C_n-E- super magic graceful labeling of some special graphs

Abstract

A graph G possess an H-covering when each edge in $E(G)$ pertaining to a subgraph of G isomorphic to H. This graph G is H-magic if there exists a total labeling $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each subgraph H' of G isomorphic to H, $\sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = M$ is a constant. An H-E-super magic graceful labeling (H-E-SMGL) is a bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ with $f(E(G)) = \{1, 2, \ldots, q\}$ so that $\sum_{v \in V(H')} f(v) - \sum_{e \in E(H')} f(e) = M$ for few positive integer M. Herein, we examine the C_n-E-SMGL of some graphs.

Key words: H-covering, H-magic labeling, H-E-super magic labeling, H-E- super magic graceful labeling.

AMS subject classification code: 05C78

1 Introduction

All graphs considered in this article are finite, simple and undirected. The vertex set and edge set of a graph G is represented as $V(G)$ and $E(G)$ correspondingly, $p = |V|$ and $q = |E|$. A graph labeling is a map that takes graph elements to numbers(typically integers). Various classes of labelings has been introduced by several experts.An excellent analysis of graph labelings is glimpsed in [5] .

During 1963, Sedl$\grave{a}\check{c}$ek [16] described magic labeling in graphs. A graph G is magic when the edges of G usually labeled with $\{1, 2, \ldots, q\}$ such that the sum over the labels of all edges incident with any vertex is equal [10] $\sum\limits_{v \in N(v)} f(uv) = M$.

A covering of G is a family of subgraphs $H_1, H_2, \ldots, H_h$ so that each edge of $E(G)$ pertaining to at least one of the subgraphs $H_i, 1 \leq i \leq h$. This results that G possess an $(H_1, H_2, \ldots, H_h)$ covering. When each H_i is isomorphic to the graph H, then G have an H-covering. Assume that G have an H-covering. A total labeling is a bijective function f from $V(G) \cup E(G)$ to $\{1, 2, 3, \ldots, |V(G)| + |E(G)|\}$ is named an H-magic labeling of G if there exists a positive integer M (termed the magic constant) so that for every subgraph $H^{'}$ of G isomorphic to H, $\sum\limits_{v \in V(H^{'})} f(v) + \sum\limits_{e \in E(H^{'})} f(e) = M$. A graph which possess such a labeling is termed H-magic. The function f is named as H-E-super magic labeling when $f(E(G)) = \{1, 2, \ldots, q\}$.

The concept of H-magic labeling was explained by Gutierrez and Llado [6] .

Llado and Moragas [8] explored few C_n-supermagic graphs.

Rosa [14] initiated a labeling known as β-valuation. Golomb [7] named that labeling as graceful. An one to one function f from the vertices of G to { 0, 1, 2, …, q } is named as graceful labeling of G when every edge uv is labeled as $|f(u) - f(v)|$, the resultant edge labels are different.

To acquire more knowledge regarding H-E-super magic graphs, read [17].

In 2019, Sindhu Murugan and S. Chandra Kumar [15] initiated an H-E-super magic graceful labeling (H-E-SMGL). An H-E-SMGL is a bijective function f from $V(G) \cup E(G)$ to $\{1, 2, \ldots, p+q\}$ with $f(E(G)) = \{1, 2, \ldots, q\}$ and $\sum_{v \in V(H')} f(v) - \sum_{e \in E(H')} f(e) = M$ for few positive integer M. Herein, we examine C_n-E-SMGL of some families of graphs. There are so many types of magic labelings in graphs, defined and studied by various authors [1–4, 9, 11–13]

2 C_n-E-Super magic graceful graphs

Theorem 2.1. *Let $n \geq 5$ be an odd integer. Then the wheel graph W_n is C_3-E-SMGL with magic constant $\frac{9n+5}{2}$.*

Proof. Denote the vertices of n-cycle of the wheel W_n as $a_1, a_2, \ldots, a_n$ and its central vertex by r. We define a total labeling $f : V(W_n) \cup E(W_n) \to \{1, 2, 3, \ldots, 3n+1\}$ as follows:

$$f(v) = \begin{cases} 2n+1 & if\ v = r \\ 2n+2 & if\ v = a_1 \\ 2n + \frac{i+3}{2} & if\ v = a_i,\ i\ is\ odd\ for\ 3 \leq i \leq n \\ \frac{5n+3+i}{2} & if\ v = a_i,\ i\ is\ even\ for\ 2 \leq i \leq n-1 \end{cases}$$

and

$$f(e) = \begin{cases} i & if\ e = ra_i\ for\ 1 \leq i \leq n \\ 2n+1-i & if\ e = a_i a_{i+1}\ for\ 1 \leq i \leq n-1 \\ n+1 & if\ e = a_n a_1. \end{cases}$$

Now, we prove that f is a $C_3 - E$-SMGL of W_n.

Let C_3^i for $1 \leq i \leq n$ be the subcycle of W_n with $V(C_3^i) = \{a_i : 1 \leq i \leq n\} \cup \{r\}$ and

$E(C_3^i) = \{a_i a_{i\oplus_n 1} : 1 \leq i \leq n\} \cup \{ra_i : 1 \leq i \leq n\} \cup \{ra_{i\oplus_n 1} : 1 \leq i \leq n\}$.

Case 1: Suppose $i = 1$.

Then $M = \sum\limits_{v\in V(C_3^i)} f(v) - \sum\limits_{e\in E(C_3^i)} f(e) = f(r) + f(a_1) + f(a_2) - [f(a_1a_2) + f(ra_1) + f(ra_2)]$
$= [2n+1] + [2n+2] + [\frac{5n+5}{2}] - [2n+1+2] = \frac{9n+5}{2}$.

Case 2: Suppose i is even for $2 \leq i \leq n-1$.

Then $M = \sum\limits_{v\in V(C_3^i)} f(v) - \sum\limits_{e\in E(C_3^i)} f(e) = f(r) + f(a_i) + f(a_{i+1}) - [f(a_ia_{i+1}) + f(ra_i) + f(ra_{i+1})]$
$= [2n+1] + [\frac{5n+3+i}{2}] + [2n+2+\frac{i}{2}] - [2n+1-i+i+i+1] = \frac{9n+5}{2}$.

Case 3: Suppose i is odd for $3 \leq i \leq n-2$.

Then $M = \sum\limits_{v\in V(C_3^i)} f(v) - \sum\limits_{e\in E(C_3^i)} f(e) = f(r) + f(a_i) + f(a_{i+1}) - [f(a_ia_{i+1}) + f(ra_i) + f(ra_{i+1})]$
$= [2n+1] + [2n+\frac{i+3}{2}] + [\frac{5n+4+i}{2}] - [2n+1-i+i+i+1] = \frac{9n+5}{2}$

Case 4: Suppose $i = n$.

Then $M = \sum\limits_{v\in V(C_3^i)} f(v) - \sum\limits_{e\in E(C_3^i)} f(e) = f(r) + f(a_n) + f(a_1) - [f(a_na_1) + f(ra_1) + f(ra_n)]$
$= [2n+1] + [\frac{5n+3}{2}] + [2n+2] - [n+1+1+n] = \frac{9n+5}{2}$.

The graph W_n is $C_3 - E$-SMG with magic constant $\frac{9n+5}{2}$. □

Example 2.1. *The Wheel W_7 admits C_3-E-SMGL with magic constant* 34.

Denote the vertices of n-cycle of the wheel W_n as $a_1, a_2, \ldots, a_7$ and its central vertex by r.

Define $f : V(W_7) \cup E(W_7) \to \{1, 2, 3, \ldots, 22\}$ as follows:

$$f(v) = \begin{cases} 15 & if\ v = r \\ 16 & if\ v = a_1 \\ 14 + \frac{i+3}{2} & if\ v = a_i,\ i\ is\ odd\ for\ 3 \leq i \leq 7 \\ 19 + \frac{i}{2} & if\ v = a_i,\ i\ is\ even\ for\ 2 \leq i \leq 6 \end{cases}$$

and

$$f(e)=\begin{cases} i & if\ e=ra_i\ for\ 1\le i\le 7\\ 15-i & if\ e=a_ia_{i+1}\ for\ 1\le i\le 6\\ 8 & if\ e=a_7a_1. \end{cases}$$

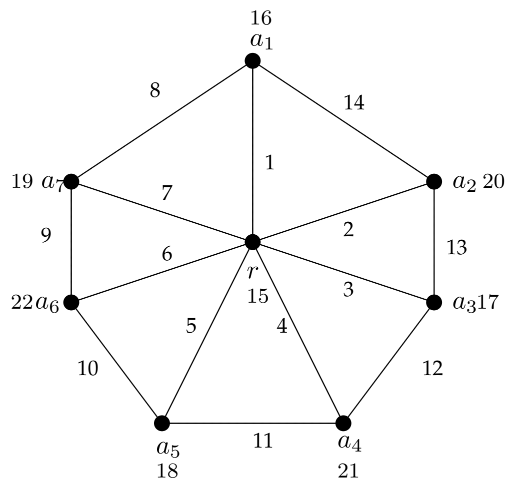

Figure 2.1 : C_3-E-SMGL of W_7

To prove that f is a $C_3 - E$-SML of W_7.

Let C_3^i for $1 \le i \le n$ be the subcycle of W_n with $V(C_3^i) = \{a_i : 1 \le i \le 7\} \cup \{r\}$ and $E(C_3^i) = \{a_i a_{i\oplus_n 1} : 1 \le i \le 7\} \cup \{ra_i : 1 \le i \le 7\} \cup \{ra_{i\oplus_7 1} : 1 \le i \le 7\}$.

Case 1: Suppose $i = 1$.

Then $M = \sum\limits_{v\in V(C_3^i)} f(v) - \sum\limits_{e\in E(C_3^i)} f(e) = f(r) + f(a_1) + f(a_2) - [f(a_1a_2) + f(ra_1) + f(ra_2)]$
$= [15] + [16] + [20] - [14 + 1 + 2] = 34.$

Case 2: Suppose i is even for $2 \le i \le 6$.

Then $M = \sum\limits_{v\in V(C_3^i)} f(v) - \sum\limits_{e\in E(C_3^i)} f(e) = f(r) + f(a_i) + f(a_{i+1}) - [f(a_ia_{i+1}) + f(ra_i) + f(ra_{i+1})]$
$= [15] + [19 + \frac{i}{2}] - [16 + \frac{i}{2}] - [15 - i + i + i + 1] = 34.$

Case 3: Suppose i is odd for $3 \le i \le 5$.

Then $M = \sum\limits_{v\in V(C_3^i)} f(v) - \sum\limits_{e\in E(C_3^i)} f(e) = f(r) + f(a_i) + f(a_{i+1}) - [f(a_ia_{i+1}) + f(ra_i) + f(ra_{i+1})]$

$= [15] + [14 + \frac{i+3}{2}] + [\frac{39+i}{2}] - [15 - i + i + i + 1] = 34$

Case 4: Suppose $i = 7$.

Then $M = \sum_{v \in V(C_3^i)} f(v) - \sum_{e \in E(C_3^i)} f(e) = f(r) + f(a_n) + f(a_1) - [f(a_n a_1) + f(ra_1) + f(ra_n)]$
$= [15] + [19] + [16] - [8 + 1 + 7] = 34.$

The graph W_n is $C_3 - E$-SMG with magic graceful constant 34.

Theorem 2.2. *Let $n \geq 1$ be an integer. Then the Ladder graph $L_n = P_2 \times P_n$ admits C_4-E-SMGL with magic constant $9n + 4$.*

Proof. Let $V(L_n) = \{a_i, b_i : 1 \leq i \leq n\}$ and $E(L_n) = \{a_i a_{i+1}, b_i b_{i+1} : 1 \leq i \leq n-1\} \cup \{a_i b_i : 1 \leq i \leq n\}$ be the vertex set and the edge set of L_n respectively.

We define a total labeling $f : V(L_n) \cup E(L_n) \to \{1, 2, \ldots, 5n - 2\}$ as follows:

$$f(v) = \begin{cases} 2n + i + 3 & if\ v = a_i\ for\ 1 \leq i \leq n \\ 5n - i - 1 & if\ v = b_i\ for\ 1 \leq i \leq n \end{cases}$$

$$f(e) = \begin{cases} i & if\ e = a_i b_i\ for\ 1 \leq i \leq n \\ 2n - i & if\ e = a_i a_{i+1}\ for\ 1 \leq i \leq n-1 \\ 3n - i - 1 & if\ e = b_i b_{i+1}\ for\ 1 \leq i \leq n-1. \end{cases}$$

Now, we prove that f is a $C_4 - E$-SMGL of L_n.

Let C_4^i for $1 \leq i \leq n-1$ be the subcycle of L_n with $V(C_4^i) = \{a_i, b_i : 1 \leq i \leq n\}$ and $E(C_4^i) = \{a_i a_{i+1} : 1 \leq i \leq n-1\} \cup \{b_i b_{i+1} : 1 \leq i \leq n-1\} \cup \{a_i b_i : 1 \leq i \leq n\}$.

Suppose $1 \leq i \leq n - 1$.

Then $M = \sum_{v \in V(C_4^i)} f(v) - \sum_{e \in E(C_4^i)} f(e) = f(a_i) + f(a_{i+1}) + f(b_i) + f(b_{i+1}) - [f(a_i b_i) + f(a_{i+1} b_{i+1}) + f(a_i a_{i+1}) + f(b_i b_{i+1})]$

$= [2n + i + 3] + [2n + i + 4] + [5n - i - 1] + [5n - i - 2] - [i + i + 1 + 2n - i + 3n - i - 1 = 9n + 4.$

The graph L_n is $C_4 - E$-SMG with magic constant $9n + 4$. □

Example 2.2. *The Ladder graph $L_5 = P_2 \times P_5$ admits C_4-E-SMGL with magic constant* 49.

Let $V(L_5) = \{a_i, b_i : 1 \leq i \leq 5\}$ and $E(L_5) = \{a_i a_{i+1}, b_i b_{i+1} : 1 \leq i \leq 4\} \cup \{a_i b_i : 1 \leq i \leq 5\}$ be the vertex set and the edge set of L_5 respectively.

Define $f : V(L_5) \cup E(L_5) \to \{1, 2, \ldots, 23\}$ as follows:

$$f(v) = \begin{cases} 13 + i & if\ v = a_i\ for\ 1 \leq i \leq 5 \\ 24 - i & if\ v = b_i\ for\ 1 \leq i \leq 5 \end{cases}$$

and

$$f(e) = \begin{cases} i & if\ e = a_i b_i\ for\ 1 \leq i \leq 5 \\ 10 - i & if\ e = a_i a_{i+1}\ for\ 1 \leq i \leq 4 \\ 14 - i & if\ e = b_i b_{i+1}\ for\ 1 \leq i \leq 4. \end{cases}$$

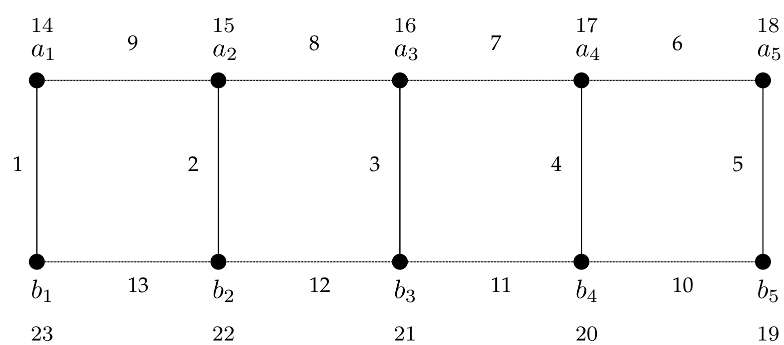

Figure 2.2 : C_4-E-SMGL of L_5

To prove that f is a $C_4 - E$-SMGL of L_5.

Let C_4^i for $1 \leq i \leq 4$ be the subcycle of L_5 with $V(C_4^i) = \{a_i, b_i : 1 \leq i \leq 5\}$ and $E(C_4^i) = \{a_i a_{i+1} : 1 \leq i \leq 4\} \cup \{b_i b_{i+1} : 1 \leq i \leq 4\} \cup \{a_i b_i : 1 \leq i \leq 5\}$.

Suppose $1 \leq i \leq 4$.

Then $M = \sum\limits_{v \in V(C_4^i)} f(v) - \sum\limits_{e \in E(C_4^i)} f(e) = f(a_i) + f(a_{i+1}) + f(b_i) + f(b_{i+1}) - [f(a_i b_i) + f(a_{i+1} b_{i+1}) +$

$f(a_i a_{i+1}) + f(b_i b_{i+1})]$

$= [13 + i] + [14 + i] + [24 - i] + [23 - i] - [i + i + 1 + 10 - i + 14 - i] = 49.$

Thus the graph L_5 is $C_4 - E$-SMG with magic constant 49.

Theorem 2.3. *Let $n \geq 2$ be an integer. Then the triangular Ladder TL_n admits C_3-E-SMGL with magic constant $M = 10n - 5$.*

Proof. Let $V(TL_n) = \{a_i, b_i : 1 \leq i \leq n\}$ and $E(TL_n) = \{a_i a_{i+1}, b_i b_{i+1} : 1 \leq i \leq n-1\} \cup \{a_i b_i : 1 \leq i \leq n\} \cup \{a_i b_{i+1} : 1 \leq i \leq n-1\}$ be the vertex set and the edge set of TL_n respectively.. We define a total labeling $f : V(TL_n) \cup E(TL_n) \rightarrow \{1, 2, \ldots, 6n-3\}$ as follows:

$$f(v) = \begin{cases} 4n + 2i - 3 & if\ v = a_i\ for\ 1 \leq i \leq n \\ 4n + 2i - 4 & if\ v = b_i\ for\ 1 \leq i \leq n \end{cases}$$

and

$$f(e) = \begin{cases} 2i - 1 & if\ e = a_i b_i\ for\ 1 \leq i \leq n \\ 2n + 2i - 2 & if\ e = a_i a_{i+1}\ for\ 1 \leq i \leq n-1 \\ 2n + 2i - 1 & if\ e = b_i b_{i+1}\ for\ 1 \leq i \leq n-1 \\ 2i & if\ e = a_i b_{i+1}\ for\ 1 \leq i \leq n-1. \end{cases}$$

To prove that f is a $C_3 - E$-SMGL of TL_n.

Let C_3^i for $1 \leq i \leq n-1$ be the subcycle of TL_n with $V(C_3^i) = \{a_i : 1 \leq i \leq n\} \cup \{b_i : 1 \leq i \leq n\}$ and $E(C_3^i) = \{a_i a_{i+1} : 1 \leq i \leq n-1\} \cup \{b_i b_{i+1} : 1 \leq i \leq n-1\} \cup \{a_i, b_i : 1 \leq i \leq n\}$. Suppose $1 \leq i \leq n-1$.

Then $M = \sum\limits_{v \in VC_3^i} f(v) - \sum\limits_{e \in EC_3^i} f(e) = f(a_i) + f(a_{i+1}) + f(b_{i+1}) - [f(a_i a_{i+1}) + f(a_{i+1} b_{i+1}) + f(b_{i+1} a_i)]$
$= [4n + 2i - 3] + [4n + 2i - 1] + [4n + 2i - 2] - [2n + 2i - 2 + 2i + 1 + 2i] = 10n - 5.$

The graph TL_n is $C_3 - E$-SMG with magic constant $10n - 5$. ∎

Example 2.3. *The triangular Ladder TL_5 admits C_3-E-SMGL with magic constant $M = 45$.*

Let $V(TL_5) = \{a_i, b_i : 1 \leq i \leq 5\}$ and $E(TL_5) = \{a_i a_{i+1}, b_i b_{i+1} : 1 \leq i \leq 4\} \cup \{a_i b_i : 1 \leq i \leq 5\} \cup \{a_i b_{i+1} : 1 \leq i \leq 4\}$ be the vertex set and the edge set of TL_5 respectively.

Define $f : V(TL_5) \cup E(TL_5) \to \{1, 2, \ldots, 27\}$ as follows:

$$f(v) = \begin{cases} 17 + 2i & if\ v = a_i\ for\ 1 \leq i \leq 5 \\ 16 + 2i & if\ v = b_i\ for\ 1 \leq i \leq 5 \end{cases}$$

and

$$f(e) = \begin{cases} 2i - 1 & if\ e = a_i b_i\ for\ 1 \leq i \leq 5 \\ 8 + 2i & if\ e = a_i a_{i+1}\ for\ 1 \leq i \leq 4 \\ 9 + 2i & if\ e = b_i b_{i+1}\ for\ 1 \leq i \leq 4 \\ 2i & if\ e = a_i b_{i+1}\ for\ 1 \leq i \leq 4. \end{cases}$$

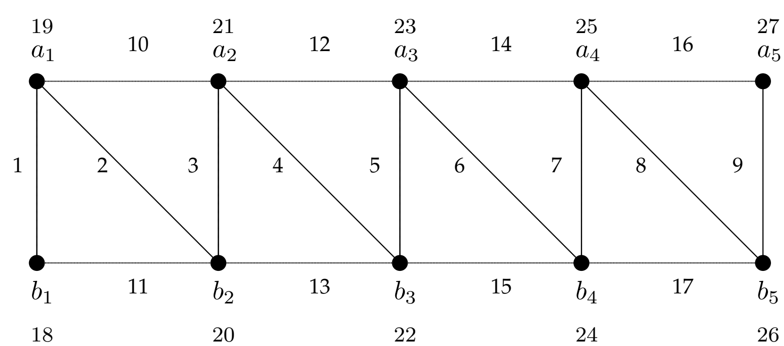

Figure 2.3 : C_3-E-SMGL of triangular ladder TL_5

To prove that f is a $C_3 - E$-SMGL of TL_5.

Let C_3^i for $1 \leq i \leq 4$ be the subcycle of TL_5 with $V(C_3^i) = \{a_i : 1 \leq i \leq 5\} \cup \{b_i : 1 \leq i \leq 5\}$ and $E(C_3^i) = \{a_i a_{i+1} : 1 \leq i \leq 4\} \cup \{b_i b_{i+1} : 1 \leq i \leq 4\} \cup \{a_i, b_i : 1 \leq i \leq 5\}$. Suppose $1 \leq i \leq n - 1$.

Then $M = \sum\limits_{v \in V(C_3^i)} f(v) - \sum\limits_{e \in E(C_3^i)} f(e) = f(a_i) + f(a_{i+1}) + f(b_{i+1}) - [f(a_i a_{i+1}) + f(a_{i+1} b_{i+1}) + f(b_{i+1} a_i)]$
$= [17 + 2i] + [17 + 2i + 2] + [16 + 2i + 2] - [2i + 1 + 8 + 2i + 2i] = 45.$

Thus the graph TL_5 is $C_3 - E$-SMG with magic constant 45.

Theorem 2.4. *Let $n \geq 2$ be an integer. Then the triangular snake graph Δ_n admit C_3-E-SMGL with magic constant $M = 7n + 2$.*

Proof. Let $V(\Delta_n) = \{a_i : 1 \leq i \leq n+1\} \cup \{b_i : 1 \leq i \leq n\}$ and $E(\Delta_n) = \{a_i a_{i+1} : 1 \leq j \leq n\} \cup \{a_i b_i : 1 \leq j \leq n\} \cup \{a_{i+1} b_i : 1 \leq i \leq n\}$ be the vertex set and the edge set of Δ_n respectively. We define a total labeling $f : V(\Delta_n) \cup E(\Delta_n) \to \{1, 2, \ldots, 6n+1\}$ as follows:

$$f(v) = \begin{cases} 3n + i & if\ v = a_i\ for\ 1 \leq i \leq n+1 \\ 5n + 2 - i & if\ v = b_i\ for\ 1 \leq i \leq n \end{cases}$$

and

$$f(e) = \begin{cases} i & if\ e = a_i a_{i+1}\ for\ 1 \leq i \leq n \\ 3n + 1 - i & if\ e = a_i b_i\ for\ 1 \leq j \leq n \\ n + i & if\ e = a_{i+1} b_i\ for\ 1 \leq j \leq n. \end{cases}$$

To prove that f is a $C_3 - E$-SMGL of Δ_n.

Let C_3^i for $1 \leq i \leq n$ be the subcycle of L_n with $V(C_3^i) = \{a_i : 1 \leq j \leq n\} \cup \{b_i : 1 \leq j \leq n\}$ and $E(C_3^i) = \{a_i a_{i+1} : 1 \leq j \leq n\} \cup \{a_i b_i : 1 \leq j \leq n\} \cup \{a_{i+1}, b_i : 1 \leq j \leq n\}$.

Suppose $1 \leq i \leq n$.

Then $M = \sum\limits_{v \in V(C_3^i)} f(v) - \sum\limits_{e \in E(C_3^i)} f(e) = f(a_i) + f(a_{i+1}) + f(b_i) - [f(a_i a_{i+1}) + f(a_i b_i) + f(a_{i+1} b_i)]$
$= [3n + i] + [3n + i + 1] + [5n + 2 - i] - [i + 3n + 1 - i + n + i] = 7n + 2.$

The graph Δ_n is $C_3 - E$-SMG with magic constant $7n + 2$. □

Example 2.4. *The triangular snake graph Δ_6 admits C_3-E-SMGL with magic constant $M = 44$.*

Let $V(\Delta_6) = \{a_i : 1 \leq i \leq 7\} \cup \{b_i : 1 \leq i \leq 6\}$ and $E(\Delta_6) = \{a_i a_{i+1} : 1 \leq i \leq 6\} \cup \{a_i b_i : 1 \leq i \leq 6\} \cup \{a_{i+1} b_i : 1 \leq i \leq 6\}$ be the vertex set and the edge set of Δ_6 respectively. Define $f : V(\Delta_6) \cup E(\Delta_6) \to \{1, 2, \ldots, 37\}$ as follows:

$$f(v) = \begin{cases} 18+i & if\ v = a_i\ for\ 1 \leq i \leq 7 \\ 32-i & if\ v = b_i\ for\ 1 \leq i \leq 6 \end{cases}$$

and

$$f(e) = \begin{cases} i & if\ e = a_i a_{i+1}\ for\ 1 \leq i \leq 6 \\ 19-i & if\ e = a_i b_i\ for\ 1 \leq i \leq 6 \\ 6+i & if\ e = a_{i+1} b_i\ for\ 1 \leq i \leq 6. \end{cases}$$

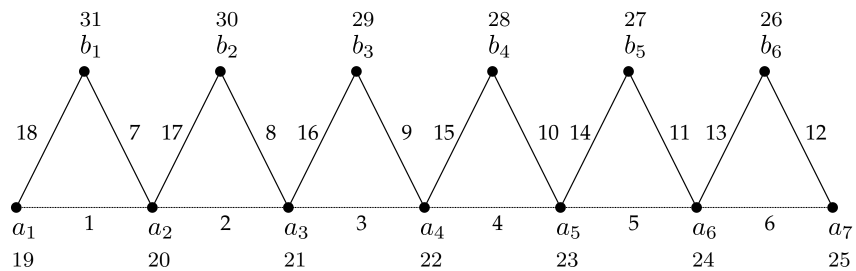

Figure 2.4 : C_3-E-SMGL of triangular snake Δ_6

To prove that f is a $C_3 - E$-SMGL of Δ_6.

Let C_3^i for $1 \leq i \leq 6$ be the subcycle of Δ_6 with $V(C_3^i) = \{a_i : 1 \leq i \leq 6\} \cup \{b_i : 1 \leq i \leq 6\}$ and

$E(C_3^i) = \{a_i a_{i+1} : 1 \leq i \leq 6\} \cup \{a_i b_i : 1 \leq i \leq 6\} \cup \{a_{i+1}, b_i : 1 \leq i \leq 6\}$.

Suppose $1 \leq i \leq 6$.

Then $M = \sum_{v \in V(C_3^i)} f(v) - \sum_{e \in E(C_3^i)} f(e) = f(a_i) + f(a_{i+1}) + f(b_i) - [f(a_i a_{i+1}) + f(a_i b_i) + f(a_{i+1} b_i)]$
$= [18+i] + [18+i+1] + [32-i] - [19-i+6+i+i] = 44$.

Thus the graph Δ_n is $C_3 - E$-SMG with magic constant 44.

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