Original Research Article

# Climate change data: use of an autoregresssive (AR) model in presence of change points under a Bayesian approach

## Abstract

Objectives: T In this study, we introduce a statistical model applied to climate change data consisting of an autoregressive times series (AR) model which represents a type of random process.
Methodology: Different existing times series models fitted for the annual mean climate variables as MA (moving average) models or ARIMA (Autoregressive Integrated Moving Average) models. We explore the behavior of the climate times series for possible change-points. A Bayesian approach using MCMC (Markov Chain Monte Carlo) methods is considered to get the inferences of interest.
Results: we observe that the proportions of pairs of years with same annual temperature are different for each climate station..

Conclusion: Was possible to detect from the adjusted model, significant differences between consecutive annual averages of two years and long periods where it was observed a sharp change in mean annual temperatures in different weather seasons using CUSUM.

*Keywords: climate data, AR models , change-points , annual mean temperature, Bayesian approach, MCMC methods.*

## 1 Introduction

Climate change is of great concern since great changes in temperature and precipitation has been observed worldwide in the last years (see, for instance, [1, 2]. See also https://www.un.org /en/sections/issues-depth/climate-change/ - accessed on 01 July 2021). In the last decades we have seen glaciers shrinkage, earlier melting of the ice in rivers and lakes, changes in plant and animal areas, and earlier owering of trees in different parts of the world ([3], [4]). As a special case, precipitation during winter and spring is projected to be higher for the northern part of the USA, and lower for the southwest. Additionally, for many regions of the planet there are predictions that heat waves will become more intense and cold waves will be less intense everywhere (see, for instance, https://www.ncdc.noaa.gov/monitoring-references/faq/indicators.php). In the last decades, the global average temperature increased approximately $0.1 - C(0.18 - F)$ per decade in the years from 1920 to 1940 (http://www.currentresults.com/Environment-Facts/changes-in-earth-temperature.php - accessed on 01 July 2021). From 2000 to 2009 the annual average global temperature has been $0.61 - C(1.1 - F)$ higher than the period ranging from 1950 to 1980. The use of different statistical models has been considered in the literature as indispensable tools in the analysis of climate data. Therefore, it is very important to introduce new statistical models to be used in the forecast of temperature (or precipitation) and also to detect years of climate change-points.

The literature introduces many studies in climate change, a subject of general interest. [5] study the impact of climate change on water resources and flooding; [6] deal with the relation between climate change and health effects; [7] perform an analysis of the changes in global temperature taking into account the time period since the pre-industrial era; [8] study the relation between global warming and climate change; [9] analyse the impact of climate change on the costal areas of Bangladesh; [10] deal with sea level changes in connection to global warming; [11] study the impact of climate change on migration; [12] describes the developments in the understanding of how temperature and humidity have changed; [13] study the impact of climate change on the marine life; [14] analyse the impact of climate change on the sub-Saharan Africa; [15] study the health effects of future food production under climate change; [16] analyse the threat posed on ecosystems by climate change; [17] present an analysis of the relation between temperature increase and crop production; [18] gives a statistical analysis of change in the global temperature; [19] study the changes in extreme temperature; [20] study the joint change in temperature and precipitation from multiple climate models under a Baysian approach. See also, [21]; [22]; [23] and [24].

The presence of specific change-points in climate time series also were studied by many authors; [25] and [26], considered Bayesian approaches to study change-point problems for climate data; [27], assumed autoregressive series in the detection of climate change-points; [28] studied change-points detection applied to temperature and precipitation data; and [29] used a genetic algorithm to detect change-points in climate data.

Given the complexity of the likelihood function considering the presence of change-points, many authors assume a Bayesian approach using Markov Chain Monte Carlo (MCMC) methods to get inferences for the parameters of the proposed models (see, for example, [30]; [31]; [25]; [26]; [32]; [33]; [34]).

In the present work, the estimation of the location of the possible change-points are obtained assuming an autoregressive model for annual mean temperatures under a Bayesian approach using MCMC methods. From the obtained Monte Carlo we obtain contrasts (diferences of the estimated means for consecutive years) and CUSUM (cumulative sum control) chats to detect possible years of climate change.

The paper is organized as follows: section 2 presents the methodology used in the data analysis. Section 3 introduces some applications with temperature data from four climate stations followed by long periods of time. Finally, Section 4 presents some concluding remarks.

# 2 Methodology

Different existing times series models could be fitted for the annual mean climate variables as MA (moving average) models or ARIMA (Autoregressive Integrated Moving Average) models (see for example, [35]; [36]; [37]). In this work, we consider the use of a simple AR (autoregressive) model in the analysis of climate (temperature) variables which gives a good fit for the data and from the fitted model we explore the behavior of the climate times series for possible change-points. Assuming temporal climate data, as for example, monthly or annual averages of temperature denoted by a random variable Y which could be transformed (e.g., logarithm transformation), we consider a autoregressive model AR(J) of order J, given by,

$$
\begin{aligned}
&\text{Model AR(1)}: \quad y_1 = \alpha_0 + \epsilon_1, \quad y_2 = \alpha_1 y_1 + \epsilon_2, \quad y_i = \alpha_1 y_{i-1} + \epsilon_i, \quad \text{for} \quad i = 3, 4, 5, \cdots, n \\
&\text{Model AR(2)}: \quad y_1 = \alpha_0 + \epsilon_1, \quad y_2 = \alpha_1 y_1 + \epsilon_2, \quad y_3 = \alpha_1 y_2 + \alpha_2 y_1 + \epsilon_3, \quad y_i = \alpha_1 + y_{i-1} + \alpha_2 y_{i-2} + \epsilon_i, \\
&\qquad \text{for} \quad i = 4, 5, \cdots, n. \\
&\text{Model AR(3)}: \quad y_1 = \alpha_0 + \epsilon_1, \quad y_2 = \alpha_1 y_1 + \epsilon_2, \quad y_3 = \alpha_1 y_2 + \alpha_2 y_1 + \epsilon_3, \quad y_4 = \alpha_1 y_3 + \alpha_2 y_2 + \alpha_3 y_1 + \epsilon_4, \\
&\qquad y_i = \alpha_1 y_{i-1} + \alpha_2 y_{i-2} + \alpha_3 y_{i-3}, \quad \text{for} \quad i = 5, 6, \cdots, n. \\
&\text{Model AR(4)}: \quad y_1 = \alpha_0 + \epsilon_1, \quad y_2 = \alpha_1 y_1 + \epsilon_2, \quad y_3 = \alpha_1 y_2 + \alpha_2 y_1 + \epsilon_3, \quad y_4 = \alpha_1 y_3 + \alpha_2 y_2 + \alpha_3 y_1 + \epsilon_4 \\
&\qquad y_5 = \alpha_1 y_4 + \alpha_2 y_3 + \alpha_3 y_2 + \alpha_4 y_1 + \epsilon_5, \quad y_i = \alpha_1 y_{i-1} + \alpha_2 y_{i-2} + \alpha_3 y_{i-3} + \alpha_4 y_{i-4} + \epsilon_i, \\
&\qquad \text{for} \quad i = 6, 7, \cdots, n.
\end{aligned} \tag{2.1}
$$

where $\epsilon_i$ is an error term (a not observed random variable) assumed to be independent, identically distributed with a normal distribution $N(0, \sigma^2)$. Similarly for other AR(J) models where $J > 4$.

We assume a Bayesian analysis for the data assuming the model defined in (2.1). Combining the joint prior distribution for the parameters of the assumed model with the likelihood function given by,

$$L(\alpha_0, \alpha_1, \cdots, \alpha_{j-1}, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\sigma^2}} \exp\left\{-\frac{\epsilon_i^2}{2\sigma^2}\right\} \tag{2.2}$$

where $\epsilon_i$ is obtained from (2.1) for $i = 1, 2, 3, \cdots, n$, the joint posterior distribution for the parameters of the model is obtained using the Bayes formula [38]. The posterior summaries of interest are obtained using Markov Chain Monte Carlo (MCMC) simulation methods as the popular Gibbs sampling algorithm or the Metropolis-Hastings algorithm ([33]; [39]) using the free existing OpenBUGS software [40]. Since the OpenBugs software only requires the likelihood function and the prior distributions for each parameter of the model, we do not present here all conditional posterior distributions $p(\theta_j / \boldsymbol{\theta}_j,$ data), where $\boldsymbol{\theta}_j$ denotes the vector of all $p$ parameters of the model except $\theta_j, j = 1, 2, \cdots, p$ needed for the Gibbs sampling or Metropolis-Hastings algorithms (see for example,[41]).

For a Bayesian analysis, we assume independent prior distributions given by, normal distributions $N(0, a^2)$ for the parameters $\alpha_0, \alpha_1, \cdots, \alpha_{J-1}$ and a uniform prior $U(0, b)$ for the parameter $\varphi = 1/\sigma^2$ . The hyperparameters $a$ and $b$, are assumed known.

## 2.1 Model Discrimination Criterion - Deviance Information Criterion (DIC)

In the discrimination of better model, we use the DIC criterion thais is very popular to discriminate Bayesian models using MCMC methods. In our case we discriminate the proposed model (2.1) considering different choices of AR(J) structures.

The DIC criterion [42] is based on the posterior mean of deviance. The deviance is defined by

$$D(\boldsymbol{\theta}) = -2\ln(L(\boldsymbol{\theta})) + C \tag{2.3}$$

where $\boldsymbol{\theta}$ is a vector of unknown model parameters; $L(\boldsymbol{\theta})$ is the likelihood and C is a constant (not always known) when comparing two models. The DIC criterion is then given by

$$D(\boldsymbol{\theta}) = D(\widehat{\boldsymbol{\theta}}) + 2p_D \tag{2.4}$$

where $D(\widehat{\boldsymbol{\theta}})$ is the deviation calculated on the posterior mean $\widehat{\boldsymbol{\theta}} = \mathbb{E}(\boldsymbol{\theta} \mid y)$ and $p_D$ is the number of model parameters, given by $p_D = \overline{D} - D(\widehat{\boldsymbol{\theta}})$ where $\overline{D} = \mathbb{E}[D(\boldsymbol{\theta} \mid y)]$ is the posterior mean of the deviation that measures the goodness of fit of the data for each model. For the conclusion, the lowest DIC values indicate the best models. DIC also could have negative values.

### 2.2 Change-point detection

To detect the climate change-points we get from the generated Gibbs samples for the joint posterior distribution obtained assuming model (2.1) the Monte Carlo estimates for the contrasts $\theta_i = \mu_i - \mu_{i-1}$, where $\mu_i = \alpha_i y_{i-1} + \alpha_2 y_{i-2} + \cdots + \alpha_{J-1} y_1$, if we assume a AR(J) model in (1), where, $y_i = \log(\text{mean.temperature}_i)$. In this way, we detect a significant mean change-point in a specified year, if a $95\%$ credible interval for $\theta_i$ does not contain the zero value (climate mean in year $i$ is statistically different of climate mean in year $i-1$).

In the detection of year periods where the climate variable start a new behavior (could be above or below a standard climate behavior in a specified period of time), we use standard CUSUM (cumulative sum control) charts usually used in statistical quality control, which is used for monitoring change detection ([43]; [44]). The CUSUM assumed in this work considers the cumulative sum up to time i from all annual mean climate diferences in the previous years. Observe that if the climate variable do not have changes in long periods of years, in general although the great volatility of the annual mean climate variables, we should have CUSUM close to zero along all years. The purpose of cumulative sum chart (CUSUM) is to monitor the small shift in the process mean of the samples collects at a time intervals. These measurements of samples at a given time interval represents the subgroups. Instead of calculating the subgroups mean independently, the CUSUM chart represents the information of current and previous samples.

## 3 Results

The data sets considered as illustrations of the proposed methodology consist of the temperature measures extracted from the Research Data Archive site managed by the Data Engineering Section of the Computational and Information Systems Laboratory at the National Center for Atmospheric Research, United States of America. This site contains a large and diverse collection of meteorological and oceanographic observations, (see the sites https://rda.ucar.edu/index.html?hash=data user&action=register and https://rda.ucar.edu/datasets/ds570.0/#!subset.html, both accessed on 01 July 2021). It contains data from more than 4700 different climate stations (2600 in more recent years) from all around the world. Different follow-up periods are given for the different climate stations, and collection of data for some of them goes as far back as the mid-1700s.The primary data sets consist of monthly average temperature. Since the data sets have many missing observations (months with no information), these were replaced by the monthly averages of the available data for that month. For instance, if in a given year we have missing data for the month of January, we fill the hole in the data by assigning to that month the mean obtained using the values for the month January from all the years in which they are available. The data used in our calculations were the annual temperature averages.

As illustrative purposes for the proposed methodology, we consider the datasets from four climate stations extracted from the file ds570 of the Research Data Archive. The climate stations, observational periods, the number T of observed data are: station 30910 in Aberdeen, United Kingdom (1872-2020; T = 149); station 67000 in Geneva, Switzerland (1826-2020; T = 195), station 722080 in Charleston, USA (1832-2020; T = 189) and station 171300 in Ankara, Turkey (1826-2020; T = 195).

Figure 1 show the plots of the annual temperature averages for the four climate stations given in logarithm scale during their corresponding observational periods. From Figure 1, we see the possible presence of change points indicating changes from increasing/decreasing trends to decreasing/increasing and that in the final years of the follow-up periods, for each climate station, there is an indication of an increasing trend in the annual temperature averages.

For a Bayesian analysis of the AR(J) regression model (2.1), we assume independent prior distributions for the parameters $\alpha_0, \alpha_k, k = 1, 2, \cdots, J, \varphi = 1/\sigma^2$ that is, $\alpha_0 \sim N(0, 10), \alpha_k \sim N(0, 1), \varphi = 1/\sigma^2 \sim U(0, 1000)$, considering the four climate stations. Observe that we are assuming approximately non-informative priors for all parameters.

The posterior summaries of interest are obtained using the OpenBugs software [45]. The convergence of the MCMC algorithm was monitored using the trace-plots of the generated Gibbs samples (see Appendix 4 in the suplementary file). In the simulation process to generate the samples of the joint posterior distribution of interest, we considered initially 200,000 Gibbs samples discarded to eliminate the effect of the initial values in the iterative process and taking a final sample of size 1,000 to get the Monte Carlo estimates for each parameter (taking every $100^{th}$ sample of a total of 100,000 simulated samples).

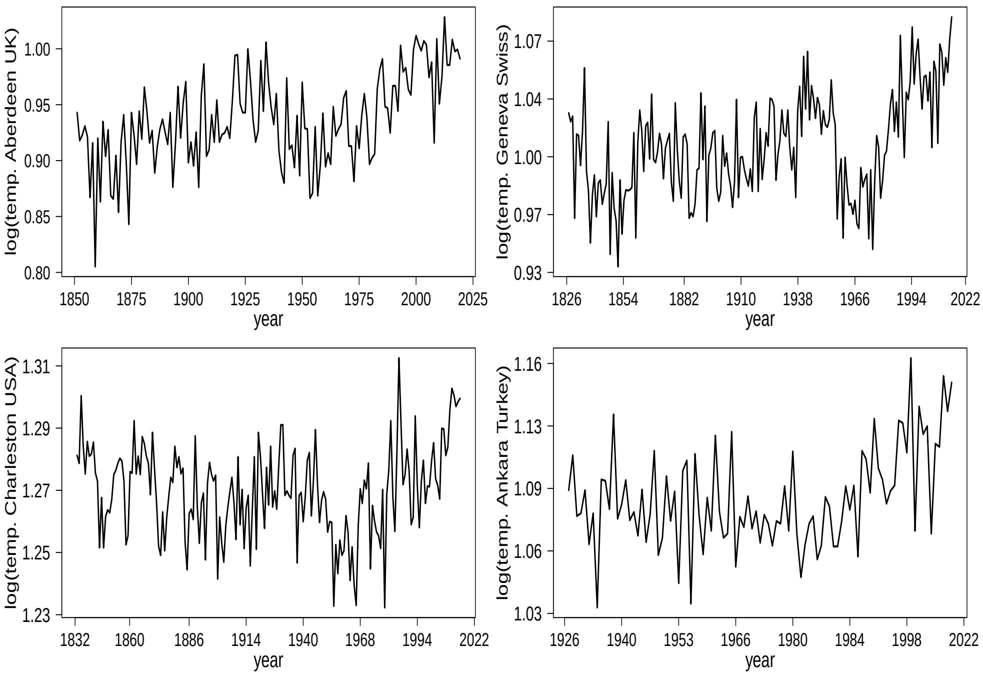


Figure 1: Annual average temperature in the logarithmic scale (Aberdeen, Geneva, Charleston and Ankara).

From the DIC discrimination criterion, we observed that AR(2) model (2.1) gives better fit (parsimony) for the annual temperature data of the four climate stations (smaller Monte Carlo estimates for DIC in all cases). The DIC values were estimated by negative values. Table 1 shows the posterior summaries (posterior mean, posterior standard deviation and $95\%$ credible intervals) for each parameter of the assumed model.

Figure 2 shows the scatter plots of the estimated means obtained for each fitted model togetter with the observed annual mean temperatures, from where, we observe good fit of the proposed model for the four climate data sets (see also Appendix 2 in the suplementary file).

Figure 3 shows the normality plots of the residuals used to check if the assumption of normality of the errors is verified. In general we observe that the normality assumption for the residuals are well verified for all cases. Appendix 3 in the suplementary file shows the autocorrelation plots of the residuals for each climate station, from where we observe noncorrelated residuals.Thus the assumptions of the proposed model are satisfied in all cases.

Table 1: Posterior summaries (Aberdeen, Geneva, Charleston, Ankara)

| Aberdeen | mean | sd | Lower $95\%$ci | Upper $95\%$ ci |
|---|---|---|---|---|
| $\alpha_0$ | 2.1040 | 0.0726 | 1.9640 | 2.2460 |
| $\alpha_1$ | 0.9014 | 0.0306 | 0.8438 | 0.9589 |
| $\alpha_2$ | 0.0990 | 0.0309 | 0.0420 | 0.1583 |
| $\phi = 1/\sigma^2$ | 191.9000 | 21.7600 | 151.4000 | 237.0000 |
| Geneva | | | | |
| $\alpha_0$ | 2.3640 | 0.0646 | 2.2410 | 2.4910 |
| $\alpha_1$ | 0.9326 | 0.0241 | 0.8873 | 0.9798 |
| $\alpha_2$ | 0.0679 | 0.0241 | 0.0220 | 0.1150 |
| $\phi = i/\sigma^2$ | 251.2000 | 25.0400 | 206.4000 | 304.2000 |
| Charleston | | | | |
| $\alpha_0$ | 2.9560 | 0.0333 | 2.8900 | 3.0180 |
| $\alpha_1$ | 0.9898 | 0.0277 | 0.9161 | 1.0240 |
| $\alpha_2$ | 0.0102 | 0.0113 | -0.1021 | 0.0325 |
| $\phi = 1/\sigma^2$ | 921.1000 | 58.2000 | 778.3000 | 997.2000 |
| Ankara | | | | |
| $\alpha_0$ | 2.5190 | 0.0745 | 2.3690 | 2.6710 |
| $\alpha_1$ | 0.9698 | 0.0277 | 0.9161 | 1.0240 |
| $\alpha_2$ | 0.0308 | 0.0278 | -0.0230 | 0.0840 |
| $\phi = 1/\sigma^2$ | 187.4000 | 26.8100 | 139.8000 | 246.2000 |

From the Monte Carlo estimates for the contrasts $\theta_i = \mu_i - \mu_{i-1}$, where $\mu_i = \alpha_1 y_{i-1} + \alpha_2 y_{i-2} + \cdots + \alpha_{j-1} y_1$, where $y_i = \log(\text{mean.temperature}_i)$, we can detect the consecutive years showing the same annual mean temperatures for each climate station ($95\%$ credible interval for $\theta_i$ containing the zer value). From Tables A.1, A.2, a.3, and A.4 in Appendix 1 in the supplementary file, we see the years with not significant differences between the annual mean temperatures for two consecutives years considering each assumed climate stations given by:

- Aberdeen, UK: $\theta_{53}$ (years 1923-1924), $\theta_{58}$ (years 1928-1929), $\theta_{85}$ (years 1955-1956), $\theta_{93}$ (years 1963-1964) and $\theta_{149}$ (years 2019-2020). All the other consecutive pairs of years have significant change in the annual mean temperatures ($95\%$ credible intervals for the associated contrasts $\theta$ does not contain the zero value).
- Geneva, Switzerland: $\theta_{32}$ (years 1856-1857), $_{64}$ (years 1888-1889), $\theta_105$ (years 1929-1930), $\theta_{145}$ (years 1969-1970), $\theta_{174}$ (years 1998-1999), $\theta_{188}$ (years 2012-2013) and $\theta_{191}$ (years 2015-2016). All the other consecutive pairs of years have significant change in the annual mean temperatures ($95\%$ credible intervals for the associated contrasts $\theta$ does not contain the zero value).

- Charleston, USA: $\theta_3$ (years 1833-1834), $\theta_{29}$ (years 1859-1860) and $\theta_{103}$ (years 1933-1934). All the other consecutive pairs of years have significant change in the annual mean temperatures ($95\%$ credible intervals for the associated contrasts $\theta$ does not contain the zero value).

- Ankara, Turkey: $\theta_3$ (years 1927-1928), $\theta_5$ (years 1929-1930), $\theta_{11}$ (years 1935-1936), $\theta_{68}$ (years 1992-1993), $\theta_{75}$ (years 1999-2000), $\theta_{84}$ (years 2008-2009) and $\theta_{93}$ (years 2017-2018). All the other consecutive pairs of years have significant change in the annual mean temperatures ($95\%$ credible intervals for the associated contrasts $\theta$ does not contain the zero value).

From these results, we observe that the proportions of pairs of years with same annual temperature are different for each climate station: 0.03356 or $3.36\%$ (5 pairs of consecutive years in a follow-up period of 149 years) for Aberdeen, U.K.; 0.035897 or $3.59\%$ (7 pairs of consecutive years in a follow-up period of 195 years) for Geneva, Switzerland; 0.01587 or $1.59\%$ (3 pairs of consecutive years in a follow-up period of 189 years) for Charleston,USA and 0.07368 or $7.37\%$ (7 pairs of consecutive years in a follow-up period of 95 years) for Ankara, Turkey.

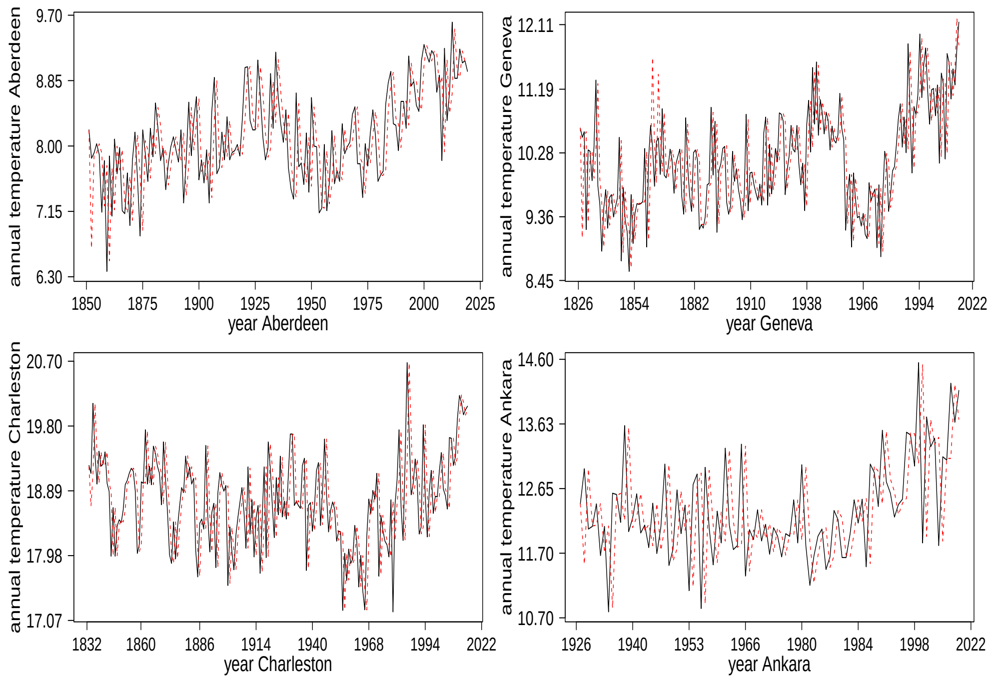


Figure 2: Fitted and observed temperature means for the data of the four climate stations (Aberdeen, Geneva, Charleston and Ankara). Plots with dotted line associated to the fitted model.

Figure 4 shows the graphs of CUSUM (partial sum of consecutive mean temperature differences) versus time order (Aberdeen, Geneva, Charleston and Ankara). Figure 4 (see also Tables A.1, A.2, A.3 and A.4 in Appendix 1 in the suplementary file) shows the following behavior for each climate station:

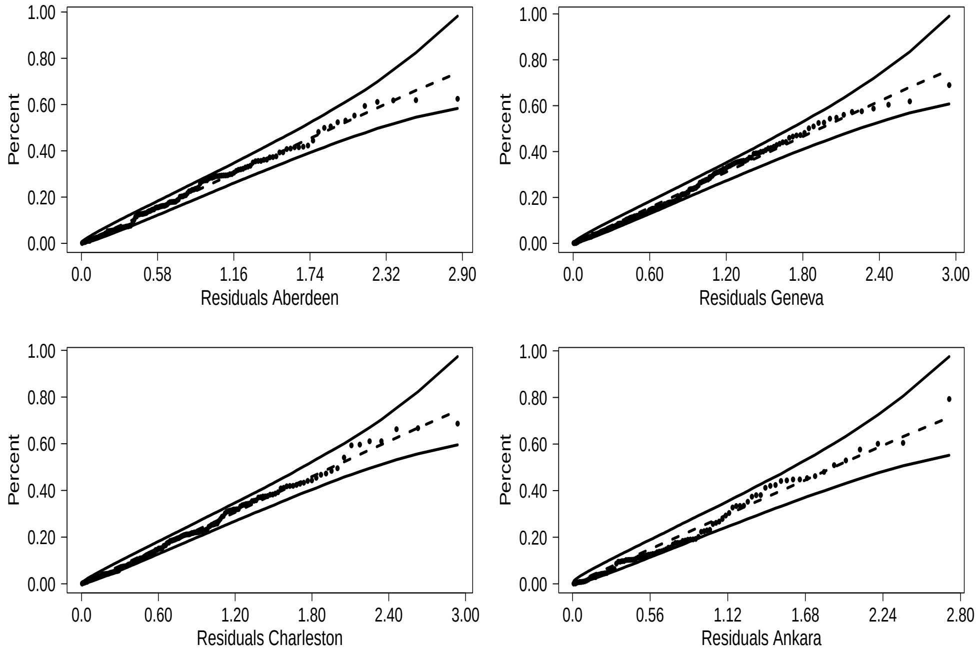


Figure 3: Residual graphs (Aberdeen, Geneva, Charleston and Ankara).

- Aberdeen, UK: the graph of CUSUM (partial sum of consecutive mean temperature differences for two consecutive years) show a cyclic behavior (up/down) but in general there is a trend in the increasing in the CUSUM of the estimated diferences from the begining of the follow-up period until the year close to 1940 ($\theta_{70}$); after the year 1940 there is a decreasing in the CUSUM of the estimated diferences of consecutive pairs of years until the year close to 1960 ($\theta_{90}$); after the year 1960 we observe a increasing in CUSUM of the estimated diferences showing that the annual mean temperatures are increasing year after year until the end of the follow-up period.
- Geneva, Switzerland: the graph of CUSUM (partial sum of consecutive mean temperature differences for two consecutive years) show a cyclic behavior (up/down) but in general there is a decreasing in the CUSUM of the estimated diferences from the begining of the follow-up period until the year close to 1845 ($\theta_{21}$); after the year 1845 there is a increasing in the CUSUM of the estimated diferences of consecutive pairs of years until the year close to 1960 ($\theta_{137}$); after the year 1960 we observe some decreasing in the CUSUM of the estimated diferences (more stability of the annual diferences).
- Charleston, USA: the graph of CUSUM (partial sum of consecutive mean temperature differences for two consecutive years) show a cyclic behavior (up/down) but with more stability around zero for the CUSUM of the estimated diferences from the begining of the follow-up period until the year close to 1960 ($\theta_{128}$); after the year 1960 we observe a increasing in the CUSUM of the estimated diferences showing that the annual temperatures of the year is becoming larger to the annual mean temperature of the previous year.
- Ankara, Turkey: the graph of CUSUM (partial sum of consecutive mean temperature differences for two consecutive years) show a cyclic behavior (up/down) but with more stability around zero for the CUSUM of the estimated diferences from the begining of the follow-up

period until the year close to 1995 ($\theta_{70}$); after the year 1995 we observe a increasing in the CUSUM of the estimated diferences showing that the annual temperatures of the year is becoming larger to the annual mean temperature of the previous year.

# 4 Conclusion

Climate changes (temperature, rainfall, etc.) have been observed all over the world. As these climate changes can have different effects in diferents locations, the detection of times when changes occur is of great interest to all. The modeling of climate time series has been considered in different ways, as observed in the literature. In this study, we assumed a simple model based on an auto-regressive structure, which in addition to leading to a good fit to the data, it was also possible to detect from the adjusted model, significant differences between consecutive annual averages of two years and long periods where it was observed a sharp change in mean annual temperatures in different weather seasons using usual CUSUM control charts usually used in industrial statistical control. The results obtained are simple to be reproduced for any climate series especially under a Bayesian approach using MCMC methods to generate samples of the joint posterior distribution for the parameters of the proposed model. It is important to point out that two-by-two comparisons for the climatic averages in two consecutive years are obtained simultaneously with the simulation of the Gibbs samples. These results can be of great interest in the study of the climatic changes observed throughout the planet.

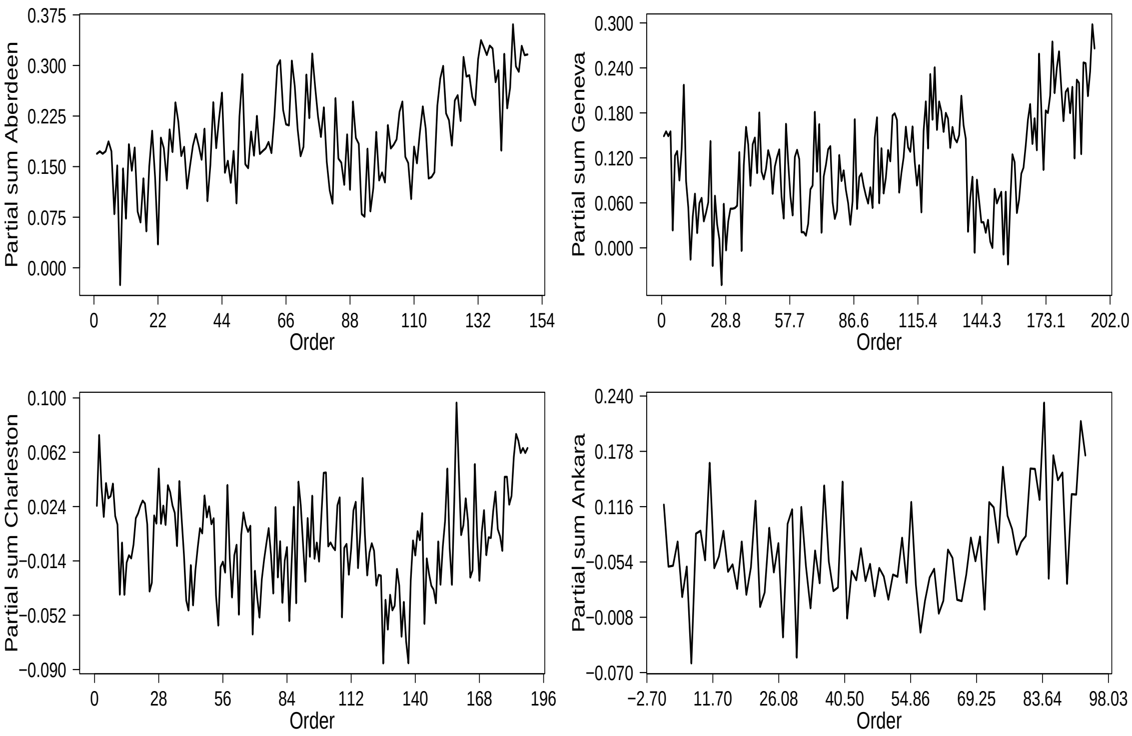


Figure 4: Graph of CUSUM (partial sum of consecutive annual mean temperature differences) versus order (Aberdeen, Geneva, Charleston and Ankara).

## References

[1] NRC. *America's climate choices*. National Academies Press, 2011.

[2] N Noaa. State of the climate: Global climate report for annual 2018. *URL www. ncdc. noaa. gov/sotc/global/201813*, 2019.

[3] IPCC. Report of the nineteenth session of the intergovernmental panel on climate change (ipcc) geneva, 17-20 (am only) april 2002. 2007.

[4] IPCC. Technical summary. In *Climate change 2013: the physical science basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, pages 33–115. Cambridge University Press, 2013.

[5] Nigel W Arnell and Ben Lloyd-Hughes. The global-scale impacts of climate change on water resources and flooding under new climate and socio-economic scenarios. *Climatic change*, 122(1):127–140, 2014.

[6] Anthony Costello, Mustafa Abbas, Adriana Allen, Sarah Ball, Sarah Bell, Richard Bellamy, Sharon Friel, Nora Groce, Anne Johnson, Maria Kett, et al. Managing the health effects of climate change: lancet and university college london institute for global health commission. *The lancet*, 373(9676):1693–1733, 2009.

[7] Ed Hawkins, Pablo Ortega, Emma Suckling, Andrew Schurer, Gabi Hegerl, Phil Jones, Manoj Joshi, Timothy J Osborn, Valérie Masson-Delmotte, Juliette Mignot, et al. Estimating changes in global temperature since the preindustrial period. *Bulletin of the American Meteorological Society*, 98(9):1841–1856, 2017.

[8] Maurice Lineman, Yuno Do, Ji Yoon Kim, and Gea-Jae Joo. Talking about climate change and global warming. *PloS one*, 10(9):e0138996, 2015.

[9] Russell Kabir, Hafiz TA Khan, Emma Ball, and Kay Caldwell. Climate change impact: the experience of the coastal areas of bangladesh affected by cyclones sidr and aila. *Journal of environmental and public health*, 2016, 2016.

[10] Anders Levermann, Peter U Clark, Ben Marzeion, Glenn A Milne, David Pollard, Valentina Radic, and Alexander Robinson. The multimillennial sea-level commitment of global warming. *Proceedings of the National Academy of Sciences*, 110(34):13745–13750, 2013.

[11] David J Kaczan and Jennifer Orgill-Meyer. The impact of climate change on migration: a synthesis of recent empirical insights. *Climatic Change*, 158(3):281–300, 2020.

[12] Tom Matthews. Humid heat and climate change. *Progress in Physical Geography: Earth and Environment*, 42(3):391–405, 2018.

[13] Elvira S Poloczanska, Christopher J Brown, William J Sydeman, Wolfgang Kiessling, David S Schoeman, Pippa J Moore, Keith Brander, John F Bruno, Lauren B Buckley, Michael T Burrows, et al. Global imprint of climate change on marine life. *Nature Climate Change*, 3(10):919–925, 2013.

[14] Olivia Serdeczny, Sophie Adams, Florent Baarsch, Dim Coumou, Alexander Robinson, William Hare, Michiel Schaeffer, Mahé Perrette, and Julia Reinhardt. Climate change impacts in sub-saharan africa: from physical changes to their social repercussions. *Regional Environmental Change*, 17(6):1585–1600, 2017.

[15] Marco Springmann, Daniel Mason-D'Croz, Sherman Robinson, Tara Garnett, H Charles J Godfray, Douglas Gollin, Mike Rayner, Paola Ballon, and Peter Scarborough. Global and regional health effects of future food production under climate change: a modelling study. *The Lancet*, 387(10031):1937–1946, 2016.

[16] Monica G Turner, W John Calder, Graeme S Cumming, Terry P Hughes, Anke Jentsch, Shannon L LaDeau, Timothy M Lenton, Bryan N Shuman, Merritt R Turetsky, Zak Ratajczak, et al. Climate change, ecosystems and abrupt change: science priorities. *Philosophical Transactions of the Royal Society B*, 375(1794):20190105, 2020.

[17] Chuang Zhao, Bing Liu, Shilong Piao, Xuhui Wang, David B Lobell, Yao Huang, Mengtian Huang, Yitong Yao, Simona Bassu, Philippe Ciais, et al. Temperature increase reduces global yields of major crops in four independent estimates. *Proceedings of the National Academy of Sciences*, 114(35):9326–9331, 2017.

[18] Gordon R Richards. Change in global temperature: a statistical analysis. *Journal of Climate*, 6(3):546–559, 1993.

[19] LV Alexander et.al. Global observed changes in daily climate extremes of temperature and precipitation. *Journal of Geophysical Research*, 111, 2006.

[20] Claudia Tebaldi and Bruno Sansó. Joint projections of temperature and precipitation change from multiple climate models: a hierarchical bayesian approach. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 172(1):83–106, 2009.

[21] Ben Powell. Statistical modelling of climate change. In *Climate Change*, pages 341–354. Elsevier, 2016.

[22] Jacqueline Diaz-Nieto and Robert L Wilby. A comparison of statistical downscaling and climate change factor methods: impacts on low flows in the river thames, united kingdom. *Climatic Change*, 69(2):245–268, 2005.

[23] Aurélien Ribes, Francis W Zwiers, Jean-Marc Azais, and Philippe Naveau. A new statistical approach to climate change detection and attribution. *Climate Dynamics*, 48(1):367–386, 2017.

[24] Erik Vanem. Joint statistical models for significant wave height and wave period in a changing climate. *Marine Structures*, 49:180–205, 2016.

[25] Daniel Barry and John A Hartigan. A bayesian analysis for change point problems. *Journal of the American Statistical Association*, 88(421):309–319, 1993.

[26] Bradley P Carlin, Alan E Gelfand, and Adrian FM Smith. Hierarchical bayesian analysis of changepoint problems. *Journal of the royal statistical society: series C (applied statistics)*, 41(2):389–405, 1992.

[27] Michael W Robbins, Colin M Gallagher, and Robert B Lund. A general regression changepoint test for time series data. *Journal of the American Statistical Association*, 111(514):670–683, 2016.

[28] Colin Gallagher, Robert Lund, and Michael Robbins. Changepoint detection in daily precipitation data. *Environmetrics*, 23(5):407–419, 2012.

[29] Shanghong Li and Robert Lund. Multiple changepoint detection via genetic algorithms. *Journal of Climate*, 25(2):674–686, 2012.

[30] Jorge Alberto Achcar and Heleno Bolfarine. Constant hazard against a change-point alternative: a bayesian approach with censored data. *Communications in Statistics-Theory and Methods*, 18(10):3801–3819, 1989.

[31] David W Bacon and Donald G Watts. Estimating the transition between two intersecting straight lines. *Biometrika*, 58(3):525–534, 1971.

[32] Pedro E Ferreira. A bayesian analysis of a switching regression model: known number of regimes. *Journal of the American Statistical Association*, 70(350):370–374, 1975.

[33] Alan E Gelfand and Adrian FM Smith. Sampling-based approaches to calculating marginal densities. *Journal of the American statistical association*, 85(410):398–409, 1990.

[34] Andrew Gelman, John B Carlin, Hal S Stern, and Donald B Rubin. *Bayesian data analysis*. Chapman and Hall/CRC, 1995.

[35] William R Bell. An introduction to forecasting with time series models. *Insurance: Mathematics and Economics*, 3(4):241–255, 1984.

[36] George EP Box, Jenkins, and Gregory C Reinsel. *Time series analysis: forecasting and control*. John Wiley & Sons, 1994.

[37] Douglas C Montgomery, Cheryl L Jennings, and M Kulahci. Introduction to time series analysis and forecasting, a john wiley & sons. *Inc., New Jersey*, 2008.

[38] George EP Box and George C Tiao. Bayesian inference in statistical analysis, addision-wesley. *Reading, MA*, 1973.

[39] Siddhartha Chib and Edward Greenberg. Understanding the metropolis-hastings algorithm. *The american statistician*, 49(4):327–335, 1995.

[40] David J Lunn, Andrew Thomas, Nicky Best, and David Spiegelhalter. Winbugs-a bayesian modelling framework: concepts, structure, and extensibility. *Statistics and computing*, 10(4):325–337, 2000.

[41] José M Bernardo and Adrian FM Smith. *Bayesian theory*, volume 405. John Wiley & Sons, 2009.

[42] David J Spiegelhalter, Nicola G Best, Bradley P Carlin, and Angelika Van Der Linde. Bayesian measures of model complexity and fit. *Journal of the royal statistical society: Series b (statistical methodology)*, 64(4):583–639, 2002.

[43] Olivia A Grigg, VT Farewell, and DJ Spiegelhalter. Use of risk-adjusted cusum and rsprtcharts for monitoring in medical contexts. *Statistical methods in medical research*, 12(2):147–170, 2003.

[44] George A Barnard. Control charts and stochastic processes. *Journal of the Royal Statistical Society: Series B (Methodological)*, 21(2):239–257, 1959.

[45] David Spiegelhalter, Andrew Thomas, Nicky Best, and Dave Lunn. Openbugs user manual. *Version*, 3(2):2007, 2007.

Fischler, R-H. (1998). A Mathematical History of the Golden Number. Dover Publications.