Short communication

A Remark on “Do triaxial Supramassive Compact Star exist ?”

Abstract : Taking into account three facts — (i) the ambipolar diffusion in Neutron Star core is expected to be the dominant mode of field decay in the early evolution of magnetar (i.e as long as the ages much less than ~ 10^4 years), (ii) magnetar field decay is negligible as long as the core temperature (T) is high i.e. few times of 10^8 K but less than 10^9 K , (iii) internal magnetic fields remain so strong at the early phase of magnetar that core temperature stays higher than several times 10^8 K for at least 10^3 years (proposed by Dall’Osso et al [5]) we show six magnetars , e.g. SGR 1806-20, Swift J1818.0-1607, IE1547.0-5408, PSRJ1846-0258, SGR 1900+14 and CX0U J171405.7-381031 are real triaxial stars . After elapse of some periods in triaxial phase these stars will enter into magnetar phase (so called magnetar). The significance of our result is —a) physically stable triaxial star is possible , b) triaxial phase is stable for a long period ranging from few years to few hundred years , c) detection of such triaxial stars through gravitational waves as well as electro-magnetic counterparts is possible.

Key Words: Gravitational Waves, Neutron Star, Magnetar, Triaxial Star

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Introduction

In the rapid communication paper : “ Do triaxial supramassive compact stars exist ? ”, Phys. Rev. D **94**, 101302 R (2016) Uryū et al [1] proposed a way in finding the triaxial star —

a) first consider quasi-equilibrium solutions of triaxially deformed rotating neutron star ,
b) then generalized it (i.e. Jacobi ellipsoid) under relativistic gravity and compressible equation of state (EoS),
c) and search the difference in the maximum masses of triaxial and axisymmetric solutions.

If this difference is only about 10% then it provides a strong evidence of triaxial star whose EoS of high density matter in its (neutron star) core is softer (i.e. phase transition of high density matter) a probe through the detection of gravitational waves under fall back accretion. This means in modeling the realistic neutron star (NS), a supra-massive triaxial neutron star would form satisfying the EoS softer for the higher densities inside the core and the EoS is stiffer at lower densities (i.e. outer core). Based on the recent successful detection of gravitational waves from a binary black hole merger they propose accretion induced neutron stars are very promising source of triaxial compact stars that can be tested through detection of emitted gravitational waves from them (neutron stars) for compactness M/R ~ 0.2 – 0.3 . In this paper we emphasis on that neutron stars (i.e. magnetars) , having ambipolar diffusion in their cores than the accreting new born neutron stars (i.e. Magnetars) are the promising source triaxial stars (Compact Stars).

Analysis of observational data [4] for newborn accreting magnetars hints that

a) the newborn millisecond magnetars can survive only a few tens of seconds (i.e. extremely short) if the initial magnetar mass = 1.4 $M_\odot$ and if the magnetar mass up to 2.9 $M_\odot$ then it would collapse to a black-hole.
b) With the increase in accretion rate the magnetar mass also increases resulting which the column height (h_{ac}) decreases rapidly (t ~ 4.3 s) and subsequently decreases slowly.
c) The magnetar spins up due to an accretion torque . This means the accretion torque is dominant for the spin up evaluation of the magnetar that requires accretion at a very high rate in comparison to the GW radiation and dipole radiation at a relatively low luminosity.
d) The observed fact is the dipole radiation luminosity is several orders of magnitude larger than the GW radiation luminosity. Hence the available period shortens from initial 1.00 ms to 0.40 ms before the magnetar collapses to a black hole. This means it is very difficult to detect when the magnetar reaches its break up limit (for example, considering deformation and general relativity effects the break up limit of a non-rotating neutron star with maximum mass and EoS is ~ 0.82 ms.

Studies [5,6] of early evolution of newly born magnetars with strong magnetic fields suggest

ambipolar diffusion in the core of the magnetar has an important role on the stability during its early phase. If magnetars are born with millisecond spin period then

i) their initial spin energy will be $E_{spin} \sim 3 \times 10^{52} (P_{initial} / ms)^{-2}$ erg.
ii) The spin-down time scale through magnetic dipole radiation will be ~ 1 day for external dipole fields in the 10^{14} G range.
iii) For newly formed magnetars with dipole fields in excess of ~ (6 – 7) x 10^{14} G and spin period of ≈ 1 ms are strongly magnetized.

iv) In the magnetar interior the possible field strength is in the range ~10^{16} G and this super-strong magnetic fields possibly have a relevant role on GW emission in the early evolutionary phase of these magnetars.

v) The generation mechanism for the super-strong internal field suggests the magnetic axis is expected to be just tilted (i.e. angle χ), initially, to the spin axis and because of this for small χ, the GW luminosity is largely suppressed as per the relation [7]

$$\dot{E}_{GW} = (- 2/5) . \{G (I \epsilon_{\beta})^2 / c^5\} . w^6 . \mathrm{Sin}^2 \chi (1 + 15 \mathrm{Sin}^2 \chi) \quad \ldots\ldots\ldots\ldots\ldots.(1)$$

vi) This orthogonalization process occurs so quick, enough for strong GW emission making it competitive with magneto-dipole radiation (i.e. electromagnetic radiation).

vii) Several models [8 – 10] have been suggested for a secular increase of the tilt angle χ between the magnetic dipole and the spin axis. But most promising model (given by Blandford and Romani [11]) that identify a secular increase in the surface magnetic field of young neutron stars (with braking index $n < 3$) for a period at least over the first ~ $10^3 – 10^4$ years.

viii) The above identified property for. $10^3 – 10^4$ years old pulsars with braking index lower than 'n' less than 3 is also valid for magnetar candidates also having similar ages and similar high glitch activities [12, 13].

ix) The identified three separate processes [14, 15] for secular magnetic field evolution in the neutron star interior are : Ohmic dissipation and ambipolar diffusion (these two affect directly the dissipation in the NS interior) while the third one is the Hall drift which com magnetic energy (indirectly affect the dissipation). Regarding the magnetar early period of evolution (i.e. as long as the age much less that ~ 10^4 years) ambipolar diffusion in the NS core is expected to be dominant mode of field decay [5].

x) Ambipolar diffusion , in fact, drives a slow motion of charged particles w.r.t. back ground neutrons, which is opposed by both particle friction and chemical potential gradients in the stably stratified medium inside neutron star. This means ambipolar diffusion has two effective modes on chemical composition: a) the solenoidal mode

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which does not perturb chemical equilibrium (i.e. counteracted only by particle friction) and b) the irrotational mode that does perturb chemical equilibrium and can not be active on time scales shorter than the β-reaction time scale. Considering the occurrence of field decay on the same time scale in both modes the investigation result [14] showed at temperature $T > 10^9$ K the field decay time scale

$$t_d^{(early)} \cong 2.2 \times 10^4 (T / 10^9 K) . (\rho_{15} / 0.7)^{2/3} . (B / 10^{16} G)^{-2} \text{ yrs} \ldots\ldots\ldots (2)$$

is much longer than the NS age or its cooling-time scale.

This means field decay is negligible as long as the temperature is this high. In respect of magnetar ultra-strong magnetic field this means field decay at temperature $T > 10^9$ K is not frozen if the interior magnetic field B_t is longer than 10^{16} G. In other words, consideration of ambipolar diffusion in the neutron star core signifies that the NS core remain at fairly high temperature (i.e. few times of 10^8 K but less than 10^9 K) for a long time if the decaying field is ~ 10^{16} G [5]. In view of magnetar early evolutionary phase Dall'Osso et al [5] show the ambipolar process can prevent the cooling of the magnetar core below a temperature ~ 10^9 K for hundreds to thousands years (the magnetic field can remain strong at least for 10^3 years).

It is required a stable triaxially phased magnetars for detection them. Ambipolar diffusion in magnetar interior (i.e. core) provides an opportunity for high temperature and strong magnetic energized magnetar core for long period ranging ~ $10^3 – 10^4$ years. At present 31 magnetars have been detected. Out of them only six magnetars e.g. SGR 1806 – 20 , Swift J1818.0 – 1607, IE1547.0 – 5408, PSR J1846 – 0258, SGR 1900 + 14 and CXOU J171405.7 – 381031, are in the newly / early born phases whose ages are less than 10^3 years. Other magnetars whose characteristic ages are more than 10^3 years.

So, we choose that magnetars whose ages (characteristic) are less than 10^3 years so that their triaxial phases remain for a long period (at least few times of 10 years) which will be helpful for observation (see table I). For example, magnetar SGR 1806 – 20 and Swift J1818.0 – 1607 are very young (or baby) magnetars having ages ~ 240 years only. So, their triaxial phases would continue for next 1000 to 10000 years, at least 760 years. Similarly, for other four magnetars i.e. IE 1547.0 – 5408 , PSR J1846 – 0258 , SGR 1900 + 14 and CXOU J171405.7 – 381031 their triaxial phases would stay for 310 years, 270 years, 100 years and 50 years, respectively. Regarding ellipticity for Swift J1818.0 – 1607 and PSR J1846 -0258 are 9.678×10^{-3} and 1.7709×10^{-3}, respectively, which are more effective for becoming good triaxial stars in comparison to other four magnetars. The internal core magnetic fields of these two magnetars are 9.67×10^{17} G and 1.770×10^{17} G, respectively i.e. one order less than that of the other four magnetars.

Table I : Various parameters of six magnetars

Magnetar	Period (P) s	$\dot{P}$ s.s^{-1}	Present Age	Dipole Field G	Internal Field ($B_{toroidal}$) G	Ellipticity ε	Remaining Period(at least) of Triaxial Phase
1E 1547-5408	2.0721255	4.77×10^{-11}	690 yrs	3.2×10^{14}	1.14×10^{18}	1.14×10^{-2}	310 yrs
CXOU							

J171405.7-381031	3.825352	6.40 x 10^{-11}	950 yrs	5.0 x 10^{14}	1.936 x 10^{18}	1.936 x 10^{-2}	50 yrs
SGR 1806 -20	7.54773	49.5 x 10^{-11}	240 yrs	20 x 10^{14}	5.439 x 10^{18}	5.4394 x 10^{-2}	760 yrs.
Swift J1818.0-1607	1.36349	9.0 x 10^{-11}	240 yrs	3.5 x 10^{14}	9.67137 x 10^{17}	9.678 x 10^{-3}	760 yrs.
SGR 1900+14	5.19987	9.2 x 10^{-11}	900 yrs	7.0 x 10^{14}	1.36778 x 10^{18}	1.367 x 10^{-2}	100 yrs
PSR J1846 - 0258	0.326571	0.71 x 10^{-11}	730 yrs	0.49 x 10^{14}	1.7709 x 10^{17}	1.7709 x 10^{-2}	270 yrs.

As these two magnetars are baby magnetars, their internal magnetic field would be ~ 10^{18} G when they become aged. Comparing the ages (characteristic), ellipticity and core magnetic field strengths of the above six magnetars one can conclude — although these six magnetars are triaxial stars but Swift J1818.0 – 1607 is the best triaxial star among them. In other words, the magnetar Swift J1818.0 – 1607 offers the astronomers an opportunity as a triaxial compact object where various properties of the magnetar can be tested.

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