Original Research Article

Number of Digits in Fractional Part of Arithmetic Operations of Two Terminating Decimals

Abstract

By Defining the valid digits and the standard fraction form of a terminating decimal, this paper makes an investigation on the number of digits in the fractional part of addition, subtraction, and multiplication of two terminating decimals. It is proved that the number of the valid digits in the fractional part of addition or subtraction of two terminating decimals does not exceed the maximal number of the valid digits of the two operands while that of multiplication does not exceed the total of the two. Several theorems are proved in detail mathematical deductions and a potential application is exhibited in integer factorization. The results in the paper are helpful to understand the change of the number of digits in the related arithmetic operations.

Keywords:Terminating decimal; decimal fraction; fractional part; number of digits; arithmetic operation.

1 Introduction

Fractions and decimal numbers (or decimals) have been impressed in our minds ever since primary school time. We have many times exercised their operations of addition, subtraction, multiplication, and division; we also have known that a terminating decimal can be converted into a fraction. Everyone is surely able to do arithmetic calculations very well. For example, we can easily calculate the multiplication of two terminating decimals, $x = 1.23$ and $y = 4.56789$, to obtain the product $x \cdot y = 5.6185047$. Likewise, it is easy to multiply $u = 1.24$ with $v = 4.56785$ to obtain $u \cdot v = 5.6641340$. However, few have noticed and cared about the difference between the fractional parts of $x \cdot y$ and $u \cdot v$ in their forms of decimal fractions. Although the fractional parts of $x \cdot y$ and $u \cdot v$ have the same number of digits, 7 digits, their decimal fractions are $\frac{6185047}{10000000} = \frac{6185047}{10^7}$ and $\frac{6641340}{10000000} = \frac{664134}{10^6}$, respectively, showing that the rightmost digit of the fraction number $\frac{6185047}{10^7}$ is at the ten-millionths place while that of the fraction number $\frac{664134}{10^6}$ is at the millionths place. Note that the fractional parts of x and u contain the same number of digits, 2 digits, and those of y and v also contain the same number of digits, 5 digits; it is natural to raise a question: in what case do the fractional parts of the products $x \cdot y$ and $u \cdot v$ have the same number of digits? Related to this question, there is another torn question: if the number of digits in the fractional part of multiplication $x \cdot y$ is known, how are those with x and y? For example, let $x \cdot y = 29.3159144$; is there a way to determine the number of digits in the fractional parts of x and y. Looking through kinds of literatures I can access, I have found no answer to the questions nor a report related to the questions. Consequently, I have to do the investigation myself. This paper presents the answers to the questions.

The paper consists of five sections. Except for this introductory section, section 2 gives preliminaries including notations and definitions, section 3 shows the main results, section 4 proposes an application to integer factorization, and the last section presents the conclusions as well as expectations.

2 Preliminaries

This section presents necessary definitions and fundamental knowledge for later investigation. Subsections 2.1 and 2.2 demonstrate classical knowledge and notations, subsection 2.3 gives new concepts and notations.

2.1 Classical Knowledge

The classical knowledge of this paper is related to the decimal, terminating decimal and decimal fractions, mainly introduced in textbooks of primary schools and their aided-teaching materials or websites. As fundamental mathematical tools, the floor function and the fractional part function are also necessary for this paper to express the integer part, *i.e.,the whole number*, and the fractional part of a decimal. For reader's convenience, we here list some sources taken from certain professional books, school textbooks, and websites.

1. Description, concept, and definition of the decimal number and terminating decimal can be seen in books[1],[2],[3],[4], and web pages [5],[6],[7],[8].

2. Description, concept, and definition of the decimal fraction can be seen in books [2],[3],[4], and web pages [9],[10].

3. Equivalence and conversation of a terminating decimal to a decimal fraction can be seen in book [2] and web page [11].

4. The definition of trailing zeros can be seen on the web page [12].

5. The floor function, fractional part function and their properties are introduced in [13],[14], and [15].

2.2 Classical Symbols

In this whole paper, symbol $A \Rightarrow B$ means statement *A* can derive out statement *B*. Symbol $x_I.a_1a_2...a_n$ means a terminating decimal with x_I being the whole number and $0.a_1a_2...a_n$ being the fractional part. Symbols $\lfloor x \rfloor$ and $\lceil x \rceil$ are respectively the floor and ceil functions such that $x - 1 < \lfloor x \rfloor \leqslant x \leqslant \lceil x \rceil < x + 1$ or $x = \lfloor x \rfloor + \{x\} = \lceil x \rceil - 1 + \{x\}$, where $\{x\}$ is the fractional part of x.

2.3 New Definitions and Notations

Definition 1. The valid digits of a terminating decimal t mean the digits of t without trailing zeros. The valid digits of the fractional part of a terminating decimal are the digits after the decimal point and without the trailing zeros.

For example, the valid digits of 2.1400 are 2, 1, and 4 while those of the fractional part are 1 and 4, ignoring two digits 00.

For the purpose of this paper, use symbol $D_f(t)$ to denote the number of valid digits in the fractional part of t. Hence by Definition 1,

$$D_f(2.1400) = 2$$

It is sure that $D_f(t) = 0$ if t is an integer; otherwise, $D_f(t) > 0$. Now consider a real number x that is expressed by

$$x = \lfloor x \rfloor + \{x\} \tag{2.1}$$

If $\{x\}$ is a terminating decimal, it is equivalent to a decimal fraction and can be converted to the form of

$$\{x\} = \frac{d_x}{10^\alpha} \tag{2.2}$$

where $\alpha = D_f(\{x\})$ and $0 \leqslant d_x < 10^\alpha$ are integers.

By such means and letting $x_I = \lfloor x \rfloor$, x can be expressed in the form

$$x = x_I + \frac{d_x}{10^\alpha} \tag{2.3}$$

Note that, $\alpha = D_f(\{x\})$ and $\{x\} = \frac{d_x}{10^\alpha}$ imply $d_x \not\equiv 0(\mod 10)$ since otherwise a contradiction $\alpha \neq D_f(\{x\})$ is led to. For example, in terminating decimal 2.14 , $\{x\} = 0.14$ and $D_f(\{x\}) = 2$. If the restriction $D_f(\{x\}) = 2$ is ignored, $\frac{d_x}{10^\alpha}$ might take $\frac{14}{10^2}$, $\frac{140}{10^3}$ or an arbitrary equivalent decimal fraction of 0.14 while actually only $\frac{14}{10^2}$ is the perfect choice.

Definition 2. For a given terminating decimal x, under the restriction of (2.2), expression (2.3) is called the standard fraction form of x. In another word, expression (2.3) is the standard fraction form of x provided that integers x_I, d_x and α satisfy $x_I = \lfloor x \rfloor$, $\alpha = D_f(x) > 0$, $d_x \not\equiv 0(\mathrm{mod}10)$, and $0 \leqslant d_x < 10^\alpha$.

From now on, the terms decimal fraction and terminating decimal are used without distinction if they are of the standard fraction form unless specially mentioned.

3 Main Results

Let x and y be decimal fractions; then $x \pm y$ and $x \cdot y$ of course lead to decimal fractions. We obtain several results on the number of digits in the fractional part of $x \pm y$ and $x \cdot y$, as shown in the following subsections.

3.1 Fundamental Properties

Lemma 1. Let x be a terminating decimal with standard fraction form $x = x_I + \frac{d_x}{10^\alpha}$; then $D_d(x) = \alpha$.

Proof. The lemma is directly derived from (2) and what is said in Definition 2.

□

Lemma 2. Given an integer $\alpha > 0$ and a terminating decimal $x = 10^\alpha q + r$, where $q \geq 0$ is an integer and $0 < r < 10^\alpha$ is a terminating decimal; then $D_f(x) = D_f(r)$.

Proof. $x = 10^\alpha q + r \Rightarrow \frac{x}{10^\alpha} = q + \frac{r}{10^\alpha}$. By definition of the floor function and the fractional part function, $\{\frac{x}{10^\alpha}\} = \{\frac{r}{10^\alpha}\}$, which says $D_f(x) = D_f(r)$.

□

Lemma 3. Given an integer $\alpha \geqslant 0$; then the inequality $0 \leqslant D_f(\frac{x}{10^\alpha}) \leqslant \alpha$ holds for an arbitrary nonnegative integer x. If $x \not\equiv 0(\mathrm{mod}10)$, $D_f(\frac{x}{10^\alpha}) = \alpha$.

Proof. Consider two cases, $\alpha = 0$ and $\alpha > 0$. For the case $\alpha = 0$, x's being an integer yields $\frac{x}{10^\alpha} = x \Rightarrow D_f(x) = 0 = \alpha$, establishing the lemma.

The case $\alpha > 0$ is also considered by two cases, $x \equiv 0(\mathrm{mod}10^\alpha)$ and $x \not\equiv 0(\mathrm{mod}10^\alpha)$. The case $x \equiv 0(\mathrm{mod}10^\alpha)$ surely yields $D_f(\frac{x}{10^\alpha}) = 0$ for an arbitrary $\alpha > 0$.

For the case $x \not\equiv 0(\mathrm{mod}10^\alpha)$, let $x = 10^\alpha q + r$,where q and r are integers satisfying $q \geq 0$ and $0 < r < 10^\alpha$; then by Lemma 2, it holds

$$D_f(\frac{x}{10^\alpha}) = D_f(\frac{r}{10^\alpha})$$

This time, we still have to consider two cases. One is that $r \not\equiv 0(\bmod 10)$; the other is that, there exists an integer σ such that $0 < \sigma < \alpha$, $r \equiv 0(\bmod 10^{\sigma})$ and $r \not\equiv 0(\bmod 10^{\sigma+1})$. The first case obviously yields $D_f(\frac{r}{10^{\alpha}}) = \alpha$ by Lemma 1. The second case yields $r = 10^{\sigma}s$ with s being a positive integer and $s \not\equiv 0(\bmod 10)$. Hence $D_f(\frac{r}{10^{\alpha}}) = D_f(\frac{s}{10^{\alpha-\sigma}})$. Note that, $0 < r < 10^{\alpha}$ leads to $0 < s < 10^{\alpha-\sigma}$. By Lemma 1 this follows $D_f(\frac{s}{10^{\alpha-\sigma}}) = \alpha - \sigma < \alpha$. Accordingly, the inequality $0 \leqslant D_f(\frac{x}{10^{\alpha}}) \leqslant \alpha$ holds and $D_f(\frac{x}{10^{\alpha}}) = \alpha$ in the case $x \not\equiv 0(\bmod 10)$.
□

Lemma 4. Given an integer $\alpha \geq 0$; then $D_f(\frac{x}{2^{\alpha}}) = \alpha$ for an arbitrary positive odd integer x.
Proof. Consider two cases, $\alpha = 0$ and $\alpha > 0$. The case $\alpha = 0$ yields that $\frac{x}{2^{\alpha}}$ is an integer and $D_f(\frac{x}{2^{\alpha}}) = 0 = \alpha$. Next use mathematical induction on α for the case $\alpha > 0$. When $\alpha = 1$, by (P31) in [13] it holds $\frac{x}{2^{\alpha}} = \lfloor \frac{x}{2^{\alpha}} \rfloor + \frac{1}{2} \Rightarrow D_f(\frac{x}{2^{\alpha}}) = 1 = \alpha$. Assume $D_f(\frac{x}{2^k}) = k$ for positive integer k; it follows $\frac{x}{2^k} = x_I.a_1a_2...a_{k-1}5$, where $0 \leqslant a_i \leqslant 9$, $1 \leqslant i \leqslant k-1$ and $x_I = \lfloor \frac{x}{2^k} \rfloor$. Then

$$\frac{x}{2^{k+1}} = \frac{x_I}{2} + \frac{0.a_1a_2...a_{k-1}5}{2} = \frac{x_I}{2} + \frac{0.a_1a_2...a_{k-1}}{2} + 0.\underbrace{0...0}_{k-1}25$$

This time, if x_I is even, it follows

$$\frac{x}{2^{k+1}} = \frac{x_I}{2} + \frac{0.a_1a_2...a_{k-1}}{2} + 0.\underbrace{0...0}_{k-1}25 \Rightarrow D_f(\frac{0.a_1a_2...a_{k-1}}{2} + 0.\underbrace{0...0}_{k-1}25) = k+1$$

and x_I being odd yields

$$\frac{x}{2^{k+1}} = \frac{x_I}{2} + \frac{0.a_1a_2...a_{k-1}}{2} + 0.\underbrace{0...0}_{k-1}25 \Rightarrow D_f(0.5 + \frac{0.a_1a_2...a_{k-1}}{2} + 0.\underbrace{0...0}_{k-1}25) = k+1$$

Consequently, $D_f(\frac{x}{2^{k+1}}) = k+1$ and the proof is established.
□

Lemma 5. Let $x = x_I + \frac{d_x}{2^{\alpha}}$ be a terminating decimal, where $0 \leqslant x_I = \lfloor x \rfloor$, $\alpha > 0$ are integers and $1 \leqslant d_x < 2^{\alpha}$ is an odd integer ; then $D_f(x) = \alpha$.
Proof. Converting x to the standard fraction form leads to

$$x = x_I + \frac{5^{\alpha}d_x}{10^{\alpha}}$$

Since d_x is odd, $5^{\alpha}d_x$ is also odd, showing $5^{\alpha}d_x \not\equiv 0(\bmod 10)$. Meanwhile $1 \leqslant d_x < 2^{\alpha}$ yields $5^{\alpha} \leqslant 5^{\alpha}d_x < 10^{\alpha}$. By Lemmas 1 and 3, it holds that $D_f(x) = \alpha$.
□

3.2 Arithmetic Operational Properties

Theorem 1. Let $x = x_I + \frac{d_x}{10^\alpha}$ and $y = y_I + \frac{d_y}{10^\beta}$ be standard fraction forms of two decimal fractions; then

$$D_f(x \pm y) \leqslant \max(\alpha, \beta)$$

Proof. Without loss of generality , consider two cases, $\alpha > \beta$ and $\alpha = \beta$. Since $\frac{d_x}{10^\alpha} = 0.a_1a_2...a_\alpha$ and $\frac{d_y}{10^\beta} = 0.b_1b_2...b_\beta$, it follows

$$x \pm y = x_I \pm y_I + \frac{d_x}{10^\alpha} \pm \frac{d_y}{10^\beta} = x_I \pm y_I + 0.a_1a_2...a_\alpha \pm 0.b_1b_2...b_\beta$$

Let $S = \frac{d_x}{10^\alpha} \pm \frac{d_y}{10^\beta}$; then for the case $\alpha > \beta$, it is known that

$$S = 0.s_1s_2...s_\beta a_{\beta+1}...a_\alpha$$

where $0 \leqslant s_i \leqslant 9, 1 \leqslant i \leqslant \beta$.

Hence

$$D_f(x \pm y) = \alpha$$

For the case $\alpha = \beta$, it holds that

$$S = 0.s_1s_2...s_\beta$$

If there are trailing zeros in S, $D_f(x \pm y) < \alpha$; otherwise $D_f(x \pm y) = \alpha$. Consequently, each case leads to $D_f(x \pm y) \leqslant \max(\alpha, \beta)$.

□

Theorem 2. Let $x = x_I + \frac{d_x}{10^\alpha}$ and $y = y_I + \frac{d_y}{10^\beta}$ be standard fraction forms of two decimal fractions; then

$$0 \leq D_f(xy) \leqslant \alpha + \beta$$

Proof. Direct calculations yield

$$xy = x_Iy_I + \frac{x_Id_y}{10^\beta} + \frac{y_Id_x}{10^\alpha} + \frac{d_xd_y}{10^{\alpha+\beta}} \tag{3.1}$$

By Lemma 3, $0 \leqslant D_f(\frac{x_Idy}{10^\beta}) \leqslant \beta$, $0 \leqslant D_f(\frac{y_Id_x}{10^\alpha}) \leqslant \alpha$ and $0 \leqslant D_f(\frac{d_xd_y}{10^{\alpha+\beta}}) \leqslant \alpha + \beta$. By Theorem 1 it is immediately known that

$$0 \leqslant D_f(xy) \leqslant \alpha + \beta$$

□

Example 1. Take $x = 0.125$ and $y = 0.8$;then $D_f(x) = 3$,$D_f(y) = 1$. Check $xy = 0.1 \Rightarrow D_f(xy) = 1$ and validate $0 \leq D_f(xy) \leq 4$.
Example 2. Take $x = 1.25$ and $y = 2.4$;then $D_f(x) = 2$,$D_f(y) = 1$. Check $xy = 3 \Rightarrow D_f(xy) = 0$ and validate $0 \leq D_f(xy) \leq 3$.
Example 3. Take $x = 2.125$ and $y = 3.8$;then $D_f(x) = 3$,$D_f(y) = 1$. Check $xy = 8.075 \Rightarrow D_f(xy) = 3$ and validate $0 \leq D_f(xy) \leq 4$.
Example 4. Take $x = 2.17865$ and $y = 13.456$;then $D_f(x) = 5$,$D_f(y) = 3$. Check $xy = 29.3159144 \Rightarrow D_f(xy) = 7$ and validate $0 \leq D_f(xy) \leq 8$.
Example 5. Take $x = 2.17867$ and $y = 13.456$;then $D_f(x) = 5$,$D_f(y) = 3$. Check $xy = 29.31618352 \Rightarrow D_f(xy) = 8$ and validate $0 \leq D_f(xy) \leq 8$.

Theorem 3. Let $x = x_I + \frac{d_x}{2^\alpha}$ and $y = y_I + \frac{d_y}{2^\beta}$ be two terminating decimals, where $0 \leqslant x_I = \lfloor x \rfloor$, $0 \leqslant y_I = \lfloor y \rfloor$, $0 \leqslant \alpha$,$0 \leq \beta$ are integers, d_x and d_y are odd integers satisfying $1 \leqslant d_x < 2^\alpha$ and $1 \leqslant d_y < 2^\beta$; then

$$0 \leqslant D_f(xy) \leqslant \alpha + \beta$$

Proof. Converting x and y to their standard fraction forms yields

$$x = x_I + \frac{5^\alpha d_x}{2^\alpha 5^\alpha} = x_I + \frac{5^\alpha d_x}{10^\alpha}$$

$$y = y_I + \frac{5^\alpha d_y}{2^\alpha 5^\alpha} = y_I + \frac{5^\alpha d_y}{10^\alpha}$$

Then it follows

$$xy = x_I y_I + x_I \frac{5^\beta d_y}{10^\beta} + y_I \frac{5^\alpha d_x}{10^\alpha} + \frac{5^{\alpha+\beta} d_x d_y}{10^{\alpha+\beta}}$$

By Theorem 2, this immediately leads to

$$0 \leqslant D_f(xy) \leqslant \alpha + \beta$$

□

Example 6. Take $x = 2.4375 = 2 + \frac{7}{2^4}$ and $y = 13.5390625 = 13 + \frac{69}{2^7}$; then $D_f(x) = 4$,$D_f(y) = 7$. Check $xy = 33.00146484375 \Rightarrow D_f(xy) = 11$ and validate $0 \leq D_f(xy) \leq 11$.

3.3 More Other Properties

Theorem 4. Let x and y be two terminating decimals; assume $\lambda = \frac{y}{x}$ is also a terminal decimal satisfying $1 < \lambda < 2$ and $0 \leqslant D_f(\lambda) = 1$; then

$$D_f(x) \leqslant D_f(y) \leqslant D_f(x) + 1$$

Proof. Without loss of generality, let $x = x_I + \frac{d_x}{10^\alpha}$ be the standard fraction form of x and $\lambda = 1 + \frac{d_\lambda}{10}$ with $0 < d_\lambda < 10$ being an integer. First consider $d_x \neq 0$; it follows $D_f(x) = \alpha$ and

$$y = \lambda x = x_I + \frac{d_\lambda x_I}{10} + \frac{d_x}{10^\alpha} + \frac{d_\lambda d_x}{10^{\alpha+1}} \tag{3.2}$$

Note that, by Lemma 2,$0 \leqslant D_f(\frac{d_\lambda x_I}{10}) \leqslant 1$, $D_f(\frac{d_x}{10^\alpha}) = \alpha$ and $0 \leqslant D_f(\frac{d_\lambda d_x}{10^{\alpha+1}}) \leqslant \alpha + 1$. Obviously, if $d_\lambda d_x \not\equiv 0(\bmod 10)$, $D_f(\frac{d_\lambda d_x}{10^{\alpha+1}}) = \alpha + 1 \Rightarrow D_f(y) = \alpha + 1$. If $d_\lambda d_x \equiv 0(\bmod 10)$, due to $d_x \not\equiv 0(\mathrm{mod}10)$ it must hold that $d_x = 2^a s$ or $d_x = 5^b t$ with s and t being odd integers, $a > 0$ and $b > 0$ being integers.

If $d_x = 2^a s$, it yields

$$D_f(\frac{d_\lambda d_x}{10^{\alpha+1}}) = \begin{cases} \alpha, d_\lambda = 5 \\ \alpha + 1, d_\lambda \neq 5 \end{cases}$$

If $d_x = 5^b t$, it follows

$$D_f(\frac{d_\lambda d_x}{10^{\alpha+1}}) = \begin{cases} \alpha - 2, d_\lambda = 8 \\ \alpha - 1, d_\lambda = 4, \\ \alpha, d_\lambda = 2, 6 \\ \alpha + 1, d_\lambda 1, 3, 5, 7, 9 \end{cases}$$

Note that, when $d_x = 0 \Rightarrow D_f(x) = 0$,equality (3.2) turns to be

$$y = \lambda x = x_I + \frac{d_\lambda x_I}{10}$$

showing $0 \leqslant D_f(y) \leqslant 1$.

Overall, it holds

$$D_f(x) \leqslant D_f(y) \leqslant D_f(x) + 1$$

□

Corollary 1. Let x and y be two terminating decimals and $\frac{y}{x} = 2$; then

$$D_f(x) - 1 \leq D_f(y) \leq D_f(x)$$

Proof. Let $x = x_I + \frac{d_x}{10^\alpha}$ be the standard fraction form of x; then $d_x = 0 \Rightarrow D_f(y) = D_f(x)$ and it follows

$$y = 2x = 2x_I + \frac{2d_x}{10^\alpha}$$

saying

$$D_f(y) = \begin{cases} \alpha - 1, d_x \equiv 0(\mathrm{mod}5) \\ \alpha, \quad\quad d_x \equiv /0(\mathrm{mod}5) \end{cases}$$

That is $D_f(x) - 1 \leq D_f(y) \leq D_f(x)$.

□

Corollary 2. Let x and y be odd positive integers; then for an arbitrary integer $\alpha \geq 0$, it holds

$$D_f(\frac{x}{2^\alpha}) = D_f(\frac{y}{2^\alpha}) = \frac{1}{2}D_f(\frac{xy}{2^{2\alpha}})$$

Proof. The case $\alpha = 0$ yields $D_f(\frac{x}{2^\alpha}) = D_f(\frac{y}{2^\alpha}) = \frac{1}{2}D_f(\frac{xy}{2^{2\alpha}}) = 0$. When $\alpha > 0$, by Lemma 4, $D_f(\frac{x}{2^\alpha}) = D_f(\frac{y}{2^\alpha}) = \alpha$ and $D_f(\frac{xy}{2^{2\alpha}}) = 2\alpha$, saying $D_f(\frac{x}{2^\alpha}) = D_f(\frac{y}{2^\alpha}) = \frac{1}{2}D_f(\frac{xy}{2^{2\alpha}})$.
□

Theorem 5. Let x_0 , y_0 be odd positive integers and $\alpha \geq 0$ be a given integer. Assume x and y are terminating decimals originated from $x = \frac{x_0}{2^\alpha}$ and $y = \frac{y_0}{2^\beta}$, where $\beta \geq 0$ is an integer ; then

$$D_f(xy) = 2\alpha \Leftrightarrow \beta = \alpha$$

Proof. The relationship $\beta = \alpha \Rightarrow D_f(xy) = 2\alpha$ is what Corollary 2 declaims. Now by Lemma 4, $D_f(x) = \alpha$, $D_f(y) = \beta$ and $D_f(xy) = \alpha + \beta$. This immediately yields

$$D_f(xy) = 2\alpha \Rightarrow \beta = \alpha$$

□

4 A Potential Application

Given a composite odd integer $c = xy$, where x and y are odd integers; by Corollary 2 and Theorem 5, x or y can be found by converting c to be of the form $t = \frac{c}{2^{2\alpha}}$,where $\alpha > 0$ is an integer because t must be a multiplication of two terminating decimals, u and v, such that $u \leq \frac{\sqrt{c}}{2^\alpha}$, $v \geq \frac{\sqrt{c}}{2^\alpha}$ and $D_f(u) = D_f(v) = \alpha$. Finding out u that satisfies $D_f(u) = \alpha$ and $(2^\alpha u)|c$ can accomplish the factorization of c. The following examples show a demonstration to such a method.

Example 7. Let $c = 4171$. Then $\frac{c}{2^4} = 260.6875 = \frac{u}{2^2} \cdot \frac{v}{2^2}$. Since $\frac{\sqrt{c}}{2^2} = 16.14581989$, it is found that $u = 11.25$ satisfies $2^2u = 43$ to be a divisor of 4171.

Example 8. Let $c = 65869$. Then $\frac{c}{2^6} = 1029.203125 = \frac{u}{2^3} \cdot \frac{v}{2^3}$. Since $\frac{\sqrt{c}}{2^3} = 32.08119581$, it is found that $u = 24.875$ satisfies $2^3u = 199$ to be a divisor of 65869.

5 Conclusions and Future Work

Knowing the relationship dealing with the number of digits in the result of an arithmetic operation and in the operands joining the operation is helpful for knowing the numbers related to the operation. The study of this issue is meaningful and valuable in the case that a multiplication is known while the operands forming the multiplication are not known. I proved in paper [16] that the two divisors of an RSA number must be of the same decimal digits via such studies. That is useful to exclude the candidates with different decimal digits in solving the problem of factoring the RSA numbers. Theorem 5 of this paper, tells us that, for a terminate decimal $t = \frac{xy}{2^{2\alpha}}$, the two factors of t must be of the same number of the valid digits in their fractional parts. This also can help us to exclude the candidates with different number of the valid digits in their fractional parts in finding the factors of t. Either paper [16] or this paper reaches the same point though the two papers focus on different kinds of numbers. The potential application introduced in section 4 might break out a new way for integer factorization, which forms our future work. Hope more younger people to concern it.

References

[1] Ivan Niven. Numbers: Rational and Irrational. Random House and The L. W. Singer Company,1961.

[2] US Flight Standards Service. Decaimals (Chapter 1): Airframe & Powerplant Mechanics General Handbook. Federal Aviation Administration, Flight Stanadards National Field Office, 1976. (also seen in SAvStop Online Magazine. http://avstop.com/ac/apgeneral/decimals.html)

[3] Douglas Downing. Dictionary of Mathematics Terms. Barron's Educational Series, Inc., 2009.

[4] Nancy Tufo, Sharyn Sweeney. Decimal Competency Packet. North Shore Community College, 2004.

[5] Vedantu Class. Decimals. Vedantu Master Classes, Retrieved at: https://byjus.com/maths/decimals/#Arithmetic Operations on Decimals. Feb., 2023.

[6] Vedantu Class. Terminating Decimal. Vedantu Master Classes, Retrieved at: https://www.vedantu.com/maths/terminating-decimal. Feb., 2023.

[7] Ashish Kumar. Decimal–Definition with Examples. Splashlearn Classes, Retrieved at: https:// www.splashlearn.com/math-vocabulary/decimals/decimal.Feb., 2023.

[8] CueMath. Terminating Decimals. CueMath Classes, Retrived at: https://www.cuemath.com/numbers/terminating-decimal. Feb., 2023.

[9] Handwiki. Decimal fraction. Handwiki Terms, Retrieved at https://handwiki.org/wiki/Decimal#Decimal_fractions. Feb., 2023.

[10] Ashish Kumar. Decimal Fraction–Definition with Examples. Splashlearn Classes, Retrieved at: https://www.splashlearn.com/math-vocabulary/decimals/decimal-fraction. Feb., 2023.

[11] Nagva. Lesson Explainer: Converting between Fractions, Decimals, and Percentages. Nagav Classes, Retrived at: https://www.nagwa.com/en/explainers/575191947410/. Feb., 2023.

[12] Wikipedia. Trailing zero. Academic Dictionaries and Encyclopedia, Retrived at: https://en-academic.com/dic.nsf/enwiki/3762076?ysclid=lernux444h989528275. Feb., 2023.

[13] Wang X. Frequently-Used Properties of the Floor Function. International Journal of Applied Physics and Mathematics. 2020, 10(4): 135-142. DOI: 10.17706/ijapm.2020.10.4.135-142.

[14] Patrick Corn, Thaddeus Abiy, Jubayer Nirjhor, etc. Floor Function. Brilliant.org, Retrived at: https://brilliant.org/wiki/floor-function/, Feb., 2023.

[15] Patrick Corn, Christopher Williams, Hamza A, etc. Fractional Part Function. Brilliant.org, Retrived at: https://brilliant.org/wiki/factional-part-function/, Feb., 2023.

[16] Wang X. Number of Digits in Two Integers and Their Multiplication. Journal of Advances in Applied Mathematics. 2019,4(2):69-74, https://dx.doi.org/10.22606 /jaam.2019.42004.