E-Contraction in Controlled Metric Spaces

ABSTRACT

The goal of this study is to prove a fixed-point theorem for E-contraction in a completely controlled metric space. Many previous findings in the literature are extended/generalized by our findings. We also present examples that demonstrate the utility of these findings.

Keywords: Fixed point theory; E-contraction; Controlled metric space.

MSC: 47H10; 54H25

1. Introduction and Preliminaries

The notion of E-contraction was introduced by Fulga and Proca [29]. Later, this concept has been improved by several authors, e.g., [30-32].

Dassand Gupta [26] established first fixed point theorem for rational contractive type conditions in metric space.

Theorem 1.1 (see [26]). Let (Y, d) be a complete metric space, and let $\mathcal{T}: Y \to Y$ be a self-mapping. If there exist $\alpha, \beta \in [0,1)$ with $\alpha + \beta < 1$ such that

$$d(\mathcal{T}\xi, \mathcal{T}\nu) \leq \alpha d(\xi, \nu) + \beta \frac{[1 + d(\xi, \mathcal{T}\xi)] d(\nu, \mathcal{T}\nu)}{1 + d(\xi, \nu)} \qquad (1.1)$$

for all $\xi, \nu \in Y$, then $\mathcal{T}$ has a unique fixed point $\xi^* \in Y$.

Nazam*et al.* [27] proved a real generalization of Dass-Gupta fixed point theorem in the frame work of dualistic partial metric spaces.

Czerwik [1] reintroduced a new class of generalized metric spaces, called as b-metric spaces, as generalizations of metric spaces.

Definition 1 (see [1]) Let Y be a nonempty set and $s \geq 1$. A function $d_b: Y \times Y \longrightarrow [0, \infty)$ is said to be a b -metric if for all $\xi, \nu, \omega \in Y$,

(b1). $d_b(\xi, \nu) = 0$ iff $\xi = \nu$
(b2). $d_b(\xi, \nu) = d_b(\nu, \xi)$for all $\xi, \nu \in Y$
(b3). $d_b(\xi, \omega) \leq s[d_b(\xi, \nu) + d_b(\nu, \omega)]$

The pair (Y, d_b) is then called a b-metric space. Subsequently, many fixed-point results on such spaces were given (see [2–7]).

Kamran et al. [8] initiated the concept of extended b-metric spaces.

Definition 2 (see [8]) Let Y be a nonempty set and $p: Y \times Y \longrightarrow [1, \infty)$be a function. A function $d_e: Y \times Y \longrightarrow [0, \infty)$is called an extended b -metric if for all $\xi, \nu, \omega \in Y$,

(e1). $d_e(\xi,\nu) = 0$ iff $\xi = \nu$
(e2). $d_e(\xi,\nu) = d_e(\nu,\xi)$ for all $\xi,\nu \in Y$
(e3). $d_e(\xi,\omega) \leq p(\xi,\omega)[d_e(\xi,\nu) + d_e(\nu,\omega)]$

The pair (Y,d_e) is called an extended b-metric space.

Recently, a new kind of a generalized b-metric space was introduced by Mlaiki et al. [9].

Definition 3 (see [9]) Let Y be a nonempty set and $p: Y \times Y \longrightarrow [1,\infty)$ be a function. A function $d_c: Y \times Y \longrightarrow [0,\infty)$ is called a controlled metric if for all $\xi,\nu,\omega \in Y$,

(c1). $d_c(\xi,\nu) = 0$ iff $\xi = \nu$
(c2). $d_c(\xi,\nu) = d_c(\nu,\xi)$ for all $\xi,\nu \in Y$
(c3). $d_c(\xi,\omega) \leq p(\xi,\nu)d_c(\xi,\nu) + p(\nu,\omega)d_c(\nu,\omega)$

The pair (Y,d_c) is called a controlled metric space (see also [10]).

The Cauchy and convergent sequences in controlled metric type spaces are defined in this way

Definition 4 (see [9]) Let (Y,d_c) be a controlled metric space and $\{\xi_n\}_{n\geq 0}$ be a sequence in D. Then,

1. The sequence $\{\xi_n\}$ converges to some ξ in Y; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\xi_n,\xi) < \varepsilon$ for all $n \geq N$. In this case, we write $\lim_{n\to\infty} \xi_n = \xi$.
2. The sequence $\{\xi_n\}$ is Cauchy; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\xi_n,\xi_m) < \varepsilon$ for all $n,m \geq N$.
3. The controlled metric space (Y,d_c) is called complete if every Cauchy sequence is convergent.

Definition 5 (see [9]) Let (Y,d_c) be a controlled metric space. Let $\xi \in Y$ and $\varepsilon > 0$.

1. The open ball $B(\xi,\varepsilon)$ is
   $B(\xi,\varepsilon) = \{\nu \in Y: d_c(\nu,\xi) < \varepsilon\}$.
2. The mapping $\Gamma: Y \longrightarrow Y$ is said to be continuous at $\xi \in Y$; if for all $\varepsilon > 0$, there exists $\delta > 0$ such that $\Gamma(B(\xi,\varepsilon)) \subseteq B(\Gamma\xi,\varepsilon)$.

This paper's main objective is to propose a fixed-point theorem for E-contractions in the context of complete controlled metric spaces. Our finding broadens and generalises a few established findings in the literature. We also provide examples to highlight the applicability of the findings made in E-contractive circumstances.

2 Main Results

The following theorem is our main result.

Theorem 2.1. Let (Y, d_c)be a complete controlled metric space. Let $\Gamma: Y \longrightarrow Y$ be a mapping such $\lambda = \frac{2\delta}{1+\delta} < 1$,

$d_c(\Gamma\xi, \Gamma\nu) \leq \delta[d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|]$ (2.1)

for all $\xi, \nu \in Y$. For $\xi_0 \in Y$, take $\xi_n = \Gamma^n\xi_0$. Assume that

$\sup_{m\geq 1} \lim_{i\to\infty} \frac{p(\xi_{i+1}, \xi_{i+2})p(\xi_{i+1}, \xi_m)}{p(\xi_i, \xi_{i+1})} < \lambda^{-1}$ (2.2)

Suppose that $\lim_{n\to\infty} p(\xi_n, \xi)$and $\lim_{n\to\infty} p(\xi, \xi_n)$ exist, are finite, and $\delta \lim_{n\to\infty} p(\xi, \xi_n) < 1$ for every$\xi \in Y$, then Γ possesses a unique fixed point.

*Proof.*Let $\xi_0 \in Y$ be initial point. Define a sequence $\{\xi_n\}$as$\xi_{n+1} = \Gamma\xi_n, \forall\, n \in \mathbb{N}$. Obviously, if $\exists n_0 \in \mathbb{N}$ for which $\xi_{n_0+1} = \xi_{n_0}$, then $\Gamma\xi_{n_0} = \xi_{n_0}$, and the proof is finished. Thus, we suppose that $\xi_{n+1} \neq \xi_n$ for each$n \in \mathbb{N}$. Thus, by (2.1), we have

$$d_c(\xi_n, \xi_{n+1}) = d_c(\Gamma\xi_{n-1}, \Gamma\xi_n)$$

$$\leq \delta d_c(\xi_{n-1}, \xi_n) + \delta|d_c(\xi_{n-1}, \Gamma\xi_{n-1}) - d_c(\xi_n, \Gamma\xi_n)|$$

$= \delta d_c(\xi_{n-1}, \xi_n) + \delta|d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})|$(2.3)

If $d_c(\xi_{n-1}, \xi_n) \leq d_c(\xi_n, \xi_{n+1})$ for some n, then from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta[d_c(\xi_{n-1}, \xi_n) - d_c(\xi_{n-1}, \xi_n) + d_c(\xi_n, \xi_{n+1})] = d_c(\xi_n, \xi_{n+1})$$

which is a contradiction. Hence$d_c(\xi_{n-1}, \xi_n) > d_c(\xi_n, \xi_{n+1})$ and so from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta[d_c(\xi_{n-1}, \xi_n) + d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})]$$

The last inequality gives

$d_c(\xi_n, \xi_{n+1}) \leq \frac{2\delta}{1+\delta} d_c(\xi_{n-1}, \xi_n) = \lambda d_c(\xi_{n-1}, \xi_n)$(2.3)

Thus, we have

$d_c(\xi_n, \xi_{n+1}) \leq \lambda d_c(\xi_{n-1}, \xi_n) \leq \lambda^2 d_c(\xi_{n-2}, \xi_{n-1}) \leq \cdots \leq \lambda^n d_c(\xi_0, \xi_1)$(2.4)

For all $n, m \in \mathbb{N}$ and $n < m$, we have

$$d_c(\xi_n, \xi_m) \leq p(\xi_n, \xi_{n+1})d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m)d_c(\xi_{n+1}, \xi_m)$$

$$\leq p(\xi_n, \xi_{n+1})d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m)p(\xi_{n+1}, \xi_{n+2})d_c(\xi_{n+1}, \xi_{n+2})$$

$$+p(\xi_{n+1},\xi_m)p(\xi_{n+2},\xi_m)d_c(\xi_{n+2},\xi_m)$$

$$\leq p(\xi_n,\xi_{n+1})d_c(\xi_n,\xi_{n+1}) + p(\xi_{n+1},\xi_m)p(\xi_{n+1},\xi_{n+2})d_c(\xi_{n+1},\xi_{n+2})$$

$$+p(\xi_{n+1},\xi_m)p(\xi_{n+2},\xi_m)p(\xi_{n+2},\xi_{n+3})d_c(\xi_{n+2},\xi_{n+3})$$

$$+p(\xi_{n+1},\xi_m)p(\xi_{n+2},\xi_m)p(\xi_{n+3},\xi_m)d_c(\xi_{n+3},\xi_m)$$

$$\leq p(\xi_n,\xi_{n+1})d_c(\xi_n,\xi_{n+1})$$

$$+\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})d_c(\xi_i,\xi_{i+1})$$

$$+\prod_{i=n+1}^{m-1} p(\xi_j,\xi_m)\, d_c(\xi_{m-1},\xi_m) \tag{2.5}$$

This implies that

$$d_c(\xi_n,\xi_m) \leq p(\xi_n,\xi_{n+1})d_c(\xi_n,\xi_{n+1})$$

$$+\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})d_c(\xi_i,\xi_{i+1})$$

$$+\prod_{i=n+1}^{m-1} p(\xi_j,\xi_m)\, d_c(\xi_{m-1},\xi_m)$$

$$\leq p(\xi_n,\xi_{n+1})\lambda^n d_c(\xi_0,\xi_1)$$

$$+\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})\lambda^i d_c(\xi_0,\xi_1)$$

$$+\prod_{i=n+1}^{m-1} p(\xi_j,\xi_m)\, \lambda^{m-1} d_c(\xi_0,\xi_1)$$

$$\leq p(\xi_n,\xi_{n+1})\lambda^n d_c(\xi_0,\xi_1)$$

$$+\sum_{i=n+1}^{m-1}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})\lambda^i d_c(\xi_0,\xi_1) \tag{2.6}$$

Let

$$\eta_r = \sum_{i=0}^{r}\left(\prod_{j=0}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})\lambda^i d_c(\xi_0,\xi_1) \tag{2.7}$$

Consider

$\mu_i = \sum_{i=0}^{r}\left(\prod_{j=0}^{i} p(\xi_j, \xi_m)\right) p(\xi_i, \xi_{i+1})\lambda^i d_c(\xi_0, \xi_1)$ (2.8)

In view of condition (2.2) and the ratio test, we ensure that the series $\sum_i \mu_i$converges. Thus, $\lim_{n\to\infty} \eta_n$exists. Hence, the real sequence $\{\eta_n\}$ is Cauchy. Now, using (2.6), we get

$d_c(\xi_n, \xi_m) \leq d_c(\xi_0, \xi_1)[\lambda^n p(\xi_n, \xi_{n+1}) + (\eta_{m-1} - \eta_n)]$(2.9)

Above, we used $p(\xi, \nu) \geq 1$. Letting $n, m \longrightarrow \infty$ in (2.9), we obtain

$\lim_{n,m\to\infty} d_c(\xi_n, \xi_m) = 0$ (2.10)

Thus, the sequence $\{\xi_n\}$is Cauchy in the complete controlled metric space (Y, d_c). So, there is some $\xi^* \in Y$.So that

$\lim_{n\to\infty} d_c(\xi_n, \xi^*) = 0;$ (2.11)

that is, $\xi_n \longrightarrow \xi^*$as $n \longrightarrow \infty$. Now, we will prove that $\xi^*$is a fixed point of Γ. By (2.1) and condition (iii), we get

$$d_c(\xi^*, \Gamma\xi^*) \leq p(\xi^*, \xi_{n+1})d_c(\xi^*, \xi_{n+1}) + p(\xi_{n+1}, \Gamma\xi^*)d_c(\xi_{n+1}, \Gamma\xi^*)$$

$$= p(\xi^*, \xi_{n+1})d_c(\xi^*, \xi_{n+1}) + p(\xi_{n+1}, \Gamma\xi^*)d_c(\Gamma\xi_n, \Gamma\xi^*)$$

$$\leq p(\xi^*, \xi_{n+1})d_c(\xi^*, \xi_{n+1})$$

$$+p(\xi_{n+1}, \Gamma\xi^*)\delta[d_c(\xi_n, \xi^*) + |d_c(\xi_n, \Gamma\xi_n) - d_c(\xi^*, \Gamma\xi^*)|]$$

$$\leq p(\xi^*, \xi_{n+1})d_c(\xi^*, \xi_{n+1})$$

$$+p(\xi_{n+1}, \Gamma\xi^*)\delta[d_c(\xi_n, \xi^*) + |d_c(\xi_n, \xi_{n+1}) - d_c(\xi^*, \Gamma\xi^*)|]$$

(2.12)

Taking the limit as $n \longrightarrow \infty$ and using (2.10), (2.11) and the fact that $\lim_{n\to\infty} p(\xi_n, \xi)$and $\lim_{n\to\infty} p(\xi, \xi_n)$ exist, are finite, we obtain that

$d_c(\xi^*, \Gamma\xi^*) \leq [\delta \lim_{n\longrightarrow\infty} p(\xi_{n+1}, \Gamma\xi^*)]d_c(\sigma^*, \mathrm{F}\sigma^*)$ (2.13)

Suppose that $\xi^* \neq \Gamma\xi^*$, having in mind that $[\delta \lim_{n\longrightarrow\infty} p(\xi_{n+1}, \Gamma\xi^*)] < 1$, so

$0 < d_c(\xi^*, \Gamma\xi^*) \leq [\delta \lim_{n\longrightarrow\infty} p(\xi_{n+1}, \Gamma\xi^*)]d_c(\sigma^*, \mathrm{F}\sigma^*) < d_c(\sigma^*, \mathrm{F}\sigma^*)$(2.14)

It is a contradiction. This yields that $\xi^* = \Gamma\xi^*$. Now, we prove the uniqueness ofξ^*.Let ν^*be another fixed point of Γ in Y, then $\Gamma\nu^* = \nu^*$. Now, by (2.1), we have

$$d_c(\xi^*, \nu^*) = d_c(\Gamma\xi^*, \Gamma\nu^*)$$

$$\leq \delta[d_c(\xi^*, \nu^*) + |d_c(\xi^*, \Gamma\xi^*) - d_c(\nu^*, \Gamma\nu^*)|]$$

$$= \delta[d_c(\xi^*, \nu^*) + |d_c(\xi^*, \xi^*) - d_c(\nu^*, \nu^*)|]$$

$= \delta d_c(\xi^*, \nu^*)$(2.15)

It is a contradiction. This yields that$\xi^* = \nu^*$. It completes the proof.

3 Example

Now we furnish some examples to demonstrate the validity of the hypotheses of generality of our result.

Example 3.1 Let$Y = \{0,1,2\}$. Take the controlled metric d_c defined as

$$d_c(0,0) = d_c(1,1) = d_c(2,2) = 0,$$

$$d_c(0,1) = d_c(1,0) = \frac{1}{2}, d_c(0,2) = d_c(2,0) = \frac{11}{20}, d_c(1,2) = d_c(2,1) = \frac{3}{20},$$

where $p: Y \times Y \longrightarrow [1, \infty)$ is symmetric such that

$$p(0,0) = p(1,1) = p(2,2) = p(1,2) = 1, p(0,2) = 2, p(0,1) = \frac{3}{2}$$

Given $\Gamma : Y \longrightarrow Y$ as

$\Gamma 0 = 2$ and $\Gamma 1 = \Gamma 2 = 1$.

If$\gamma = \frac{2}{3}$. Then

$$\lambda = \frac{2\gamma}{1+\gamma} == \frac{\frac{4}{3}}{1+\frac{2}{3}} = \frac{4}{5} < 1,$$

Take $\xi_0 = 0$, then $\xi_1 = 2$, and $\xi_n = 1$, for all $n \geq 2$, we have$\lim_{n\to\infty} p(\xi_n, \xi)$and $\lim_{n\to\infty} p(\xi, \xi_n)$ exist, are finite, and $\gamma \lim_{n\to\infty} p(\xi, \xi_n) < 1$ for every $\xi \in Y$. Also

$$\sup_{m\geq 1} \lim_{i\to\infty} \frac{p(\xi_{i+1}, \xi_{i+2}) p(\xi_{i+1}, \xi_m)}{p(\xi_i, \xi_{i+1})} = 1 < \frac{5}{4} = \lambda^{-1}$$

We consider the following cases.

(1) Let$\xi = \nu = 0$,then

$$d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(2) Let $\xi = \nu = 1$, then

$$d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(3) Let $\xi = \nu = 2$, then

$$d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(4) Let $\xi = 0, \nu = 1$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 0, \Gamma 1) = d_c(2,1) = \frac{3}{20}$$

$$\leq \frac{2}{3}\left[\left(\frac{1}{2}\right) + \left|\left(\frac{11}{20}\right) - (0)\right|\right]$$

$$= \frac{2}{3}[d_c(0,1) + |d_c(0,2) - d_c(1,1)|]$$

$$= \gamma[d_c(0,1) + |d_c(0,\Gamma 0) - d_c(1,\Gamma 1)|]$$

$$= \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(5) Let $\xi = 1, \nu = 0$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 1, \Gamma 0) = d_c(1,2) = \frac{3}{20}$$

$$\leq \frac{2}{3}\left[\left(\frac{1}{2}\right) + \left|(0) - \left(\frac{11}{20}\right)\right|\right]$$

$$= \delta[d_c(1,0) + |d_c(1,1) - d_c(0,2)|]$$

$$= \delta[d_c(1,0) + |d_c(1,\Gamma 1) - d_c(0,\Gamma 0)|]$$

$$= \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(6) Let $\xi = 0, \nu = 2$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 0, \Gamma 2) = d_c(2,1) = \frac{3}{20}$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(7) Let $\xi = 2, \nu = 0$, then

$$d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 2,\Gamma 0) = d_c(1,2) = \frac{3}{20}$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(8) Let $\xi = 1, \nu = 2$, then

$$d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 1,\Gamma 2) = d_c(1,1) = 0$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(9) Let $\xi = 2, \nu = 1$, then

$$d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 2,\Gamma 1) = d_c(1,1) = 0$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

Clearly, (2.2) is satisfied. On the other hand, note that (2.1) holds for all $\xi, \nu \in Y$. All other hypotheses of Theorem 2.1 are verified, and so Γ has a unique fixed point, which is $\xi^* = 1$.

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