Fixed Point Results in Complete Random Cone Metric Spaces over Banach Algebra

Abstract

We define fixed point results in Random cone metric space (RCMS) over Banach algebra. Also we give related corollaries and illustrate the examples.

Keywords: Banach algebra, Random cone metric space(RCMS), Random fixed point(FP), Cyclic (α, β) − admissible mapping.

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INTRODUCTION

A cone metric spaces over Banach algebras was first developed by Liu and Xu [1]. Itoh [3] used random differential equations to demonstrate random fixed point theorems in Banach spaces. In Cone random metric space, fixed point outcomes were constructed by Smriti Mehta et al. [2]. By utilising complete cone metric spaces, Seong-Hoon Cho [4] established the idea of C-class functions and cyclic (α, β) -admissible mappings in Banach algebras.

In this paper, we use reference article [4] for Random cone metric spaces (RCMS) instead of cone metric spaces. In this work unique fixed point is referred to as "UFP" and fixed point is referred to as "FP".

1 PRELIMINARIES

Definition 1

Let (M, d_B) be a RCMS and Φ set of all continuous family of mapping $\zeta: int(\mathcal{P}) \cup int(\mathcal{P}) \rightarrow int(\mathcal{P}) \cup \{0\}$ it satisfies $\zeta^{-1}(0) = 0$ and $\zeta(t) \ll t$ for all $t \in int(\mathcal{P})$.

Definition 2

Let (M, d_B) be a RCMS and $\{x_n(\omega)\} \subset \Omega \times M$ be a sequence and $x(\omega) \in \Omega \times M$, it satisfy the following:

1. $\lim_{n\to\infty} x_n(\omega) = x(\omega) \Leftrightarrow \forall\, r \in int(\mathcal{P})$, there is $\mathbb{N}$:$\forall n > \mathbb{N}, r - d_B\big(x_n(\omega), x(\omega)\big) \in int(\mathcal{P})$.
2. $\{x_n(\omega)\}$ is a Cauchy sequence $\Leftrightarrow \forall\ r \in int(\mathcal{P})$, there is $\mathbb{N}$, $\forall n, m > \mathbb{N}, r - d_B\big(x_n(\omega), x_m(\omega)\big) \in int(\mathcal{P})$.

Note that if $\lim_{n\to\infty} d_B\big(x_n(\omega), x(\omega)\big) = 0$, then $\lim_{n\to\infty} x_n(\omega) = x(\omega)$. Converse is also true when $\mathcal{P}$ normal cone is. If $\mathcal{P}$ is normal then $x_n(\omega)$ is a Cauchy in $\Omega \times M$ $\Leftrightarrow$ $\lim_{n\to\infty} d_B\big(x_n(\omega), x(\omega)\big) = 0$.

Definition 3

Suppose Ψ is set of all continuous family of function $\xi: \mathcal{P} \to \mathcal{P}$ such that ξ is strictly increasing, $x < y \Leftrightarrow \xi(x) < \xi(y)$ and if $\xi^{-1}(\{0\}) = 0$, if $\xi(x) \le \xi(y) \Longrightarrow x \le y$.

Definition 5

Suppose $\mathcal{C}$ is set of all continuous family of function $f: \mathcal{P} \times \mathcal{P} \to \mathcal{P}$ such that $\forall s, t \in \mathcal{P}$, $f(s,t) \le s$, and if $f(s,t) = t \Longrightarrow$ Either $s = 0$ or $t = 0$. Then $f \in \mathcal{C}$ is a C-class function.

Definition 6

Define $\alpha, \beta: \Omega \times M \to \mathcal{P}$, where M is a non-empty set. $T: \Omega \times M \to M$ is cyclic $(\alpha, \beta) -$ admissible mapping if

a) $\alpha\big(x(\omega)\big) - e \in \mathcal{P}, x \in M \Longrightarrow \beta\big(Tx(\omega)\big) - e \in \mathcal{P}$

b) $\beta\big(x(\omega)\big) - e \in \mathcal{P}, x \in M \Longrightarrow \beta\big(Tx(\omega)\big) - e \in \mathcal{P}$.

2 MAIN RESULT

Theorem 1

Consider a complete RCMS (M, d_B) and let $T: \Omega \times M \to M$ be such that

$$\xi\left(d_B\big(Tx(\omega), Ty(\omega)\big)\right) \le f(\xi\left(d_B\big(x(\omega), y(\omega)\big)\right), \zeta(d_B(x(\omega), y(\omega)))) \qquad (1)$$

for all $x(\omega), y(\omega) \in \Omega \times M$ with $\alpha\big(x(\omega)\big)\beta\big(x(\omega)\big) - e \in \mathcal{P}$, where $f \in \mathcal{C}, \xi \in \Psi, \zeta \in \Phi$. Additionally, the following are fulfilled

(i) T is cyclic (α, β) −admissible;

(ii) $\exists x_0(\omega) \in \Omega \times M$ such that $\alpha\big(x_0(\omega)\big) - e \in \mathcal{P}$ and $\beta\big(x_0(\omega)\big) - e \in \mathcal{P}$;

(iii) Either T is continuous or if $x_n(\omega) \subset \Omega \times M$ is a sequence with $\beta\big(x_n(\omega)\big) - e \in \mathcal{P}$ $\forall$ $n = 1,2,3, ...$ and $x(\omega)$ is a cluster point of $\{x_n(\omega)\}$, then $\beta\big(x(\omega)\big) - e \in \mathcal{P}$.

After that, T has FP. As well, if

$$\alpha\big(x(\omega)\big) - e \in \mathcal{P} \text{ and } \beta\big(x(\omega)\big) - e \in \mathcal{P} \quad (2)$$

Likewise, T has a UFP.

Proof

Consider $\{x_n(\omega)\}$ in $\Omega \times M$ by $x_n(\omega) = Tx_{n-1}(\omega) = T^n x_0(\omega)$ for all $n = 1,2,3, ...$, if it exist N such that $x_N(\omega) = x_{N+1}(\omega)$ is a FP of T. As a result, we presume $x_n(\omega) \neq x_{n+1}(\omega) \forall n = 1,2,3, ...$

For any $x_0(\omega) \in \Omega \times M$, then by (α, β) −admissible mapping we get $\alpha\big(x_0(\omega)\big) - e \in \mathcal{P}$, it implies that $\beta\big(Tx_0(\omega)\big) - e = \beta\big(x_1(\omega)\big) - e \in \mathcal{P}$. Again if $x_1(\omega) \in \Omega \times M$, then by (α, β) −admissible mapping we get $\alpha\big(x_2(\omega)\big) - e = \alpha\big(Tx_1(\omega)\big) - e \in \mathcal{P}$, it implies that $\beta\big(Tx_2(\omega)\big) - e = \beta\big(x_3(\omega)\big) - e \in \mathcal{P}$, inductively we have

$\forall n = 0,1,2, ...$

$$\alpha\big(x_{2n}(\omega)\big) - e \in \mathcal{P} \text{ and } \beta\big(x_{2n+1}(\omega)\big) - e \in \mathcal{P} \quad (3)$$

Similarly, $\beta\big(x_0(\omega)\big) - e \in \mathcal{P}, \alpha\big(x_1(\omega)\big) - e \in \mathcal{P}$. On continuing this process we have

$$\beta\big(x_{2n}(\omega)\big) - e \in \mathcal{P} \text{ and } \alpha\big(x_{2n+1}(\omega)\big) - e \in \mathcal{P} \quad (4)$$

Hence

$$\alpha\big(x_n(\omega)\big) - e \in \mathcal{P} \text{ and } \beta\big(x_n(\omega)\big) - e \in \mathcal{P} \quad (5)$$

Since, $\alpha\big(x_{n+1}(\omega)\big) - e \in \mathcal{P}$ and $\beta\big(x_n(\omega)\big) - e \in \mathcal{P}$

$$\alpha\big(x_{n+1}(\omega)\big)\beta\big(x_n(\omega)\big) - \alpha\big(x_{n+1}(\omega)\big) - \beta\big(x_n(\omega)\big) + e$$
$$= (\alpha\big(x_{n+1}(\omega)\big) - e)(\beta\big(x_n(\omega)\big) - e) \quad (6)$$

Also,

$$\alpha\big(x_{n+1}(\omega)\big) + \beta\big(x_n(\omega)\big) - 2e = (\alpha\big(x_{n+1}(\omega)\big) - e)(\beta\big(x_n(\omega)\big) - e) \quad (7)$$

Hence,

$$\alpha\big(x_{n+1}(\omega)\big)\beta\big(x_n(\omega)\big) - e \in \mathcal{P}\ \forall n = 1,2,3 \ldots \qquad (8)$$

From equation (1), we have

$$\xi\left(d_B\big(x_n(\omega), x_{n+1}(\omega)\big)\right) = \xi(d_B(Tx_{n-1}(\omega), Tx_n(\omega)))$$

$$\leq f(\xi\left(d_B\big(x_{n-1}(\omega), x_n(\omega)\big)\right), \zeta(d_B(x_{n-1}(\omega), x_n(\omega))))$$

$$\leq \xi\left(d_B\big(x_{n-1}(\omega), x_n(\omega)\big)\right) \forall n = 1,2,3, \ldots \qquad (9)$$

Because ξ is increasing,

$$d_B\big(x_n(\omega), x_{n+1}(\omega)\big) \leq d_B\big(x_{n-1}(\omega), x_n(\omega)\big) \qquad (10)$$

Hence $\{d_B\big(x_n(\omega), x_{n+1}(\omega)\big)\}$ is decreasing and $\mathcal{P}$ is regular $\exists s \in \mathcal{P}$,

$$\lim_{n\to\infty} d_B\big(x_{n-1}(\omega), x_n(\omega)\big) = s \qquad (11)$$

Then $s \in int(\mathcal{P}) \cup \{0\}$ (12)

Assume $s \in int(\mathcal{P})$. Take limit $n \to \infty$ in (9) and using continuity of f, ξ and ζ

$$f\big(\xi(s), \zeta(s)\big) = \xi(s) \qquad (13)$$

Either if, $\xi(s) = 0$ or $\zeta(s) = 0$. Hence $s = 0$, which is a contradiction to our assumption

$$s \notin int(\mathcal{P}) \qquad (14)$$

$$\lim_{n\to\infty} d_B\big(x_n(\omega), x_{n-1}(\omega)\big) = 0 \qquad (15)$$

We demonstrate $\{x_n(\omega)\}$ is a Cauchy. Suppose assume $\{x_n(\omega)\}$ is not Cauchy.

By lemma 6 (SH Cho 2018) there exist $r \in int(\mathcal{P})$, a subsequences $\{x_{m(k)}(\omega)\}$ and $\{x_{n(k)}(\omega)\}$ of $\{x_n(\omega)\}$. Furthermore $m(k)$ is the smallest index number $\forall k \in \mathbb{N}, m(k) > n(k) > k$ from (5) we have $\alpha\big(x_{m(k)}\big)\beta\big(x_{n(k)}\big) - e \in \mathcal{P}$ with $x(\omega) = x_{m(k)}(\omega)$ and $y(\omega) = x_{n(k)}(\omega)$, we have

$$\xi(d_B(Tx_{m(k)}(\omega), Tx_{n(k)}(\omega))) \leq$$

$$f(\xi(d_B(x_{m(k)}(\omega), x_{n(k)}(\omega))), \zeta(d_B(x_{m(k)}(\omega), x_{n(k)}(\omega)))) \qquad (16)$$

Using continuity of ζ, ξ, f and $k \to \infty$ we have

$$\xi\big(\zeta(r)\big) - f\left(\xi\big(\zeta(r)\big), \zeta\big(\zeta(r)\big)\right) \in \mathcal{P} \cap (-\mathcal{P})$$

$$f\left(\xi\big(\zeta(r)\big), \zeta\big(\zeta(r)\big)\right) = \xi\big(\zeta(r)\big)$$

(17)

Either $\zeta\big(\zeta(r)\big) = 0$ or $\xi\big(\zeta(r)\big) = 0$. Thus $\zeta(r) = 0$ and $r = 0$, it is not true for our assumption. Therefore $\{x_n(\omega)\}$ is Cauchy as well as from the completeness

$$x(\omega) = \lim_{n\to\infty} x_n(\omega) \in \Omega \times M \tag{18}$$

T is continuous, $\lim_{n\to\infty} x_n(\omega) = Tx(\omega)$ and $x(\omega) = Tx(\omega)$. Assume that (iii) hold,

$$\beta x(\omega) - e \in \mathcal{P} \tag{19}$$

Because T is (α, β) −admissible, $\alpha\big(x_n(\omega)\big) - e \in \mathcal{P}$

Using (5), it becomes, $\alpha\big(x_n(\omega)\big)\beta\big(x(\omega)\big) - e \in \mathcal{P}\ \forall\ n = 1,2,3, \ldots$

From (1),

$$\xi(d_B(Tx_{n+1}(\omega), Tx(\omega))) \le f(\xi(d_B(x_n(\omega), Tx(\omega))), \zeta(d_B(x_n(\omega), x(\omega)))) \tag{20}$$

Let $n \to \infty$ in (20), using continuity of f, ξ and ζ, then $\xi(d_B\big(x(\omega), Tx(\omega)\big) \in \mathcal{P} \cap (-\mathcal{P})$

Thus $\xi\left(d_B\big(x(\omega), Tx(\omega)\big)\right) = 0$

Hence $x(\omega) = Tx(\omega)$.

The Uniqueness Part:

Suppose we have some other fixed point $\rho(\omega)$ then from (2),

$\alpha\big(x\ (\omega)\big) - e \in \mathcal{P}$ and $\beta\big(\rho(\omega)\big) - e \in \mathcal{P}$. Hence $\alpha\big(x\ (\omega)\big)\beta\big(\rho(\omega)\big) - e \in \mathcal{P}$.

From (1)

$$\begin{aligned}\xi(d_B(x(\omega), \rho(\omega))) &= \xi(d_B(Tx, T\rho)) \\ &\le f(\xi(d_B(x(\omega), \rho(\omega))), \zeta(d_B(x(\omega), \rho(\omega)))) \\ &\le \xi(d_B(x(\omega), \rho(\omega)))\end{aligned}$$

Hence

$$f(\xi(d_B(x(\omega), \rho(\omega))), \zeta(d_B(x(\omega), \rho(\omega)))) = \xi(d_B(x(\omega), \rho(\omega)))$$

Either $\xi(d_B(x(\omega), \rho(\omega))) = 0$ or $\zeta(d_B(x(\omega), \rho(\omega))) = 0$.

$\Rightarrow d_B\big(x(\omega), \rho(\omega)\big) = 0$ and $x(\omega) = \rho(\omega)$.

Example 1

Let $\mathfrak{A} = \mathbb{R}^2, \mathcal{P} = \big\{\big(\wp_1(\omega), \wp_2(\omega)\big) \in \mathfrak{A}: \wp_1(\omega), \wp_2(\omega) \ge 0\big\}$, $||\daleth(\omega)|| = |\wp_1(\omega) + \wp_2(\omega)|\ \forall\ \daleth = \big(\wp_1(\omega), \wp_2(\omega)\big) \in \mathfrak{A}$. Define multiplication $\daleth\gimel$ of $\daleth = \big(\wp_1(\omega), \wp_2(\omega)\big)$ and $\gimel = \big(\ell_1(\omega), \ell_2(\omega)\big)$,

$$\daleth\gimel = (\daleth_1(\omega)\gimel_1(\omega) - \daleth_2(\omega)\gimel_2(\omega) - \daleth_2(\omega)\gimel_1(\omega))$$

A unit $e = (0,1)$, $\mathcal{P}$ is regular cone,

$int(\mathcal{P}) = \{(\wp_1(\omega), \wp_2(\omega)) \in \mathfrak{A}: \wp_1(\omega), \wp_2(\omega) > 0\}$.

Let $M = \mathbb{R}^2$ and $d_B: \Omega \times M \to \mathcal{P}$ defined as
$d(\daleth, \gimel) = \max\{|\wp_1(\omega) - w_2(\omega)|, |\wp_2(\omega) - w_1(\omega)|\}$. Then (M, d_B) is a complete RCMS and $d_B(\daleth(\omega), \gimel(\omega)) \in int(\mathcal{P})$ with $\daleth(\omega) \neq \gimel(\omega)$. Define $T: \Omega \times M \to \mathcal{P}$ by

$$T\daleth = \begin{cases} \frac{1}{10}\daleth, & \daleth = (\wp_1(\omega), \wp_2(\omega)) \in \mathcal{P}\ , ||\daleth|| < 1 \\ e^{(\daleth_1(\omega), \daleth_2(\omega))}, & otherwise \end{cases}$$

Let $\zeta(t) = \frac{1}{3}t$ and $\xi(t) = t \ \forall \ t = (t_1, t_2) \in \mathcal{P}$. Define $\alpha, \beta: M \to \mathcal{P}$ by

$$\alpha(\daleth) = \beta(\daleth) = \begin{cases} e, & \daleth = (\wp_1, \wp_2) \in \mathcal{P} \ and \ ||\daleth|| < 1 \\ 0, & otherwise \end{cases}$$

Let $f(s,t) = s - \zeta(s) \ \forall \ s, t \in \mathcal{P}$, we have

$$f(\xi(d_B(\daleth(\omega), \gimel(\omega))), \zeta(d_B(\daleth(\omega), \gimel(\omega)))) - \xi(d_B(T\daleth(\omega), T\gimel(\omega)))$$

$$= \frac{17}{30}\max\{|\wp_1(\omega) - \ell_2(\omega)|, |\wp_2(\omega) - \ell_1(\omega)|\}$$

Hence $\xi(d_B(T\daleth(\omega), T\gimel(\omega))) \leq f(\xi(d_B(\daleth(\omega), \gimel(\omega))), \zeta(d_B(\daleth(\omega), \gimel(\omega))))$

$\forall \ \daleth(\omega), \gimel(\omega) \in \Omega \times M$ With $\alpha(\daleth(\omega))\beta(\daleth(\omega)) - e \in \mathcal{P}$.

Let $\daleth_n(\omega) = \left|\frac{1}{n+6}\right| + \left|\frac{3}{n+6}\right| \ \forall \ n = 1,2,3 \ldots$ Then $\daleth_n \in \mathcal{P}$ and $||x_n|| < 1$. Hence $\beta(\daleth_n) - e = (0,0) \in \mathcal{P}$. $\lim_{n\to\infty} \daleth_n = (0,0)$ and $(0,0)$ is a cluster point of $\{x_n\}$. $\beta(\daleth_n) - e = (0,0) \in \mathcal{P}$.

Hence (0,0) is a FP of T.

Corollary 1

Let M be complete RCMS, define $T: \Omega \times M \to M$ such that

$\xi(d_B(\alpha(\daleth(\omega))\beta(\daleth(\omega))\, T\daleth(\omega), T\gimel(\omega))) \ \leq \ f(\xi\ (d_B(\daleth(\omega), T\daleth(\omega))), \zeta(d_B(\gimel(\omega), T\gimel(\omega))))$ (21)

$\forall \ \daleth(\omega), \gimel(\omega) \in \Omega \times M, \xi \in \Psi$ and $\zeta \in \Phi$. If it holds (i),(ii) and (iii) of theorem (1) and equation (2). Then T has a UFP.

Corollary 2

Let M be complete RCMS, define $T: \Omega \times M \rightarrow M$ such that

$$\xi(d_B(\alpha(\daleth(\omega))\beta(\daleth(\omega))\, T\daleth(\omega), T\gimel(\omega))) \leq f(\xi(d_B(\daleth(\omega), T\daleth(\omega))), \zeta(d_B(\gimel(\omega), T\daleth(\omega)))) \quad (22)$$

$\forall\ \daleth(\omega), \gimel(\omega) \in \Omega \times M, \xi \in \Psi$ and $\zeta \in \Phi$. If it holds (i),(ii) and (iii) of theorem (1) and equation (2). Then T has a UFP.

3 APPLICATIONS

Theorem 2

Consider a complete RCMS (M, d_B) and let $T: \Omega \times M \rightarrow M$ be (α, β) −admissible mapping

$$\xi\left(d_B\left(T\daleth(\omega), T\gimel(\omega)\right)\right) \leq \xi(d_B(\daleth(\omega), y(\omega))) - \zeta(d_B(\daleth(\omega), y(\omega))) \qquad (23)$$

for all $\daleth(\omega), \gimel(\omega) \in \Omega \times M$ with $\alpha(\daleth(\omega))\beta(\daleth(\omega)) - e \in \mathcal{P}$,where $\xi \in \Psi$ and $\zeta \in \Phi$ such that

$$\zeta(\xi(t)) \leq \zeta(t) \text{ for all } t > 0 \qquad (24)$$

$\daleth_0(\omega) \in \Omega \times M$ such that $\alpha(\daleth_0(\omega)) - e \in \mathcal{P}$ and $\beta(\daleth_0(\omega)) - e \in \mathcal{P}$. Suppose that $\alpha(\daleth_0(\omega)) - e \in \mathcal{P}$ and $\beta(\daleth_0(\omega)) - e \in \mathcal{P}$, where $\daleth_0(\omega) \in \Omega \times M$ such that

If either

(1) T is continuous or

(2) if $\{\daleth_n(\omega)\} \subset \Omega \times M$ is a sequence such that $lim_{n\rightarrow\infty}\, d_B(\daleth_n(\omega), \daleth(\omega)) = 0$ and $\beta(\daleth_n(\omega)) - e \in \mathcal{P}\ \forall\ n = 1,2,3, \ldots$ then $\beta(\daleth(\omega)) - e \in \mathcal{P}$.

Then T has a FP in M. If $\alpha(\daleth(\omega))\beta(\gimel(\omega)) - e \in \mathcal{P}$ for all fixed points $\daleth(\omega), \gimel(\omega)$ of T, then T has UFP.

Proof

Let $f(s,t) = s - \zeta(s),\ \forall\ s, t \in \mathcal{P}$ where $k < \frac{3}{5}$. Then f is the $C -$ class function.

For any $\daleth(\omega), \gimel(\omega) \in \Omega \times M$ with $\alpha(\daleth(\omega))\beta(\daleth(\omega)) - e \in \mathcal{P}$

$$0 \leq \xi(d_B(\daleth(\omega), \gimel(\omega))) - \xi(d_B(\daleth(\omega), \gimel(\omega))) - \xi(d_B(T\daleth(\omega), T\gimel(\omega)))$$

$$\leq \xi(d_B(\daleth(\omega),\gimel(\omega))) - \zeta(\ \xi(d_B(\daleth(\omega),\gimel(\omega)))) - \xi(d_B(T\daleth(\omega),T\gimel(\omega)))\backslash\backslash$$

$$= f(\xi(d_B(\daleth(\omega),\gimel(\omega)),\zeta(d_B(\daleth(\omega),\gimel(\omega))) - \xi(d_B(T\daleth(\omega),T\gimel(\omega))).$$

This result is same as from (1) and also it satisfies all the conditions of theorem (1).

Corollary 3

Consider a complete RCMS (M,d_B) and let $T:\Omega\times M\to M$ be (α,β) −admissible mapping

$$d_B\big(T\daleth(\omega),T\gimel(\omega)\big) \leq d_B(\daleth(\omega),\gimel(\omega)) - \zeta(d_B(\daleth(\omega),\gimel(\omega))) \qquad (25)$$

for all$\daleth(\omega),\gimel(\omega)\in\Omega\times M$ with $\alpha\ \big(\daleth(\omega)\big)\beta\big(\daleth(\omega)\big) - e\in\mathcal{P}$, where $\phi\in\Phi$

Suppose that $\alpha\big(\daleth_0(\omega)\big) - e\in\mathcal{P}$ $ and $\beta\big(\daleth_0(\omega)\big) - e\in\mathcal{P}$, where $\daleth_0(\omega)\in\Omega\times M$.

Assume that the condition of (iii) in theorem (1) and $\alpha\big(\daleth(\omega)\big)\beta\big(\daleth(\omega)\big) - e\in\mathcal{P}$ for all fixed points of T. Then it has a UFP of T.

Take $\alpha\big(\daleth(\omega)\big) = \beta\big(\daleth(\omega)\big) = e$ in theorem 2, it has a following results.

Corollary 4

Consider a complete RCMS (M,d_B) and define $T:\Omega\times M\to M$ such that

$$\xi(d_B(T\daleth(\omega),T\gimel(\omega)))\ \leq \xi(d_B(\daleth(\omega),\gimel(\omega))) - \zeta(d_B(\daleth(\omega),\gimel(\omega)))$$

$\forall\ \daleth(\omega),\gimel(\omega)\in\Omega\times M$, $\xi\in\Psi$ and $\zeta\in\Phi$ with $\zeta\big(\xi(t)\big)\leq\zeta(t)$ for all $t>0$.

Then T has UFP in $\Omega\times M$.

Corollary 5

Consider a complete RCMS (M,d_B) and define $T:\Omega\times M\to M$ such that

$$d_B(T\daleth(\omega),T\gimel(\omega))\ \leq d_B(\daleth(\omega),\gimel(\omega)) - \zeta(d_B(\daleth(\omega),\gimel(\omega)))$$

$\forall\ \daleth(\omega),\gimel(\omega)\in\Omega\times M$, where $\zeta\in\Phi$. Then T has UFP in $\Omega\times M$.

CONCLUSION

In this article, we discussed fixed point(FP) results in complete random cone metric space via Banach algebra. Also, we gave an important example and corollaries related to our main result.

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