Original Research Article

An Alternative Estimator for the Estimation of Polynomial Regression Model (PRM)

Abstract

The maximum likelihood (ML) approach is used to fit the polynomial regression model (PRM) in the presence of small sample sizes. The ML technique is applied to the data of PPP, GDP, and output/total production cost in Nigeria between 1989 and 1999. The results of the analyses (by ML approach and that of the OLS) are presented for comparison. The analysis shows that the ML gives parameter estimates of 128.889, -5.24, -29.208, 10.523 and the OLS resulted in 128.009, 5.196, -30.376, 11.009. The analysis of the first data set (of iron content and weight loss of some specimen tested in a corrosion wheel set-up) shows that both estimators accounted for good fit because they both have high R2 values and significant t-ratios. The result of the model fit of the four data sets using ML results in reasonable parameter estimates (with lesser S.E relative to the parameter estimates), lower MSE, and very high R^2-values. Although both methods were generally well adapted, ML was more effective than OLS because it led to a smaller sample size's MSE.

Keywords: Polynomial regression model (PRM)**,** Maximum Likelihood (ML), Ordinary Least Squares (OLS)**,** Mean square error (MSE), Unbiasedness, Robustness, Efficiency**.**

1.0 Introduction

Polynomial regression is a type of regression where the relationship between dependent γ and the independent χ variables is expressed as a K^{th} degree polynomial. Polynomial regression can be used to model situations in which the relationship between the independent and dependent variables is not linear. The idea behind polynomial regression is to fit a polynomial curve to the observed data and use this curve to make predictions. The distribution of carbon isotopes in lake sediments, the growth rate of tissues, and the transmission of disease epidemics have all been described using polynomial regression. Between the value of χ and the associated conditional mean of γ, abbreviated as $E(\gamma \mid \chi)$, it fits a nonlinear connection. Even though a nonlinear model is used in polynomial regression to analyze data, the regression function $E(\gamma \mid \chi)$ is linear with respect to the unknown parameters that are estimated from the data. As a result, polynomial regression can be considered a linear statistical estimation problem. This perspective makes multiple linear regression a specialized form of polynomial regression. In a 1974 study by Gergonne, the design of an experiment for polynomial regression was presented. Regression analysis evolved significantly in the 20th century with the substantial impact of polynomial regression, which placed more attention on design and inference-related concerns.

In polynomial regression, a dependent variable is regressed on the powers of the independent variables. It can also be used when the study and the explanatory factors have a curved relationship. In some cases, a nonlinear relationship in a limited range of explanatory variables can be modelled using polynomials. The relationship between variables in a data collection is frequently better represented by an equation. The most popular representation is a polynomial of degree K^{th} that has the formula

$$Y = a_k x^k + ... + a_1 x + a_0 + \varepsilon \quad (1)$$

The error serves as a reminder that, for any given value of χ, the polynomial will typically produce an estimate rather than an implied value of the dataset. The general polynomial regression model is a common name for the equation above. The predictors that result from the polynomial expansion of the “baseline” predictors are interaction features. These predictors or qualities are also applied in categorization settings. The quantity of data points used to generate the polynomial determines its maximum order. The polynomial has a maximum order of k=N-1 for a set of N data points. Though higher order polynomials travel through each data point directly, they can behave erratically between them due to a phenomenon known as polynomial wiggle, hence it is typically best practice to use the order as low as feasible to effectively represent the dataset.

Statistical inference, systematic risk estimation, and production are just a few of the numerous tasks for which polynomial regression analysis has been utilized in the business world. The exploration of data sets for users using approaches that allow for description and inference is current practice in its teaching. However, there are numerous options available for the real learner when computing regression coefficients and summary statistics. One of them was described in Kmenta (1990) as a computational design that enables users to do calculations with only a pencil and paper. It was also recommended that students may easily build a scatter plot and a ruler to visually approximate the regression line. It was also advised to employ statistical software, which is currently available to users of mainframe and microcomputers (Mundrake & Brown,1989). The ordinary least squares (OLS) or nave estimator of the relevant slope parameter will be biased if a regressor of linear regression is measured with errors, and the bias will typically attenuate the true value of the slope parameter (Cheng & Van Ness, 1999).

Under the constraints of the Gauss-Markov theorem, the least squares approach in the Polynomial Regression Model (PR)M, like other regression models, minimizes the variance of the unbiased estimators of the coefficients. More recently, the estimation of polynomial models has been complemented by different methods such as adjusted least squares, structural least squares and many more with certain inadequacies which are dealt with differently. It has been established that most of these methods present estimators with notable inadequacies. The most notable of these inadequacies are poor handling of outliers and small sample sizes. The latter leads to the violation of the OLS assumption on large samples and makes the validity of the procedure questionable. This motivates the application of the maximum likelihood (ML) estimator for modelling polynomial regression models in the presence of small samples.

2.0 Materials and Methods

The application of ML technique for first order polynomial regression model of degree 2 and 3 polynomial regression models. The development of the ML technique entails the construction of the likelihood function on the response variable Y in (1) by assuming that it is Gaussian in nature. The log-likelihood will be maximized with respect to each partial slope coefficient and the resulting system of equations will be solved simultaneously to obtain the ML estimate of each of the partial slope coefficients. The ML estimators will be fitted to some data sets which are characterized by small sample sizes. These data were would be subjected to exploratory analysis to affirm that they follow the appropriate polynomial (regression) order. The data sets are namely data of corrosion wheel set up, data of Nigerian PPP and GDP and data of output, the total production cost of a commodity and electricity consumption data. The ML approach is used to fit the appropriate polynomial order to each of the data sets. The validity of the model fit (by ML approach) shall be tested by comparing its results with that of the OLS using their parameter estimates (partial slope coefficients), standard error estimates (of each partial slope), coefficients of determination and mean square error (MSE) values. In particular, the efficiency of the methods shall be tested using the coefficient of determination, the test of significance (of parameters or variables) and MSE. The illustration and implementation shall be done by considering the problem of ten home sizes (sq ft) and their power consumption (KWh/month) reported in McClave & Deitrich (1991) as well as the data of total production cost and output of a commodity reported in Gujarati (2004) to mention a few. The analysis will be carried out using SAS (version 9.4).

3.0 Application of the Maximum Likelihood Approach to Polynomial Regression Model

Polynomial regression is a specific form of multiple regression that involves only a single independent (predictor) variable X. The one-variable polynomial regression model can be represented as:

$$Y_i = \beta_o + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + \ldots + \beta_k X_i^k + \varepsilon \quad for\ i = 1, 2, \ldots, n \qquad (2)$$

where Y_i is the response variable, X_i*'s are* the control variables, β_i*'s* are the partial slope parameters, ε_i is the stochastic disturbance (error) term which is Gaussian with expected value zero and common variance σ_i^2 and k is the degree of the polynomial. (2) is called the first order k^{th} degree polynomial which in fact is synonymous with (1).

Effectively, (2) is the same as having a multiple model with $X_1 = X$, $X_2 = X^2$, $X_3 = X^3$e.t.c

From (2), a first-order polynomial regression model of degree (3) is

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + \varepsilon_i \qquad (3)$$

Where the random error term is tagged as the random noise ε_i which is assumed to have a Gaussian distribution with mean zero and variance σ_i^2 i.e. N $(0, \sigma_i^2)$, so that four unknown parameters, β_0, β_1, $\beta_2$and β_3 are to be estimated using any given sample (data). Since X_i*'s* are thought of as fixed points and non-random, their randomness is dealt with using the noise variables ε_i, then for fixed X_i*'s*, the distribution of Y_i is also equal to $N\,(E(Y_i), \sigma^2)$ with p.d.f.

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(Y_i - \mu_i)^2 / 2\sigma^2} \qquad (4)$$

Since $\mu_i = E(Y_i) = \hat{Y}_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3$ (unbiasedness)

Then (4) can be written as

$$f(Y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(Y-E(Y_i))^2/2\sigma^2} \tag{5}$$

so that the likelihood function of the random sample $Y_1, \ldots, Y_n$ can be written as

$$L = f(Y_1, \ldots, Y_n) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}[Y_i - \hat{Y}(X_i)]^2} \tag{6}$$

So that (4) becomes

$$L = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 X_i - \beta_2 X_i^2 - \beta_3 X_i^3)^2} \tag{7}$$

Taking the natural logarithm of (5) yields the log-likelihood

$$\ln L = \frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 + \frac{1}{2\sigma^2}\sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_i - \beta_2 X_i^2 - \beta_3 X_i^3)^2 \tag{8}$$

The estimates of the parameters β_0, β_1, β_2 and β_3 are obtained by maximizing (8) w.r.t the parameters. The maximization of the log-likelihood function w.r.t parameters β_0, β_1, β_2 *and* β_3 yields

$$\sum_{i=1}^{n} Y_i = n\beta_0 + \beta_1 \sum_{i=1}^{n} X_i + \beta_2 \sum_{i=1}^{n} X_i^2 + \beta_3 \sum_{i=1}^{n} X_i^3 \tag{8a}$$

$$\sum_{i=1}^{n} X_i Y_i = \beta_0 \sum_{i=1}^{n} X_i + \beta_1 \sum_{i=1}^{n} X_i^2 + \beta_2 \sum_{i=1}^{n} X_i^3 + \beta_3 \sum_{i=1}^{n} X_i^4 \tag{8b}$$

$$\sum_{i=1}^{n} X_i^2 Y_i = \beta_0 \sum_{i=1}^{n} X_i^2 + \beta_1 \sum_{i=1}^{n} X_i^3 + \beta_2 \sum_{i=1}^{n} X_i^4 + \beta_3 X_i^5 \tag{8c}$$

$$\sum_{i=1}^{n} X_i^3 Y_i = \beta_0 \sum_{i=1}^{n} X_i^3 + \beta_1 \sum_{i=1}^{n} X_i^4 + \beta_2 \sum_{i=1}^{n} X_i^5 + \beta_3 \sum_{i=1}^{n} X_i^6 \tag{8d}$$

(8a) to (8d) are solved simultaneously by transforming into the following matrix form

$$\begin{pmatrix} N & \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 \\ \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 \\ \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 \\ \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 & \sum_{i=1}^{n} X_i^6 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} Y_i \\ \sum_{i=1}^{n} X_i Y_i \\ \sum_{i=1}^{n} X_i^2 Y_i \\ \sum_{i=1}^{n} X_i^3 Y_i \end{pmatrix} \qquad (9)$$

Similarly, the parameters β_0 , the intercept, β_1 the linear effect parameter, β_2 the quadratic effect parameter and β_3 the cubic effect parameter can then be obtained from (9) using Crammer's rule by seeking the determinants of a, a_0, a_1, a_2 and a_3 as follows

$$a = \begin{pmatrix} N & \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 \\ \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 \\ \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 \\ \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 & \sum_{i=1}^{n} X_i^6 \end{pmatrix}$$

$$a_0 = \begin{pmatrix} \sum_{i=1}^{n} Y_i & \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 \\ \sum_{i=1}^{n} X_i Y_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 \\ \sum_{i=1}^{n} X_i^2 Y_i & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 \\ \sum_{i=1}^{n} X_i^3 Y_i & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 & \sum_{i=1}^{n} X_i^6 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} N & \sum_{i=1}^{n} Y_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 \\ \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i Y_i & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 \\ \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^2 Y_i & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 \\ \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^3 Y_i & \sum_{i=1}^{n} X_i^5 & \sum_{i=1}^{n} X_i^6 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} N & \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} Y_i & \sum_{i=1}^{n} X_i^3 \\ \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i Y_i & \sum_{i=1}^{n} X_i^4 \\ \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^2 Y_i & \sum_{i=1}^{n} X_i^5 \\ \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^3 Y_i & \sum_{i=1}^{n} X_i^6 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} N & \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 \\ \sum_{i=1}^{n} X_i & \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 \\ \sum_{i=1}^{n} X_i^2 & \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 \\ \sum_{i=1}^{n} X_i^3 & \sum_{i=1}^{n} X_i^4 & \sum_{i=1}^{n} X_i^5 & \sum_{i=1}^{6} X_i^6 \end{pmatrix}$$

yielding $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ *and* $\hat{\beta}_3$ as

$$\hat{\beta}_0 = \frac{\det(a_0)}{\det(a)}, \quad \hat{\beta}_1 = \frac{\det(a_1)}{\det(a)}, \quad \hat{\beta}_2 = \frac{\det(a_2)}{\det(a)} \quad \text{and} \quad \hat{\beta}_3 = \frac{\det(a_3)}{\det(a)}$$

which are estimates of $\beta_0, \beta_1, \beta_2$ and β_3. Similarly, for quadratic form, $estimates\ \hat{\beta}_0, \hat{\beta}_1$, and $\hat{\beta}_2$ can be obtained

4.0 Analysis

Exploratory analysis involving the construction of a scatter plot will be done. This allows visual approximation of each regression line and informs the development of the computational design that allows us to carry out its appropriate estimation (Kmenta, 1990). Each of the plots is presented as follows:

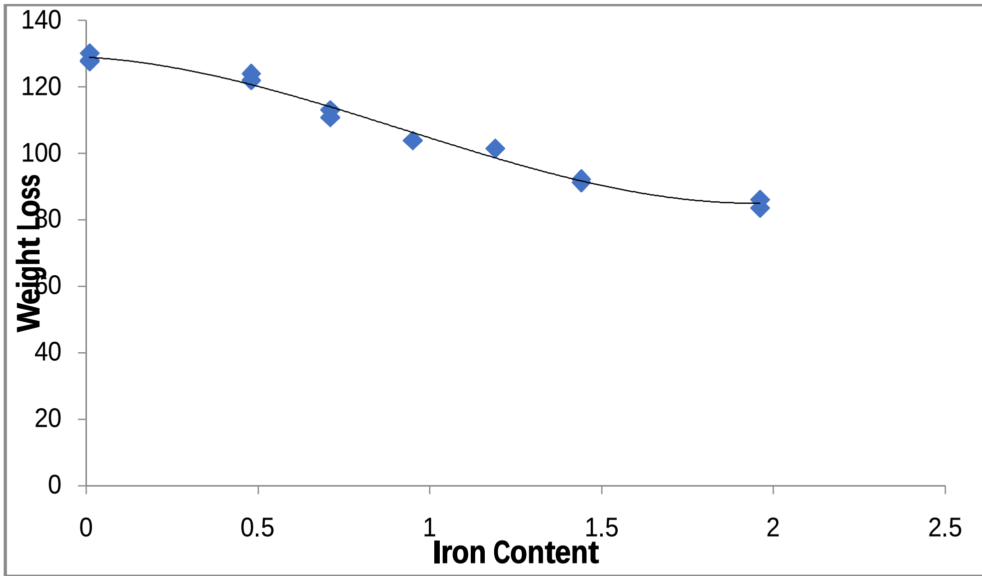

Figure 1- Graph of Iron corrosion data

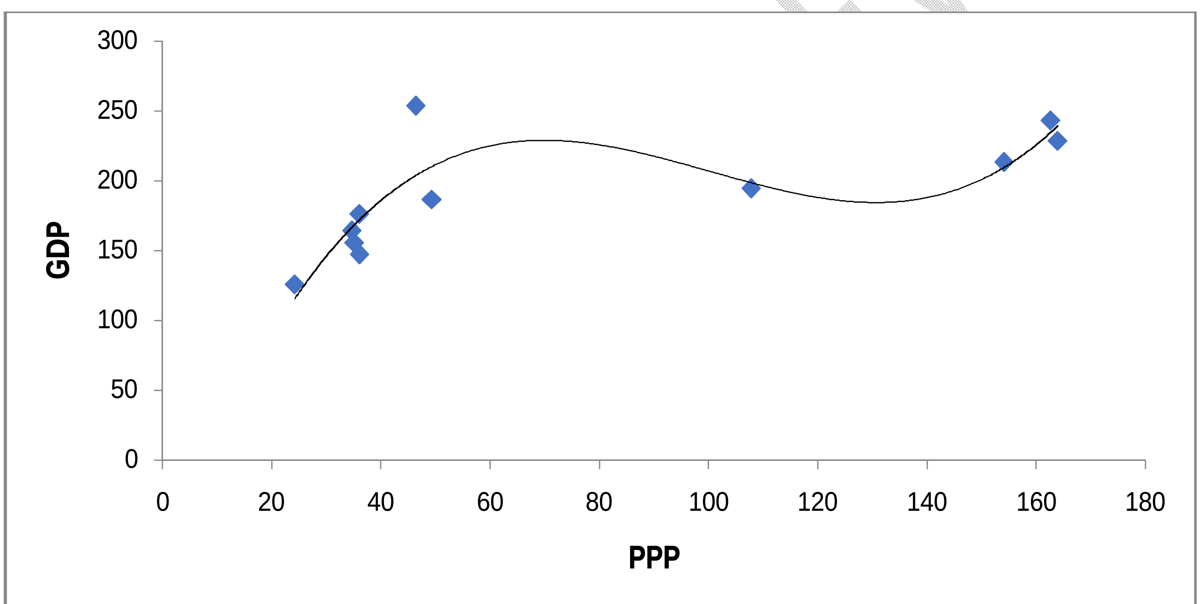

Figure 2 – Graph of Nigerian Gross Domestic Product (GDP) and Power Purchasing Parity (PPP) between 1989 and 1999

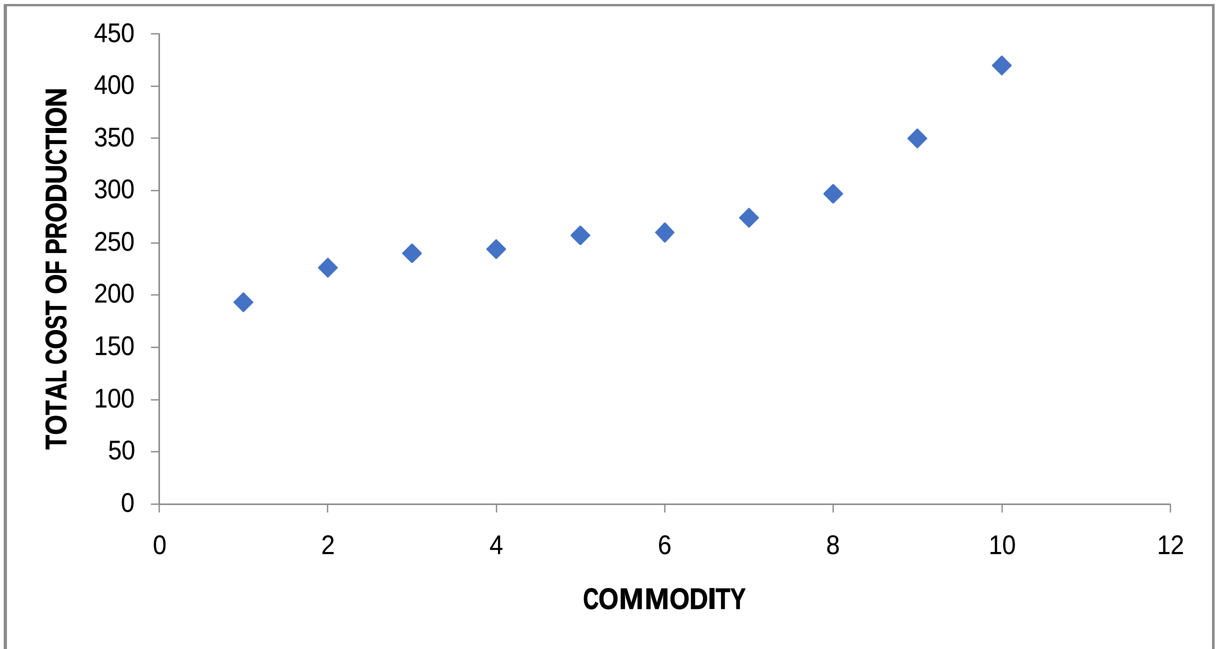

Figure 3-Graph of Total Production Cost and Output

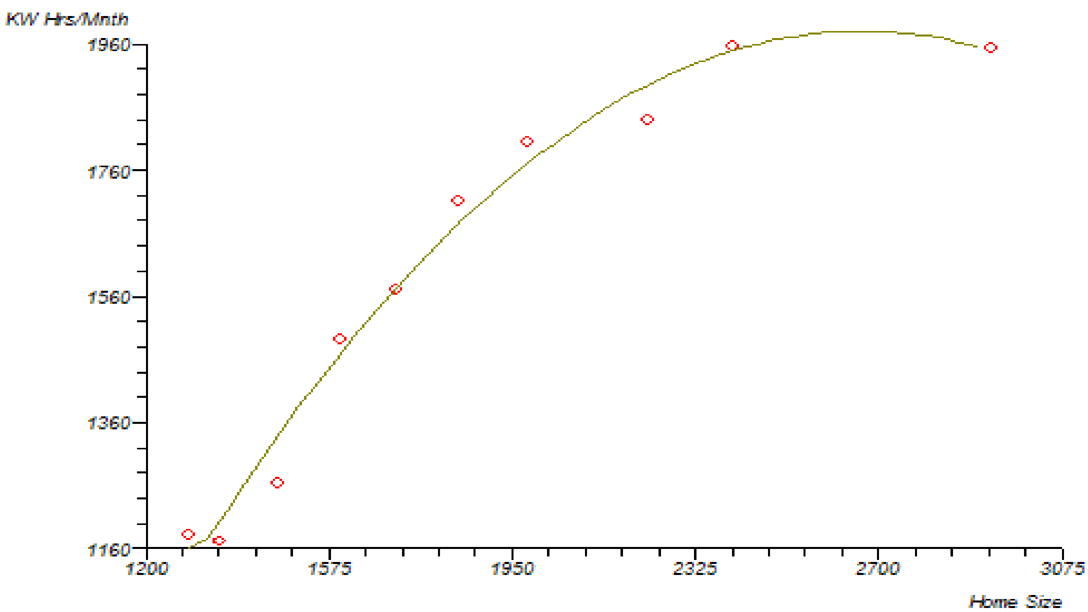

Figure 4-Graph of monthly Electricity consumption and home sizes

The exploratory analysis shows that each of these data sets either follows a quadratic or cubic order since their graphs show a sigmoid shape or parabola. This informs the use of the cubic regression model for the iron corrosion data, PPP and GDP data, and output/total production cost data while the quadratic form was used for the data of home sizes and electricity consumption.

The ML technique was applied as an alternative to the traditional OLS since the OLS assumption of large samples is not met. The results of the analyses (by ML technique and that of the OLS) are presented for comparison. The parameter estimates, standard error (S.E.), t-ratios (for the test of significance of each parameter), coefficient of determination R^2 value and the mean square error values for ML and OLS are presented in the table. These estimates give insights into the efficiency of the ML approach and the overall goodness of fit. The table of comparison is presented below:

Tab 1-Table of Comparison

S/N	Datasets	No of samples	Estimates	MLE				OLS			
1.	Data of Corrosion wheel set up	13	Parameters	β_0 128.889	β_1 -5.5244	β_2 -29.2076	β_3 10.5226	β_0 128.9489	β_1 -5.1956	β_2 -30.376	β_3 11.0086
			S.E	S.E(β_0) 1.325	S.E(β_1) 6.972	S.E(β_2) 9.288	S.E(β_3) 3.174	S.E(β_0) 1.381	S.E(β_1) 7.641	S.E(β_2) 10.473	S.E(β_3) 3.602
			P-value of /t/	0.0001	0.0485	0.0118	0.0090	0.0001	0.0436	0.0176	0.01337
			R^2 value	0.9867				0.9858			
			MSE value	5.001				5.429			
2.	Data of PPP and GDP in Nigeria between 1989 and 1999	11	Parameters	β_0 3538.061	β_1 -61.6291	β_2 0.3478	β_3 -0.000625	β_0 3538.061	β_1 -61.6291	β_2 0.3478	β_3 -0.000625
			S.E	S.E(β_0) 1117.598	S.E(β_1) 18.5044	S.E(β_2) 0.0998	S.E(β_3) 0.00018	S.E(β_0) 1117.59	S.E(β_1) 18.504	S.E(β_2) 0.0998	S.E(β_3) 0.00018
			P-value of /t/	0.0158	0.0126	0.0102	0.0092	0.0158	0.0126	0.0102	0.0092
			R^2 value	0.8245				0.8245			
			MSE value	831.2				831.2			
3.	Data of output and total production cost of a commodity	10	Parameters	β_0 141.767	β_1 63.4777	β_2 -12.9615	β_3 0.9396	β_0 141.767	β_1 63.4777	β_2 -12.9615	β_3 0.9396
			S.E	S.E(β_0) 6.3753	S.E(β_1) 4.778	S.E(β_2) 0.986	S.E(β_3) 0.059	S.E(β_0) 6.3753	S.E(β_1) 4.778	S.E(β_2) 0.986	S.E(β_3) 0.059
			P-value of /t/	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
			R^2 value	0.9983				0.9983			
			MSE value	1.789				1.8124			
4	Data of home sizes and electricity consumption	10	Parameters	β_0 -1216.14389	β_1 2.39893	β_2 -0.00045		β_0 -1217.2341	β_1 2.41333	β_2 -0.0005	
			S.E	S.E(β_0) 43.235	S.E(β_1) 0.8973	S.E(β_2) 0.00002		S.E(β_0) 44.576	S.E(β_1) 0.9148	S.E(β_2) 0.00005	
			P-value of /t/	0.0012	0.0001	0.0001		0.0016	0.0001	0.0001	
				0.9875				0.98125			

			R^2 value MSE value	1425.67	2190.365

5.0 Discussion of Results

Table 1 presents the comparison between the MLE and OLS techniques the first order cubic regression and the first order quadratic regression model using four data sets with small sample sizes. The comparison was carried out between the two methods using the parameter estimates, standard error estimates of each parameter, the p-value of the student's t-ratios, coefficients of determination and the Mean Square error(MSE) values. The parameter estimates will give insight into the bias of the estimates while the coefficient of determination and t-ratios will provide insight into the overall goodness of fit and detection of multicollinearity while the MSE values provide insight into the efficiency of the estimation technique.

The result of the analysis of the first data set (of iron content and weight loss of some specimen tested in a corrosion wheel set-up) show that the ML gave parameter estimates of 128.889, -5.524, -29.208, 10.523 and the OLS resulted in 128.949, -5.196, -30.376, 11.009. Also, the ML resulted in standard error (SE) estimates of 1.325, 6.972, 9.288, 3.174 while the OLS resulted in S.E estimates of 1.381, 7.641, 10.473, 3.602. The ML has a slightly greater R^2 value of 0.987 (when compared to that of the OLS with 0.986) while a slightly lower MSE of 5.005 of the ML indicated that the ML is more efficient than the OLS with an MSE value of 5.429 with identical (exactly the same) decisions in their tests of significance of each of the parameters.

Furthermore, the analysis of the second data set (of Nigerian PPP and GDP between 1989 and 1999) showed that both methods gave identical parameter estimates 3538.06, -61.629, 0.3478, -0.000625, 3538.06, identical S.E estimates 1117.59, 18.504, 0.099, 0.00018, identical R^2 value 0.8285 and identical MSE value of 831.2 with identically significant t-ratios.

The analysis of the third data set(on total production cost and output of a commodity) showed identical parameter estimates, and different MSE values 141.767,63.4777, -12.9615, 0.9396, identical S.E 6.3753, 4.778,0.986, 0.059, the identical R^2 value of 0.9983 but lower MSE 1.789 (than 1.8124 for the OLS

The analysis of the last data set (on electricity consumption in kilowatt-hours per month and home size of ten houses in square feet) showed that the MLE resulted in parameter estimates of -1216.1439, 2.3989, -0.000045, while the OLS resulted in parameter estimates-1217.2341, 2.41333, -0.0005. The ML has S.E. estimates 43.235, 0.8973,0.00002 while the OLS resulted in S.E. 44.576, 0.9148, 0.00005, while the ML has a slightly higher R^2 value of 0.9875 (than the OLS with 0.9813) and a lower MSE 1425.67 (than 2190.365 for the OLS). Both resulted in identically significant t-ratios.

6.0 Conclusion

The result of the model fit of the four data sets using the ML resulted in reasonable parameter estimates (with lesser S.E relative to the parameter estimates), lower MSE, significant t-ratios and very high R^2 values. The latter two results indicated the absence of multicollinearity problems and overall goodness of fit. The ML produced unbiased estimators for the OLS for three (out of the four) data sets considered since the ML estimates all coincided with that of the OLS in the three cases. The ML provided slightly different estimates from that of the OLS for the fourth data set which implies that there is a small bias in the fourth data set. However, the smaller the bias, the better the accuracy of the estimator and the estimator with the least bias is considered the best. The resulting parameter estimates by the ML showed little (in the fourth data) or no bias (in the other three data sets) which leads to the conclusion that the ML estimators are good estimators for the PRM.

In terms of goodness of fit, both estimators accounted for good fit because they both have high R^2 values and significant t-ratios but the ML gave a better fit since it resulted in higher R^2 values and lower MSE for all the data sets than the OLS technique. This latter quality ultimately leads to the conclusion that ML is a more efficient technique for small samples than OLS.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

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