Original Research Article

TREATMENT AND SLIP EFFECT ON MHD BLOOD FLOW THROUGH A STENOTIC ARTERY: A MATHEMATICAL MODEL.

Abstract

Theoretical study of the treatment and slip effect on Magneto-hydrodynamic (MHD) blood flow through a stenotic artery where the porous wall and the boundary slip at the wall of the artery have a significant effect on the blood flow was investigated. The magnetic field is applied radially on the stenotic region of the artery with blood flowing axially. Dimensionless governing equations was solved using Frobenius series method to obtain a solution for the velocity of the blood flow, blood concentration, wall shear stress, fluid acceleration and the volumetric flow rate. The Results showed that, increase in the chemical reaction increases the blood viscosity whereby decreasing the blood velocity, acceleration and shear stress at the wall but increases the volumetric flow rate of the blood while increased slip at the wall with stenosis decreases the blood velocity, acceleration and volumetric flow rate but causes an increase in the wall shear stress.

Keywords: Magneto-hydrodynamic (MHD), Slip at the Wall, Chemical Reaction, Pulsatile Pressure, Permeability of Porous Wall.

Nomenclature

P_s	Steady State Pressure Gradient	Φ	Phase Difference Angle
P_p	Pulsatile State Pressure Gradient	t	Time
w_h	Amplitude of Frequency Pulse Rate	C	Concentration of Blood
w_b	Amplitude of Frequency of the Body	θ	Temperature of Blood
f_h	Frequency of the Heart Pulse	B_m	Applied Magnetic field
f_b	Frequency of Body Acceleration	g	Acceleration due to gravity
G_b	Body Acceleration	D	Coefficient of Mass Diffusity
$G(t)$	Time Dependent Body acceleration	K	Coefficient of Porosity
$\emptyset$	Inclination of stenosed artery	Re	Reynolds number

C_p	Heat capacity
Q	Heat source term
Pe	Peclet number
S_c	Schmidt number
μ	Coefficient of Blood viscosity
ρ	Density of the Blood
Kr	Chemical Reaction
M	Magnetic Parameter
σ	Electrical Conductivity of Blood
C_w	Concentration of the Artery wall
T_w	Temperature of the Artery wall
G_r	Grashof Temperature number
G_c	Grashof Diffusion number
β_T	Coefficient of Volume expansion by temperature
β_C	Coefficient of Volume expansion by concentration
δ	Height of Stenosis
l_0	Length of Stenosis
R(z)	Stenotic Vessel Radius
R_0	Normal Artery Radius
s	Shape of Stenosis
ξ	Tapering
d	Position of Stenosis
z	Axial Direction for Flow of Blood
r	Radial Direction for Flow of Blood
$\vec{J}$	Current Density

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1. Introduction.

This theoretical work considers the flow of blood through an inclined artery having stenosis at the artery walls with the effect of heat source and body acceleration which has immense significance and importance in the growth of tumor and cardiovascular disease. Certain factors can affect the viscosity of the blood where the shear stress could reduce its viscosity. Pulsatile blood flow past an artery has caught the attention from researchers because of the relevant applicability in the biotechnological, biomedical and medical sciences. Blood circulation takes place when blood is pumped from the heart to different muscles of the body through the arteries which transports the blood with the pressure gradient creating a pulsatile flow of the blood. The application of induced chemical reaction to the areas where the stenosis is present due to plaques of cholesterols, could help to increase blood flow and treat diseases resulting to hypotension and cardiac failure. Furthermore the application of induced chemicals, drugs and slip at the wall can help treat ailments such as cancer and tumor growth.

Stenosis in the artery affects blood flowing from the heart through the artery Rabbi et al. [1] and Ellahi et al. [2]. Pralhad et al. [3] studied the blood flow through the artery with stenosis with the wall shear stress and the resistance at the wall while Ellahi et al. [4] did a mathematical model explaining the blood flow through an artery with a composite stenosis. Magnetic field applied on bio fluids have effect on the dynamics of the fluid hence this fluids are bio magnetic fluids with rich application in medical sciences and biomedical engineering. Haik et al. [5] gave a clear distinction between bio magnetic fluid (BFD) and hydro magnetic fluid (MHD). Abdullah et al. [6] studied the effect of magneto-hydrodynamic on the flow of blood through a stenosis that is irregular with the results obtained from the study showing that, the rate of flow of blood reduced as a result of magnetic field applied to the arterial segment.

Srivastava [8] did an analysis of the blood flow motion that is steady through an artery inclined with applied magnetic field with the conclusion that velocity of the blood flow decreases as a result of the increase in the magnetic field.

Sinha et al. [11] did a study on the slip and periodic body acceleration effect on pulsatile flow of blood passing through a segmented stenosed artery. Tripathi and Sharma [13] did a study on heat and mass transfer effects of a blood flow two phases which is pulsatile past a stenosed artery that is narrow with chemical reaction and radiation. Karthikeyan and Jeevitha [14] did a study analysis on the effect of heat and mass transfer on a model in two phases for unsteady blood flow that is pulsatile past an artery with stenosis having a wall that is permeable with radiation and chemical reaction effects. Amos and Ogulu [16] studied the magnetic field effect on Pulsatile Blood Flow through an axis-symmetric channel that is constricted. Bunonyo and Amos [17] studied the effect of lipid concentration on the blood that flows through an artery channel inclined with magnetic field present.

Thermal radiation had a mixed effect on the blood flow and heat source increase increased the blood flow which was helpful in treating tumor and low blood pressure as discussed by Omamoke et al. [18]. The adoption of Frobenuis method and effects of pertinent parameters on the behavior of flow was considered in the research done by Omamoke and Amos [19], Amos et al. [20], Amos et al. [21], Chinedu and Amadi [22].

This study has theoretically analyzed, showing the effects of slip, heat source, body acceleration, inclined artery angle and pulsatile pressure on the non-Newtonian unsteady blood flow through a stenotic artery. The artery walls is porous and permeable with the analysis done by solving the problem of the governing equation using the Frobenuis power series method to obtain the velocity of the blood flow, blood acceleration, wall shear stress and volumetric flow rate solutions with a graph illustrating the behavior of the effect of slip, heat source, body acceleration, magnetic field, pulsatile pressure gradient and inclined artery on the blood flow velocity, blood acceleration, shear stress and volumetric flow rate.

2. Formulation of the problem

The axisymmetric flow of blood has the coordinate (r', θ', z'), where the blood flows horizontally in the axial direction of z' . The blood flow assumed to be one dimensional flows through a rigid stenotic artery that is cylindrical with permeability at the porous wall of the artery. The blood is assumed to be non-Newtonian, electrically conducting, incompressible and viscous fluid influenced by a magnetic field applied in the radial direction perpendicular to the axial direction of the blood flow. The constriction at the artery wall is affected by the stenosis position and height.

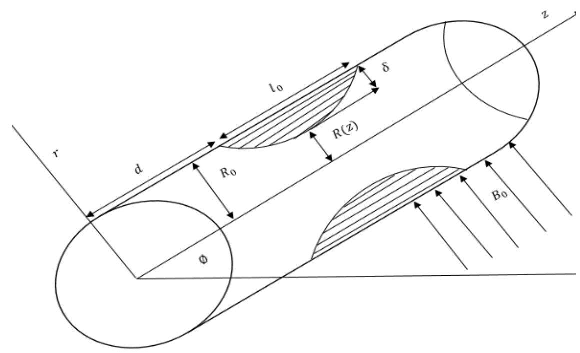

Figure 1 Flow geometry of the stenotic artery

The geometry of the one dimensional blood flow through the segmented stenotic artery, symmetrical in shape was proposed by Eldesoky [23] and Kumar et al. [24] as,

$$R(z') = \begin{cases} d'(z) - \frac{\delta'}{2}\left[1 + \cos\frac{2\pi}{l_0}\left\{z' - d' - \frac{l'_0}{2}\right\}\right] \\ d'(z) \end{cases}, d' \leq z' \leq d' + l'_0 \quad (1)$$

The greatest stenosis height (R) happens at the center, Nadeem et al. [25].

$$z = d + \frac{L_0}{s^{\left(\frac{1}{s-1}\right)}} \quad \text{For } s \geq 2 \text{ and } l_1 = \frac{d'}{R'_0}$$

3. Governing equation

There is a force that is created when the magnetic field acts on the blood which conducts electricity called the electromagnetic force $\vec{F}$.

$$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B}) \quad (1)$$

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}) \quad (2)$$

The overall intensity of the magnetic field intensity $B = B_0 + B_l$. A Lorentz force is produced or formed when the electric force combines with the magnetic force on the flow of blood where the intensity of the electric field vector $\vec{E}$ is negligible.

$$\vec{J} = \sigma(\vec{V} \times \vec{B}) \quad (3)$$

F becomes for small Reynolds number

$$\vec{J} \times \vec{B} = -\sigma B_0^2 u \quad (4)$$

$|B_0| = B_0$, F acts in the axial direction, $\vec{u} = (0,0,u)$ is the distribution of the velocity vector, $\vec{B} = (0, B_0, 0)$ is the distribution of the magnetic field vector.

The concentration of the blood and velocity of blood flow is steady and pulsatile flowing through the stenosed artery that is inclined with the magnetic field applied radially to the axial direction is of the blood flow through the artery.

$$\rho \frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial z'} + \rho G(t) + \frac{\mu}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u'}{\partial r'}\right) - \sigma_c B_0^2 u' + g\sin\emptyset - \frac{\mu}{k_p'}u' + \rho g B_C(C' - C_0) \quad (5)$$

$$\frac{\partial C'}{\partial t'} = D'\left[\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right] - E'(C' - C_0) \quad (6)$$

Dimensional form of the pressure gradient is expressed as

$$-\frac{\partial p'}{\partial z'} = P_s' + P_p' \cos(w_h t'); \ \ t \geq 0 \quad (7)$$

$w_h = 2\pi f_h$ and $w_b = 2\pi f_b$

$$G(t) = G_b' \cos(w_b t' + \varphi); \ \ t \geq 0 \quad (8)$$

The dimensionless flow geometry with stenosis, Eldesoky [23] and Kumar et al. [24].

$$R(z) = \begin{cases} (1+\ \xi z) - \frac{\delta}{2}\left[1 + \cos\frac{2\pi}{l_0}\left\{z - l_1 - \frac{l_0}{2}\right\}\right] \\ (1+\ \xi z) \end{cases}, l_1 \leq z \leq l_1 + l_0 \quad (9)$$

The dimensionless body acceleration and Pressure gradient

$$G(t) = G_0 \cos(bt + \varphi); t \geq 0 \quad (10)$$

$$-\frac{\partial p}{\partial z} = P_0 + P_l \cos(w_p t); \ \ t \geq 0 \quad (11)$$

The momentum equation in dimensionless form is written as for first consideration:

$$Re\frac{\partial u}{\partial t} = P_0 + P_L \cos t + G_0 \cos(bt + \varphi) + \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \left(M^2 + \frac{1}{K}\right)u + \frac{\sin\emptyset}{Fr} + G_c C \quad (12)$$

$$ScRe\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} - KrC \quad (13)$$

Dimensional initial and boundary slip conditions are

$$\begin{Bmatrix} \frac{\partial u'}{\partial r'} = -h'u', C' = C'_a \text{ at } r' = R'(z) \\ \frac{\partial u'}{\partial r'} = 0, \frac{\partial C'}{\partial r'} = 0 \quad \text{at } r' = 0 \end{Bmatrix} \quad (14)$$

Dimensionless initial and boundary slip conditions are

$$\begin{Bmatrix} \frac{\partial u}{\partial r} = -hu, C = C_a \text{ at } r = R(z) \\ \frac{\partial u}{\partial r} = 0, \frac{\partial C}{\partial r} = 0 \text{ at } r = 0 \end{Bmatrix} \quad (15)$$

η depends on the porous material

$$a = \frac{R(z)}{R_0 + \xi z}; \xi = \tan\emptyset, h = -\frac{\eta}{R_0\sqrt{K}}$$

Non dimensional parameters in the governing equations and boundary conditions are transformed to dimensionless form.

$$u = \frac{u'}{u_0}; \delta = \frac{\delta'}{R_0'};\ d(z') = R_0 +\ \xi z';\ r = \frac{r'}{R_0'}; z = \frac{z'}{R_0'}; b = \frac{w_b}{w_h};\ t = wt'; R(z) = \frac{R'(z)}{R_0'}; P = \frac{R_0'\rho'}{u_0\mu}; Re = \frac{\rho w {R_0'}^2}{\mu}; C = \frac{C' - C_\infty}{C_w' - C_\infty}; C_a = \frac{{C_a}' - C_\infty}{C_w' - C_\infty}; \delta = \frac{\delta'}{R_0'};\ M^2 = \frac{\sigma {R_0'}^2 B_0^2}{\mu};\ S_c = \frac{\vartheta}{D'}; Kr = \frac{E'{R_0'}^2}{\vartheta}; G_C =$$

$\frac{g\rho {R'_0}^2 \beta_C (C_w - C_0)}{u_0 \mu}; P_p = \frac{P'_p {R'_0}^2}{u_0\mu}; P_s = \frac{P'_s {R'_0}^2}{u_0\mu};\ G_b = \frac{\rho G'_b {R'_0}^2}{u_0\mu};\ f_r = \frac{u_0\mu}{g{R'_0}^2}; D = \frac{D'}{D_0}; k = \frac{k'_p}{{R'_0}^2};$ (16)

4. Method of Solution

The Frobenius method is used to solve the differential equation analytically with the solutions for the governing partial differential equation of the steady and pulsatile blood velocity and blood concentration expressed as function of time

$$u(r,t) = u_0(r) + u_p(r)\varepsilon e^{iwt} \tag{17}$$

$$C(r,t) = C_0(r) + C_p(r)\varepsilon e^{iwt} \tag{18}$$

5. Solution to the Governing equation

The concentration for the steady and pulsatile state is expressed below as

$$\frac{\partial^2 C_0}{\partial r^2} + \frac{1}{r}\frac{\partial C_0}{\partial r} - KrC_0 = 0 \tag{19}$$

$$\frac{\partial^2 C_p}{\partial r^2} + \frac{1}{r}\frac{\partial C_p}{\partial r} - \alpha C_p = 0 \tag{20}$$

Where $\alpha = Kr - i\omega Pe$

The expression of the steady and pulsatile state blood flow velocity is given below as

$$\frac{\partial^2 u_0}{\partial r^2} + \frac{1}{r}\frac{\partial u_0}{\partial r} - \beta u_0 = -G - G_r C_0 \tag{21}$$

$$\frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r}\frac{\partial u_p}{\partial r} - \gamma u_p = -L - G_r C_p \tag{22}$$

$$\beta = M^2 + \frac{1}{K};\ G = P_0 + \frac{\sin\emptyset}{Fr}; \gamma = M^2 + \frac{1}{K} + Rei\omega$$

and $L = P_p \cos t + G_b \cos(bt + \varphi)$

Applying the Funch's theorem, called the Frobenius series on equation (19) – (22) expressed as

$$C_0(r) = \sum_{n=0}^{\infty} x_n r^{n+k} \text{ Where } x_n, k \in A_1 \tag{23}$$

$$C_p(r) = \sum_{n=0}^{\infty} y_m r^{m+k} \text{ Where } y_m, k \in A_2 \tag{24}$$

$$u_0 = \sum_{n=0}^{\infty} p_n r^{n+k} \text{ Where } p_n, k \in A_3 \tag{25}$$

$$u_p = \sum_{m=0}^{\infty} q_m r^{m+k} \text{ Where } q_m, k \in A_4 \tag{26}$$

Applying the boundary condition from equation (15) into equation (19) the concentration in steady state is expressed as

$$C_0 = A_1\left[1 + \frac{Kr\, r^2}{2^2} + \frac{Kr^2\, r^4}{2^2 4^2} + \frac{Kr^3\, r^6}{2^2 4^2 6^2} + \frac{Kr^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] \tag{27}$$

$$A_1 = \frac{C_R}{\left[1 + \frac{Kr\, R^2}{2^2} + \frac{Kr^2\, R^4}{2^2 4^2} + \frac{Kr^3\, R^6}{2^2 4^2 6^2} + \frac{Kr^4\, R^8}{2^2 4^2 6^2 8^2} + \cdots\right]} \tag{28}$$

Applying the boundary condition from equation (15) into equation (20) the concentration in pulsatile state is expressed as

$$C_P = A_2\left[1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \cdots\right] \tag{29}$$

$$A_2 = \frac{C_R}{\left[1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \cdots\right]} \tag{30}$$

The concentration solution from equation (18) is obtained when equation (27) and (29) are combined and then expressed as

$$\mathrm{C(r,t)} = A_1\left[1 + \frac{\mathrm{Kr}\, r^2}{2^2} + \frac{\mathrm{Kr}^2\, r^4}{2^2 4^2} + \frac{\mathrm{Kr}^3\, r^6}{2^2 4^2 6^2} + \frac{\mathrm{Kr}^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] + \left(A_2\left[1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \cdots\right]\right)\varepsilon e^{iwt} \qquad (31)$$

Applying the boundary condition in equation (15) to equation (21) the complementary solution for the steady state blood velocity is expressed as

$$u_{0c} = A_3\left[1 + \frac{\beta\, r^2}{2^2} + \frac{\beta^2\, r^4}{2^2 4^2} + \frac{\beta^3\, r^6}{2^2 4^2 6^2} + \frac{\beta^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] \qquad (32)$$

The particular solution for the steady state blood velocity in equation (21) is expressed as

$$u_{0p} = P_0 + P_1 r^2 + P_2 r^4 + P_3 r^6 + P_4 r^8 \qquad (33)$$

The blood velocity solution in equation (17) is the combination of equation (32) and (33).

$$u_0 = A_3\left[1 + \frac{\beta\, r^2}{2^2} + \frac{\beta^2\, r^4}{2^2 4^2} + \frac{\beta^3\, r^6}{2^2 4^2 6^2} + \frac{\beta^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] + P_0 + P_1 r^2 + P_2 r^4 + P_3 r^6 + P_4 r^8 \qquad (34)$$

Where $A_3 = -$

$$\frac{\left[h\left(P_0+P_1R^2+P_2R^4+P_3R^6+P_4R^8\right)+2P_1R+4P_2R^3+6P_3R^5+8P_4R^7\right]}{\left[\frac{\beta R^2}{2}+\frac{\beta^2 R^3}{2^2 4}+\frac{\beta^3 R^5}{2^2 4^2 6}+\frac{\beta^4 R^7}{2^2 4^2 6^2 8}+\cdots\right]+h\left[1+\frac{\beta R^2}{2^2}+\frac{\beta^2 R^4}{2^2 4^2}+\frac{\beta^3 R^6}{2^2 4^2 6^2}+\frac{\beta^4 R^8}{2^2 4^2 6^2 8^2}+\cdots\right]}$$

Applying the boundary condition in equation (15) to equation (22) the complementary solution for the pulsatile state blood velocity is expressed as

$$u_{pc} = A_4\left[1 + \frac{\gamma\, r^2}{2^2} + \frac{\gamma^2\, r^4}{2^2 4^2} + \frac{\gamma^3\, r^6}{2^2 4^2 6^2} + \frac{\gamma^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] \qquad (35)$$

The particular solution for the pulsatile state blood velocity in equation (22) is expressed as

$$u_{pp} = Q_0 + Q_1 r^2 + Q_2 r^4 + Q_3 r^6 + Q_4 r^8 \qquad (36)$$

The blood velocity solution in equation (17) is the combination of equation (35) and (36).

$$u_P = A_4\left[1 + \frac{\gamma\, r^2}{2^2} + \frac{\gamma^2\, r^4}{2^2 4^2} + \frac{\gamma^3\, r^6}{2^2 4^2 6^2} + \frac{\gamma^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] + Q_0 + Q_1 r^2 + Q_2 r^4 + Q_3 r^6 + Q_4 r^8 \qquad (37)$$

Where $A_4 = -$

$$\frac{\left[h\left(Q_0+Q_1R^2+Q_2R^4+Q_3R^6+Q_4R^8\right)+2L_1R+4L_2R^3+6L_3R^5+8L_4R^7\right]}{\left[\frac{\beta_2 R^2}{2}+\frac{\beta_2{}^2 R^3}{2^2 4}+\frac{\beta_2{}^3 R^5}{2^2 4^2 6}+\frac{\beta_2{}^4 R^7}{2^2 4^2 6^2 8}+\cdots\right]+h\left[1+\frac{\beta_2 R^2}{2^2}+\frac{\beta_2{}^2 R^4}{2^2 4^2}+\frac{\beta_2{}^3 R^6}{2^2 4^2 6^2}+\frac{\beta_2{}^4 R^8}{2^2 4^2 6^2 8^2}+\cdots\right]}$$

The blood velocity solution from equation (12) is obtained by substituting equation (34) and (37) into equation (17) which is expressed as

$$\mathrm{u(r,t)} = A_3\left[1 + \frac{\beta\, r^2}{2^2} + \frac{\beta^2\, r^4}{2^2 4^2} + \frac{\beta^3\, r^6}{2^2 4^2 6^2} + \frac{\beta^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] + P_0 + P_1 r^2 + P_2 r^4 + P_3 r^6 + P_4 r^8 + \left(A_4\left[1 + \frac{\gamma\, r^2}{2^2} + \frac{\gamma^2\, r^4}{2^2 4^2} + \frac{\gamma^3\, r^6}{2^2 4^2 6^2} + \frac{\gamma^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] + Q_0 + Q_1 r^2 + Q_2 r^4 + Q_3 r^6 + Q_4 r^8\right)\varepsilon e^{iwt} \quad (38)$$

The Solution for the Fluid Acceleration equation

$$\mathrm{F(r,t)} = \frac{du}{dt} = iw\varepsilon e^{iwt}\left(A_4\left[1 + \frac{\gamma\, r^2}{2^2} + \frac{\gamma^2\, r^4}{2^2 4^2} + \frac{\gamma^3\, r^6}{2^2 4^2 6^2} + \frac{\gamma^4\, r^8}{2^2 4^2 6^2 8^2} + \cdots\right] + \frac{4Q_1}{\gamma} + \frac{GrA_2}{\gamma} + Q_1 r^2 + Q_2 r^4 + Q_3 r^6 + Q_4 r^8\right) + \varepsilon e^{iwt}\frac{P_p}{\gamma}(iw\cos t - \sin t) + \varepsilon e^{iwt}\frac{G_b}{\gamma}(iw\cos(bt+\varphi) - b\sin(bt+\varphi)) \quad (39)$$

The Solution for the Wall Shear Stress equation

$$\frac{du}{dr} = A_3\left[\frac{\beta\, r}{2} + \frac{\beta^2\, r^3}{2^2 4} + \frac{\beta^3\, r^5}{2^2 4^2 6} + \frac{\beta^4\, r^7}{2^2 4^2 6^2 8} + \cdots\right] + 2P_1 r + 4P_2 r^3 + 6P_3 r^5 + 8P_4 r^7 + \left(A_4\left[\frac{\beta\, r}{2^2} + \frac{\beta^2\, r^3}{2^2 4} + \frac{\beta^3\, r^5}{2^2 4^2 6} + \frac{\beta^4\, r^7}{2^2 4^2 6^2 8} + \cdots\right] + 2Q_1 r + 4Q_2 r^3 + 6Q_3 r^5 + 8Q_4 r^7\right)\varepsilon e^{iwt} \quad (40)$$

The Solution for the Volumetric Flow Rate equation

$$\mathrm{Q(r,t)} = 2\pi\int_0^a r\, u(r,t)dr = 2\pi\left\{A_3\left[\frac{a^2}{2} + \frac{\beta\, a^4}{2^2 4} + \frac{\beta^2\, a^6}{2^2 4^2 6} + \frac{\beta^3\, a^8}{2^2 4^2 6^2 8} + \frac{\beta^4\, a^{10}}{2^2 4^2 6^2 8^2 10} + \cdots\right] + \frac{P_0 a^2}{2} + \frac{P_1 a^4}{4} + \frac{P_2 a^6}{6} + \frac{P_3 a^8}{8} + \frac{P_4 a^{10}}{10} + \left(A_4\left[\frac{a^2}{2} + \frac{\beta\, a^4}{2^2 4} + \frac{\beta^2\, a^6}{2^2 4^2 6} + \frac{\beta^3\, a^8}{2^2 4^2 6^2 8} + \frac{\beta^4\, a^{10}}{2^2 4^2 6^2 8^2 10} + \cdots\right] + \frac{Q_0 a^2}{2} + \frac{Q_1 a^4}{4} + \frac{Q_2 a^6}{6} + \frac{Q_3 a^8}{8} + \frac{Q_4 a^{10}}{10}\right)\varepsilon e^{iwt}\right\} \quad (41)$$

6. Graphical Results and Discussion.

From Figure 2.0, it was noticed that increased chemical reaction Kr from $0.5 \leq \mathrm{Kr} \leq 2$, caused a decrease in the blood velocity. This occurred because chemical reaction increases the internal viscosity of the blood resulting to a decrease in the blood flow. The blood flow decrease causes a decrease in the blood acceleration and volumetric flow rate in figure 3.0 and figure 4.0 but increases the wall shear stress in figure 5.0.

From figure 6.0, an increase in magnetic field M causes a decrease in the blood velocity due to an increase in Lorentz force which inhibits blood flow. This further results to a decrease in the blood acceleration, wall shear stress and volumetric flow rate from figure 7.0 to figure 9.0.

From figure 10.0, an increase in slip causes a decrease in the blood velocity. This is as a result of the relative motion between the surfaces of the blood and the stenosis wall of the artery which inhibits the blood flow. This reduces the blood acceleration and the volumetric flow rate in figure 11.0 and figure 13.0 but increases the wall shear stress in figure 12.0.

From figure 14.0, the increase in the radius of stenosis increases the blood velocity. When the radius of the stenosis was reduced the blood velocity in the artery was reduced due to stenotic arterial blockage. Furthermore, an increase in the radius of stenosis causes an increase in the blood acceleration and volumetric flow rate in figure 15.0 and figure 17.0 but a decrease in the wall shear stress in figure 16.0.

Finally, from figure 18.0 to figure 20.0, increasing values of Schmidt number, chemical reaction and magnetic field reduces the concentration of the blood due to a reduction in the internal viscosity of the blood.

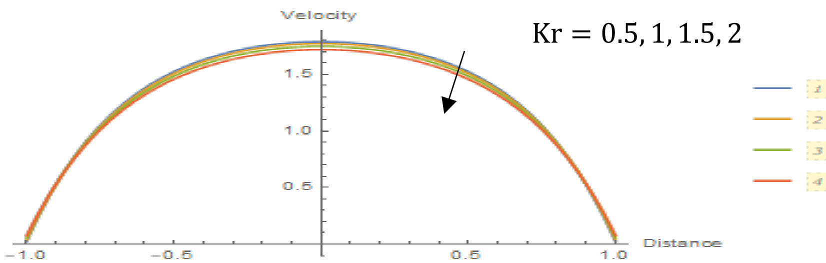

Figure 2.0 ***Graph for the velocity of Blood flow with increasing values of chemical reaction*** Kr ***when*** $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 30^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1.$

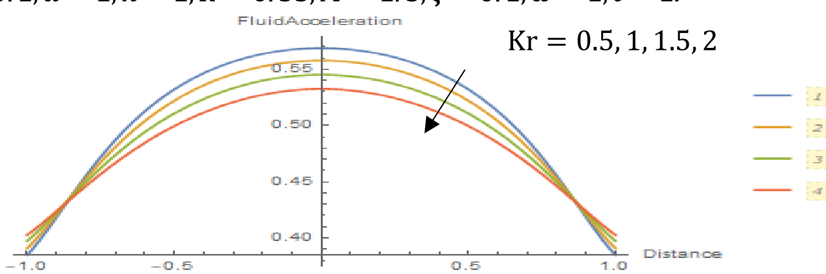

Figure 3.0 ***Graph for the Blood acceleration with increasing values of chemical reaction*** Kr ***when*** $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1.$

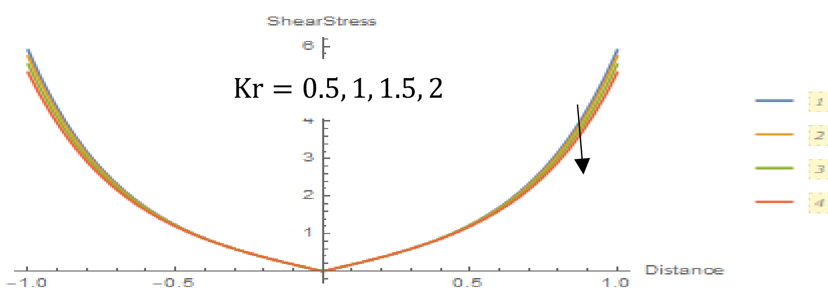

Figure 4.0 ***Graph for the shear stress at the wall with increasing values of chemical reaction*** Kr ***when*** $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 30^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1.$

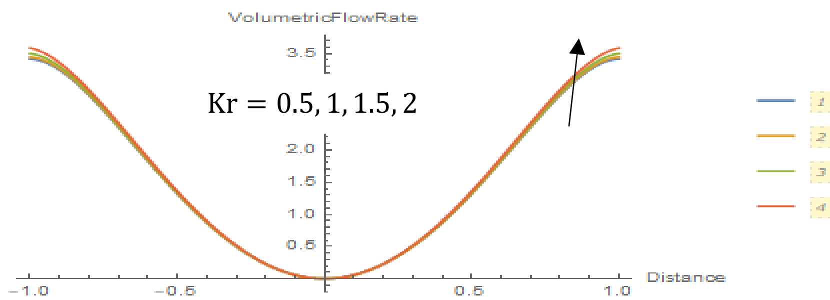

Figure 5.0 ***Graph for the Volumetric Flow rate with increasing values of chemical reaction*** $\boldsymbol{Kr}$ ***when*** $\boldsymbol{Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 30^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1.}$

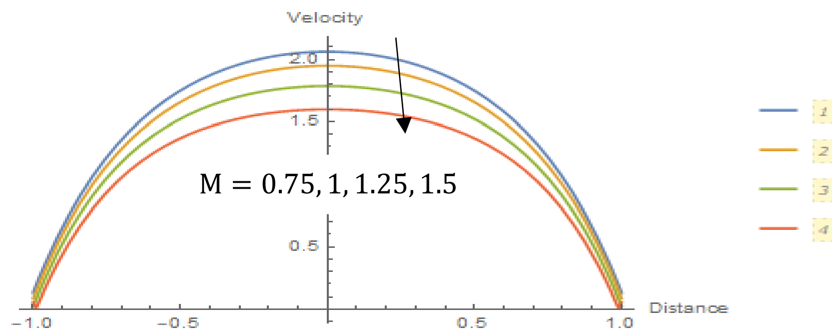

Figure 6.0 ***Blood flow velocity distribution for increase in the Magnetic field*** $\boldsymbol{M}$ ***when*** $\boldsymbol{Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 45^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, \xi = 0.1, \omega = 1, t = 1.}$

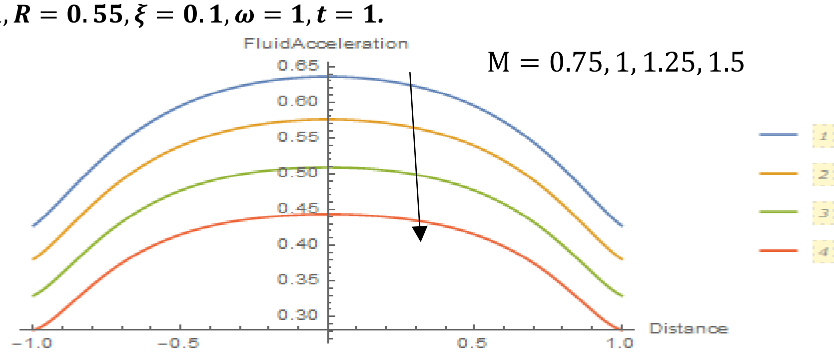

Figure 7.0 ***Blood acceleration profile for increase in the Magnetic field*** $\boldsymbol{M}$ ***when*** $\boldsymbol{Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, \xi = 0.1, \omega = 1, t = 1.}$

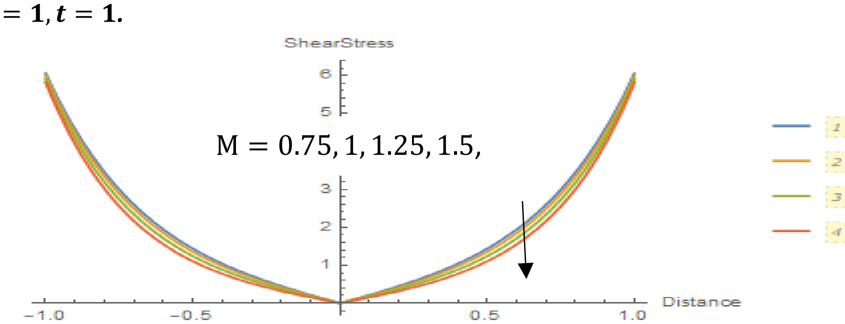

Figure 8.0 ***Wall shear stress profile for increase in the Magnetic field*** $\boldsymbol{M}$ ***when*** $\boldsymbol{Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 45^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, a = 1, \xi = 0.1, \omega = 1, t = 1.}$

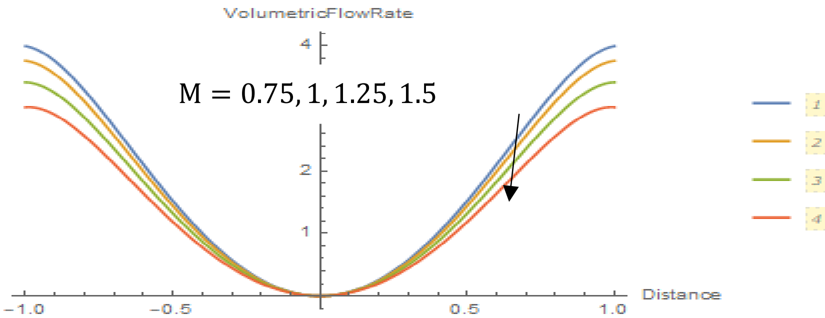

Figure 9.0 ***Volumetric flow rate profile for increase in the Magnetic field M when $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 45^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, R = 0.55, a = 1, \xi = 0.1, \omega = 1, t = 1.$***

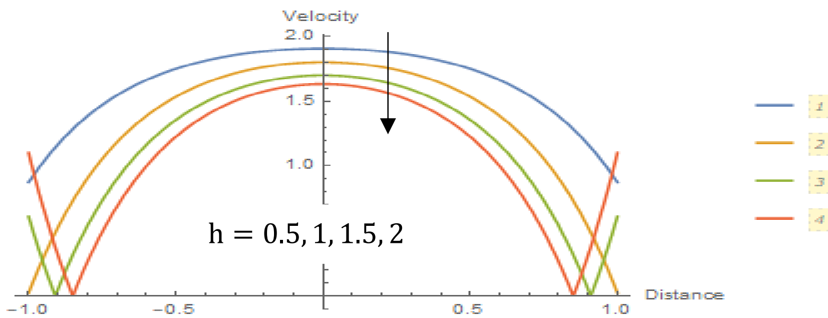

Figure 10.0 ***Blood flow velocity distribution for increase in the Slip h when $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 45^0, \beta = 30^0, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1.$***

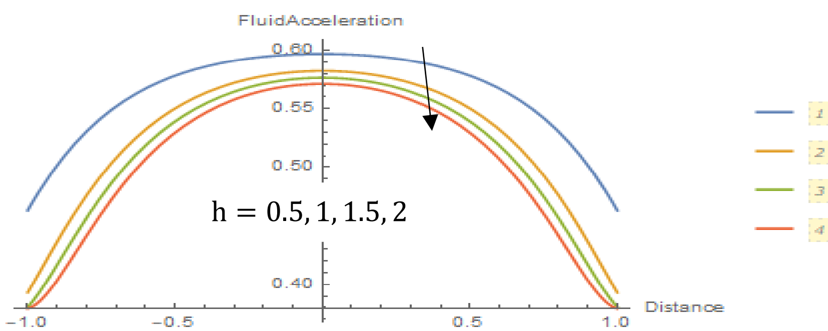

Figure 11.0 ***Blood acceleration profile for increase in the Slip h when $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \beta = 30^0, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, \xi = 0.1, \omega = 1, t = 1.$***

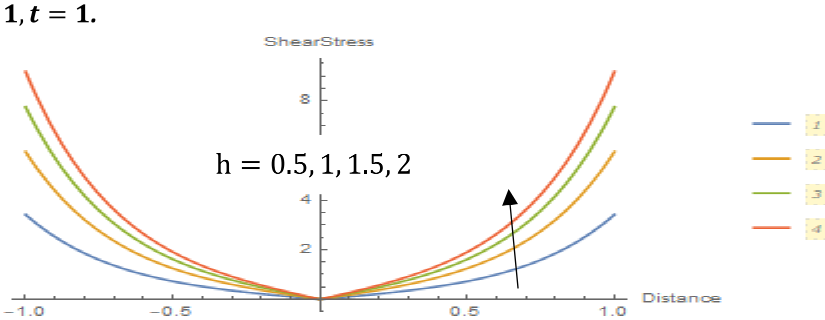

Figure 12.0 ***Wall shear stress profile for increase in the Slip h when $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 45^0, \beta = 30^0, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1.$***

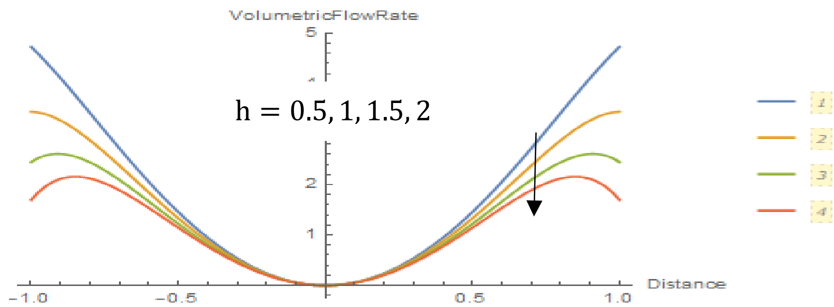

Figure 13.0 *Volumetric flow rate profile for increase in the Slip* h *when* $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 45^0, \beta = 30^0, k = 0.1, \alpha = 1, R = 0.55, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1.$

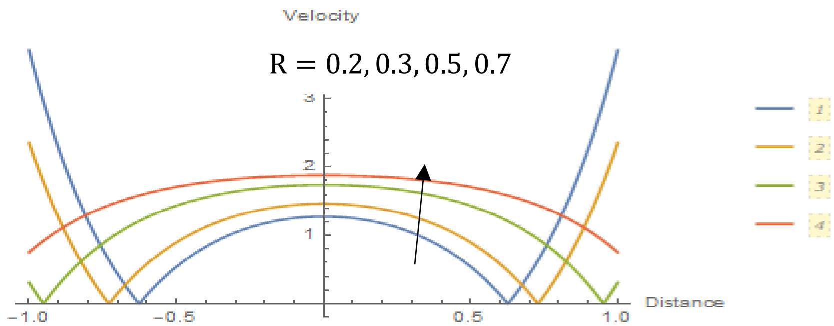

Figure 14.0 *Graph for the velocity of Blood flow with increasing values of Radius of stenosis* R *when* $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, Fr = 0.05, b = 2, \phi = 30^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, M = 1.5, \xi = 0.1, \omega = 1, t = 1.$

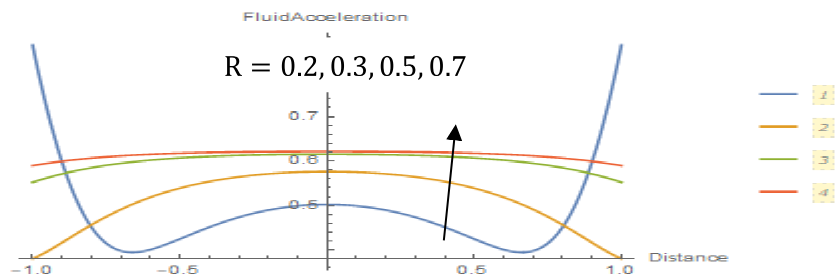

Figure 15.0 *Graph for the Blood acceleration with increasing values of Radius of stenosis* R *when* $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, M = 1.5, \xi = 0.1, \omega = 1, t = 1.$

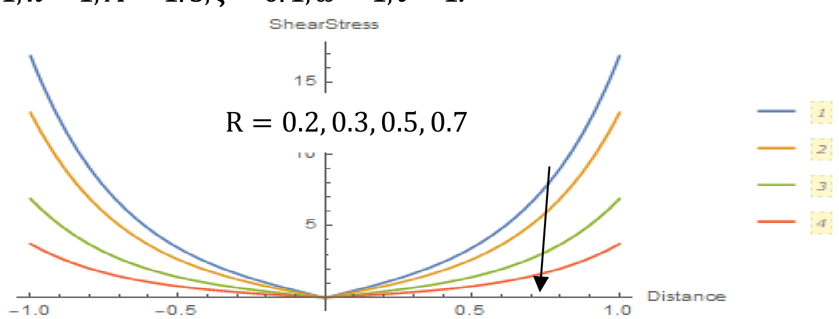

Figure 16.0 *Graph for the shear stress at the wall with increasing values of Radius of stenosis* R *when* $Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 30^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1.$

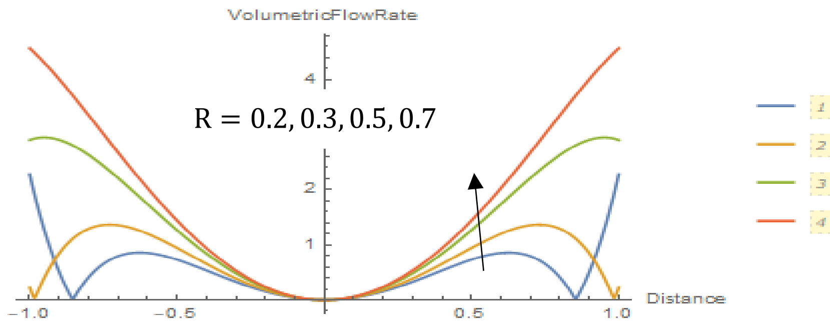

Figure 17.0 ***Graph for the Volumetric Flow rate with increasing values of Radius of stenosis*** $\boldsymbol{R}$ ***when*** $\boldsymbol{Gc = 2, Kr = 0.5, Sc = 1, Ps = 2, Pp = 4, Gb = 3, Fr = 0.05, b = 2, \phi = 30^0, \beta = 30^0, k = 0.1, \alpha = 1, h = 1, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1.}$

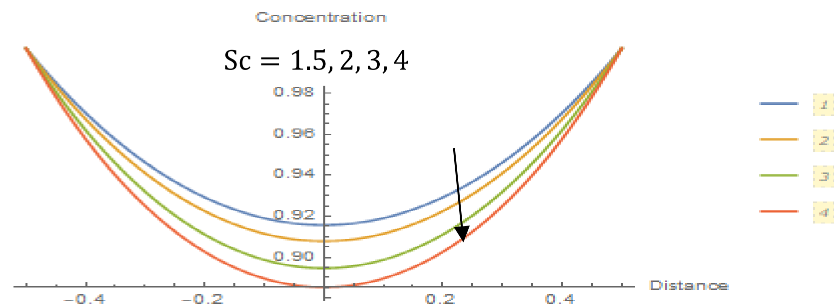

Figure 18.0 ***Graph for the Concentration with increasing values of Schmidt Number*** $\boldsymbol{Sc}$ ***when*** $\boldsymbol{Kr = 0.5, Sc = 1, a = 1, M = 1.5, \xi = 0.1, \omega = 1, t = 1.}$

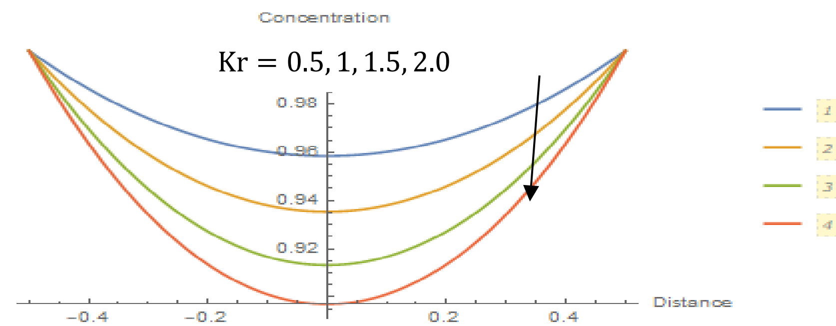

Figure 19.0 ***Graph for the Concentration with increasing values of Chemical Reaction*** $\boldsymbol{Kr}$ ***when*** $\boldsymbol{Sc = 1, a = 2, M = 1.5, b = 1, \xi = 0.1, \omega = 1, t = 1.}$

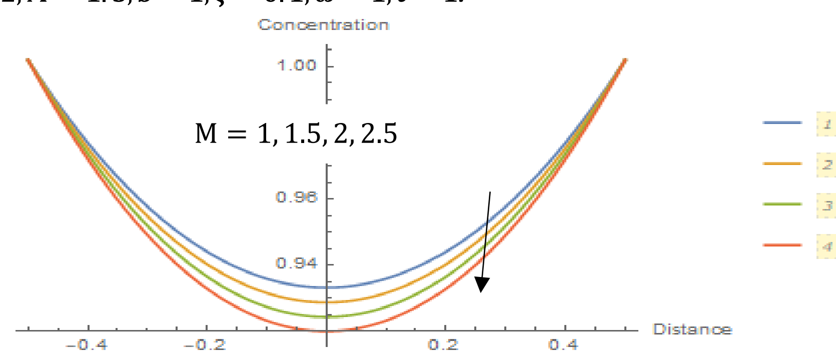

Figure 20.0 ***Graph for the Concentration with increasing values of Magnetic Field*** $\boldsymbol{M}$ ***when*** $\boldsymbol{Kr = 0.5, Sc = 1, M = 1.5, a = 1, \xi = 0.1, \omega = 1, t = 1.}$

7.0 Conclusion

This study shows the theoretical analysis of treatment effect which include chemical reaction, slip and magnetic field on MHD blood flow through a stenotic artery. The result summary from the theoretical study showed that,

(i) The increase in the Chemical reaction increases the blood viscosity whereby decreasing the blood velocity, acceleration and shear stress at the wall but increases the volumetric flow rate of the blood.

(ii) Increased magnetic field M causes a decrease in blood velocity, acceleration, shear stress at the wall and volumetric flow rate. Health practitioners could adopt this in treatment of patients with hypertension where the flow of blood to the body muscles from the heart is reduced.

(iii) Increased slip h at the wall with stenosis decreases the blood velocity, acceleration and volumetric flow rate but causes an increase in the wall shear stress. This could treat tumor growth when the slip is introduced at the stenotic wall causing a starvation in blood supply to the stenotic growth at the artery wall. This cleanses the arteries, heart valves and cavities causing the increase of blood velocity, acceleration and volumetric flow rate but a decrease in the shear stress at the wall.

(iv) Increased radius of stenosis R at the wall with increases the blood velocity, acceleration and volumetric flow rate but causes a decrease in the wall shear stress. This could treat tumor growth and hypertension. This cleanses the arteries, heart valves and cavities hence improving the blood circulation.

(v) Increase in the values of Schmidt number, chemical reaction and magnetic field reduces the concentration of the blood due to a reduction in the internal viscosity of the blood.

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APPENDIX

$P_4 = \frac{G_c A_2 \alpha^4}{\gamma 2^2 4^2 6^2 8^2}$; $P_3 = \frac{1}{\gamma}\left(64P_4 - \frac{G_c A_2 \alpha^3}{2^2 4^2 6^2}\right)$;

$P_2 = \frac{1}{\gamma}\left(36P_3 + \frac{G_c A_2 \alpha^2}{2^2 4^2}\right)$; $P_1 = \frac{1}{\gamma}\left(16P_2 - \frac{G_c A_2 \alpha}{2^2}\right)$;

$P_0 = \frac{1}{\gamma}(4P_1 + L + G_c A_2)$;

$Q_4 = \frac{G_c A_1 K r^8}{\beta 2^2 4^2 6^2 8^2}$; $Q_3 = \frac{1}{\beta}\left(64Q_4 - \frac{G_c A_1 K r^6}{2^2 4^2 6^2}\right)$; $Q_2 = \frac{1}{\beta}\left(36Q_3 + \frac{G_c A_1 K r^4}{2^2 4^2}\right)$;

$Q_1 = \frac{1}{\beta}\left(16Q_2 - \frac{G_c A_1 K r^2}{2^2}\right)$; $Q_0 = \frac{1}{\beta}(4Q_1 + G + G_c A_1)$