# A Mathematical Model of Optimal Manufacturer-Retailer Trade Credit Policy with End-User Credit Support


## ABSTRACT

The formulation and incorporation of credit function into trade credit models is a much recent development. In addition extension of this innovation to the end-user has not been achieved. This work models trade credit interaction extension from a manufacturer to a consumer through the retailer. It considers a Stackelberg game-theoretic model setting in which the manufacturer provides channel trade credit through the retailer to the end-user, while the retailer engages in promotion of the product. The paper uses backward induction to obtain the Stackelberg equilibrium for the promotion effort, the retailer's credit period and the manufacturer's credit period. It also obtains the long-run Stackelberg equilibria for the decision variables. The paper shows that the retailer is more liberal with allowable credit period than the manufacturer. In general, the players are credit period-liberal with retail margin, and ungenerous with credit period with manufacturer's margin. It further shows that the manufacturer's credit period increases more rapidly than the retailer's credit period with retailer's margin. On the other hand the manufacturer's credit period decreases more rapidly than the retailer's credit period with the manufacturer's margin..



*Keywords: Trade Credit, Supply Chain, Stackelberg Game, Credit Function, Promotion, Mathematical Model*

**2010 Mathematics Subject Classification:** 91A65, 91A10.


## 1. INTRODUCTION

Trade credit is a short term business strategy in a market supply channel in which suppliers (or manufacturers) offer credit terms to their buyers and allow delayed payment [1]. This is one of the most employed financing strategies, especially where there are financial constraints [2]. Trade credit provision depends on a number of market variables. The adoption of functional dependence between credit provision and these variables has only been recently designed by Ezimadu and Ezimadu [3]. This paper adopts this function extending it to the provision of credit to the end-user.

Trade credit is an important financial strategy which has influenced a lot of research interest using mathematical models. For instance, considering some recent trade credit papers we observe that Panda *et al.* [4] and Mahata and Mahata [5] considered product deterioration; Wang *et al.* [6] studied replenishment in relation to product deterioration; Mashud *et al.* [7] looked at preservation technology; Mahata *et al.* [8] developed a model on default risk; Das *et al.* [9] modelled product reliability; Ding *et al.*[10] proposed a mathematical model aimed at optimizing trade credit agreements and safety stock; Tiwari *et al.* [11] developed a trade credit model using warehouses.

Game theory plays a very crucial role in supply chain studies [12,13]. Considering the risk of bankruptcy for a financially constrained retailer and a supplier, both needing finance in the short term, [14] used Stackelberg game to model their relationship. They used the early payment discount scheme given to the retailer as a framework for analysing optimal result. Zhou and Zhou [15] considered a supplier-retailer channel with stable consumer demand. They examined a conditional provision of trade credit which they observed to be advantageous to the retailer, and an unconditional giving of trade credit which is

disadvantageous to the supplier, and observed that a win-win situation is achievable with an appropriate design. In a study of a remanufacturing setting where a manufacturer remanufactures a product using his consumed product [16] examined five situations which includes Nash and Stackelberg games. In bid to study the possibility of the manufacturer persuading the retailer to disclose his actual holding cost [17] developed game-theoretic models on the effectiveness of trade credit in a situation where information is asymmetric. They determined appropriate trade credit contracts which is optimal enough using mathematical programming approach and obtained the sales and order quantity. In a consideration of inventory replenishment supply chain setting with uncooperative channel members, [18] employed Stackelberg game theory to examine the conditions for the use of trade credit. Considering capital-constrained supply channel members, [19] used Stackelberg game to illustrate that a supplier's financial limitation translates to less provision of credit to the retailer. Using a non-cooperative Stackelberg game [20] examined the sale of defective items through discount, and a situation where demand depends on promotion. Li et al. [21] examined how trade credit insurance affects a financially constrained supply channel involving retailers and manufacturers. They used Stackelberg game theory to model the interaction between these channel players, and analysed financial and managerial decisions. Employing non-cooperative game theory, [22] modelled a bank-supplier-retailer interaction in which either the bank or the supplier plays the role of a Stackelberg leader with the retailer as the follower. They examined how the availability of capital influences the extent to which a retailer can engage in borrowing. In a study to analyse the performance of the members of a supply channel in a situation where payment is made in advance, on time, and where it is delayed, [23] developed three models using Stackelberg game theory involving a manufacturer and a retailer in a decentralised channel setting. They obtained optimal decisions and observed that the payment plan influences the decisions and parameters employed.

This work examines a bilateral monopolistic situation in which a manufacturer sells his product to the consumer through a retailer who in turn sells to the consumer. The retailer sells only the manufacturer's brand. The manufacturer sells credit goods to the retailer. The retailer in turn sells credit goods to the consumer, and also engages in the promotion of the product. The work will determine the optimal promotion effort. It will determine the optimal credit period for the both the supplier and the retailer, and consider how each credit period affects the other. It will also consider how the channel members' price margins affect the credit periods and the promotion efforts.

## 2. MODEL FOMULATION

This paper considers a manufacturer-retailer channel in which the supplier provides the retailer with credit fund $f_{CM}$ in the form of goods, and the retailer provides the end-user with credit $f_{CR}$, also in the form of good. The credits $f_{CM}$ and $f_{CR}$ are provided based on the repayment times $t_M$ and $t_R$, the credit repayment allowable period the manufacturer gives the retailer and the that which the retailer gives the consumer respectively. The retailer also engages in product promotion with the effort $\varphi_R$. Thus, the manufacturer's decision variable is $t_M$, while the retailer's decision variables are $\varphi_R$, and $t_R$.

### 2.1 List of Notations

We use the following notions in this work

| | |
|---|---|
| $m$ | Retailer's price margin to the end-user |
| $M$ | Manufacturer's price margin to the retailer |
| $\varphi_R$ | Retailer's promotion effort |
| $\theta$ | Retailer's promotion effectiveness parameter |
| $t_R$ | Retailer's allowable credit period given to the consumer |
| $t_M$ | Manufacturer's allowable credit period given to the retailer |

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$f_{CR}$ Retailer's credit function
$f_{CM}$ Manufacturer's credit function
$\Pi_R$ Retailer's profit function
$\Pi_M$ Manufacturer's profit function

## 2.2 Promotion-Demand and Credit Function

We recall that while advertising is a long term strategy, promotion is a short term strategy. Thus with appropriate refinement and definition the two market concept can be substituted for other. Thus we employ the well-known advertising-demand function

$$f(\varphi_R) = \theta\sqrt{\varphi_R} \tag{1}$$

as can be found in [24 – 26], where $\theta$ represents the effectiveness of retail promotion effort. Equation (1) is agreement with the usually observed saturation effect which leads to diminishing returns on advertising, and also promotion [27 - 30].
Based on the credit function proposed by [3], we adopt the credit functions

$$f_{CR}(\varphi_R, t_R, m) = \frac{K_R m\sqrt{\varphi_R}}{t_R} \tag{2}$$

and

$$f_{CM}(\varphi_R, t_M, M) = \frac{K_M M\sqrt{\varphi_R}}{t_M} \tag{3}$$

as the retailer and the manufacturer's credit functions, respectively. Also $K_R$ and $K_M$ are their respective proportionality constants.

## 2.3 The Profit Function

We design this study as a Stackelberg game where the manufacturer unveils his allowable credit $f_{CM}$ to the retailer, and the retailer in turn unveils his allowable credit $f_{CR}$ to the end-user, and also engages in product promotion over time by engaging in the expenditure $\varphi_R t_R$
The profit can be expressed as

$$Profit = Price\ Margin \times Demand + Credit\ Support\ Received - Expenditure - Credit\ Support\ Given\ Out. \tag{4}$$

Thus the retailer and the manufacturer's profit functions can be expressed as

$$\Pi_R = mf(\varphi_R) + \varphi_R t_R + f_{CM}(\varphi_R, t_M, M) - f_{CR}(\varphi_R, t_R, m) \tag{5}$$

and

$$\Pi_M = Mf(\varphi_R) - f_{CM}(\varphi_R, t_M, M) \tag{6}$$

respectively.

# 3. CREDIT PROVISION SCENARIO

## 3.1 Optimal Strategies

Both players aim to maximize their profits. We obtain the game equilibrium using backward induction approach. Thus, given the manufacturer's unveiled decisions, we first solve the retailer's problem

$$\max_{\varphi_R, t_R > 0} \Pi_R = m\vartheta\sqrt{\varphi_R} - \varphi_R t_R + \frac{K_M M\sqrt{\varphi_R}}{t_M} - \frac{K_R m\sqrt{\varphi_R}}{t_R}. \tag{7}$$

Maximizing (7) with respect to $\varphi_R$ we have

$$\frac{\partial \Pi_R}{\partial \varphi_R} = \frac{1}{2\sqrt{\varphi_R}}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right] - t_R = 0,$$

implying that

$$\varphi_R = \left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\}^2. \tag{8}$$

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Also maximizing (7) with respect to $t_R$ we have

$$\frac{\partial \Pi_R}{\partial t_R} = -\varphi_R + \frac{K_R m\sqrt{\varphi_R}}{t_R^2} = 0.$$

implying that

$$t_R = \sqrt{\frac{K_R m}{\sqrt{\varphi_R}}}, \tag{9}$$

which also implies that

$$\varphi_R = \frac{K_R^2 m^2}{t_R^4}. \tag{10}$$

From (9) and (10) we have

$$\frac{K_R^2 m^2}{t_R^4} = \left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\}^2$$

Implying

$$t_R = \frac{3K_R m t_M}{\vartheta m t_M + K_M M}. \tag{11}$$

From (11) we have that

$$\frac{\partial t_R}{\partial t_M} = \frac{(\vartheta m t_M + K_M M)(3K_R m) - 3K_R m t_M(\vartheta m)}{[\vartheta m t_M + K_M M]^2}$$
$$= \frac{3K_R m K_M M}{[\vartheta m t_M + K_M M]^2} > 0$$

This means that as the manufacturer increases the repayment period, the retailer also follows suit by extending the end-user repayment time. In essence the manufacturer's gesture extends to the end-user.

Using (11) in (9) we have

$$\frac{3K_R m t_M}{\vartheta m t_M + K_M M} = \sqrt{\frac{K_R m}{\sqrt{\varphi_R}}}$$

implying that

$$t_M = \frac{K_M M\sqrt{K_R m}}{m\left[3K_R\sqrt{\sqrt{\varphi_R}} - \vartheta\sqrt{K_R m}\right]}.$$

Now, the manufacturer's optimal problem can be expressed as

$$\max_{t_M>0} \Pi_M = M\vartheta\sqrt{\varphi_R} - \frac{K_M M\sqrt{\varphi_R}}{t_M} \tag{12}$$

Using (8) in (12) we have

$$\max_{t_M>0} \Pi_M = M\vartheta\left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\} - \frac{K_M M}{t_M}\left\{\frac{1}{2t_R}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\}$$

Maximizing with respect to $t_M$ we have

$$\frac{\partial \Pi_M}{\partial t_M} = \frac{M\vartheta}{2t_R}\left[-\frac{K_M M}{t_M^2}\right] - \frac{1}{2t_R}\left\{\frac{K_M M}{t_M}\left[-\frac{K_M M}{t_M^2}\right] - \frac{K_M M}{t_M^2}\left[m\vartheta + \frac{K_M M}{t_M} - \frac{K_R m}{t_R}\right]\right\} = 0$$

implying that

$$t_M = \frac{2K_M M t_R}{\vartheta(M-m)t_R + K_R m}. \tag{13}$$

But from (11) we have that

$$t_M = \frac{K_M M t_R}{3K_R m - m a t_R}. \tag{14}$$

Thus from (13) and (14) we have that at optimal $t_M$

$$\frac{K_M M t_R}{3K_R m - m a t_R} = \frac{2K_M M t_R}{\vartheta(M-m)t_R + K_R m}$$

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implying that

$$t_R = \frac{5K_R m}{\vartheta(M+m)} \tag{15}$$

From (14) and (15) we have

$$t_M = \frac{K_M M\left(\frac{5K_R m}{\vartheta(M+m)}\right)}{m\left[3K_R - \vartheta\left(\frac{5K_R m}{\vartheta(M+m)}\right)\right]}$$

$$= \frac{5K_M M}{\vartheta[3(M+m)-5m]} \tag{16}$$

**Proposition 3.1** Given the players' optimal problems (7) and (12) we have that promotion effort, the retailer's credit period and the manufacturer's credit period are given by (8), (15) and (16) respectively.

### 3.2 Long-run Repayment Periods

We note that (11) can be expressed as

$$t_R = \frac{3K_R m}{\vartheta m + \frac{K_M M}{t_M}},$$

so that as $t_M \longrightarrow \infty$, we have that $t_M \longrightarrow 0$, so that $\vartheta m + \frac{K_M M}{t_M} \longrightarrow \vartheta m$, and hence $t_R \longrightarrow \frac{3K_R m}{\vartheta m} = \frac{3K_R}{\vartheta}$. That is

$$t_{R(\infty)} = \frac{3K_R}{\vartheta} \tag{17}$$

Also we note that (13) can be expressed as

$$t_M = \frac{2K_M M}{\vartheta(M-m) + \frac{K_R m}{t_R}},$$

which means that as $t_R \longrightarrow \infty$ we have that $\frac{K_R m}{t_R} \longrightarrow 0$, so that $\vartheta(M-m) + \frac{K_R m}{t_R} \longrightarrow \vartheta(M-m)$, and hence $t_M \longrightarrow \frac{2K_M M}{\vartheta(M-m)}$. That is

$$t_{M(\infty)} = \frac{2K_M M}{\vartheta(M-m)} \tag{18}$$

In the nutshell both the retailer and the manufacturer's credit repayment periods are bounded above by $\frac{3K_R}{\vartheta}$ and $\frac{2K_M M}{\vartheta(M-m)}$ respectively. Thus a player can allow for increase in the credit repayment period only to an extent irrespective of the extent of the other player's repayment time relaxation gesture.

From (8), (17) and (18) we have that

$$\varphi_{R(\infty)} = \left\{\frac{1}{2\left(\frac{3K_R}{\vartheta}\right)}\left[m\vartheta + \frac{K_M M}{\frac{2K_M M}{\vartheta(M-m)}} - \frac{K_R m}{\frac{3K_R}{\vartheta}}\right]\right\}^2.$$

$$= \left\{\frac{\vartheta^2(3M-m)}{36K_R}\right\}^2 \tag{19}$$

This implies that in the long it would only be rational for the retailer to adopt (19) as the optimal promotion effort. Any effort exceeding this level means overspending which will lead to excessive strain on the retailer's resources.

Thus:

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**Proposition 3.2** Given the problems (7) and (12), the long-run promotion effort, retailer's credit period and the manufacturer's credit periods are given by (19), (17) and (18) respectively.

## 4. DISCUSSION

We now consider the results using numerical values. We let the promotion effectiveness parameter $\theta$ = 0.25. Since the manufacturer is the channel leader with first-mover advantage we have that $M > m$. Thus we let $M$ = 5500 and $m$ = 5000. Further we let constants $K_R$ = 0.2 and $K_M$ = 0.3.

### 4.1 Players' Liberality with Credit Period

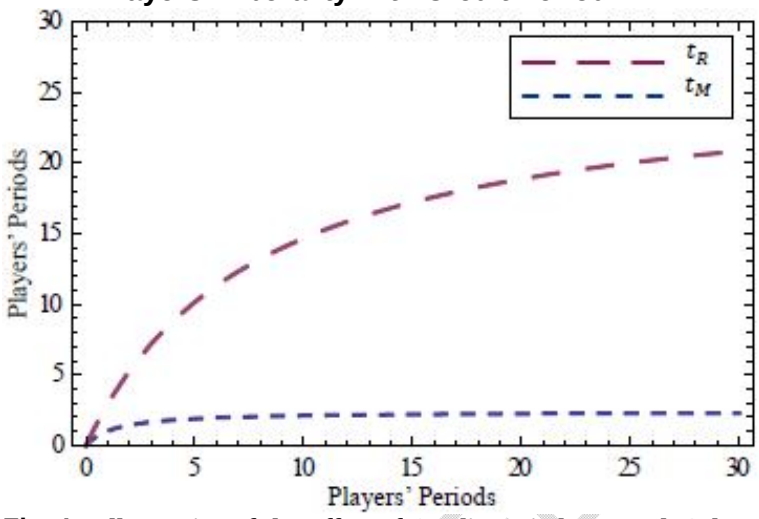


**Fig. 1. Illustration of the Effect of Credit Periods on Each Other**

Fig 1 illustrates the dependency between the retailer and the manufacturer's allowable periods. As the manufacturer's allowable credit period increases, the retailer increases his allowable credit period to the consumer. Similarly, the manufacturer's allowable credit period increases with the retailer's credit period. However, we observe that the retailer's credit period increases more rapidly than the manufacturer's credit period. Clearly, for any specific credit period we observe that the retailer's credit period is larger than that of the manufacturer, suggesting that the retailer appears to be more liberal with credit period than the manufacturer.

### 4.2 Effect of the Retailer's Margin on Credit Periods

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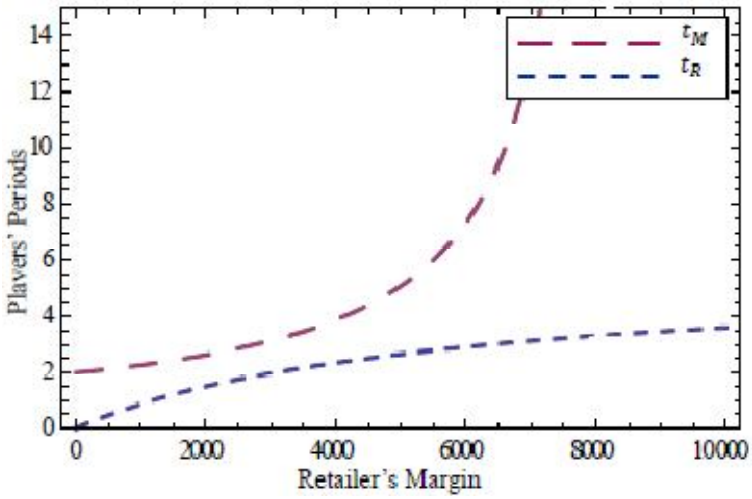


**Fig. 2. Illustration of the Effect of Retailer's Margin on Credit Periods**

Fig 2 shows that as the retailer increases his margin, he (the retailer) increases his allowable repayment time. This is because increase in price margin will result in low patronage leading to delay in selling off the product. This will provide more repayment time. Further, an envisaged increase in the retailer's credit period to the end-user will lead to increase in the manufacturer's credit period to the retailer. Clearly the manufacturer appears to be more liberal then the retailer in increasing credit period with price margin.

### 4.3 Effect of the Manufacturer's Margin on Credit Periods

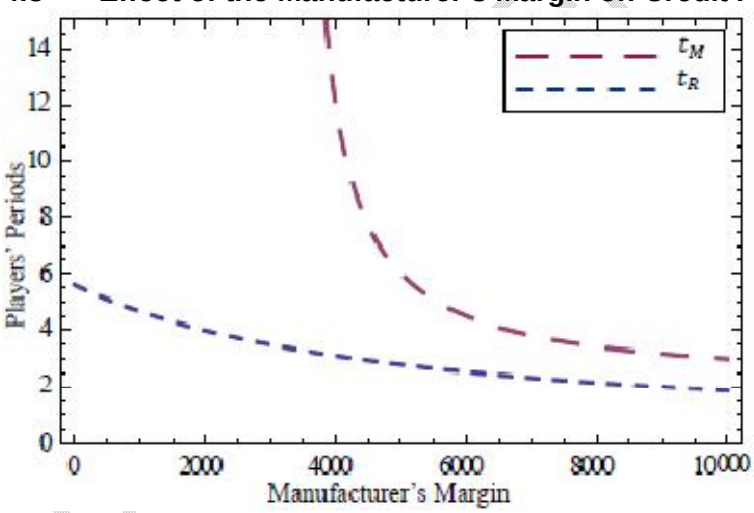


**Fig 3 Illustration of the Effect of Manufacturer's Margin on Credit Periods**

From Fig .3 we observe that an increase in the manufacturer's margin will lead to increase in credit. That is, the manufacturer now has enough revenue to finance the retailer. However, a large credit will require earlier repayment (less allowable credit period). Thus considering the effect of the manufacturer's credit on the retailer's credit, it follows that the constraint of small allowable credit period will force the retailer to also reduce his allowable credit period to the end-user.

### 4.4 Effect of the Retailer's Margin on the Promotion Effort

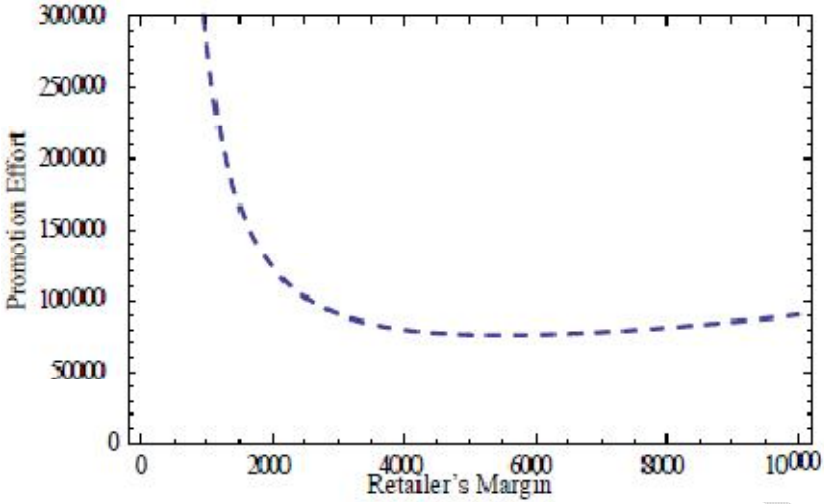


**Fig 4 Illustration of the Effect of the Manufacturer's Margin on the Promotion Effort**

At first sight it is disturbing as it appears that the retailer's promotion effort reduces with his price margin instead of increasing with it. To comprehend why this is so, we resort to Fig 2. In Fig 2 it is clear that both credit periods increase with the retailer's price margin. Such elongation of the credit periods provide the retailer with enough business or sales time. Thus he has little to worry about, and hence reduces promotion effort as can be seen in Fig 4.

### 4.5 Effect of the Manufacturer's Margin on the Promotion Effort

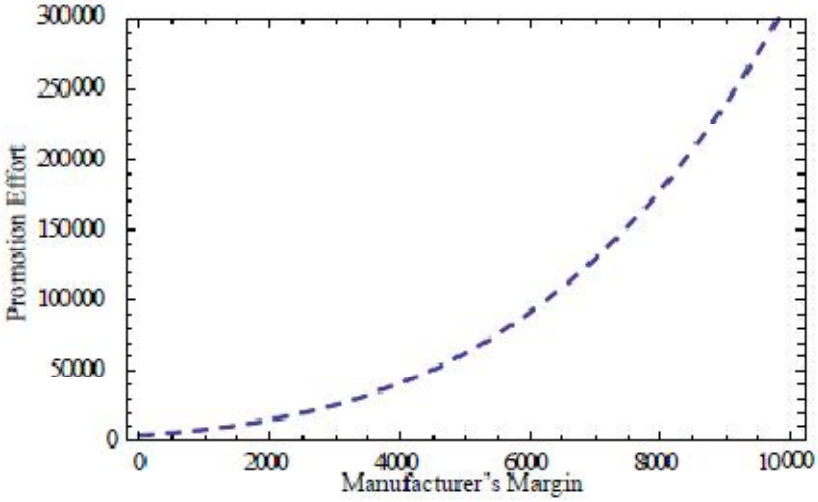


**Fig 5 Illustration of the Effect of the Manufacturer's Margin on the Promotion Effort**

Similar to the observation in Fig 4 above, we see that it appears irrational that the retailer increases effort as he pays more for the goods. To see why this is the case, we consider Fig 3 which shows that credit periods reduce with the manufacturer's margin. Such a reduction in the allowable credit period no doubt means that the retailer has very little time to repay his

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credit. Thus he is constrained to engage more in promotion to ensure increase in sales, and meet-up with deadline.

## 6. CONCLUSION

This paper used credit function in a Stackelberg game setting to consider credit provision from a manufacturer through the retailer to the consumer. It involved a manufacturer – the channel leader – who provides trade credit to the retailer, and a retailer – the follower – who in turn also provides credit to the consumer and also engages in the promotion of the product. The paper evolved a closed-form solution for the optimal strategies and long-run strategies of the players. It shows that the retailer is more liberal with credit periods than the manufacturer. In general, credit period increases with retail margin and decreases with the manufacturer's margin. The manufacturer's credit period increases more rapidly than the retailer's credit period with retail margin. On the other hand, the manufacturer's credit period decreases more rapidly than the retailer's credit period with manufacturer's margin. It further shows that while promotion increases with the manufacturer's margin, it reduces with retailer's margin.

The work was considered on a Stackelberg game setting. A consideration using A Nash game in which neither of the channel members leads the channel can provide additional insight. A model considering both players' engagement in promotion can be used to extend the work. Further, an extension and modification can examine a three-level channel situation in which the channel leader and the first follower provides credit to the retailer as was consider in [30, 31].

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