A Mathematical Model of Optimal Manufacturer-Retailer Trade Credit Policy with End-User Credit Support

ABSTRACT

The formulation and incorporation of credit function into trade credit models is a much recent development. In addition extension of this innovation to the end-user has not been achieved. This work models trade credit interaction extension from a manufacturer to a consumer through the retailer. It considers a Stackelberg game-theoretic model setting in which the manufacturer provides channel trade credit through the retailer to the end-user, while the retailer engages in promotion of the product. The paper uses backward induction to obtain the Stackelberg equilibrium for the promotion effort, the retailer's credit period and the manufacturer's credit period. It also obtains the long-run Stackelbergequilibria for the decision variables. The paper shows that the retailer is more liberal with allowable credit period than the manufacturer. In general, the players are credit period-liberal with retail margin, and ungenerous with credit period with manufacturer's margin. It further shows that the manufacturer's credit period increases more rapidly than the retailer's credit period with retailer's margin. On the other hand the manufacturer's credit period decreases more rapidly than the retailer's credit period with the manufacturer's margin..

Keywords: Trade Credit, Supply Chain, Stackelberg Game, Credit Function, Promotion, Mathematical Model

2010 Mathematics Subject Classification: 91A65, 91A10.

1. INTRODUCTION

Trade credit is a short term business strategy in a market supply channel in which suppliers (or manufacturers) offer credit terms to their buyers and allow delayed payment [1]. This is one of the most employed financing strategies, especially where there are financial constraints [2]. Trade credit provision depends on a number of market variables. The adoption of functional dependence between credit provision and these variables has only been recently designed by Ezimadu and Ezimadu [3]. This paper adopts this function extending it to the provision of credit to the end-user.

Trade credit is an important financial strategy which has influenced a lot of research interest using mathematical models. For instance, considering some recent trade credit papers we observe that Panda *et al.* [4] and Mahata and Mahata [5] considered product deterioration; Wang *et al.* [6] studied replenishment in relation to product deterioration; Mashud*et al.* [7] looked at preservation technology; Mahata*et al.* [8] developed a model on default risk; Das *et al.* [9] modelled product reliability; Ding *et al.*[10] proposed a mathematical model aimed at optimizing trade credit agreements and safety stock; Tiwari *et al.* [11] developed a trade credit model using warehouses.

Game theory plays a very crucial role in supply chain studies [12,13]. Considering the risk ofbankruptcy for a financially constrained retailer and a supplier, both needing finance in the short term, [14] used Stackelberg game to model their relationship. They used the early payment discount scheme given to the retailer as a framework for analysing optimal result. Zhou and Zhou [15] considered a supplier-retailer channel with stable consumer demand. They examined a conditional provision of trade credit which they observed to be advantageous to the retailer, and an unconditional giving of trade credit which is

Comment [Ma1]:

disadvantageous to the supplier, and observed that a win-win situation is achievable with an appropriate design. In a study of a remanufacturing setting where a manufacturer remanufactures a product using his consumed product [16] examined five situations which includes Nash and Stackelberg games. In bid to study the possibility of the manufacturer persuading the retailer to disclose his actual holding cost [17] developed game-theoretic models on the effectiveness of trade credit in a situation where information is asymmetric. They determined appropriate trade credit contracts which is optimal enough using mathematical programming approach and obtained the sales and order quantity. In a consideration of inventory replenishment supply chain setting with uncooperative channel members, [18] employed Stackelberg game theory to examine the conditions for the use of trade credit. Considering capital-constrained supply channel members, [19] used Stackelberg game to illustrate that a supplier's financial limitation translates to less provision of credit to the retailer. Using a non-cooperative Stackelberg game [20] examined the sale of defective items through discount, and a situation where demand depends on promotion. Li et al. [21] examined how trade credit insurance affects a financially constrained supply channel involving retailers and manufacturers. They used Stackelberg game theory to model the interaction between these channel players, and analysed financial and managerial decisions. Employing non-cooperative game theory, [22] modelled a bank-supplier-retailer interaction in which either the bank or the supplier plays the role of a Stackelberg leader with the retailer as the follower. They examined how the availability of capital influences the extent to which a retailer can engage in borrowing. In a study to analyse the performance of the members of a supply channel in a situation where payment is made in advance, on time, and where it is delayed, [23] developed three models using Stackelberg game theory involving a manufacturer and a retailer in a decentralised channel setting. They obtained optimal decisions and observed that the payment plan influences the decisions and parameters employed.

This work examines a bilateral monopolistic situation in which a manufacturer sells his product to the consumer through a retailer who in turn sells to the consumer. The retailer sells only the manufacturer's brand. The manufacturer sells credit goods to the retailer. The retailer in turn sells credit goods to the consumer, and also engages in the promotion of the product. The work will determine the optimal promotion effort. It will determine the optimal credit period for the both the supplier and the retailer, and consider how each credit period affects the other. It will also consider how the channel members' price margins affect the credit periods and the promotion efforts.

2. MODEL FOMULATION

This paper considers a manufacturer-retailer channel in which the supplier provides the retailer with credit fund f_{CM} in the form of goods, and the retailer provides the end-user with credit f_{CR}, also in the form of good. The credits f_{CM} and f_{CR} are provided based on the repayment times t_M and t_R, the credit repayment allowable period the manufacturer gives the retailer and the that which the retailer gives the consumer respectively. The retailer also engages in product promotion with the effort φ_R. Thus, the manufacturer's decision variable is t_M, while the retailer's decision variables are φ_R, and t_R.

2.1 List of Notations

We use the following notions in this work

m Retailer's price margin to the end-user
M Manufacturer's price margin to the retailer
φ_R Retailer's promotion effort
θ Retailer's promotion effectiveness parameter
t_R Retailer's allowable credit period given to the consumer
t_M Manufacturer's allowable credit period given to the retailer

f_{CR} Retailer's credit function
f_{CM} Manufacturer's credit function
Π_R Retailer's profit function
Π_M Manufacturer's profit function

2.2 Promotion-Demand and Credit Function

We recall that while advertising is a long term strategy, promotion is a short term strategy. Thus with appropriate refinement and definition the two market concept can be substituted for other. Thus we employ the well-known advertising-demand function

$$f(\varphi_R) = \theta\sqrt{\varphi_R} \qquad (1)$$

as can be found in [24 – 26], where θ represents the effectiveness of retail promotion effort. Equation (1) is agreement with the usually observed saturation effect which leads to diminishing returns on advertising, and also promotion [27 - 30].

Based on the credit function proposed by [3], we adopt the credit functions

$$f_{CR}(\varphi_R, t_R, m) = \frac{K_R m\sqrt{\varphi_R}}{t_R} \qquad (2)$$

and

$$f_{CM}(\varphi_R, t_M, M) = \frac{K_M M\sqrt{\varphi_R}}{t_M} \qquad (3)$$

as the retailer and the manufacturer's credit functions, respectively. Also K_R and K_M are their respective proportionality constants.

2.3 The Profit Function

We design this study as a Stackelberg game where the manufacturer unveils his allowable credit f_{CM} to the retailer, and the retailer in turn unveils his allowable credit f_{CR} to the end-user, and also engages in product promotion over time by engaging in the expenditure $\varphi_R t_R$

The profit can be expressed as

$$Profit = Price\ Margin \times Demand + Credit\ Support\ Received - Expenditure - Credit\ Support\ Given\ Out. \qquad (4)$$

Thus the retailer and the manufacturer's profit functions can be expressed as

$$\Pi_R = mf(\varphi_R) + \varphi_R t_R + f_{CM}(\varphi_R, t_M, M) - f_{CR}(\varphi_R, t_R, m) \qquad (5)$$

and

$$\Pi_M = Mf(\varphi_R) - f_{CM}(\varphi_R, t_M, M) \qquad (6)$$

respectively.

3. CREDIT PROVISION SCENARIO

3.1 Optimal Strategies

Both players aim to maximize their profits. We obtain the game equilibrium using backward induction approach. Thus, given the manufacturer's unveiled decisions, we first solve the retailer's problem

$$\max_{\varphi_R, t_R > 0} \Pi_R = m\vartheta\sqrt{\varphi_R} - \varphi_R t_R + \frac{K_M M\sqrt{\varphi_R}}{t_M} - \frac{K_R m\sqrt{\varphi_R}}{t_R}. \qquad (7)$$

Maximizing (7) with respect to φ_R we have

$$\frac{\partial \Pi_R}{\partial \varphi_R} = \frac{1}{2\sqrt{\varphi_R}}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right] - t_R = 0,$$

implying that

$$\varphi_R = \left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\}^2. \qquad (8)$$

Also maximizing (7) with respect to t_R we have

$$\frac{\partial \Pi_R}{\partial t_R} = -\varphi_R + \frac{K_R m\sqrt{\varphi_R}}{t_R^2} = 0.$$

implying that

$$t_R = \sqrt{\frac{K_R m}{\sqrt{\varphi_R}}}, \tag{9}$$

which also implies that

$$\varphi_R = \frac{K_R^2 m^2}{t_R^4}. \tag{10}$$

From (9) and (10) we have

$$\frac{K_R^2 m^2}{t_R^4} = \left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\}^2$$

Implying

$$t_R = \frac{3K_R m t_M}{\vartheta m t_M + K_M M}. \tag{11}$$

From (11) we have that

$$\frac{\partial \mathrm{t}_R}{\partial t_M} = \frac{(\vartheta m t_M + K_M M)(3K_R m) - 3K_R m t_M(\vartheta m)}{[\vartheta m t_M + K_M M]^2}$$
$$= \frac{3K_R m K_M M}{[\vartheta m t_M + K_M M]^2} > 0$$

This means that as the manufacturer increases the repayment period, the retailer also follows suit by extending the end-user repayment time. In essence the manufacturer's gesture extends to the end-user.

Using (11) in (9) we have

$$\frac{3K_R m t_M}{\vartheta m t_M + K_M M} = \sqrt{\frac{K_R m}{\sqrt{\varphi_R}}}$$

implying that

$$t_M = \frac{K_M M\sqrt{K_R m}}{m\left[3K_R\sqrt{\sqrt{\varphi_R}} - \vartheta\sqrt{K_R m}\right]}.$$

Now, the manufacturer's optimal problem can be expressed as

$$\max_{t_M>0}\Pi_M = M\vartheta\sqrt{\varphi_R} - \frac{K_M M\sqrt{\varphi_R}}{t_M} \tag{12}$$

Using (8) in (12) we have

$$\max_{t_M>0}\Pi_M = M\vartheta\left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\} - \frac{K_M M}{t_M}\left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\}$$

Maximizing with respect to t_M we have

$$\frac{\partial \Pi_M}{\partial t_M} = \frac{M\vartheta}{2t_R}\left[-\frac{K_M M}{t_M^2}\right] - \frac{1}{2t_R}\left\{\frac{K_M M}{t_M}\left[-\frac{K_M M}{t_M^2}\right] - \frac{K_M M}{t_M^2}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\} = 0$$

implying that

$$t_M = \frac{2K_M M t_R}{\vartheta(M-m)t_R + K_R m}. \tag{13}$$

But from (11) we have that

$$t_M = \frac{K_M M t_R}{3K_R m - mat_R}. \tag{14}$$

Thus from (13) and (14) we have that at optimal t_M

$$\frac{K_M M t_R}{3K_R m - mat_R} = \frac{2K_M M t_R}{\vartheta(M-m)t_R + K_R m}$$

implying that

$$t_R = \frac{5K_R m}{\vartheta(M+m)} \quad (15)$$

From (14) and (15) we have

$$t_M = \frac{K_M M\left(\frac{5K_R m}{\vartheta(M+m)}\right)}{m\left[3K_R - \vartheta\left(\frac{5K_R m}{\vartheta(M+m)}\right)\right]}$$
$$= \frac{5K_M M}{\vartheta[3(M+m)-5m]} \quad (16)$$

Proposition 3.1 Given the players' optimal problems (7) and (12) we have that promotion effort, the retailer's credit period and the manufacturer's credit period are given by (8), (15) and (16) respectively.

3.2 Long-run Repayment Periods

We note that (11) can be expressed as

$$t_R = \frac{3K_R m}{\vartheta m + \frac{K_M M}{t_M}},$$

so that as $t_M \longrightarrow \infty$, we have that $t_M \longrightarrow 0$, so that $\vartheta m + \frac{K_M M}{t_M} \longrightarrow \vartheta m$, and hence $t_R \longrightarrow \frac{3K_R m}{\vartheta m} = \frac{3K_R}{\vartheta}$. That is

$$t_{R(\infty)} = \frac{3K_R}{\vartheta} \quad (17)$$

Also we note that (13) can be expressed as

$$t_M = \frac{2K_M M}{\vartheta(M-m) + \frac{K_R m}{t_R}},$$

which means that as $t_R \longrightarrow \infty$ we have that $\frac{K_R m}{t_R} \longrightarrow 0$, so that $\vartheta(M-m) + \frac{K_R m}{t_R} \longrightarrow \vartheta(M-m)$, and hence $t_M \longrightarrow \frac{2K_M M}{\vartheta(M-m)}$. That is

$$t_{M(\infty)} = \frac{2K_M M}{\vartheta(M-m)} \quad (18)$$

In the nutshell both the retailer and the manufacturer's credit repayment periods are bounded above by $\frac{3K_R}{\vartheta}$ and $\frac{2K_M M}{\vartheta(M-m)}$ respectively. Thus a player can allow for increase in the credit repayment period only to an extent irrespective of the extent of the other player's repayment time relaxation gesture.

From (8), (17) and (18) we have that

$$\varphi_{R(\infty)} = \left\{\frac{1}{2\left(\frac{3K_R}{\vartheta}\right)}\left[m\vartheta + \frac{K_M M}{\frac{2K_M M}{\vartheta(M-m)}} - \frac{K_R m}{\frac{3K_R}{\vartheta}}\right]\right\}^2.$$
$$= \left\{\frac{\vartheta^2(3M-m)}{36K_R}\right\}^2 \quad (19)$$

This implies that in the long it would only be rational for the retailer to adopt (19) as the optimal promotion effort. Any effort exceeding this level means overspending which will lead to excessive strain on the retailer's resources.

Thus:

Proposition 3.2 Given the problems (7) and (12), the long-run promotion effort, retailer's credit period and the manufacturer's credit periods are given by (19), (17) and (18) respectively.

4. DISCUSSION

We now consider the results using numerical values. We let the promotion effectivenessparameterθ = 0.25. Since the manufacturer is the channel leader with first-mover advantage we have that $M > m$. Thus we let M = 5500and m = 5000. Further we let constants K_R = 0.2andK_M = 0.3.

4.1 Players' Liberality with Credit Period

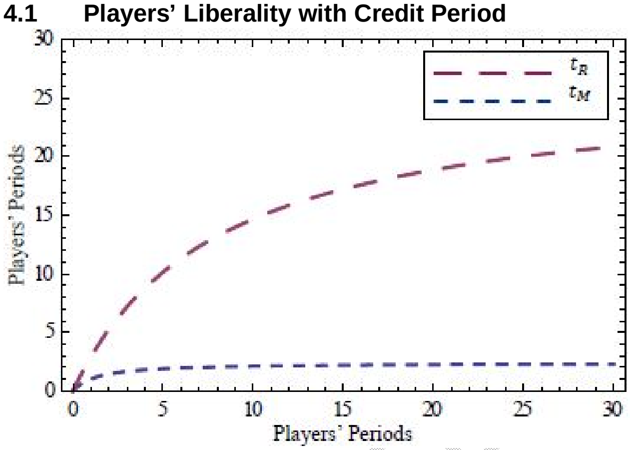

Fig.1. Illustration of the Effect of Credit Periods on Each Other

Fig1 illustrates the dependency between the retailer and the manufacturer's allowable periods. As the manufacturer's allowable credit period increases, the retailer increases his allowable credit period to the consumer. Similarly, the manufacturer's allowable credit period increases with the retailer's credit period. However, we observe that the retailer's credit period increases more rapidly than the manufacturer's credit period. Clearly, for any specific credit period we observe that the retailer's credit period is larger than that of the manufacturer, suggesting that the retailer appears to be more liberal with credit period than the manufacturer.

4.2 Effect of the Retailer's Margin on Credit Periods

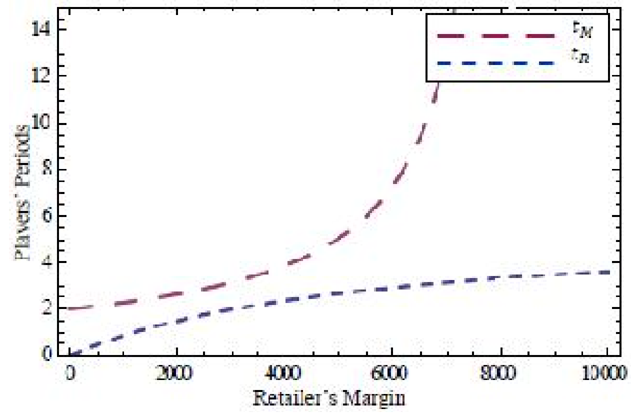

Fig.2. Illustration of the Effect of Retailer's Margin on Credit Periods

Fig2 shows that as the retailer increases his margin, he (the retailer) increases his allowable repayment time. This is because increase in price margin will result in low patronage leading to delay in selling off the product. This will provide more repayment time. Further, an envisaged increase in the retailer's credit period to the end-user will lead to increase in the manufacturer's credit period to the retailer. Clearly the manufacturer appears to be more liberal then the retailer in increasing credit period with price margin.

4.3 Effect of the Manufacturer's Margin on Credit Periods

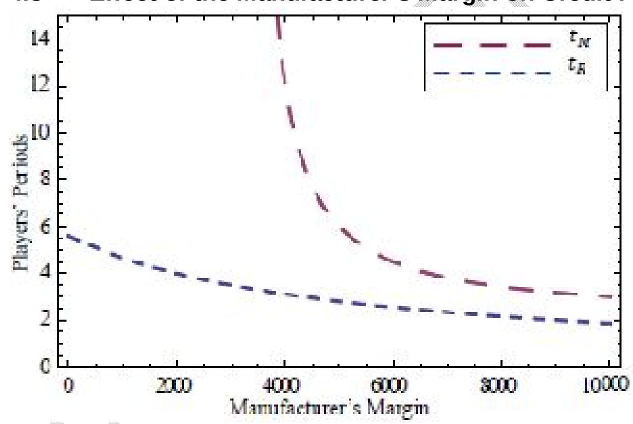

Fig3 Illustration of the Effect of Manufacturer's Margin on Credit Periods

From Fig.3 we observe that an increase in the manufacturer's margin will lead to increase in credit. That is, the manufacturer now has enough revenue to finance the retailer. However, a large credit will require earlier repayment (less allowable credit period). Thus considering the effect of the manufacturer's credit on the retailer's credit, it follows that the constraint of small allowable credit period will force the retailer to also reduce his allowable credit period to the end-user.

4.4 Effect of the Retailer's Margin on the Promotion Effort

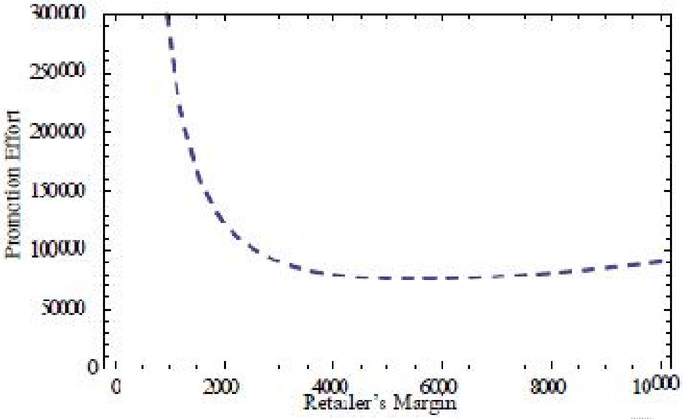

Fig4 Illustration of the Effect of the Manufacturer's Margin on the Promotion Effort

At first sight it is disturbing as it appears that the retailer's promotion effort reduces with his price margin instead of increasing with it. To comprehend why this is so, we resort to Fig2. In Fig2 it is clear that both credit periods increase with the retailer's price margin. Such elongation of the credit periods provide the retailer with enough business or sales time. Thus he has little to worry about, and hence reduces promotion effort as can be seen in Fig4.

4.5 Effect of the Manufacturer's Margin on the Promotion Effort

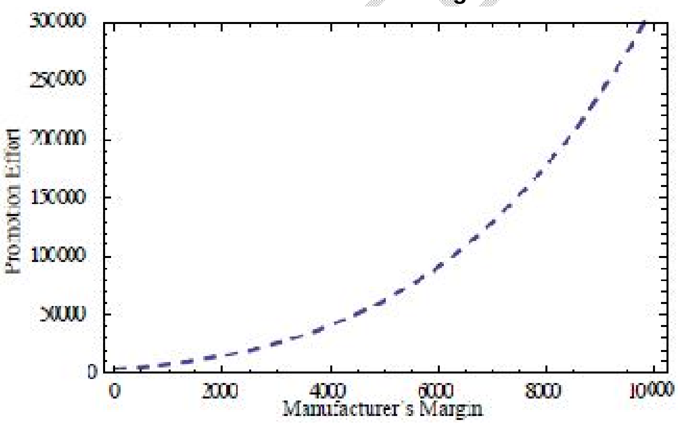

Fig5 Illustration of the Effect of the Manufacturer's Margin on the Promotion Effort

Similar to the observation in Fig4 above, we see that it appears irrational that the retailer increases effort as he pays more for the goods. To see why this is the case, we consider Fig3 which shows that credit periods reduce with the manufacturer's margin. Such a reduction in the allowable credit period no doubt means that the retailer has very little time to repay his credit. Thus he is constrained to engage more in promotion to ensure increase in sales, and meet-up with deadline.

6. CONCLUSION

This paper used credit function in a Stackelberg game setting to consider credit provision from a manufacturer through the retailer to the consumer. It involved a manufacturer – the channel leader – who provides trade credit to the retailer, and a retailer – the follower – who in turn also provides credit to the consumer and also engages in the promotion of the product. The paper evolved a closed-form solution for the optimal strategies and long-run strategies of the players. It shows that the retailer is more liberal with credit periods than the manufacturer. In general, credit period increases with retail margin and decreases with the manufacturer's margin. The manufacturer's credit period increases more rapidly than the retailer's credit period with retail margin. On the other hand, the manufacturer's credit period decreases more rapidly than the retailer's credit period with manufacturer's margin. It further shows that while promotion increases with the manufacturer's margin, it reduces with retailer's margin.

The work was considered on a Stackelberg game setting. A consideration using A Nash game in which neither of the channel members leads the channel can provide additional insight. A model considering both players' engagement in promotion can be used to extend the work. Further, an extension and modification can examine a three-level channel situation in which the channel leader and the first follower provides credit to the retailer as was consider in [30, 31].

REFERENCES

[1] Cuñat V, Garcia-Appendini E, Trade credit and its role in entrepreneurial finance, In: Cumming, D. (Ed.), 2012, Oxford Handbook of Entrepreneurial Finance, Oxford University Press, New York, 2012: pp. 526-557.

[2] Berger AN, Udell GF, The Economics of Small Business Finance: The Roles of Private Equity and Debt Markets in the Financial Growth Cycle, *Journal of Banking and Finance*, 1998:22: 613-673.

[3] Ezimadu PE, Ezimadu SO. A Game-Theoretic Credit Period and Promotion Model in a Supplier-Retailer Channel, *Asian Research Journal of Mathematics*, 2022:18(11): 351-361.

[4] Panda GC, Khan MA-A, Shaikh AA. Acredit policy approach in a two-warehouse inventory model for deteriorating items with price- and stock-dependent demand under partial backlogging. *Journal of Industrial Engineering International,* 2019:15: 147-170.

[5] Mahata P, Mahata GC. Two-echelon trade credit with default risk in an EOQ model for deteriorating items under dynamic demand, *Journal of Industrial and Management Optimization,* 2021:17(6): 3659-3684. Doi:10.3934/jimo.2020138

[6] Wang L, Chen Z, Chen M, Zhang R. Inventory policy for deteriorating item with time-varying demand under trade credit and inflation, *Journal of Systems Science and Information*, 2019:7(2): 115-133

[7] Mashud AH, Wee H-M, Huang C-V. Preservation technology investment, trade credit and partial backordering model for a non-instantaneous deteriorating inventory, *RAIRO Operations Research,* 2021:55: S51-S77. https://doi.org/10.1051/ro/2019095

[8] Mahata P, Mahata GC. De SK. Optimal replenishment and credit policy in supply chain inventory model under two levels of trade credit with time- and credit-sensitive demand involving default risk, *Journal of Industrial Engineering International,* 2018:14(1): 31-42.

[9] Das S, Khan MA, Mahmoud EE, Abdel-Aty A, Abualnaja KM, Shaikh AA. A production inventory model with partial trade credit policy and reliability. *Alexandria Engineering Journal*, 2021:60(1): 1325-1338.

[10] Ding Y, Jiang Y, Wu L, Zhou Z. Two-echelon supply chain network design with trade credit, *Computers and Operations Research*,131(c). https://doi.org/10.1016/j.cor.2021.105270

[11] Tiwari S, Cardenas-Barron LE, Malik AI, Jaggi CK. Retailer's credit and inventory decisions for imperfect quality and deteriorating items under two-level trade credit, *Computer & Operations Research,*2022: 138. https://doi.org/10.1016/j.cor.2021.105617

[12] Ezimadu PE, Nwozo CR. Modeling Dynamic Cooperative Advertising in a Decentralized Channel, *Yugoslav Journal of Operations Research*, 2018:28(4): 539-566.

[13] Rzeczycki A. Supply chain decision making with use of game theory, Procedia Computer Science, 2022:207: 3988-3997.

[14] Kouvelis P, Zhao W. Financing the Newsvendor: Supplier vs. Bank, and the Structure of Optimal Trade Credit Contracts, *Operations Research,*INFORMS2012:60(3): 566-580.

[15] Zhou Y-W, Zhou D. Determination of the optimal trade credit policy: A supplier-Stackelberg model, Journal of the Operational Research Society, 2013;64(7): 1030-1048. https://doi.org/10.1057/jors.2012.102

[16] Maiti T, Giri BC. A closed loop supply chain under retail price and product quality dependent demand, *Journal of Manufacturing Systems*, 2015:25(3): 624-637.

[17] Wang Z, Zhang Y, Guo J, Xia Q. Trade Credit Contract in a Supply Chain under Dual Asymmetric Information, *Chinese Journal of Management Science*, 2017:25(9): 148-158.

[18] Wu D, Zhang B, Baron O. A trade credit model with asymmetric competing retailers, *Production and Operations Management*, 2018:28(1): 206-231. https://doi.org/10.1111/poms.12882

[19] Jin W, Wang C. Modeling and simulation for supply chain finance under uncertain environment. *Technological and Economic Development of Economy,* 2020:26(4): 725-750. https://doi.org/10.3846/tede.2020.12054

[20] Yadav R, Jayaswal MK, Mittal M, Sangal I, Pareek S. A Game Theoretic Approach: Impact of Learning on the Optimality Ordering Policies for Imperfect Quality Items, *RevistaInvestigatcionOperacional*, 2020:41(2): 200-213.

[21] Li H, Bi G, Xiaoyong Y, Wang D. Trade credit insurance in a capital-constrained supply chain, *International Transactions in Operational Research,* 2020:27(5): 2340-2369.

[22] Hovelaque V, Viviani J-L, Mansour MA. Trade and Bank Credit in a Non-Cooperative chain with a Price-Sensitive Demand, *International Journal of Production Research*, 2022:60(5): 1553-1568.

[23] Cao B-B, You T-H, Ou CXJ, Zhu H, Liu C-H. Optimizing payment schemes in a decentralized supply chain: A Stackelberg game with quality investment and bank credit, *Computers & Industrial Engineering*, 2022: 168. https://doi.org/10.1016/j.cie.2022.108077

[24] Ezimadu PE. Modelling subsidy as a cooperative advertising channel coordination mechanism. Nigerian Journal of Basic and Applied Science. 2019;27(2): 127-135.

[25] Xie J, Wei JC. Coordination advertising and pricing in a manufacturer-retailer channel, *European Journal of Operational Research*, 2009:197(2): 785-791.

[26] Ezimadu PE. A Game-Theoretic Cooperative Advertising Model: The Feasibility of the Distributor's Involvement in a Manufacturer-Distributor-Retailer Channel, *FUW Trends in Science & Technology Journal*, 2019:4(2): 416 – 421.

[27] He X, Prasad A. Sethi SP. Cooperative Advertising and Pricing in a Dynamic Stochastic Supply Chain: Feedback Stackelberg Strategies, *Production and Operations Management*, 2009:18(1): 78-94.

[28] He Y, Liu Z, Usman K. Coordination of Cooperative advertising in a two-period fashion and textiles supply chain, *Mathematical Problems in Engineering*, Article ID 2014: 356726, 10 pages. https://doi.org/10.1155/2014/356726

[29] Simon JL, Arndt J. The shape of the advertising function. Journal of Advertising Research. 1980; 20:11–28.

[30] Ezimadu PE. Cooperative Advertising in a Manufacturer-Distributor-Retailer Supply Chain, *Transactions of the Nigerian Association of Mathematical Physics,* 2016:2: 205-216.

[31] Ezimadu PE. Modelling Cooperative Advertising Decisions in a Manufacturer-Distributor-Retailer Supply Chain Using Game Theory, *Yugoslav Journal of Operations Research*, 2020:30(2): 147-176.