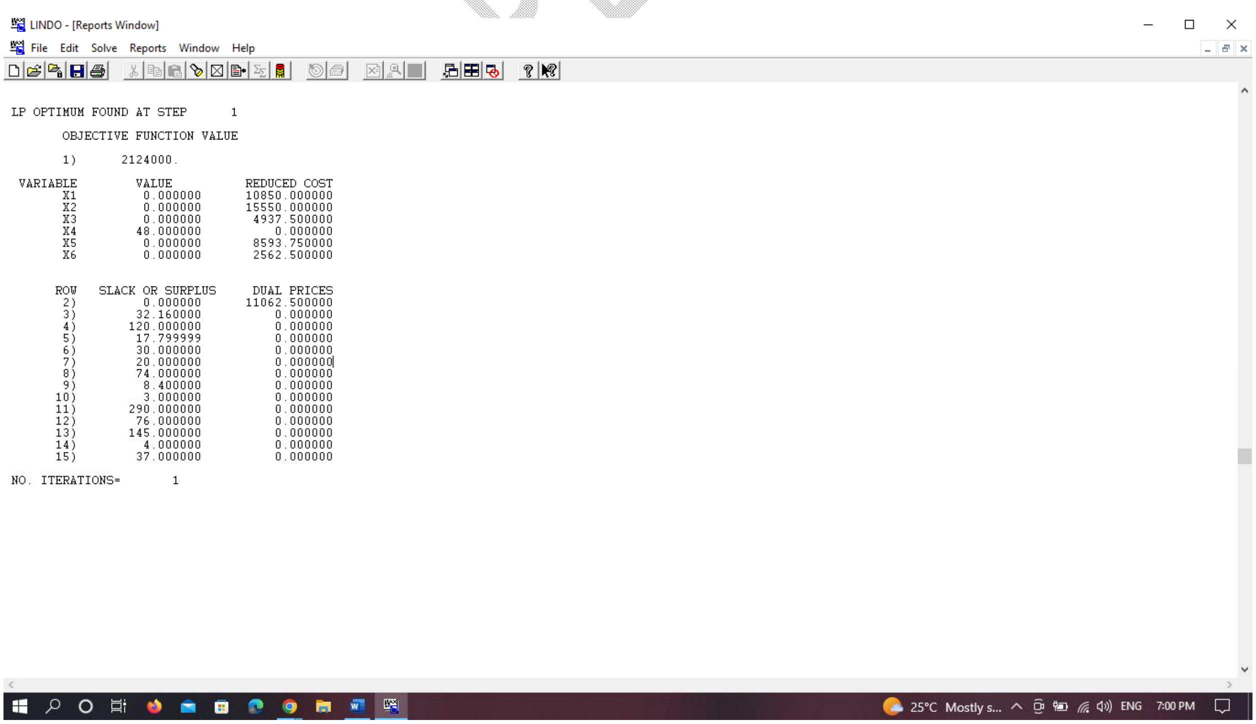
OBJECTIVE FUNCTION VALUE

1) 2124000.

VARIABLE	VALUE	REDUCED COST
X1	0.000000	10850.000000
X2	0.000000	15550.000000
X3	0.000000	4937.500000
X4	48.000000	0.000000
X5	0.000000	8593.750000
X6	0.000000	2562.500000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	11062.500000
3)	32.160000	0.000000
4)	120.000000	0.000000
5)	17.799999	0.000000
6)	30.000000	0.000000
7)	20.000000	0.000000
8)	74.000000	0.000000
9)	8.400000	0.000000
10)	3.000000	0.000000
11)	290.000000	0.000000
12)	76.000000	0.000000
13)	145.000000	0.000000
14)	4.000000	0.000000
15)	37.000000	0.000000

NO. ITERATIONS= 1



The solution shows that X4 = 48

So, we check for additional constraint x1 >38, and applying branch and bound technique that gives a new problem of

Maximize Z=33400x1+28700x2+72500x3+44250x4+8000x5+8500x6

Subject to

4.0x1+4.0x2+7.0x3+4.0x4+ 1.5x5+1.0x6 <192 (Labour Constraints)
1.25x1+1.25x2+3.50x3+1.33x4+ 0.58x5+0.58x6 <96 (Machinery/Diesel Constraints)
2.50x1+2.00x2+6.00x3+2.50x4+ 0.25x5+0.25x6 <240 (Wood Constraints)
0.25x1+0.20x2+0.27x3+0.15x4+ 0.50x5+0.50x6 <25 (Edge Tape Constraints)
0.25x1+0.25x2+ 0.15x5+0.15x6 <30 (3x30 Screw nail Constraints)
0.25x1+0.25x2 <20 (5x30 Screw nail Constraints)
x1+x2 <74 (Bed Hook Constraints)
0.3x3+0.2x4 <18 (3x16 Screw Nail Constraints)
0.25x3+0.25x4+ 0.17x5+0.17x6 <15 (Bracket Iron Constraints)
8x3+2x4+ x5 <386 (Wardrobe Handles Constraints)
6x3+3x4 <220 (Wardrobe Hinges Constraints)
2x3+x5 <145 (Drawer Runner Constraints)
2x3+x4+ 0.25x5+0.25x6 <52 (Back Cover Constraints)
X1 > 38

Solving gives

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 1711700.

VARIABLE	VALUE	REDUCED COST
X1	38.000000	0.000000
X2	0.000000	15550.000000
X3	0.000000	4937.500000
X4	10.000000	0.000000
X5	0.000000	8593.750000
X6	0.000000	2562.500000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	11062.500000
3)	35.200001	0.000000
4)	120.000000	0.000000
5)	14.000000	0.000000
6)	20.500000	0.000000
7)	10.500000	0.000000
8)	36.000000	0.000000
9)	16.000000	0.000000
10)	12.500000	0.000000
11)	366.000000	0.000000
12)	190.000000	0.000000
13)	145.000000	0.000000
14)	42.000000	0.000000

15) 0.000000 -10850.000000

NO. ITERATIONS= 1

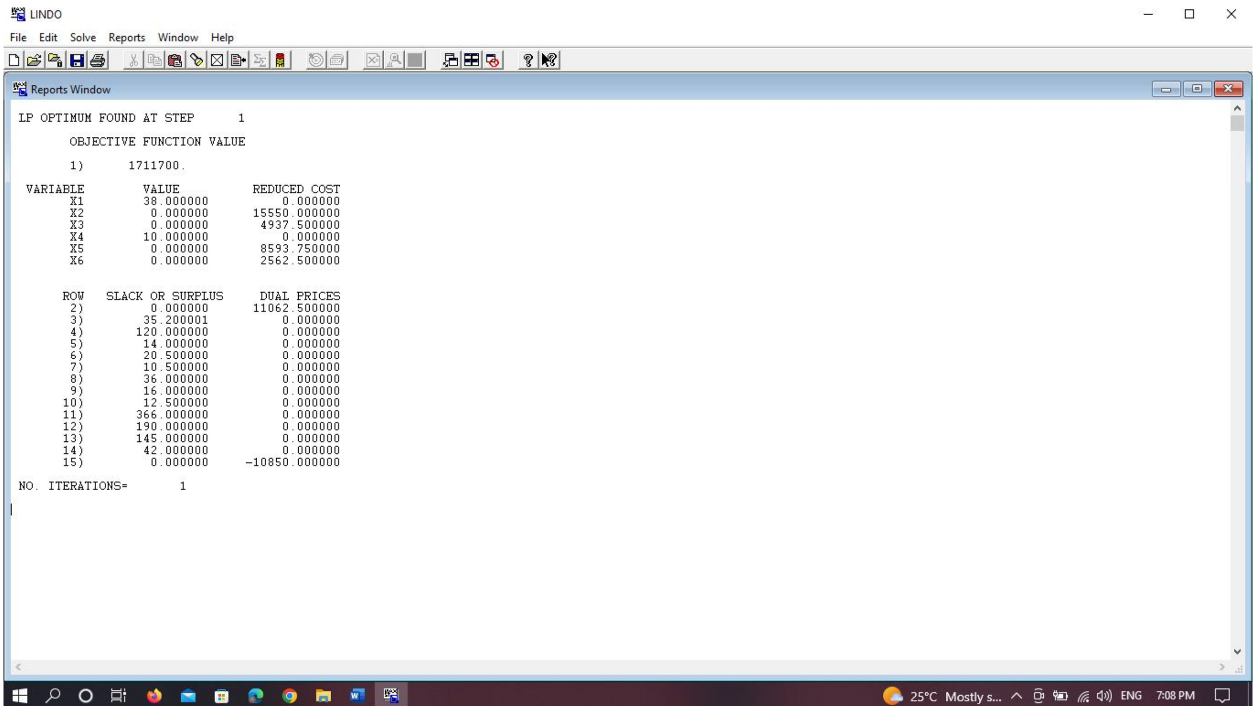


Fig.3: The report window in LINDO showing sensitivity analysis

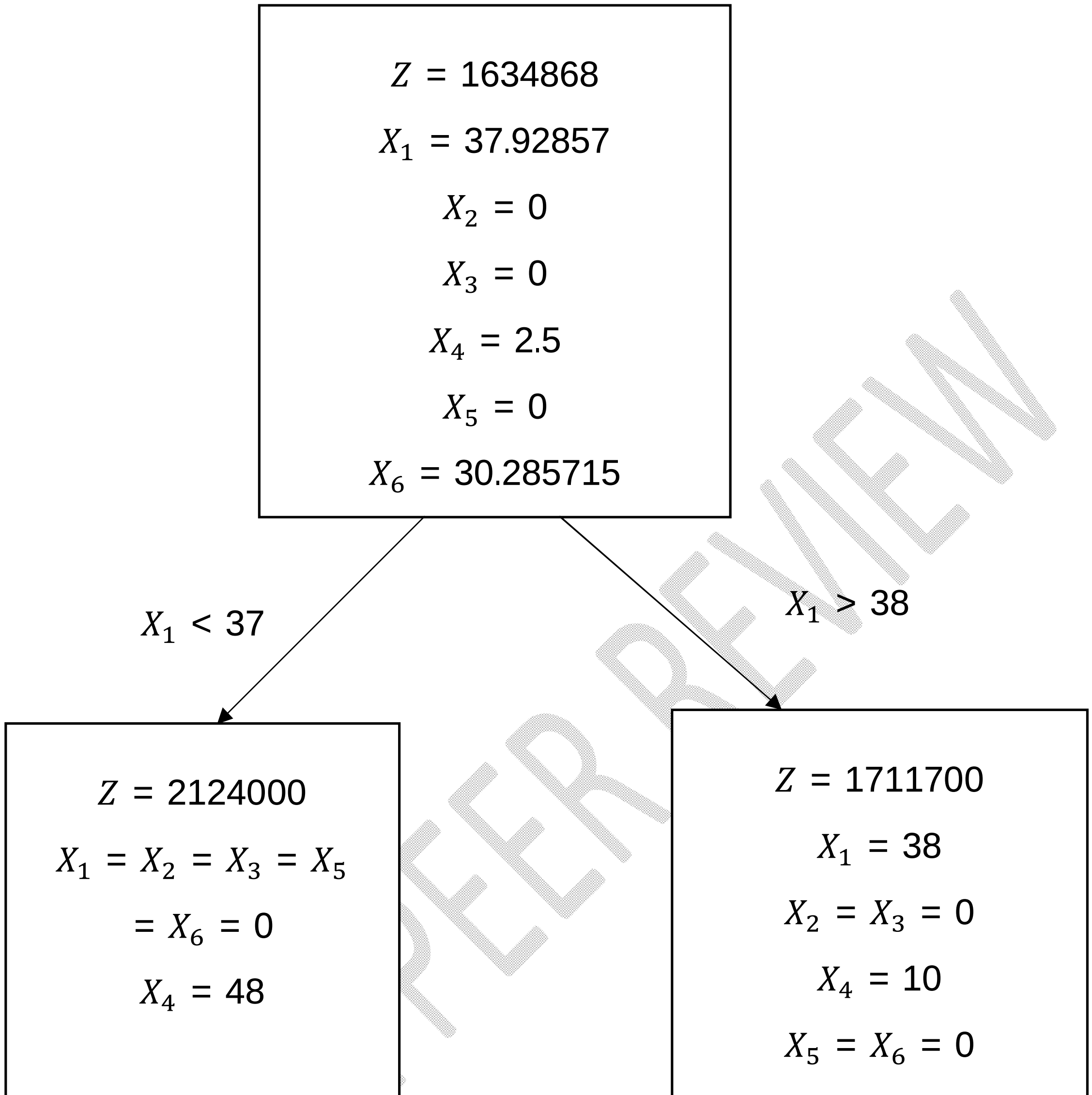


fig 5 integer results using branch and bound

4.3 RESULT INTERPRETATION

From Fig. 1 and 2, the optimum value is obtained after the third iteration, producing an optimum profit of N1634868.; and that 37.92857 x_1 : Bed (6x6ft), 2.5 (x_4) wardrobes (6x4ft), and 30.285715 x_6 : Shoe Rack contribute to the maximum profit of N1634868monthly.

Since, there are fractions in the results, the concept of branch and bound method of solving integer programming is introduced. This yield the results in figure 3 and 4 respectively.

The out come of the whole integer programming is represented in figure 5 which gave both optimal solution and fathomed solution.

Thus, fig.5 optimum solution shows that optimal solution of ₦ 1711700 will be reached only when the company produces only 38 pieces of x_1, $and\ 10\ pieces\ of\ x_4$ in the mix without producing $x_2 x_3, x_5\ and\ x_6$.

SUMMARY, CONCLUSION AND RECOMMENDATION

5.0 SUMMARY

The objective of this study was to apply linear programming in obtaining the product mix for the furniture production process of Mudiame Business Concept Enterprise. The decision variables in this research work are the six different types of furniture (bed (6x6ft), bed (6x4ft), wardrobe (6x6ft), wardrobe (6x4ft), side drawer, shoe rack) produced by the factory. The researcher focused mainly on thirteen resources (labour, machinery, wood, edge tape, screw nails, bed hook, bracket iron, wardrobe handles, wardrobe hinges, drawer runner, back cover) used in the production and the amount of resource required of each variable (furniture). The result shows that 38 units of the Bed (6x6ft), 10 units of Wardrobe (6x4ft), and 0 units for the rest of the products should be produced to yield a maximum profit of ₦ 1711700 monthly.

5.1 CONCLUSION

Based on the analysis carried out in this study and the results shown, Mudiame Business Concept Enterprise should produce the six different furniture in order to satisfy her customers. Also, more of the the Bed (6x6ft), and Wardrobe (6x4ft), should be produced in order to attain a maximum profit, because it contributes mostly to the profit earned by the company.

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