

# A Remark on “Do Triaxial Supermassive Compact Star Exist?”

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## ABSTRACT

Taking into account three facts — (i) the ambipolar diffusion in Neutron Star core is expected to be the dominant mode of field decay in the early evolution of magnetar ( i.e as long as the ages much less than  $\sim 10^4$  years), (ii) magnetar field decay is negligible as long as the core temperature (T) is high i.e. few times of  $10^8$  K but less than  $10^9$  K , (iii) internal magnetic fields remain so strong at the early phase of magnetar that core temperature stays higher than several times  $10^8$  K for at least  $10^3$  years (proposed by Dall'Osso et al) we show six magnetars, e.g., SGR1806-20, SwiftJ1818.0-1607, IE1547.0-5408, PSRJ1846-0258, SGR 1900+14 and CXOU J171405.7-381031 are real triaxial stars. These stars will enter into magnetar phase (so called magnetar) after elapse of some periods in triaxial phase. The significance of our result is —a) physically stable triaxial star is possible, b) triaxial phase is stable for a long period ranging from few years to few hundred years, c) detection of such triaxial stars through gravitational waves as well as electro-magnetic counterparts is possible.

*Keywords: Gravitational waves; neutron star; magnetar; triaxial star.*

## 1. INTRODUCTION

In the rapid communication paper : “ Do triaxial supermassive compact stars exist ? ”, Phys. Rev. D 94, 101302 R (2016) Uryū et al [1] proposed a way in finding the triaxial star —

- a) first consider quasi-equilibrium solutions of triaxially deformed rotating neutron star ,
- b) then generalized it (i.e. Jacobi ellipsoid) under relativistic gravity and compressible equation of state (EoS),
- c) and search the difference in the maximum masses of triaxial and axisymmetric solutions.

If this difference is only about 10% then it provides a strong evidence of triaxial star whose EoS of high density matter in its (neutron star) core is softer (i.e. phase transition of high density matter) a probe through the detection of gravitational waves under fall back accretion.

This means in modeling the realistic neutron star (NS), a supra-massive triaxial neutron star would form satisfying the Equation of State (EoS) softer for the higher densities inside the core and the EoS is stiffer at lower densities (i.e. outer core) [2,3]. Based on the recent successful detection of gravitational waves from a binary black hole merger they propose accretion induced neutron stars are very promising source of triaxial compact stars that can be tested through detection of emitted gravitational waves from them (neutron stars ) for compactness  $M/R \sim 0.2 - 0.3$  . In this paper we emphasis on that neutron stars (i.e., magnetars) , having ambipolar diffusion in their cores than the accreting new born neutron stars (i.e., magnetars) are the promising source triaxial stars (Compact Stars).

## 2. METHODOLOGY

Analysis of observational data mentioned in [4] for newborn accreting magnetars hints that

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- a) the newborn millisecond magnetars can survive only a few tens of seconds (i.e. extremely short) if the initial magnetar mass =  $1.4 M_{\odot}$  and if the magnetar mass up to  $2.9 M_{\odot}$  then it would collapse to a black-hole.
- b) With the increase in accretion rate the magnetar mass also increases resulting which the column height ( $h_{ac}$ ) decreases rapidly ( $t \sim 4.3$  s) and subsequently decreases slowly.
- c) The magnetar spins up due to an accretion torque. This means the accretion torque is dominant for the spin up evaluation of the magnetar that requires accretion at a very high rate in comparison to the GW radiation and dipole radiation at a relatively low luminosity.
- d) The observed fact is the dipole radiation luminosity is several orders of magnitude larger than the GW radiation luminosity. Hence the available period shortens from initial 1.00 ms to 0.40 ms before the magnetar collapses to a black hole. This means it is very difficult to detect when the magnetar reaches its break up limit (for example, considering deformation and general relativity effects the break up limit of a non-rotating neutron star with maximum mass and EoS is  $\sim 0.82$  ms).

“Studies [5,6] of early evolution of newly born magnetars with strong magnetic fields suggest ambipolar diffusion in the core of the magnetar has an important role on the stability during its early phase”. If magnetars are born with millisecond spin period then

- i) their initial spin energy will be  $E_{spin} \sim 3 \times 10^{52} (P_{initial} / ms)^{-2}$  erg.
- ii) The spin-down time scale through magnetic dipole radiation will be  $\sim 1$  day for external dipole fields in the  $10^{14}$  G range.
- iii) For newly formed magnetars with dipole fields in excess of  $\sim (6-7) \times 10^{14}$  G and spin period of  $\approx 1$  ms are strongly magnetized.
- iv) In the magnetar interior the possible field strength is in the range  $\sim 10^{16}$  G and this super-strong magnetic fields possibly have a relevant role on GW emission in the early evolutionary phase of these magnetars.
- v) The generation mechanism for the super-strong internal field suggests the magnetic axis is expected to be just tilted (i.e. tilt angle  $\chi$ ), initially, to the spin axis and because of this for small  $\chi$ , the GW luminosity is largely

suppressed and the total rate of energy emission ( $\dot{E}$ ) [7] is

$$\dot{E}_{GW} = (-2/5) \cdot \{G (I \epsilon_{\beta})^2 / c^5\} \cdot \omega^6 \cdot \text{Sin}^2 \chi (1 + 15 \text{Sin}^2 \chi) \dots \dots \dots (1)$$

where  $\epsilon_{\beta}$  = magnetic deformation,  $I$  = moment of inertia of spherical neutron star with its spin down ( $\omega$ ).

- vi) This orthogonalization process occurs so quick, enough for strong GW emission making it competitive with magneto-dipole radiation (i.e., electromagnetic radiation).
- vii) “Several models [8–10] have been suggested for a secular increase of the tilt angle  $\chi$  between the magnetic dipole and the spin axis”. “But most promising model (given by Blandford and Romani [11]) that identify a secular increase in the surface magnetic field of young neutron stars (with braking index  $n < 3$ ) for a period at least over the first  $\sim 10^3 - 10^4$  years”.
- viii) The above identified property for  $10^3 - 10^4$  years old pulsars with braking index lower than  $n$  less than 3 (i.e.,  $n < 3$ ) is also valid for magnetar candidates also having similar ages and similar high glitch activities [12, 13].
- ix) The identified three separate processes [14,15] for secular magnetic field evolution in the neutron star interior are : Ohmic dissipation and ambipolar diffusion (these two affect directly the dissipation in the NS interior) while the third one is the Hall drift which conserves magnetic energy (indirectly affect the dissipation). Regarding the magnetar early period of evolution (i.e. as long as the age much less than  $\sim 10^4$  years) ambipolar diffusion in the NS core is expected to be dominant mode of field decay [5].
- x) Ambipolar diffusion, in fact, drives a slow motion of charged particles w.r.t. background neutrons, which is opposed by both particle friction and chemical potential gradients in the stably stratified medium inside neutron star. This means ambipolar diffusion has two effective modes on chemical composition: a) the solenoidal mode

which does not perturb chemical equilibrium (i.e. counteracted only by particle friction) and b) the irrotational mode that does perturb chemical equilibrium and cannot be active on time scales shorter than the  $\beta$ -reaction time scale.

Considering the occurrence of field decay on the same time scale in both modes the investigation result [14] showed at temperature  $T > 10^9$  K the field decay time scale

$$t_d^{(early)} \cong 2.2 \times 10^4 (T / 10^9 \text{ K}) \cdot (\rho_{15} / 0.7)^{2/3} \cdot (B / 10^{16} \text{ G})^{-2} \text{ years} \dots\dots\dots (2)$$

where  $T$ ,  $\rho$  and  $B$  are the temperature, density and magnetic field inside the core, respectively, is much longer than the NS age or its cooling-time scale.

This means field decay is negligible as long as the temperature is this high. In respect of magnetar ultra-strong magnetic field this means field decay at temperature  $T > 10^9$  K is not frozen if the interior magnetic field  $B_i$  is longer than  $10^{16}$  G. In other words, consideration of ambipolar diffusion in the neutron star core signifies that the NS core remain at fairly high temperature (i.e. few times of  $10^8$  K but less than  $10^9$  K) for a long time if the decaying field is  $\sim 10^{16}$  G [5]. In view of magnetar early evolutionary phase Dall’Osso et al [5] show the ambipolar process can prevent the cooling of the magnetar core below a temperature  $\sim 10^9$  K for hundreds to thousands years ( the magnetic field can remain strong at least for  $10^3$  years ).

It is required a stable triaxially phased magnetars for detection them. Ambipolar diffusion in magnetar interior (i.e. core) provides an opportunity for high temperature and strong magnetic energized magnetar core for long

period ranging  $\sim 10^3 - 10^4$  years. At present 31 magnetars have been detected. Out of them only six magnetars e.g., SGR 1806 – 20 , Swift J1818.0 – 1607, IE1547.0 – 5408, PSR J1846 – 0258, SGR 1900 + 14 and CXOU J171405.7 – 381031, are in the newly / early born phases whose ages are less than  $10^3$  years. Other magnetars whose characteristic ages are more than  $10^3$  years.

### 3. OBSERVATION

We choose that magnetars whose ages (characteristic) are less than  $10^3$  years so that their triaxial phases remain for a long period (at least few times of 10 years) for observational purpose (see Table 1). For example, magnetar SGR 1806 – 20 and Swift J1818.0 – 1607 are very young (or baby) magnetars having ages  $\sim 240$  years only. So, their triaxial phases would continue for next 1000 to 10000 years, at least 760 years. Similarly, for other four magnetars i.e. IE 1547.0 – 5408 , PSR J1846 – 0258 , SGR 1900 + 14 and CXOU J171405.7 – 381031 their triaxial phases would stay for 310 years, 270 years, 100 years and 50 years, respectively. Regarding ellipticity for Swift J1818.0 – 1607 and PSR J1846 -0258 are  $9.678 \times 10^{-3}$  and  $1.7709 \times 10^{-3}$ , respectively, which are more effective for becoming good triaxial stars in comparison to other four magnetars. The internal core magnetic fields of these two magnetars are  $9.67 \times 10^{17}$  G and  $1.770 \times 10^{17}$  G, respectively i.e. one order less than that of the other four magnetars.

**Table 1. Various parameters of six magnetars**

<b>Magnetar</b>	<b>Period (P) s</b>	<b><math>\dot{P}</math> s.s<sup>-1</sup></b>	<b>Present Age</b>	<b>Dipole Field G</b>	<b>Internal Field (B<sub>toroidal</sub>) G</b>	<b>Ellipticity <math>\epsilon</math></b>	<b>Remaining Period(at least) of Triaxial Phase</b>
1E 1547-5408	2.0721255	$4.77 \times 10^{-11}$	690 yrs	$3.2 \times 10^{14}$	$1.14 \times 10^{18}$	$1.14 \times 10^{-2}$	310 yrs
CXOU J171405.7-381031	3.825352	$6.40 \times 10^{-11}$	950 yrs	$5.0 \times 10^{14}$	$1.936 \times 10^{18}$	$1.936 \times 10^{-2}$	50 yrs
SGR 1806 -20	7.54773	$49.5 \times 10^{-11}$	240 yrs	$20 \times 10^{14}$	$5.439 \times 10^{18}$	$5.4394 \times 10^{-2}$	760 yrs.
Swift J1818.0-1607	1.36349	$9.0 \times 10^{-11}$	240 yrs	$3.5 \times 10^{14}$	$9.67137 \times 10^{17}$	$9.678 \times 10^{-3}$	760 yrs.
SGR 1900+14	5.19987	$9.2 \times 10^{-11}$	900 yrs	$7.0 \times 10^{14}$	$1.36778 \times 10^{18}$	$1.367 \times 10^{-2}$	100 yrs
PSR J1846 - 0258	0.326571	$0.71 \times 10^{-11}$	730 yrs	$0.49 \times 10^{14}$	$1.7709 \times 10^{17}$	$1.7709 \times 10^{-2}$	270 yrs.

As these two magnetars are baby magnetars, their internal magnetic field would be  $\sim 10^{18}$  G when they become aged. Comparing the ages (characteristic), ellipticity and core magnetic field strengths of the above six magnetars one can conclude — although these six magnetars are triaxial stars but Swift J1818.0 – 1607 is the best triaxial star among them. In other words, the magnetar Swift J1818.0–1607 offers the astronomers an opportunity as a triaxial compact object where various properties of the magnetar can be measured.

#### 4. CONCLUSION

We conclude that the above six magnetars e.g., six magnetars i.e., SGR 1806 – 20, Swift J1818.0 – 1607, IE1547.0 – 5408, PSR J1846 – 0258, SGR 1900 + 14 and CXOU J171405.7 – 381031, are real Triaxial Stars and precisely, they are at present passing through their triaxial phases. The magnetar Swift J1818.0-1607 is a very young, baby triaxial star can be treated as the most promising source for observation.

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#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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