E-Contraction in Controlled Metric Spaces

ABSTRACT

The goal of this study is to prove a fixed-point theorem for E-contraction in a completely controlled metric space. Many previous findings in the literature are extended/generalized by our findings. We also present examples that demonstrate the utility of these findings.

Keywords: Fixed point theory; E-contraction; Controlled metric space.

MSC: 47H10; 54H25

1. Introduction and Preliminaries

The notion of E-contraction was introduced by Fulga and Proca [29]. Later, this concept has been improved by several authors, e.g., [30-32].

Dassand Gupta [26] established first fixed point theorem for rational contractive type conditions in metric space.

Theorem 1.1 (see [26]). Let (Y, d) be a complete metric space, and let $\mathcal{T}: Y \rightarrow Y$ be a self-mapping. If there exist $\alpha, \beta \in [0,1)$ with $\alpha + \beta < 1$ such that

$$d(\mathcal{T}\xi, \mathcal{T}\nu) \leq \alpha d(\xi, \nu) + \beta \frac{[1 + d(\xi, \mathcal{T}\xi)]d(\nu, \mathcal{T}\nu)}{1 + d(\xi, \nu)} \qquad (1.1)$$

for all $\xi, \nu \in Y$, then $\mathcal{T}$ has a unique fixed point $\xi^* \in Y$.

Nazam*et al.* [27] proved a real generalization of Dass-Gupta fixed point theorem in the frame work of dualistic partial metric spaces.

Czerwik [1] reintroduced a new class of generalized metric spaces, called as b-metric spaces, as generalizations of metric spaces.

Definition 1 (see [1]) Let Y be a nonempty set and $s \geq 1$. A function $d_b: Y \times Y \longrightarrow [0, \infty)$ is said to be a b -metric if for all $\xi, \nu, \omega \in Y$,

(b1). $d_b(\xi, \nu) = 0$ iff $\xi = \nu$
(b2). $d_b(\xi, \nu) = d_b(\nu, \xi)$ for all $\xi, \nu \in Y$
(b3). $d_b(\xi, \omega) \leq s[d_b(\xi, \nu) + d_b(\nu, \omega)]$

The pair (Y, d_b) is then called a b-metric space. Subsequently, many fixed-point results on such spaces were given (see [2–7]).

Kamran et al. [8] initiated the concept of extended b-metric spaces.

Definition 2 (see [8]) Let Y be a nonempty set and $p: Y \times Y \longrightarrow [1, \infty)$ be a function. A function $d_e: Y \times Y \longrightarrow [0, \infty)$ is called an extended b -metric if for all $\xi, \nu, \omega \in Y$,

(e1). $d_e(\xi, \nu) = 0$ iff $\xi = \nu$
(e2). $d_e(\xi, \nu) = d_e(\nu, \xi)$for all $\xi, \nu \in Y$
(e3). $d_e(\xi, \omega) \leq p(\xi, \omega)[d_e(\xi, \nu) + d_e(\nu, \omega)]$

The pair$(Y, d_e)$is called an extended b-metric space.

Recently, a new kind of a generalized b-metric space was introduced by Mlaiki et al. [9].

Definition 3 (see [9]) Let$Y$ be a nonempty set and $p: Y \times Y \longrightarrow [1, \infty)$be a function. A function $d_c: Y \times Y \longrightarrow [0, \infty)$is called a controlled metric if for all $\xi, \nu, \omega \in Y$,

(c1). $d_c(\xi, \nu) = 0$ iff $\xi = \nu$
(c2). $d_c(\xi, \nu) = d_c(\nu, \xi)$for all $\xi, \nu \in Y$
(c3). $d_c(\xi, \omega) \leq p(\xi, \nu) d_c(\xi, \nu) + p(\nu, \omega) d_c(\nu, \omega)$

The pair$(Y, d_c)$is called a controlled metric space (see also [10]).

The Cauchy and convergent sequences in controlled metric type spaces are defined in this way

Definition 4 (see [9]) Let(Y, d_c)be a controlled metric space and $\{\xi_n\}_{n \geq 0}$be a sequence in D. Then,

1. The sequence $\{\xi_n\}$ converges to some ξ in Y; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\xi_n, \xi) < \varepsilon$ for all $n \geq N$. In this case, we write$\lim_{n \to \infty} \xi_n = \xi$.
2. The sequence $\{\xi_n\}$ is Cauchy; if for every $\varepsilon > 0$,there exists $N = N(\varepsilon) \in \mathbb{N}$such that $d_c(\xi_n, \xi_m) < \varepsilon$ for all $n, m \geq N$.
3. The controlled metric space $(Y, d_c)$is called complete if every Cauchy sequence is convergent.

Definition 5 (see [9]) Let(Y, d_c)be a controlled metric space. Let $\xi \in Y$and $\varepsilon > 0$.

1. The open ball $B(\xi, \varepsilon)$ is
   $B(\xi, \varepsilon) = \{\nu \in Y: d_c(\nu, \xi) < \varepsilon\}$.
2. The mapping $\Gamma: Y \longrightarrow Y$ is said to be continuous at $\xi \in Y$; if for all $\varepsilon > 0$, there exists $\delta > 0$ such that$\Gamma\big(B(\xi, \varepsilon)\big) \subseteq B(\Gamma\xi, \varepsilon)$.

This paper's main objective is to propose a fixed-point theorem for E-contractions in the context of complete controlled metric spaces. Our finding broadens and generalises a few established findings in the literature. We also provide examples to highlight the applicability of the findings made in E-contractive circumstances.

2 Main Results

The following theorem is our main result.

Theorem 2.1. Let (Y, d_c)be a complete controlled metric space, and $\Gamma: Y \longrightarrow Y$ be a mapping with$\lambda = \frac{2\delta}{1+\delta} < 1$(please qualify lambda and delta), and

$d_c(\Gamma\xi, \Gamma\nu) \leq \delta[d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|]$ (2.1)

for all $\xi, \nu \in Y$. For each$\xi_0 \in Y$ and each n, we let$\xi_n = \Gamma^n \xi_0$.If

$\sup_{m\geq 1} \lim_{i\to\infty} \frac{p(\xi_{i+1},\xi_{i+2})p(\xi_{i+1},\xi_m)}{p(\xi_i,\xi_{i+1})} < \lambda^{-1}$, (2.2)

$\lim_{n\to\infty} p(\xi_n, \xi)$and $\lim_{n\to\infty} p(\xi, \xi_n)$are finite, and$\delta \lim_{n\to\infty} p(\xi, \xi_n) < 1$ for every$\xi \in Y$, then Γ possesses a unique fixed point.

*Proof.*Let $\xi_0 \in Y$be an initial point. Define the sequence $\{\xi_n\}$by$\xi_{n+1} = \Gamma\xi_n, \forall\, n \in \mathbb{N}$. Obviously, if there exists$n_0 \in \mathbb{N}$such that$\xi_{n_0+1} = \xi_{n_0}$, then $\Gamma\xi_{n_0} = \xi_{n_0}$, and we are done. Thus, we assume that $\xi_{n+1} \neq \xi_n$ for each$n \in \mathbb{N}$.Thus, by (2.1), we have

$$d_c(\xi_n, \xi_{n+1}) = d_c(\Gamma\xi_{n-1}, \Gamma\xi_n)$$

$$\leq \delta d_c(\xi_{n-1}, \xi_n) + \delta|d_c(\xi_{n-1}, \Gamma\xi_{n-1}) - d_c(\xi_n, \Gamma\xi_n)|$$

$= \delta d_c(\xi_{n-1}, \xi_n) + \delta|d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})|$(2.3)

If $d_c(\xi_{n-1}, \xi_n) \leq d_c(\xi_n, \xi_{n+1})$ for some n, then from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta[d_c(\xi_{n-1}, \xi_n) - d_c(\xi_{n-1}, \xi_n) + d_c(\xi_n, \xi_{n+1})] = d_c(\xi_n, \xi_{n+1}),$$

which is a contradiction. Hence$d_c(\xi_{n-1}, \xi_n) > d_c(\xi_n, \xi_{n+1})$, and so from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta[d_c(\xi_{n-1}, \xi_n) + d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})]$$

The last inequality gives

$d_c(\xi_n, \xi_{n+1}) \leq \frac{2\delta}{1+\delta} d_c(\xi_{n-1}, \xi_n) = \lambda d_c(\xi_{n-1}, \xi_n)$(2.3)

Thus, we have

$d_c(\xi_n, \xi_{n+1}) \leq \lambda d_c(\xi_{n-1}, \xi_n) \leq \lambda^2 d_c(\xi_{n-2}, \xi_{n-1}) \leq \cdots \leq \lambda^n d_c(\xi_0, \xi_1)$(2.4)

For all $n, m \in \mathbb{N}$and $n < m$, we have

$$d_c(\xi_n, \xi_m) \leq p(\xi_n, \xi_{n+1}) d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m) d_c(\xi_{n+1}, \xi_m)$$

$$\leq p(\xi_n, \xi_{n+1}) d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m) p(\xi_{n+1}, \xi_{n+2}) d_c(\xi_{n+1}, \xi_{n+2})$$

$$+p(\xi_{n+1},\xi_m)p(\xi_{n+2},\xi_m)d_c(\xi_{n+2},\xi_m)$$

$$\leq p(\xi_n,\xi_{n+1})d_c(\xi_n,\xi_{n+1}) + p(\xi_{n+1},\xi_m)p(\xi_{n+1},\xi_{n+2})d_c(\xi_{n+1},\xi_{n+2})$$

$$+p(\xi_{n+1},\xi_m)p(\xi_{n+2},\xi_m)p(\xi_{n+2},\xi_{n+3})d_c(\xi_{n+2},\xi_{n+3})$$

$$+p(\xi_{n+1},\xi_m)p(\xi_{n+2},\xi_m)p(\xi_{n+3},\xi_m)d_c(\xi_{n+3},\xi_m)$$

$$\leq p(\xi_n,\xi_{n+1})d_c(\xi_n,\xi_{n+1})$$

$$+\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})d_c(\xi_i,\xi_{i+1})$$

$$+\prod_{i=n+1}^{m-1} p(\xi_j,\xi_m)\, d_c(\xi_{m-1},\xi_m) \qquad (2.5)$$

This implies that

$$d_c(\xi_n,\xi_m) \leq p(\xi_n,\xi_{n+1})d_c(\xi_n,\xi_{n+1})$$

$$+\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})d_c(\xi_i,\xi_{i+1})$$

$$+\prod_{i=n+1}^{m-1} p(\xi_j,\xi_m)\, d_c(\xi_{m-1},\xi_m)$$

$$\leq p(\xi_n,\xi_{n+1})\lambda^n d_c(\xi_0,\xi_1)$$

$$+\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})\lambda^i d_c(\xi_0,\xi_1)$$

$$+\prod_{i=n+1}^{m-1} p(\xi_j,\xi_m)\, \lambda^{m-1} d_c(\xi_0,\xi_1)$$

$$\leq p(\xi_n,\xi_{n+1})\lambda^n d_c(\xi_0,\xi_1)$$

$$+\sum_{i=n+1}^{m-1}\left(\prod_{j=n+1}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})\lambda^i d_c(\xi_0,\xi_1) \qquad (2.6)$$

Let

$$\eta_r = \sum_{i=0}^{r}\left(\prod_{j=0}^{i} p(\xi_j,\xi_m)\right)p(\xi_i,\xi_{i+1})\lambda^i d_c(\xi_0,\xi_1) \qquad (2.7)$$

Consider

$\mu_i = \sum_{i=0}^{r}\left(\prod_{j=0}^{i} p(\xi_j,\xi_m)\right) p(\xi_i,\xi_{i+1})\lambda^i d_c(\xi_0,\xi_1).$ (2.8)

Then by condition (2.2) and by the *ratio test*, the series $\sum_i \mu_i$ is convergent, that is $\lim_{n\to\infty}\eta_n$ exists. Hence, the sequence $\{\eta_n\}$ is Cauchy. Now, by (2.6), we have

$d_c(\xi_n,\xi_m) \le d_c(\xi_0,\xi_1)[\lambda^n p(\xi_n,\xi_{n+1}) + (\eta_{m-1} - \eta_n)]$ (2.9)

Note that $p(\xi,\nu) \ge 1$. Letting $n, m \longrightarrow \infty$ in (2.9), we obtain

$\lim_{n,m\to\infty} d_c(\xi_n,\xi_m) = 0$ (2.10)

This shows that the sequence $\{\xi_n\}$ is Cauchy in the complete controlled metric space (Y, d_c). Thus, there exists $\xi^* \in Y$ such that

$\lim_{n\to\infty} d_c(\xi_n,\xi^*) = 0,$ (2.11)

that is, $\xi_n \longrightarrow \xi^*$ as $n \longrightarrow \infty$. Now, we show that ξ^* is a fixed point of Γ. By (2.1) and condition (iii), we have

$$d_c(\xi^*,\Gamma\xi^*) \le p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1}) + p(\xi_{n+1},\Gamma\xi^*)d_c(\xi_{n+1},\Gamma\xi^*)$$

$$= p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1}) + p(\xi_{n+1},\Gamma\xi^*)d_c(\Gamma\xi_n,\Gamma\xi^*)$$

$$\le p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1})$$

$$+p(\xi_{n+1},\Gamma\xi^*)\delta[d_c(\xi_n,\xi^*) + |d_c(\xi_n,\Gamma\xi_n) - d_c(\xi^*,\Gamma\xi^*)|]$$

$$\le p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1})$$

$$+p(\xi_{n+1},\Gamma\xi^*)\delta[d_c(\xi_n,\xi^*) + |d_c(\xi_n,\xi_{n+1}) - d_c(\xi^*,\Gamma\xi^*)|].$$

(2.12)

Since $\lim_{n\to\infty} p(\xi_n,\xi)$ and $\lim_{n\to\infty} p(\xi,\xi_n)$ are finite, by (2.10), (2.11) we have

$d_c(\xi^*,\Gamma\xi^*) \le [\delta \lim_{n\to\infty} p(\xi_{n+1},\Gamma\xi^*)]d_c(\sigma^*,\mathrm{F}\sigma^*).$ (2.13)

Suppose that $\xi^* \ne \Gamma\xi^*$, having in mind that $[\delta \lim_{n\to\infty} p(\xi_{n+1},\Gamma\xi^*)] < 1$, we have

$0 < d_c(\xi^*,\Gamma\xi^*) \le [\delta \lim_{n\to\infty} p(\xi_{n+1},\Gamma\xi^*)]d_c(\sigma^*,\mathrm{F}\sigma^*) < d_c(\sigma^*,\mathrm{F}\sigma^*)$ (2.14)

This is a contradiction. Thus, we must have $\xi^* = \Gamma\xi^*$. Next, we show that ξ^* is unique. Let ν^* be another fixed point of Γ in Y, then $\Gamma\nu^* = \nu^*$. And so, by (2.1), we have

$$d_c(\xi^*,\nu^*) = d_c(\Gamma\xi^*,\Gamma\nu^*)$$

$$\leq \delta[d_c(\xi^*,\nu^*) + |d_c(\xi^*,\Gamma\xi^*) - d_c(\nu^*,\Gamma\nu^*)|]$$

$$= \delta[d_c(\xi^*,\nu^*) + |d_c(\xi^*,\xi^*) - d_c(\nu^*,\nu^*)|]$$

$= \delta d_c(\xi^*,\nu^*)$(2.15)

This is a contradiction. Thus,$\xi^* = \nu^*$. This completes the proof.

3 Example

Now we furnish some examples to demonstrate the validity of the hypotheses of generality of our result.

Example 3.1 Let$Y = \{0,1,2\}$. Take the controlled metric d_c defined as

$$d_c(0,0) = d_c(1,1) = d_c(2,2) = 0,$$

$$d_c(0,1) = d_c(1,0) = \frac{1}{2}, d_c(0,2) = d_c(2,0) = \frac{11}{20}, d_c(1,2) = d_c(2,1) = \frac{3}{20},$$

where$p: Y \times Y \longrightarrow [1,\infty)$ is symmetric such that

$$p(0,0) = p(1,1) = p(2,2) = p(1,2) = 1, p(0,2) = 2, p(0,1) = \frac{3}{2}$$

Given $\Gamma : Y \longrightarrow Y$ as

$\Gamma 0 = 2$and $\Gamma 1 = \Gamma 2 = 1$.

If$\gamma = \frac{2}{3}$. Then

$$\lambda = \frac{2\gamma}{1+\gamma} == \frac{\frac{4}{3}}{1+\frac{2}{3}} = \frac{4}{5} < 1,$$

Take $\xi_0 = 0$, then $\xi_1 = 2$, and $\xi_n = 1$, for all $n \geq 2$, we have$\lim_{n\to\infty} p(\xi_n,\xi)$and $\lim_{n\to\infty} p(\xi,\xi_n)$ exist, are finite, and $\gamma \lim_{n\to\infty} p(\xi,\xi_n) < 1$ for every $\xi \in Y$. Also

$$\sup_{m\geq 1} \lim_{i\to\infty} \frac{p(\xi_{i+1},\xi_{i+2})p(\xi_{i+1},\xi_m)}{p(\xi_i,\xi_{i+1})} = 1 < \frac{5}{4} = \lambda^{-1}$$

We consider the following cases.

(1) Let$\xi = \nu = 0$,then

$$d_c(\Gamma\xi,\Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(2) Let $\xi = \nu = 1$, then

$$d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(3) Let $\xi = \nu = 2$, then

$$d_c(\Gamma\xi, \Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(4) Let$\xi = 0, \nu = 1$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 0, \Gamma 1) = d_c(2,1) = \frac{3}{20}$$

$$\leq \frac{2}{3}\left[\left(\frac{1}{2}\right) + \left|\left(\frac{11}{20}\right) - (0)\right|\right]$$

$$= \frac{2}{3}[d_c(0,1) + |d_c(0,2) - d_c(1,1)|]$$

$$= \gamma[d_c(0,1) + |d_c(0,\Gamma 0) - d_c(1,\Gamma 1)|]$$

$$= \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(5) Let$\xi = 1, \nu = 0$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 1, \Gamma 0) = d_c(1,2) = \frac{3}{20}$$

$$\leq \frac{2}{3}\left[\left(\frac{1}{2}\right) + \left|(0) - \left(\frac{11}{20}\right)\right|\right]$$

$$= \delta[d_c(1,0) + |d_c(1,1) - d_c(0,2)|]$$

$$= \delta[d_c(1,0) + |d_c(1,\Gamma 1) - d_c(0,\Gamma 0)|]$$

$$= \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(6) Let$\xi = 0, \nu = 2$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 0, \Gamma 2) = d_c(2,1) = \frac{3}{20}$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(7) Let$\xi = 2, \nu = 0$, then

$$d_c(\Gamma\xi, \Gamma\nu) = d_c(\Gamma 2, \Gamma 0) = d_c(1,2) = \frac{3}{20}$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(8) Let $\xi = 1, \nu = 2$, then

$$d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 1,\Gamma 2) = d_c(1,1) = 0$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(9) Let $\xi = 2, \nu = 1$, then

$$d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 2,\Gamma 1) = d_c(1,1) = 0$$

$$\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

Clearly, (2.2) is satisfied. On the other hand, note that (2.1) holds for all $\xi, \nu \in \Upsilon$. All other hypotheses of Theorem 2.1 are verified, and so Γ has a unique fixed point, which is $\xi^* = 1$.

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