

Fixed Points of E-Contraction in Controlled Metric Spaces

Abstract

The goal of this study is to prove a fixed-point theorem for E-contraction in a completely controlled metric space. Many previous findings in the literature are extended/ generalized by our findings. We also present examples that demonstrate the utility of these findings.

Keywords: Fixed point theory; E-contraction; controlled metric space.

MSC: 47H10; 54H25

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1 Introduction and Preliminaries

“The notion of E-contraction was introduced by Fulga and Proca [1]. Later, this concept has been improved by several authors, e.g.”, [2-4].

Dass and Gupta [5] established “first fixed point theorem for rational contractive type conditions in metric space”.

Theorem 1.1 (see [5]) Let (Y, d) be a complete metric space, and let $\mathcal{T}: Y \rightarrow Y$ be a self-mapping. If there exist $\alpha, \beta \in [0, 1)$ with $\alpha + \beta < 1$ such that

$$d(\mathcal{T}\xi, \mathcal{T}\nu) \leq \alpha d(\xi, \nu) + \beta \frac{[1 + d(\xi, \mathcal{T}\xi)]d(\nu, \mathcal{T}\nu)}{1 + d(\xi, \nu)} \quad (1.1)$$

for all $\xi, \nu \in Y$, then \mathcal{T} has a unique fixed point $\xi^* \in Y$.

Nazam et al. [6] proved “a real generalization of Dass-Gupta fixed point theorem in the frame work of dualistic partial metric spaces”.

Czerwik [7] reintroduced “a new class of generalized metric spaces, called as b-metric spaces, as generalizations of metric spaces”.

Definition 1 (see [7]) Let Y be a nonempty set and $s \geq 1$. A function $d_b: Y \times Y \rightarrow [0, \infty)$ is said to be a b-metric if for all $\xi, \nu, \omega \in Y$,

- (b1). $d_b(\xi, \nu) = 0$ iff $\xi = \nu$
- (b2). $d_b(\xi, \nu) = d_b(\nu, \xi)$ for all $\xi, \nu \in Y$
- (b3). $d_b(\xi, \omega) \leq s[d_b(\xi, \nu) + d_b(\nu, \omega)]$

“The pair (Y, d_b) is then called a b-metric space. Subsequently, many fixed-point results on such spaces were given” (see [8–13]).

Kamran et al. [14] initiated “the concept of extended b-metric spaces”.

Definition 2 (see [8]) Let Y be a nonempty set and $p: Y \times Y \rightarrow [1, \infty)$ be a function. A function $d_e: Y \times Y \rightarrow [0, \infty)$ is called an extended b-metric if for all $\xi, \nu, \omega \in Y$,

- (e1). $d_e(\xi, \nu) = 0$ iff $\xi = \nu$
- (e2). $d_e(\xi, \nu) = d_e(\nu, \xi)$ for all $\xi, \nu \in Y$

$$(e3). d_e(\xi, \omega) \leq p(\xi, \omega)[d_e(\xi, \nu) + d_e(\nu, \omega)]$$

The pair (Y, d_e) is called an extended b-metric space.

“Recently, a new kind of a generalized b-metric space was introduced” by Mlaiki et al. [15].

Definition 3 (see [15]) Let Y be a nonempty set and $p: Y \times Y \rightarrow [1, \infty)$ be a function. A function $d_c: Y \times Y \rightarrow [0, \infty)$ is called a controlled metric if for all $\xi, \nu, \omega \in Y$,

- (c1). $d_c(\xi, \nu) = 0$ iff $\xi = \nu$
- (c2). $d_c(\xi, \nu) = d_c(\nu, \xi)$ for all $\xi, \nu \in Y$
- (c3). $d_c(\xi, \omega) \leq p(\xi, \nu)d_c(\xi, \nu) + p(\nu, \omega)d_c(\nu, \omega)$

The pair (Y, d_c) is called a controlled metric space (see also [16]).

The Cauchy and convergent sequences in controlled metric type spaces are defined in this way

Definition 4 (see [15]) Let (Y, d_c) be a controlled metric space and $\{\xi_n\}_{n \geq 0}$ be a sequence in D . Then,

1. The sequence $\{\xi_n\}$ converges to some ξ in Y ; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\xi_n, \xi) < \varepsilon$ for all $n \geq N$. In this case, we write $\lim_{n \rightarrow \infty} \xi_n = \xi$.
2. The sequence $\{\xi_n\}$ is Cauchy; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\xi_n, \xi_m) < \varepsilon$ for all $n, m \geq N$.
3. The controlled metric space (Y, d_c) is called complete if every Cauchy sequence is convergent.

Definition 5 (see [15]) Let (Y, d_c) be a controlled metric space. Let $\xi \in Y$ and $\varepsilon > 0$.

1. The open ball $B(\xi, \varepsilon)$ is

$$B(\xi, \varepsilon) = \{\nu \in Y: d_c(\nu, \xi) < \varepsilon\}.$$

2. The mapping $\Gamma: Y \rightarrow Y$ is said to be continuous at $\xi \in Y$; if for all $\varepsilon > 0$, there exists $\delta > 0$ such that $\Gamma(B(\xi, \varepsilon)) \subseteq B(\Gamma\xi, \delta)$.

This paper's main objective is to propose a fixed-point theorem for E-contractions in the context of complete controlled metric spaces. Our finding broadens and generalises a few established findings in the literature. We also provide examples to highlight the applicability of the findings made in E-contractive circumstances.

2 Main Results

The following theorem is our main result.

Theorem 2.1 Let (Y, d_c) be a complete controlled metric space and $\Gamma: Y \rightarrow Y$ be a mapping such that

$$d_c(\Gamma\xi, \Gamma\nu) \leq \delta[d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|] \tag{2.1}$$

for all $\xi, \nu \in Y$, where $0 \leq \delta < 1$. For $\xi_0 \in Y$ and each n , we let $\xi_n = \Gamma^n \xi_0$. If

$$\sup_{m \geq 1} \lim_{i \rightarrow \infty} \frac{p(\xi_{i+1}, \xi_{i+2})p(\xi_{i+1}, \xi_m)}{p(\xi_i, \xi_{i+1})} < \frac{1+\delta}{2\delta} \tag{2.2}$$

and $\lim_{n \rightarrow \infty} p(\xi_n, \xi)$ and $\lim_{n \rightarrow \infty} p(\xi, \xi_n)$ exist, are finite, and $\delta \lim_{n \rightarrow \infty} p(\xi, \xi_n) < 1$ for every $\xi \in Y$, then Γ possesses a unique fixed point.

Proof Let $\xi_0 \in Y$ be an initial point. Define the sequence $\{\xi_n\}$ by $\xi_{n+1} = \Gamma\xi_n, \forall n \in \mathbb{N}$. Obviously, if there exists $n_0 \in \mathbb{N}$ such that $\xi_{n_0+1} = \xi_{n_0}$, then $\Gamma\xi_{n_0} = \xi_{n_0}$, and the proof is finished. Thus, we assume that $\xi_{n+1} \neq \xi_n$ for each $n \in \mathbb{N}$. Thus, by (2.1), we have

$$\begin{aligned} d_c(\xi_n, \xi_{n+1}) &= d_c(\Gamma\xi_{n-1}, \Gamma\xi_n) \\ &\leq \delta d_c(\xi_{n-1}, \xi_n) + \delta |d_c(\xi_{n-1}, \Gamma\xi_{n-1}) - d_c(\xi_n, \Gamma\xi_n)| \\ &= \delta d_c(\xi_{n-1}, \xi_n) + \delta |d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})| \end{aligned} \tag{2.3}$$

If $d_c(\xi_{n-1}, \xi_n) \leq d_c(\xi_n, \xi_{n+1})$ for some n , then from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta [d_c(\xi_{n-1}, \xi_n) - d_c(\xi_{n-1}, \xi_n) + d_c(\xi_n, \xi_{n+1})] = d_c(\xi_n, \xi_{n+1})$$

which is a contradiction. Hence $d_c(\xi_{n-1}, \xi_n) > d_c(\xi_n, \xi_{n+1})$, and so from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta [d_c(\xi_{n-1}, \xi_n) + d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})]$$

The last inequality gives

$$d_c(\xi_n, \xi_{n+1}) \leq \frac{2\delta}{1+\delta} d_c(\xi_{n-1}, \xi_n) \tag{2.3}$$

Let $\lambda = \frac{2\delta}{1+\delta} < 1$. Thus, we have

$$d_c(\xi_n, \xi_{n+1}) \leq \lambda d_c(\xi_{n-1}, \xi_n) \leq \lambda^2 d_c(\xi_{n-2}, \xi_{n-1}) \leq \dots \leq \lambda^n d_c(\xi_0, \xi_1) \tag{2.4}$$

For all $n, m \in \mathbb{N}$ and $n < m$, we have

$$\begin{aligned} d_c(\xi_n, \xi_m) &\leq p(\xi_n, \xi_{n+1})d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m)d_c(\xi_{n+1}, \xi_m) \\ &\leq p(\xi_n, \xi_{n+1})d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m)p(\xi_{n+1}, \xi_{n+2})d_c(\xi_{n+1}, \xi_{n+2}) \\ &\quad + p(\xi_{n+1}, \xi_m)p(\xi_{n+2}, \xi_m)d_c(\xi_{n+2}, \xi_m) \\ &\leq p(\xi_n, \xi_{n+1})d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m)p(\xi_{n+1}, \xi_{n+2})d_c(\xi_{n+1}, \xi_{n+2}) \\ &\quad + p(\xi_{n+1}, \xi_m)p(\xi_{n+2}, \xi_m)p(\xi_{n+2}, \xi_{n+3})d_c(\xi_{n+2}, \xi_{n+3}) \\ &\quad + p(\xi_{n+1}, \xi_m)p(\xi_{n+2}, \xi_m)p(\xi_{n+3}, \xi_m)d_c(\xi_{n+3}, \xi_m) \\ &\leq p(\xi_n, \xi_{n+1})d_c(\xi_n, \xi_{n+1}) \\ &\quad + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^i p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1})d_c(\xi_i, \xi_{i+1}) \\ &\quad + \prod_{i=n+1}^{m-1} p(\xi_j, \xi_m) d_c(\xi_{m-1}, \xi_m) \end{aligned} \tag{2.5}$$

This implies that

$$\begin{aligned} d_c(\xi_n, \xi_m) &\leq p(\xi_n, \xi_{n+1})d_c(\xi_n, \xi_{n+1}) \\ &\quad + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^i p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1})d_c(\xi_i, \xi_{i+1}) \\ &\quad + \prod_{i=n+1}^{m-1} p(\xi_j, \xi_m) d_c(\xi_{m-1}, \xi_m) \\ &\leq p(\xi_n, \xi_{n+1})\lambda^n d_c(\xi_0, \xi_1) \\ &\quad + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^i p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1})\lambda^i d_c(\xi_0, \xi_1) \end{aligned}$$

$$\begin{aligned}
 &+ \prod_{i=n+1}^{m-1} p(\xi_j, \xi_m) \lambda^{m-1} d_c(\xi_0, \xi_1) \\
 &\leq p(\xi_n, \xi_{n+1}) \lambda^n d_c(\xi_0, \xi_1) \\
 &+ \sum_{i=n+1}^{m-1} (\prod_{j=n+1}^i p(\xi_j, \xi_m)) p(\xi_i, \xi_{i+1}) \lambda^i d_c(\xi_0, \xi_1)
 \end{aligned} \tag{2.6}$$

Let

$$\eta_r = \sum_{i=0}^r (\prod_{j=0}^i p(\xi_j, \xi_m)) p(\xi_i, \xi_{i+1}) \lambda^i d_c(\xi_0, \xi_1) \tag{2.7}$$

Consider

$$\mu_i = \sum_{i=0}^r (\prod_{j=0}^i p(\xi_j, \xi_m)) p(\xi_i, \xi_{i+1}) \lambda^i d_c(\xi_0, \xi_1) \tag{2.8}$$

Then by condition (2.2) and by the ratio test, the series $\sum_i \mu_i$ is convergent. That is, $\lim_{n \rightarrow \infty} \eta_n$ exists. Hence, the sequence $\{\eta_n\}$ is Cauchy. Now, by (2.6), we have

$$d_c(\xi_n, \xi_m) \leq d_c(\xi_0, \xi_1) [\lambda^n p(\xi_n, \xi_{n+1}) + (\eta_{m-1} - \eta_n)] \tag{2.9}$$

Note that $p(\xi, \nu) \geq 1$. Letting $n, m \rightarrow \infty$ in (2.9), we obtain

$$\lim_{n, m \rightarrow \infty} d_c(\xi_n, \xi_m) = 0 \tag{2.10}$$

This shows that the sequence $\{\xi_n\}$ is Cauchy in the complete controlled metric space (Y, d_c) . Thus, there is some $\xi^* \in Y$. So that

$$\lim_{n \rightarrow \infty} d_c(\xi_n, \xi^*) = 0; \tag{2.11}$$

that is, $\xi_n \rightarrow \xi^*$ as $n \rightarrow \infty$. Now, we will prove that ξ^* is a fixed point of Γ . By (2.1) and condition (iii), we have

$$\begin{aligned}
 d_c(\xi^*, \Gamma \xi^*) &\leq p(\xi^*, \xi_{n+1}) d_c(\xi^*, \xi_{n+1}) + p(\xi_{n+1}, \Gamma \xi^*) d_c(\xi_{n+1}, \Gamma \xi^*) \\
 &= p(\xi^*, \xi_{n+1}) d_c(\xi^*, \xi_{n+1}) + p(\xi_{n+1}, \Gamma \xi^*) d_c(\Gamma \xi_n, \Gamma \xi^*) \\
 &\leq p(\xi^*, \xi_{n+1}) d_c(\xi^*, \xi_{n+1}) \\
 &+ p(\xi_{n+1}, \Gamma \xi^*) \delta [d_c(\xi_n, \xi^*) + |d_c(\xi_n, \Gamma \xi_n) - d_c(\xi^*, \Gamma \xi^*)|] \\
 &\leq p(\xi^*, \xi_{n+1}) d_c(\xi^*, \xi_{n+1}) \\
 &+ p(\xi_{n+1}, \Gamma \xi^*) \delta [d_c(\xi_n, \xi^*) + |d_c(\xi_n, \xi_{n+1}) - d_c(\xi^*, \Gamma \xi^*)|]
 \end{aligned} \tag{2.12}$$

Since $\lim_{n \rightarrow \infty} p(\xi_n, \xi)$ and $\lim_{n \rightarrow \infty} p(\xi, \xi_n)$ exist, are finite, by (2.10), (2.11), we have

$$d_c(\xi^*, \Gamma \xi^*) \leq [\delta \lim_{n \rightarrow \infty} p(\xi_{n+1}, \Gamma \xi^*)] d_c(\xi^*, \Gamma \xi^*) \tag{2.13}$$

Suppose that $\xi^* \neq \Gamma \xi^*$, having in mind that $[\delta \lim_{n \rightarrow \infty} p(\xi_{n+1}, \Gamma \xi^*)] < 1$, so

$$0 < d_c(\xi^*, \Gamma \xi^*) \leq [\delta \lim_{n \rightarrow \infty} p(\xi_{n+1}, \Gamma \xi^*)] d_c(\xi^*, \Gamma \xi^*) < d_c(\xi^*, \Gamma \xi^*) \tag{2.14}$$

This is a contradiction. Thus, we must have $\xi^* = \Gamma \xi^*$. Next, we show that ξ^* is unique. Let ν^* be another fixed point of Γ in Y , then $\Gamma \nu^* = \nu^*$. And so, by (2.1), we have

$$\begin{aligned}
 d_c(\xi^*, \nu^*) &= d_c(\Gamma \xi^*, \Gamma \nu^*) \\
 &\leq \delta [d_c(\xi^*, \nu^*) + |d_c(\xi^*, \Gamma \xi^*) - d_c(\nu^*, \Gamma \nu^*)|] \\
 &= \delta [d_c(\xi^*, \nu^*) + |d_c(\xi^*, \xi^*) - d_c(\nu^*, \nu^*)|] \\
 &= \delta d_c(\xi^*, \nu^*) \tag{2.15}
 \end{aligned}$$

This is a contradiction. Thus, $\xi^* = \nu^*$. It completes the proof.

3 Examples

Now we furnish some examples to demonstrate the validity of the hypotheses of generality of our result.

Example 3.1 Let $Y = \{0,1,2\}$. Take the controlled metric d_c defined as

$$d_c(0,0) = d_c(1,1) = d_c(2,2) = 0, \\ d_c(0,1) = d_c(1,0) = \frac{1}{2}, d_c(0,2) = d_c(2,0) = \frac{11}{20}, d_c(1,2) = d_c(2,1) = \frac{3}{20}$$

where $p: Y \times Y \rightarrow [1, \infty)$ is symmetric such that

$$p(0,0) = p(1,1) = p(2,2) = p(1,2) = 1, p(0,2) = 2, p(0,1) = \frac{3}{2}$$

Given $\Gamma : Y \rightarrow Y$ as

$$\Gamma 0 = 2 \text{ and } \Gamma 1 = \Gamma 2 = 1.$$

If $\gamma = \frac{2}{3}$. Then

$$\lambda = \frac{2\gamma}{1 + \gamma} = \frac{\frac{4}{3}}{1 + \frac{2}{3}} = \frac{4}{5} < 1,$$

Take $\xi_0 = 0$, then $\xi_1 = 2$, and $\xi_n = 1$, for all $n \geq 2$, we have $\lim_{n \rightarrow \infty} p(\xi_n, \xi)$ and $\lim_{n \rightarrow \infty} p(\xi, \xi_n)$ exist, are finite, and $\gamma \lim_{n \rightarrow \infty} p(\xi, \xi_n) < 1$ for every $\xi \in Y$. Also

$$\sup_{m \geq 1} \lim_{i \rightarrow \infty} \frac{p(\xi_{i+1}, \xi_{i+2}) p(\xi_{i+1}, \xi_m)}{p(\xi_i, \xi_{i+1})} = 1 < \frac{5}{4} = \lambda^{-1}$$

We consider the following cases.

- (1) Let $\xi = \nu = 0$, then $d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|]$
- (2) Let $\xi = \nu = 1$, then $d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|]$
- (3) Let $\xi = \nu = 2$, then $d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|]$
- (4) Let $\xi = 0, \nu = 1$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 0, \Gamma 1) = d_c(2, 1) = \frac{3}{20} \\ \leq \frac{2}{3} \left[\left(\frac{1}{2} \right) + \left| \left(\frac{11}{20} \right) - (0) \right| \right] \\ = \frac{2}{3} [d_c(0,1) + |d_c(0,2) - d_c(1,1)|] \\ = \gamma [d_c(0,1) + |d_c(0, \Gamma 0) - d_c(1, \Gamma 1)|] \\ = \gamma [d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|]$$

- (5) Let $\xi = 1, \nu = 0$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 1, \Gamma 0) = d_c(1, 2) = \frac{3}{20} \\ \leq \frac{2}{3} \left[\left(\frac{1}{2} \right) + \left| (0) - \left(\frac{11}{20} \right) \right| \right]$$

$$\begin{aligned}
 &= \delta[d_c(1,0) + |d_c(1,1) - d_c(0,2)|] \\
 &= \delta[d_c(1,0) + |d_c(1,\Gamma 1) - d_c(0,\Gamma 0)|] \\
 &= \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma \xi) - d_c(\nu, \Gamma \nu)|]
 \end{aligned}$$

(6) Let $\xi = 0, \nu = 2$, then

$$\begin{aligned}
 d_c(\Gamma \xi, \Gamma \nu) &= d_c(\Gamma 0, \Gamma 2) = d_c(2,1) = \frac{3}{20} \\
 &\leq \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma \xi) - d_c(\nu, \Gamma \nu)|]
 \end{aligned}$$

(7) Let $\xi = 2, \nu = 0$, then

$$\begin{aligned}
 d_c(\Gamma \xi, \Gamma \nu) &= d_c(\Gamma 2, \Gamma 0) = d_c(1,2) = \frac{3}{20} \\
 &\leq \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma \xi) - d_c(\nu, \Gamma \nu)|]
 \end{aligned}$$

(8) Let $\xi = 1, \nu = 2$, then

$$\begin{aligned}
 d_c(\Gamma \xi, \Gamma \nu) &= d_c(\Gamma 1, \Gamma 2) = d_c(1,1) = 0 \\
 &\leq \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma \xi) - d_c(\nu, \Gamma \nu)|]
 \end{aligned}$$

(9) Let $\xi = 2, \nu = 1$, then

$$\begin{aligned}
 d_c(\Gamma \xi, \Gamma \nu) &= d_c(\Gamma 2, \Gamma 1) = d_c(1,1) = 0 \\
 &\leq \gamma[d_c(\xi, \nu) + |d_c(\xi, \Gamma \xi) - d_c(\nu, \Gamma \nu)|]
 \end{aligned}$$

Clearly, (2.2) is satisfied. On the other hand, note that (2.1) holds for all $\xi, \nu \in Y$. All other hypotheses of Theorem 2.1 are verified, and so Γ has a unique fixed point, which is $\xi^* = 1$.

4 Conclusion

It is concluded that broadens and generalises a few established findings in the literature. We also provide examples to highlight the applicability of the findings made in E-contractive circumstances.

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Competing Interests

Authors have declared that no competing interests exist.

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