

ON THE USE OF ARIMA AND GARCH IN MODELLING NIGERIA'S NAIRA – US DOLLAR MONTHLY EXCHANGE RATES

ABSTRACT

This paper aimed at modelling the volatility of monthly average official exchange rate (Naira/USD) using the Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) for the period January, 1981 to December, 2021. The data for the study was obtained from Central Bank of Nigeria 2021 Statistical Bulletin. The time plot, Augmented Dickey Fuller (ADF) and Phillip's Perron (PP) were used to check for the Stationarity of the Series. It was discovered that the series is not stationary, thus the need for differencing to make it stationary. Based on the findings of the study, it was concluded that the ARIMA (0, 2,2) and GARCH (1,1) with Student's t-distribution are the optimal models for modeling monthly average official exchange rates return (Naira/USD) in Nigeria.

Keywords: Exchange Rate, GARCH, Heteroscedasticity, ARIMA, Volatility.

I INTRODUCTION

A time series is a record of a variable's results through time; the results may be recorded every day, every week, every month, every three months, every year, or at any other interval of time that is defined. Time series data are known to be unstable. Without a doubt, the daily rate of exchange between two currencies is a time series. According to David et al. (2016), "a time series can be investigated in order to create a suitable model that might be required for planning purposes. For instance, knowing how much one currency will be worth in the future might help a country avoid inflation, calculate its balance of payments, and create workable economic policies, among other advantages".

The volatility of financial time series appears to change over time. Bollerslev (1986) extended "the autoregressive conditional heteroscedasticity (ARCH) model to include generalized autoregressive conditional heteroscedasticity (GARCH), which is one class of models that have been developed with a feature that accommodates the dynamics of conditional heteroscedasticity. The use of money as a medium of exchange makes it easier for different people interacting in a marketplace to complete transactions. However, because there are several forms of exchange,

dealings between persons who reside in other nations are more challenging”. The exchange rate, according to Obafemi (2017), “is the value of one country's currency in respect to another. It is the cost involved in exchanging one currency for another. In terms of the currencies of the majority of industrialized nations, such as United States Dollars and British Pound Sterling, it calculates the internal value of an economy”.

One of the endogenous factors that might impact a country's economic success is exchange rate policy. Following the introduction of the structural adjustment program policy in 1986, Nigeria switched from a pegged or hard exchange rate regime to a more flexible regime, with the help of the Central Bank of Nigeria. No exchange rate is "clean or pure float," which is a situation where the exchange rate is completely determined by market forces of demand and supply, as highlighted by Esam (2017) in his work. Instead, the dominant system is the managed float, in which the monetary authorities periodically intervene in the foreign exchange market of a country in order to achieve some strategic goals.

Empirically, Shahla *et al*, (2012) studied and modelled “Monthly average foreign exchange rates of Pakistan using ARIMA and GARCH models. The findings of their study revealed ARMA (1,2) and GARCH (1,2) to be best models in terms of fitting and forecasting power of the exchange rate return”. Bala et al, (2013), “uses variants GARCH volatility models to examines monthly exchange–rate return for three major currencies in the Nigerian: FX markets: The Naira/USD, Euro and BPS. Their findings reveal that TGARCH is the best fitting model for Euro, while ARCH and PARCH (1,1) are the best fitting models for BPS return and Naira/USD returns and IGARCH was suitable for the USD return model with volatility breaks”. Umar et al (2019) applied “ARIMA and GARCH models in modelling Naira/Pounds exchange rate volatility and found that ARIMA (2, 1, 1) and GARCH (1,1) are the optimal with the highest log-likelihood and lowest AIC and BIC”. Similarly, Odukoya and Adio (2022) carried out “study with title: Time Series Analysis of Exchange Rate Nigerian Naira to Us Dollar. Their study revealed that ARIMA (1, 1, 0) as an appropriate model for forecasting Nigerian Naira to Us Dollar exchange rate”. Etuk (2012) applied “a seasonal ARIMA model for daily Nigerian Naira-US Dollar exchange rates and found SARIMA (2, 1, 0)×(0,1,1)₇ as the optimal model that best fitted Nigerian Naira to Us Dollar series”.

Oyengu, Oyenkunle and Agona (2019) “model exchange rate of Nigeria against four major currencies using ARIMA model. The result of the analysis revealed that the four major

currencies (Dollar, Pounds, Euro and Swiss Franz) times series data were not stationary at their original form and followed ARIMA (1,2,1); ARIMA (2,2,1); ARIMA (2,2,1) and ARIMA (2,2,2) respectively”. Gabriel (2019) “models Naira/1 Rupee exchange rate (NREXR) using ARIMA framework using a spanning from 2008 to 2020 and found that ARIMA (1, 1, 2) model is the best performing model”. Nwankwo (2014) used “Autoregressive Integrated Moving Average Model for Exchange Rate (Naira to Dollar) from 1982 – 2011 and concluded that AR(1) is the most preferred model”. Adetunji, Adejumo and Omowaye (2016) examined “ARIMA (0, 0, 0 to 2, 2, 2) using Square Root Transformation (SRT), Natural Log Transformation (NLT) and original series without transformation (WT) of average exchange rate of Nigeria Naira to US Dollar from 1960 to 2015 and found that ARIMA (1, 0, 0) when SRT is utilized provide optimal output model for forecasting average yearly exchange rate of Nigeria Naira to US Dollar”. Adepoju, Yaya, and Ojo (2013) applied “variants of GARCH models under non-normal innovations-t-distribution and Generalized Error Distribution (GED) on selected Nigeria exchange rates and found that the Asymmetric GARCH model with t-distribution and Generalized error distribution are selected in most cases and both distribution showed evidence of leptokurtic in Naira – USD exchange rate”. Atoi and Nwambeke (2021) examines “money market and foreign exchange market dynamics in Nigeria by estimating the dynamic correlation and volatility spillovers between Nigeria Naira/US Dollar with data from January 2007 to August 2019 and found that the dynamic conditional correlation GARCH (1, 1) and Baba, Engle, Kraft and Kroner GARCH (1,1) were the best model for estimating the dynamic correlation and volatility spillovers between Naira and USD Bureau De change and interbank call rate”. Therefore, the goal of this paper is to fit a time series models (ARIMA and GARCH models) for the average official Naira/US Dollar exchange rates.

II MATERIALS AND METHODS

A) Data

The data for this study is monthly average official exchange rates (Naira/US Dollar). The data was obtained from Central Bank of Nigeria 2021 Statistical Bulletin for the period January, 1981 – December, 2021).

B) Autoregressive Integrated Moving Average (ARIMA)

Box and Jenkin (1978) proposed Autoregressive Integrated Moving Average with general notation ARIMA (p, d, q). The p denotes the order of autoregressive term, d order of integrated term (that is the number of terms a series need to be difference to induce stationarity) and q is the order of moving average terms.

$$\phi(B)(1 - B)^d y_t = \mu + \theta(B)\varepsilon_t \quad (1)$$

Where $\phi(B)$ and $\theta(B)$ are the polynomials of degree p and q respectively. $(1 - B)^d = \nabla^d$ stand for the differencing operator and ε_t is white noise process which is independently and normally distributed with zero mean and constant variance.

C) Autoregressive Conditional Heteroscedasticity (ARCH) Model

Engle (1982) proposed a systematic framework for volatility modelling called Autoregressive Conditional Heteroscedasticity. In this model, the conditional variance of a time series is a function of past shocks. The model is capable if investigating the issues involving volatility of economic variables.

The ARCH model assumes that:

$$y_t = \sigma_t \varepsilon_t \quad (2)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^m \alpha_i a_{t-i}^2 \quad (3)$$

Where ε_t is a sequence of independent and identically distributed random variables with mean zero and variance one $\varepsilon_t \sim N(0,1)$. α_0 represents the average values of σ_t^2 . The ARCH coefficients α_i must satisfy stationary condition to ensure that the unconditional variation exists.

If $\sum_{i=1}^p \alpha_i < 1$ the ARCH model is weakly stationary with constant unconditional variables:

$$\sigma^2 = \frac{\omega}{1 - \sum_{i=1}^p \alpha_i} \quad (4)$$

D) The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

Despite Engle's widely adopted ARCH approach, the model has a lot of drawbacks. For instance, the ARCH model assumes that the conditional variance is affected equally by both positive and negative shocks. Bollerslev (1986) proposed the generalized ARCH, or GARCH, model to address this issue by letting the current volatility to depend on both the first q past volatility and the p past squared innovations. This model can be written as:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (5)$$

where $\omega > 0$, $\alpha_i \geq 0$, $i = 1, \dots, p$ and $\beta_j \geq 0$, $j = 1, \dots, q$, are sufficient conditions to ensure that the conditional variance $\sigma_t^2 > 0$. ω represent the average values of σ_t^2 , ε_t is a white noise with mean zero and variance 1. The parameters α_i represents the ARCH effect and β_j represents the GARCH effect. In addition, to achieve the stationarity requirement in GARCH models the summation of $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j$ must be less than one. This summation reflects the persistence of innovations (shocks) to the volatility, meaning that the impact of a volatility shock disappears over time at an exponential rate.

The first order, GARCH (1,1) model is said to be weakly stationary if $\alpha_1 + \beta_1 < 1$, in this situation the unconditional variance is:

$$var(\varepsilon_t) = \frac{\omega}{1 - \alpha_1 - \beta_1}. \quad (6)$$

When $\alpha_1 + \beta_1 = 1$, it implies that the unconditional variance is infinite. Therefore, the GARCH model is named integrated GARCH or IGARCH model.

It has been demonstrated that the low-order GARCH (1,1) model can accurately represent both volatility clustering and thick tails of data. The volatility model family's GARCH (p, q) models are typically regarded as its most reliable members (Bollerslev et al., 1992; Angelidis et al., 2004).

III RESULTS AND DISCUSSION

The previous section presents the theory of ARIMA and GARCH models. In this section, the empirical results were presented.

EXR

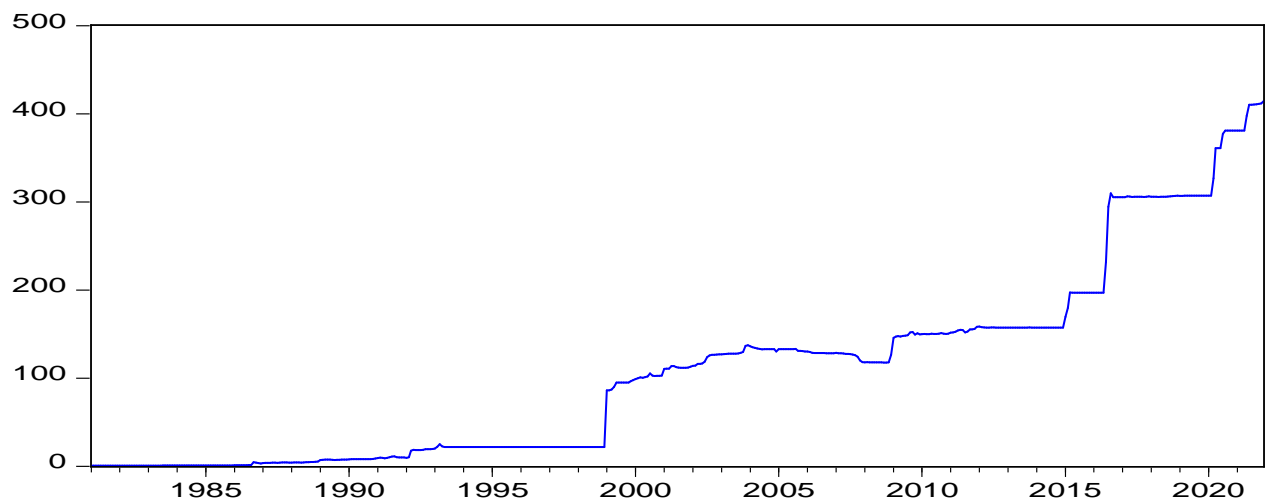


Fig.1: Time Plot of Monthly Average Official Exchange Rate (Naira/USD) at Level
D(EXR)

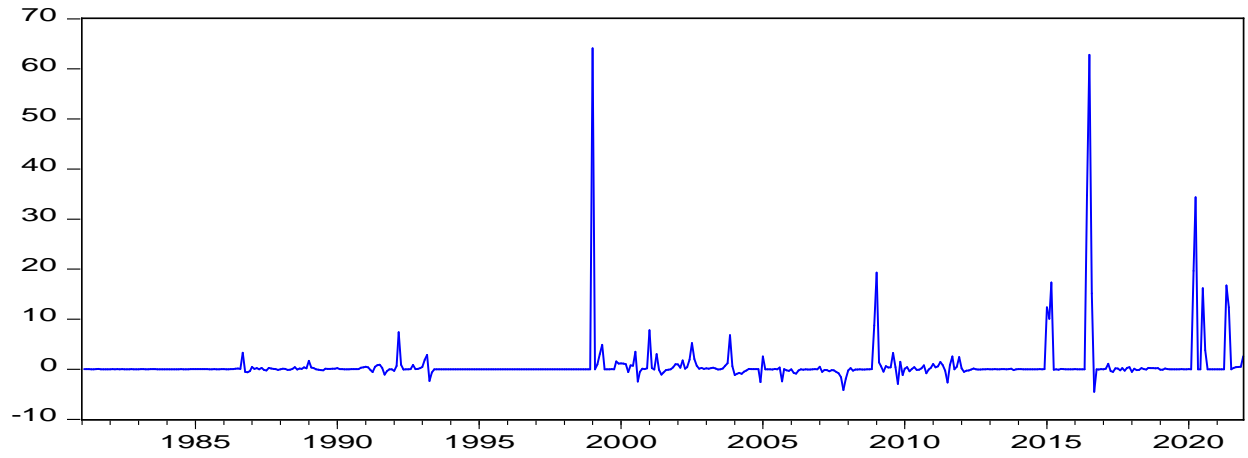


Fig. 2: Time Plot of Monthly Average Official Exchange Rate (Naira/USD) after first Difference

Date: 10/16/22 Time: 11:30
 Sample: 1981M01 2021M12
 Included observations: 492

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.990	0.990	485.01	0.000
		2	0.979	-0.037	960.47	0.000
		3	0.968	-0.008	1426.4	0.000
		4	0.957	-0.010	1882.8	0.000
		5	0.946	-0.011	2329.6	0.000
		6	0.935	-0.005	2766.9	0.000
		7	0.924	-0.007	3194.8	0.000
		8	0.914	0.025	3613.9	0.000
		9	0.904	0.035	4025.1	0.000
		10	0.894	-0.008	4428.5	0.000
		11	0.885	-0.009	4823.9	0.000
		12	0.875	-0.008	5211.6	0.000
		13	0.865	-0.006	5591.4	0.000
		14	0.855	-0.010	5963.4	0.000
		15	0.845	-0.011	6327.6	0.000
		16	0.835	-0.009	6683.8	0.000
		17	0.825	-0.017	7031.9	0.000
		18	0.814	-0.008	7372.0	0.000
		19	0.805	0.027	7704.7	0.000
		20	0.795	-0.008	8030.0	0.000
		21	0.785	-0.006	8348.0	0.000
		22	0.777	0.083	8660.1	0.000
		23	0.770	0.040	8967.4	0.000
		24	0.763	-0.009	9269.7	0.000

Fig 3. Correlogram of Exchange Rate

The correlogram plot was used to test for the presents of seasonal effect in the data set. The plot as presented in figure 3 indicates that there is no seasonal effect in the data set. Thus, providing the justification for using non-seasonal ARIMA model.

Table 1: Descriptive Statistic of the Returns of Exchange Rate

Mean	0.8428	Std Dev.	5.1266	Jarque-Bera	200565
Maximum	64.1100	Skewness	9.1047	Sig. of J-B	0.00000
Minimum	-4.5000	Kurtosis	100.3243		

Sample January, 1981 to December, 2021

Table 2: Unit Root Test using ADF and PP

Method	Difference Order	Statistic	P-value	Remark
ADF	0	2.0289	0.9999	Not Stationary
	1	-14.1653	0.0000	Stationary
PP	0	2.2110	1.0000	Not Stationary
	1	-15.5428	0.0000	Stationary

Source: Extracted from EVIEWS Output

Table 1 presents the summary statistic of returns of exchange rate. The result revealed a mean close to 1 with high standard deviation of 5.1266 which indicates high level of fluctuations in the returns of monthly official average exchange rate. The skewness and kurtosis values indicate that the series is leptokurtic and positively skewed suggesting that exchange rate returns is not normally distributed as confirmed by Jarque-Bera statistic.

The Augmented Dickey Fuller (ADF) and Phillips Perron (PP) results presented in Table 2 indicates that exchange rate returns are not stationary (p-values > 0.05). However, the series becomes stationary after first difference (p-values < 0.05). The parameters of interest were not identified after first difference, thus the need for second differencing.

ARIMA Model Fitting

Table 3: Results of ARIMA (0, 2, 2)

Parameter	Coefficient	Std Error	t-statistic	P-value
C	0.0050	0.0039	1.2715	0.2042
MA(1)	-0.6437	9.5450	-0.0674	0.9463
MA(2)	-0.3563	8.2515	-0.0432	0.9656

Source: Extracted from EVIEWS Output

The Grid Search method where the appropriate model is one which is both stationary and invertible is employed. Among the possible ARIMA models applied, the ARIMA (0, 2, 2) emerges as the optimal model with the least Akaike Information Criterion (AIC). The estimates of the parameters are presented in Table 3.

RESIDUALS DIAGNOSTIC OF ARIMA MODEL

Date: 10/16/22 Time: 11:53
 Sample: 1981M01 2021M12
 Included observations: 490
 Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.002	0.002	0.0012	
		2	-0.002	-0.002	0.0024	
		3	-0.006	-0.006	0.0212	0.884
		4	0.032	0.032	0.5255	0.769
		5	-0.045	-0.046	1.5499	0.671
		6	-0.018	-0.018	1.7099	0.789
		7	-0.022	-0.022	1.9581	0.855
		8	-0.014	-0.015	2.0501	0.915
		9	-0.035	-0.033	2.6757	0.913
		10	0.003	0.002	2.6806	0.953
		11	-0.010	-0.010	2.7264	0.974
		12	-0.035	-0.037	3.3386	0.972
		13	0.007	0.007	3.3607	0.985
		14	0.020	0.015	3.5533	0.990
		15	-0.009	-0.011	3.5934	0.995
		16	0.073	0.073	6.2692	0.959
		17	0.035	0.030	6.8899	0.961
		18	0.054	0.051	8.3647	0.937
		19	-0.052	-0.051	9.7529	0.914
		20	-0.012	-0.017	9.8243	0.937
		21	-0.022	-0.019	10.071	0.951
		22	-0.014	-0.012	10.166	0.965
		23	-0.037	-0.026	10.876	0.965
		24	0.035	0.035	11.495	0.967

Figure 4: Correlogram of residuals

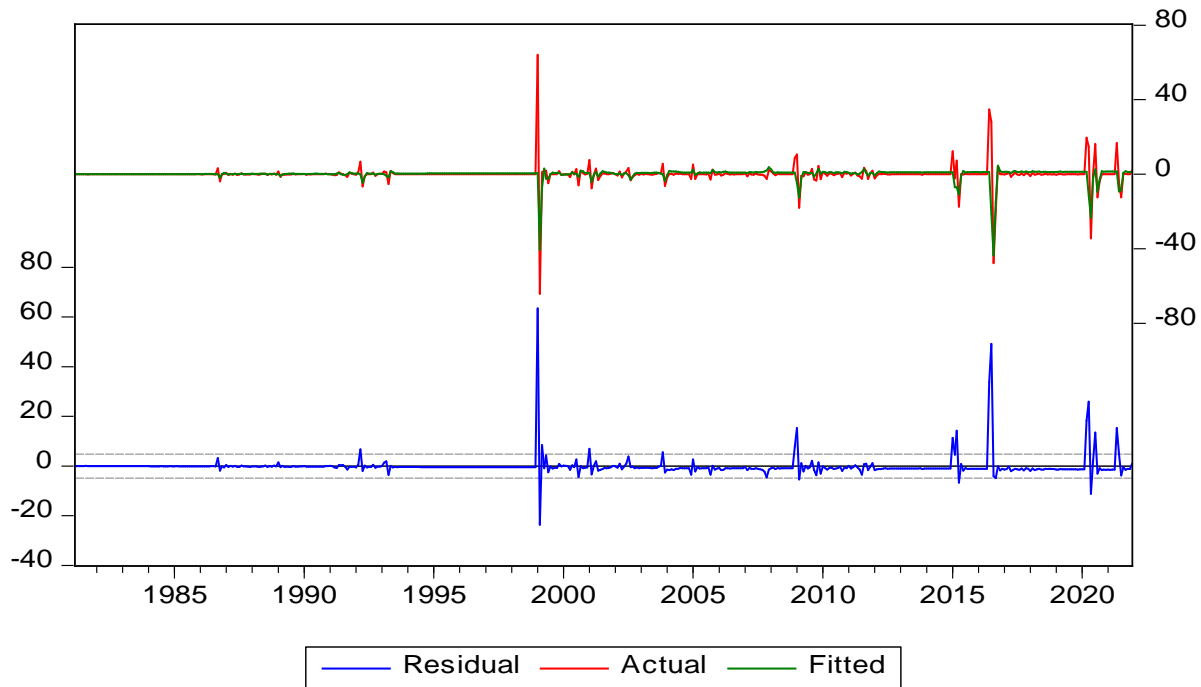


Fig 5: Residual Versus Actual Graph of the fitted model

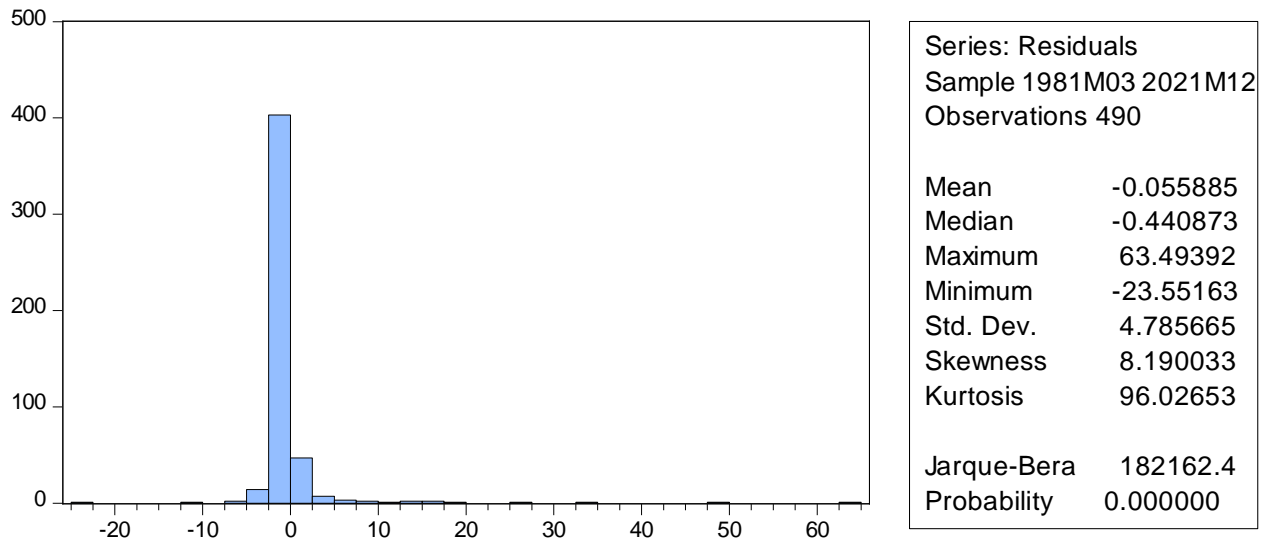


Figure 6: Histogram of residuals

The results in Figure 4 was used to test the null hypothesis which state that residuals are white noise. The residuals of the ACF and the PACF for the estimated model is relatively small or approximately equal to zero that is, the spikes are within the two-error bound or 95% confidence interval. In addition, the p-values for all the lag period were found to be greater than 5%. This

provide enough evidence to accept the null hypothesis suggesting that the residuals are white noise.

In Figure 5, we can see some spiky residuals in high volatile periods such as months of 1999, 2016 and 2021. The residuals plots are quite similar to the one for difference series. Thus, suggesting that the estimated model is adequate.

In Figure 6, the histogram and normality test for residuals are plotted. The mean value of the residuals is -0.055885 is approximately zero. The values of skewness and kurtosis are 8.190033 and 96.02653 respectively. This means that the residuals have excessive kurtosis and slightly skewed to the right. Jarque-Bera test shows that the residuals series is not normally distributed at 5% significance level.

The out of sample forecast validation graph was presented in Fig. 7 and the forecast values, actual and the errors were presented in table 4. The forecast values were closer to the actual values with increasing pattern over time. This suggest that the model is good for forecast.

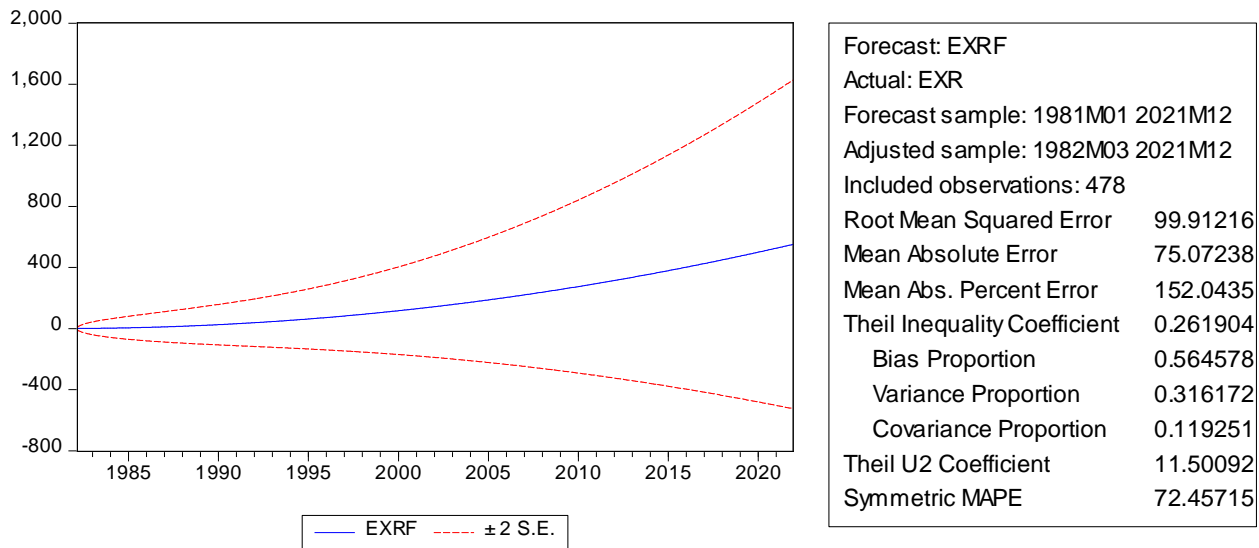


Fig. 7: Forecast Graph

Table 4: Out of Sample Forecast Performance

Years	Actual	Forecast Value	Error
Jan-2021	381.00	526.43	145.43
Feb-2021	381.00	528.65	147.65
Mar-2021	381.00	530.87	149.87
Apr-2021	381.00	533.09	152.09
May-2021	397.75	535.32	137.57
Jun-2021	410.12	537.55	127.43
Jul-2021	410.12	539.78	129.66

Aug-2021	410.39	542.03	131.64
Sep-2021	410.80	544.27	133.47
Oct-2021	411.25	546.52	135.27
Nov-2021	411.74	548.78	137.04
Dec-2021	414.34	551.04	136.70

Source: Extracted from EViews Output

GARCH Model Fitting

Table 5: Heteroskedasticity Test: ARCH

F-statistic	11.06821	Prob. F(1,488)	0.0009
Obs*R-squared	10.86710	Prob. Chi-Square(1)	0.0010

Source: Extracted from EViews Output

Table 6: Model Selection Criterion

Order of GARCH	Criterion	Error Distributions		
		Normal Dist.	Student's t Dist.	GED
(1,1)	Log Likelihood	-1761.428	-84.1563	-187.0649
	SIC	7.2527	0.4194	0.8394
	AIC	7.2099	0.3680	0.7880
	HQIC	7.2267	0.3882	0.8082
(1,2)	Log Likelihood	-1773.550	-209.5288	-209.5288
	SIC	7.3148	0.9437	0.9437
	AIC	7.2635	0.8837	0.8838
	HQIC	7.2836	0.9073	0.9073
(2, 1)	Log Likelihood	-1737.983	-108.4288	-968.700
	SIC	7.1697	1.9321	4.0424
	AIC	7.1183	2.3245	3.9825
	HQIC	7.1385	0.4351	4.0060

Source: Extract from EViews Results

Table 7: GARCH (1,1) Model with Student's t-distribution

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-1.42E-05	0.000382	-0.037148	0.9704
REXR(-1)	0.212667	0.030154	7.052706	0.0000
Variance Equation				
C	3.73E-05	0.000422	0.088534	0.9295
RESID(-1)^2	335.8282	3838.147	0.087497	0.9303
GARCH(-1)	0.051246	0.006363	8.054194	0.0000

T-DIST. DOF	2.004169	0.047890	41.84953	0.0000
R-squared	-0.051441	Mean dependent var		1.351944
Adjusted R-squared	-0.053596	S.D. dependent var		9.204078
S.E. of regression	9.447508	Akaike info criterion		0.367985
Sum squared resid	43556.64	Schwarz criterion		0.419345
Log likelihood	-84.15634	Hannan-Quinn criter.		0.388156
Durbin-Watson stat	2.336011			

Source: Extracted from EViews Output

From Table 5, it was observed that F-statistic = 11.06821, Prob(1,488) = 0.0009 < 0.05 and Obs*R-squared = 10.86710, Prob(chi-square) = 0.0010 < 0.05 indicates that the series was heteroskedastic indicating that there is presence of ARCH effect in the model. Thus, providing the justification for using GARCH models.

From Table 6 it can be observed that the GARCH (1,1) model with student's t error distribution construct has the least value of AIC, SIC, HQIC and high value of log likelihood. Thus, the GARCH (1,1) model with student's t-distribution error construct was considered as the optimal GARCH model for monthly average official exchange rate returns.

Table 7 presents the results of GARCH (1,1) models based on students' t-distribution error construct. The mean equation is $0.000042 + 0.212667REXR(-1)$. The average exchange rate returns is 0.000042 and its past values significantly predict the current series by 0.212667.

The variance equation is $\hat{h}_t = 0.0000373 + 0.051246\hat{h}_{t-1} + 335.8282\hat{\mu}_{t-1}^2$. The GARCH coefficient is positive and statistically significant at 5% level. The time-varying volatility includes a constant 0.0000373 plus its past $0.051246\hat{h}_{t-1}$ and a component which depend on past errors $335.8282\hat{\mu}_{t-1}^2$. The findings clearly establish the presence of time-varying conditional volatility of returns of exchange rate. The results also indicate that the persistence of volatility shocks as represented by the sum of the ARCH and GARCH parameters is large. This implies that effect of this month shock remains in the forecast of variance for many periods in the future.

GARCH (1,1) Model Residuals Diagnostics

Date: 09/18/22 Time: 18:06
 Sample: 1981M01 2021M12
 Included observations: 490
 Q-statistic probabilities adjusted for 1 dynamic regressor

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*
		1 -0.003	-0.003	0.0040	0.949
		2 -0.003	-0.003	0.0081	0.996
		3 -0.003	-0.003	0.0121	1.000
		4 -0.003	-0.003	0.0162	1.000
		5 -0.003	-0.003	0.0203	1.000
		6 -0.003	-0.003	0.0245	1.000
		7 -0.003	-0.003	0.0286	1.000
		8 -0.003	-0.003	0.0326	1.000
		9 -0.003	-0.003	0.0366	1.000
		10 -0.002	-0.002	0.0393	1.000
		11 -0.003	-0.003	0.0433	1.000
		12 -0.003	-0.003	0.0474	1.000

*Probabilities may not be valid for this equation specification.

Figure 8: Correlogram Q-statistic

Date: 09/18/22 Time: 18:10
 Sample: 1981M01 2021M12
 Included observations: 490

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*
		1 -0.002	-0.002	0.0021	0.964
		2 -0.002	-0.002	0.0042	0.998
		3 -0.002	-0.002	0.0062	1.000
		4 -0.002	-0.002	0.0084	1.000
		5 -0.002	-0.002	0.0105	1.000
		6 -0.002	-0.002	0.0126	1.000
		7 -0.002	-0.002	0.0148	1.000
		8 -0.002	-0.002	0.0169	1.000
		9 -0.002	-0.002	0.0191	1.000
		10 -0.002	-0.002	0.0213	1.000
		11 -0.002	-0.002	0.0235	1.000
		12 -0.002	-0.002	0.0257	1.000

*Probabilities may not be valid for this equation specification.

Figure 9: Correlogram of Standard Residual Squared

From figure 8 and 9, it can be observed that there is no evidence of serial correlation in the residuals of the estimated model since all the p-values are greater than 0.05 level of significance. Also, all the spikes of ACF and PACF lies within the 95% confidence intervals indicates that the model is good.

Table 8: Heteroskedasticity Test: ARCH

F-statistic	0.002045	Prob. F(1,487)	0.9639
Obs*R-squared	0.002053	Prob. Chi-Square(1)	0.9639

Source: Extracted from EViews Output

Table 8 presents the results of heteroscedasticity test. The F-statistic = 0.002045 with p-value = 0.9639 indicates that there is no evidence of heteroscedasticity in the residuals of the estimated model. Thus, there is no ARCH effect in the estimated model.

IV CONCLUSION

This study employed ARIMA and GARCH to model the monthly average exchange rate of Nigeria Naira and US Dollar. The results of the unit root test indicate that the data was not stationary however becomes stationary after first difference but the ARIMA model parameters was identified after second differencing. Using the model selection criterion, the ARIMA (0, 2, 2) and GARCH(1,1) with Student's t-distribution were chosen as the optimal models for forecasting the future values and modelling the volatility of monthly average exchange rate returns of Nigeria (Naira/US Dollar).

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