

**Energy losses by the interaction of charged particles with a
graphene sheet**

Abstract

Using the hydrodynamic model in the electrostatic approximation, we describe the formation of surface plasma on graphene when the charge is tilted towards the graphene sheet for motion. We calculated the electron energy loss spectrum of electrons at different angles and proved that the resonance in the spectrum is related to the frequency of graphene surface plasma, as the electron velocity decreases, the dispersion shifts to higher energies. An increase in the tilt angle will also shift the dispersion energy in a higher range.

Key words:, graphene; plasmonic; energy losses

INTRODUCTION

The hydrodynamic model for plasmonics is a macroscopic approach to a microscopic problem^[1-4]. This model combines Maxwell's equations, Euler's equation of hydrodynamics supplemented with a term due to the statistical pressure of an electron gas, and the continuity equation^[5, 6].

Graphene plasmons have recently emerged as a new research branch in photonics that has attracted substantial research efforts^[5-12]. The study of plasmons in graphene is the subject of a vast experimental and theoretical effort in the last years, due to its remarkable properties that make it preferable to other plasmonic materials like noble metals^[13-17]. Researchers have explored its capabilities taking advantage of the long plasmon lifetimes, low losses, high spatial confinement, and versatile tunability, together with the large electro-optical response provided by its two-dimensional (2D) geometry and peculiar electronic structure^[18-21]. Surface plasmons are the collective oscillations of charge at the interface of metal and medium, which belong to the electromagnetic response of free electrons on the metal surface to the incident electric field. However, the metal surface plasmon has a limited tuning range and weak field constraint. The graphene surface plasmon makes up for these deficiencies.

In studying planar surfaces with electron energy loss (EEL), a particularly instructive situation is when the electron is directed parallel to the surface. In order to further explore the relationship

between incident electron and electron energy loss in graphene plane, it is necessary to study the energy loss at oblique incidence. In this paper, we investigated the effect of mobile charges on surface plasmons of graphene, the EEL spectrum is also discussed from different perspectives, we discuss the influence of electron velocity and tilt angle on dispersion.

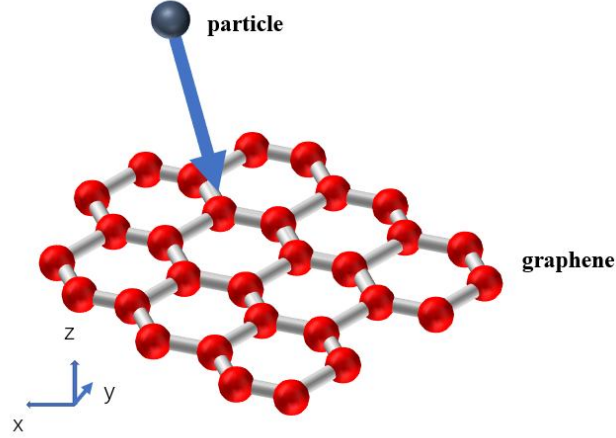


Fig.1. System considered in this paper: The electrons are tilted with respect to graphene. The effect of the interaction of the charge with the electron gas in graphene is studied using the hydrodynamic model, which has built in nonlocal corrections due to the statistical pressure of the electron gas. The charge induces surface plasmons in graphene which can be probed by EEL spectroscopy.

INDUCED ELECTROSTATIC POTENTIAL DUE TO A MOVING CHARGE

In this section we shall consider the calculation of the induced electrostatic potential $\varphi_{in}(\mathbf{r}, z, t)$ and induced electric field in graphene. the induced potential is given by^[22]

$$\varphi_{in}(\mathbf{k}, z, \omega) = \frac{n_0 e^3}{4\epsilon_0 m_g} \frac{e^{-k|z|}}{\omega^2 - \omega_{spp}^2} \int dz' e^{-k|z'|} \rho_{ex}(\mathbf{k}, z', \omega), \quad (1)$$

where n_0 is the 2D partical density. For computing these quantities in real space an inverse Fourier transform has to be performed,

$$\varphi_{in}(\mathbf{r}, z, t) = \int d\mathbf{k} d\omega e^{i(\mathbf{k}\mathbf{r} - \omega t)} \varphi_{in}(\mathbf{k}, z, \omega). \quad (2)$$

We shall consider the calculation of the induced electrostatic potential $\varphi_{in}(\mathbf{k}, z, \omega)$ and induced electric field in graphene, $E_{in}(\mathbf{k}, z, \omega) = -\nabla \varphi_{in}(\mathbf{k}, z, \omega)$, due to a charge Ze moving at the speed v .

We consider this calculation model

$$\rho_{ex}(\mathbf{r}, z, t) = Z\delta(y)\delta(x - v_x t)\delta(z - v_z t). \quad (3)$$

This equation represents the motion of a mobile charge tilted towards the graphene plane. For the sake of our later calculations, we now introduce a common prefactor $\varphi_0 = \frac{n_0 e^3}{4\epsilon_0 m_g}$ that will serve to make many integrals dimensionless. Note that φ_0 has units of electric potential. Since $k_F = \frac{2\pi}{\lambda_F}$, where λ_F is the Fermi wavelength, φ_0 can be interpreted as the average Coulomb energy between two particles in the electron gas. Substitute equation (3) into equation (1) and we get

$$\varphi_{in}(r, z, t) = \frac{\varphi_0 Z}{4\pi} \int dr' dz' dt' dk \rho_{ex}(r+r', z', t') k J_0(kr') e^{-k(|z|-|z'|)} \frac{\sin \omega_{spp} |t-t'|}{\omega_{spp}}. \quad (4)$$

And after performing integral we obtain

$$\varphi_{in}(r, z, t) = \frac{\varphi_0 Z}{4\pi} \int dt' dk k J_0(k\sqrt{y^2 + (x-v_x(t+t'))^2}) e^{-k(|z|-v_z|t+t'|)} \frac{\sin \omega_{spp} |t'|}{\omega_{spp}}, \quad (5)$$

where $v_x = v \cos \theta$, $v_z = v \sin \theta$, and let's say the Angle is $\frac{\pi}{6}$ and $\frac{\pi}{3}$. In figure 2 we represent $\varphi_{in}(r, 0, t)$ as function of the distance to the origin for six different times. For shorter times we see the formation of the surface plasmon wave. Over a longer period of time, the surface plasma has traveled a certain distance. It is clear that the electrostatic disturbance is not monochromatic since a single wavelength cannot be identified from the figure. As we will see in the next section this will translate into a non trivial spectrum for the energy loss of a charged particle when it transverses a graphene sheet.

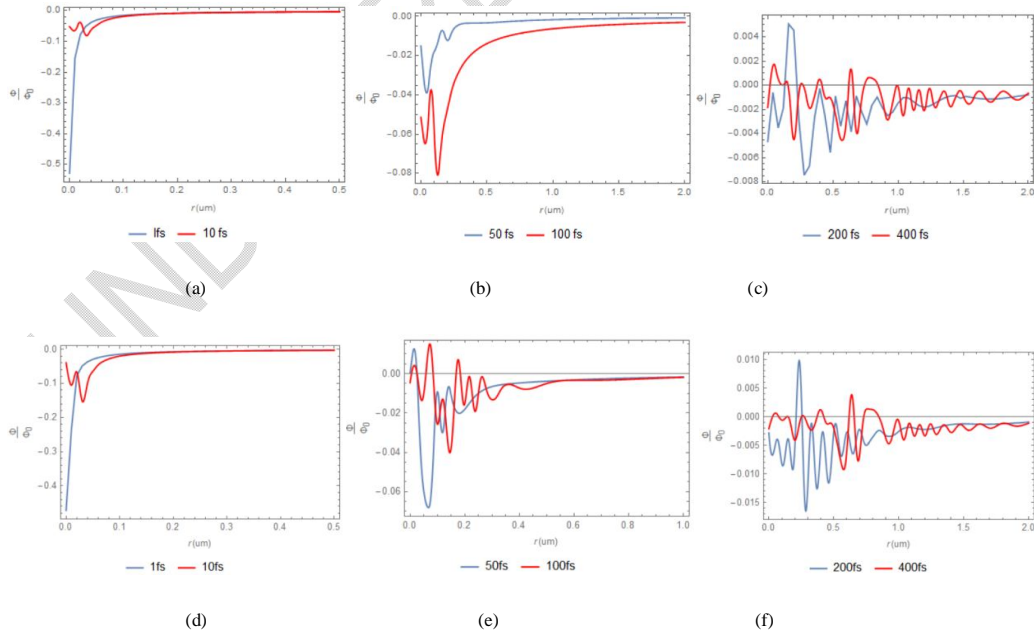


Fig.2. (a) The potential at $\frac{\pi}{6}$ for $t = 1, 10$ fs, (b) The potential at $\frac{\pi}{6}$ for $t = 50, 100$ fs, (c) The potential at $\frac{\pi}{6}$ for $t = 200,$

400 fs , (d) The potential at $\frac{\pi}{3}$ for t = 1, 10 fs ,(e) The potential at $\frac{\pi}{3}$ for t = 50, 100 fs , (f)The potential at $\frac{\pi}{3}$ for t = 200,

400 fs .And for a particle speed $v = 0.01c$. At large distances the induced potential approaches zero. At shorter time, we witness the formation of the surface plasmon polariton. The moving electric charge starts at the graphene sheet. FIG. c and FIG. f show that the charge modulation-related induced potential oscillates in the graphene sheet over time.

The EEL spectrum

Next we want to compute the EEL spectrum, to goal in view we need the quantity $E_z(\mathbf{r}, z, \omega) = -\partial \varphi_{in}(\vec{r}, z, \omega) / \partial z$ since by definition the EEL spectrum reads^[23]

$$\Gamma(\omega) = \frac{Ze}{\pi \hbar \omega} \int dt \Re \{ e^{i\omega t} \vec{v} \cdot \vec{E}(\mathbf{r}, v_z t, \omega) \}, \quad (6)$$

where $\vec{v} = (v_x, 0, v_z)$ and $Z = 1$ for the electron, and the symbol \Re stands for the real part. The

Fourier transform in \vec{r} and t of the charge distributions gives $\rho_{ex}(\mathbf{k}, z, \omega) = \frac{Z}{v_z} e^{i(\omega - k_x v_x) \frac{z}{v_z}}$. The

induced potential is given by

$$\varphi_{in}(\mathbf{k}, z, \omega) = \varphi_0 \frac{e^{-k|z|}}{\omega^2 - \omega_{spp}^2} \frac{2kz v_z}{(k v_z)^2 + (\omega - k_x v_x)^2}, \quad (7)$$

Fourier transforming to real space we have

$$\varphi_{in}(\mathbf{r}, z, \omega) = \varphi_0 \int_0^\infty \frac{dk_x dk_y}{4\pi^2} \frac{e^{-k|z|} e^{ik_x x}}{\omega^2 - \omega_{spp}^2} \frac{2kz v_z}{(k v_z)^2 + (\omega - k_x v_x)^2}. \quad (8)$$

Writing $\omega_{spp} = \sqrt{ak}$, where the parameter a is

$$a = \frac{2\alpha E_F c}{h}. \quad (9)$$

Therefore it follows that the EEL spectrum can be written as

$$\Gamma(\omega) = \frac{Ze}{\pi \hbar \omega} \varphi_0 (v \sin \theta - v \cos \theta) \int 2k \frac{dk_x dk_y}{4\pi^2} \frac{\omega + k \sin \theta v \sin \theta}{(\omega + k \sin \theta v \sin \theta)^2 - (\cos \theta kv)^2} \frac{2k}{(kv \cos \theta)^2 + (\omega - k \sin \theta v \sin \theta)^2}, \quad (10)$$

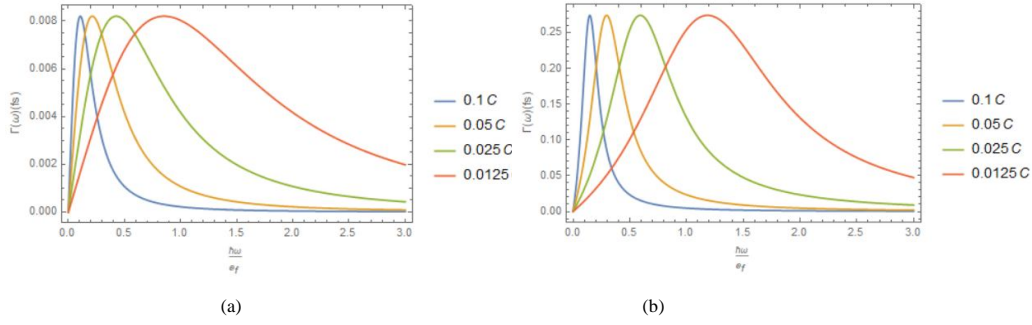


Fig.3. Loss spectrum as function of the energy for five speeds to the electron at $\frac{\pi}{6}$ and $\frac{\pi}{3}$. The peaks disperse as

function of the speed of the electrons. The long tail as function of frequency suggests that a continuum of surface plasmons is excited by the moving charge.

We plot $\Gamma(\omega)$ in figure 3. From this figure we see the loss spectrum shifts towards higher energies as the speed of the moving electron decreases. An increase in the tilt angle will also shift the dispersion energy in a higher range.

CONCLUSIONS

We have considered the problem of the excitation of surface plasmons in graphene by a fast moving charge. We analyzed the formation of surface plasmons when electrons are tilted towards graphene. As time evolves oscillatory behavior develops in the induced potential associated with the modulation of the electronic charge in the graphene sheet. We calculated the EEL spectrum at different angles, as we can see, as \vec{v} decreases and increase of tilt angle, the frequency maximum shifts to a higher frequency. This provides a new way for us to explore the energy loss of graphene.

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