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## Evolutionary sequence of spacetime and intrinsic spacetime and associated sequence of geometries in metric force fields IV

### Abstract

The flat two-dimensional relative proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s \emptyset t')$  underlying the flat four-dimensional relative proper metric spacetime  $(\mathbb{E}^3, c_s t')$ , which emerges at the first stage of evolutions of metric spacetimes and intrinsic metric spacetimes in long-range metric force fields, isolated in the first three parts of this paper, endures for no moment before transforming into a curved two-dimensional relative proper intrinsic metric spacetime with pseudo-orthogonal curvilinear intrinsic dimensions,  $\emptyset\rho'$  and  $\emptyset c_s \emptyset t'$ , on the vertical intrinsic metric spacetime hyperplane, on the larger spacetime/intrinsic spacetime of combined positive (or our) universe and the negative universe. It therefore possesses intrinsic Lorentzian metric tensor at every point. It projects an underlying flat relativistic intrinsic metric spacetime  $(\emptyset\rho, \emptyset c_s \emptyset t)$ , which is made manifested outwardly in a flat four-dimensional relativistic metric spacetime  $(\mathbb{E}^3, c_s t)$ , at the second (and final) stage of evolutions of metric spacetime and intrinsic metric spacetime in long-range metric force fields. The conclusion that the four-dimensional metric spacetime is everywhere flat in every long-range metric force field is reached.

The curved 'two-dimensional' absolute intrinsic metric spacetime  $(\emptyset\hat{\rho}, \emptyset\hat{c}_s \emptyset\hat{t})$  with absolute intrinsic sub-Riemannian metric tensor  $\emptyset\hat{g}_{ik}$ , which evolves at the first stage is brought forward to the second stage. The basic aspects of the theory of relativity on the flat relativistic metric spacetime, intrinsic theory of relativity on the underlying flat relativistic intrinsic metric spacetime and absolute intrinsic metric theory on the curved absolute intrinsic metric spacetime, associated with the presence of a metric force field in spacetime and intrinsic metric force field in intrinsic spacetime, are developed in terms of certain derived geometrical parameters, referred to as relative proper static geodetic flow speed, relative proper intrinsic geodetic static flow speed and absolute intrinsic static geodetic flow speed respectively. Particularization to the gravitational field will be a straight forward process, while using the results of this paper as template.

*long-range metric force fields, first stage of evolution of spacetime, numerical evolution of spacetime, absolute intrinsic Riemannian spacetime geometry, coexisting absolute intrinsic metric spacetimes, superposition procedure, resultant absolute intrinsic metric tensor, resultant absolute intrinsic Ricci tensor*

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## 1 Introduction

The flat four-dimensional relative proper metric spacetime, denoted by  $(\mathbb{E}'^3, c_s t')$  for convenience in the first, second and third parts of this paper [1–3] and this fourth part, where  $\mathbb{E}'^3$  is the flat three-dimensional relative proper metric space, contains the ‘four-dimensional’ rest masses  $(m_0, \varepsilon'/c_s^2)$  of particles and objects, with the assumed absence of strong (or relativistic) gravitational field. The flat  $(\mathbb{E}'^3, c_s t')$ , with this assumption, supports the special theory of relativity (SR), involving the motions of the rest masses  $(m_0, \varepsilon'/c_s^2)$  of particles, which become their special-relativistic masses  $(\gamma m_0, \gamma \varepsilon'/c_s^2)$  relative to observers.

The presence of a strong (or relativistic) gravitational field into the flat four-dimensional relative proper metric spacetime  $(\mathbb{E}'^3, c_s t')$ , will transform it into a curved four-dimensional metric spacetime  $(\mathbb{E}^3, c_s t)$  (usually denoted by  $(x^0, x^1, x^2, x^3)$ ), with Riemannian metric tensor  $g_{\mu\nu}$ , in the context of the general theory of relativity (GR). It is to be recalled however that, although curvature of four-dimensional spacetime in a metric force field is a well thought-out prescription, see pages 111 – 149 of [4], which has not been falsified until now, it nevertheless remains an unproven fundamental postulate of the general theory of relativity.

As for the isolation of the first stage of evolutions of spacetime and intrinsic spacetime and the associated geometry in long-range metric force fields in [1–3], on the other hand, only the ‘two-dimensional’ absolute intrinsic metric spacetime  $(\mathcal{O}\hat{\rho}, \mathcal{O}\hat{c}_s\mathcal{O}\hat{t})$ , isolated in those articles and illustrated in Fig. 11 of [3], reproduced as Fig. 1 of this article, is curved with absolute intrinsic sub-Riemannian metric tensor  $\mathcal{O}\hat{g}_{ik}$ , while the four-dimensional relative proper metric spacetime  $(\mathbb{E}'^3, c_s t')$  is flat in long-range metric force fields, as Fig. 1 shows.

The developments in [1–3], which leads to the geometry of Fig. 11 of [3], reproduced as Fig. 1 of this article, is preceded by the isolation of co-existing four symmetrical universes in separate spactimes with event horizon separations, referred to as four-world picture in [5–8]. The four universes constitute four-world background for the special theory of relativity (SR) on flat relative proper spacetime in each universe, as developed in those articles.

By starting with the flat four-dimensional relative proper metric spacetimes,  $(\mathbb{E}'^3, c_s t')$  of our universe in Fig. 1 and  $(-\mathbb{E}'^{3*}, -c_s t'^*)$  of the negative universe (not shown in Fig. 1), as the spacetimes of the special theory of relativity, with the inherent assumption of the absence of strong (or relativistic) gravitational field in the respective universes in [5], the flat relative proper intrinsic metric spacetime  $(\mathcal{O}\rho', \mathcal{O}c_s\mathcal{O}t')$  that underlies  $(\mathbb{E}'^3, c_s t')$  in our universe and  $(-\mathcal{O}\rho'^*, -\mathcal{O}c_s\mathcal{O}t'^*)$  that underlies  $(-\mathbb{E}'^{3*}, -c_s t'^*)$  in the negative universe, were introduced as *ansatz* in section 4 of that article.

A new set of affine spacetime/intrinsic affine spacetime diagrams involving the rotations of straight line primed intrinsic affine spacetime coordinates,  $\mathcal{O}\tilde{x}'$  and  $\mathcal{O}c\mathcal{O}\tilde{t}'$ , (of the particle’s primed intrinsic affine frame), relative to their projective



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spacetime coordinates embedded in the flat two-dimensional relative proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s \emptyset t')$ , with the inherent assumption of the absence of strong gravitational field.

Eventually the origin of the flat two-dimensional relative proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s \emptyset t')$  underlying the flat 4-dimensional relative proper metric spacetime  $(\mathbb{E}'^3, c_s t')$  in our universe (and indeed in every one of the other three universes of the four-world picture isolated in [5–8]), was derived formally in section 1 of [8], thereby demystifying the (non-observable or hidden) intrinsic metric dimensions,  $\emptyset\rho'$  and  $\emptyset c_s \emptyset t'$ , and validating their actual presence in nature.

The special theory of relativity (SR) cannot alter the extended flat 4-dimensional relative proper metric spacetime  $(\mathbb{E}'^3, c_s t')$  on which it operates, with the assumed absence of strong (or relativistic) gravitational field. The intrinsic special theory of relativity ( $\emptyset$ SR) can likewise not alter the extended flat two-dimensional proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s \emptyset t')$  on which it operates, with the assumed absence of relativistic gravitational field. These, as explained under the Summary and Conclusion section of [8], is due to the fact that the spacetime/intrinsic spacetime coordinates (or spacetime/intrinsic spacetime geometry) associated with SR/ $\emptyset$ SR are affine space-time/intrinsic affine spacetime coordinates (or affine spacetime/intrinsic affine spacetime geometry) with no metric quality.

It is by introducing the source of a long-range relativistic metric force-field (such as a source of relativistic gravitational field), at a point on the flat 3-dimensional relative proper metric space  $\mathbb{E}'^3$  and, consequently, the source of a long-range relativistic intrinsic metric force-field (such as a source of a relativistic intrinsic gravitational field), at the same point in the straight line relative proper intrinsic metric space  $\emptyset\rho'$  in Fig. 11 of [3], reproduced as Fig. 1 of this article, the extended flat relative proper metric spacetime  $(\mathbb{E}', c_s t')$  and its underlying flat relative proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s \emptyset t')$  can be made to transform into four-dimensional relativistic metric spacetime  $(\Sigma, c_s t)$ , which is underlay by two-dimensional relativistic intrinsic metric spacetime  $(\emptyset\rho, \emptyset c_s \emptyset t)$  in all finite neighborhood of the source of the long-range metric force field. The geometry associated with this at the second stage of evolutions of spacetime/intrinsic spacetime in long range relativistic metric force-fields shall be developed in the rest of this article.

## 2 Geometrical background in the four-world picture for a new flat spacetime theory of relativity associated with the presence of a long-range metric force field

There are the theory of relativity and theory of intrinsic relativity, which are associated with the presence of a long-range relativistic metric force field in the four-dimensional metric spacetime and its underlying long-range relativistic intrinsic metric force field in the two-dimensional relative intrinsic metric spacetime. These will convert the

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extended flat four-dimensional relative proper metric spacetime  $(\mathbb{E}'^3, c_s t')$  and its underlying flat two-dimensional relative proper intrinsic metric spacetime  $(\mathcal{O}\rho', \mathcal{O}c_s \mathcal{O}t')$  to extended four-dimensional relativistic metric spacetime  $(\mathbb{E}^3, c_s t)$  and its underlying extended two-dimensional relativistic intrinsic metric spacetime  $(\mathcal{O}\rho, \mathcal{O}c_s \mathcal{O}t)$ , within the long-range relative metric force field, at the second stage of evolutions of metric spacetime and intrinsic metric spacetimes in a long-range relativistic metric force field, as shall be developed in the rest of this article.

Now the non-uniform absolute proper intrinsic static flow speed  $\mathcal{O}V'_{mab}$  projected along the projective straight absolute proper intrinsic metric spacetime ‘dimensions’,  $\mathcal{O}\rho'_{ab}$  and  $\mathcal{O}c_{sab}\mathcal{O}t'_{ab}$ , along the horizontal and vertical, by the non-uniform absolute intrinsic static flow speed  $\mathcal{O}\hat{V}_m$  of the curved absolute intrinsic metric spacetime dimensions,  $\mathcal{O}\hat{\rho}$  and  $\mathcal{O}\hat{c}_s\hat{\mathcal{O}}t$ , in Fig. 1, constitute identical non-uniform absolute proper intrinsic static flow speed  $\mathcal{O}V'_{mab}$  along the straight line relative proper intrinsic metric spacetime ‘dimensions’,  $\mathcal{O}\rho'$  and  $\mathcal{O}c_{sab}\mathcal{O}t'_{ab}$ , in which  $\mathcal{O}\rho'_{ab}$  and  $\mathcal{O}c_{sab}\mathcal{O}t'_{ab}$  are embedded respectively in that figure. However non-uniform absolute proper intrinsic static flow speed  $\mathcal{O}V'_{mab}$ , cannot cause the curvature of the relative proper intrinsic metric spacetime dimensions,  $\mathcal{O}\rho'$  and  $\mathcal{O}c_s\mathcal{O}t'$ . This is so, because absolute proper intrinsic static flow speed, being an absolute intrinsic parameter, cannot produce any effect whatever on the relative proper intrinsic metric dimensions,  $\mathcal{O}\rho'$  and  $\mathcal{O}c_s\mathcal{O}t'$ .

It is by also establishing non-uniform relative proper intrinsic static flow speed  $\mathcal{O}V'_m$  along the straight line relative proper intrinsic metric space  $\mathcal{O}\rho'$  and straight line relative proper intrinsic metric time dimension  $\mathcal{O}c_s\mathcal{O}t'$  in Fig. 1 that  $\mathcal{O}\rho'$  and  $\mathcal{O}c_s\mathcal{O}t'$  can be made to evolve into curved  $\mathcal{O}\rho'$  and curved  $\mathcal{O}c_s\mathcal{O}t'$  within a long-range relativistic metric force-field, at the second stage of evolutions of metric spacetime and intrinsic metric spacetime, as shall be developed in this section. Else, the geometry of Fig. 1 will persist and evolution of metric spacetime/intrinsic metric spacetime will terminate at the first stage within a long-range metric force-field (with the geometry of Fig. 1).

It is by introducing the source of a long-range relativistic metric force-field (such as a source of relativistic gravitational field), at a point on the relative proper metric Euclidean 3-space  $\mathbb{E}'^3$  and, consequently a source of a long-range relativistic intrinsic metric force-field (such as a source of a relativistic intrinsic gravitational field), at the same point in the straight line relative proper intrinsic metric space  $\mathcal{O}\rho'$  in Fig. 1 that, non-uniform relative proper intrinsic static flow speed  $\mathcal{O}V'_m$  is established along the straight line relative proper intrinsic metric spacetime dimensions,  $\mathcal{O}\rho'$  and  $\mathcal{O}c_s\mathcal{O}t'$  and non-uniform relative proper static flow-speed  $\mathcal{O}V'_m$  is established in  $\mathbb{E}'^3$  and along  $c_s t'$ . This will cause the extended straight line  $\mathcal{O}\rho'$  and  $\mathcal{O}c_s\mathcal{O}t'$  to become curved. projecting straight line relativistic intrinsic metric spacetime dimensions,  $\mathcal{O}\rho$  and  $\mathcal{O}c_s\mathcal{O}t$ , which are made manifested in flat three-dimensional relativistic metric space  $\mathbb{E}^3$  and straight line relativistic metric time dimension  $c_s t$  respectively.

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Thus the extended flat relative proper metric spacetime  $(\mathbb{E}'^3, c_s t')$  and its underlying flat relative proper intrinsic metric spacetime  $(\mathcal{O}\rho', \mathcal{O}c_s \mathcal{O}t')$ , with the assumption of the absence of a long-range relativistic metric force field, will naturally transform into flat four-dimensional relativistic metric spacetime  $(\mathbb{E}^3, c_s t)$ , which is underlay by two-dimensional flat relativistic intrinsic metric spacetime  $(\mathcal{O}\rho, \mathcal{O}c_s \mathcal{O}t)$  in all finite neighborhood of the source of a long-range relativistic metric force field. The geometry associated with this at the second stage of evolutions of spacetime/intrinsic spacetime in a long range relativistic metric force-field shall be developed in the rest of this article.

## 2.1 The global curved relative proper intrinsic metric spacetime and its underlying projective flat relativistic intrinsic metric spacetime and its outward manifestation flat relativistic metric spacetime in a metric force field

As follows from the preceding two paragraphs, let us introduce non-uniform relative proper intrinsic static flow speed  $\mathcal{O}V'_m$  along the straight line relative proper intrinsic metric space  $\mathcal{O}\rho'$  and of the straight line relative proper intrinsic metric time dimension  $\mathcal{O}c_s \mathcal{O}t'$  in Fig. 1, which evolves at the first stage of evolutions of metric spacetime and intrinsic metric spacetimes, such that  $\mathcal{O}V'_m$  has its largest magnitude at a point  $(S, S^0)$  in  $(\mathcal{O}\rho', \mathcal{O}c_s \mathcal{O}t')$  and decreases continuously until it vanishes virtually at point O in  $(\mathcal{O}\rho', \mathcal{O}c_s \mathcal{O}t')$ , which is far removed from point  $(S, S^0)$ . These will be made manifested outwardly in non-uniform relative proper static flow speed  $V'_m$  in the relative proper metric Euclidean 3-space  $\mathbb{E}'^3$  and along the relative proper metric time dimension  $c_s t'$ , such that  $V'_m$  has its largest magnitude at the corresponding point  $(S, S^0)$  in  $(\mathbb{E}'^3, c_s t')$  and decreases in magnitude continuously until it vanishes virtually at point O in  $(\mathbb{E}'^3, c_s t')$ , which is far removed from point  $(S, S^0)$  in that figure.

The foregoing is quite apart from the non-uniform absolute proper intrinsic static flow speed  $\mathcal{O}V'_{mab}$  constituted along the straight line relative proper intrinsic metric space  $\mathcal{O}\rho'$  and the straight line relative proper intrinsic metric time dimension  $\mathcal{O}c_s \mathcal{O}t'$  by the non-uniform  $\mathcal{O}V'_{mab}$  projected along the projective absolute proper intrinsic metric dimensions,  $\mathcal{O}\rho'_{ab}$  and  $\mathcal{O}c_{sab} \mathcal{O}t'_{ab}$ , embedded in  $\mathcal{O}\rho'$  and  $\mathcal{O}c_s \mathcal{O}t'$  respectively, and the outward manifestations of these namely, the non-uniform absolute proper static flow speed  $V'_{mab}$  in  $\mathbb{E}'^3$  and along  $c_s t'$  in Fig. 1.

However the presence of non-uniform absolute proper intrinsic static flow speed  $\mathcal{O}V'_{mab}$  along the relative proper intrinsic metric space  $\mathcal{O}\rho'$  and relative proper intrinsic metric time dimension  $\mathcal{O}c_s \mathcal{O}t'$ , cannot cause the curvatures of  $\mathcal{O}c_s \mathcal{O}t'$  and  $\mathcal{O}\rho'$ , or produce any other effect on them. The presence of absolute proper static flow speed  $V'_{mab}$  in the relative proper metric space  $\mathbb{E}'^3$  and relative proper metric time dimension  $c_s t'$  can likewise produce no detectable effect on  $\mathbb{E}'^3$  and  $c_s t'$ , as mentioned earlier.

Let us recall the evolution of Fig. 11 of [3], reproduced as Fig. 1 of this article, from Fig. 6 of [3], reproduced as Fig. 2 of this article. The introduction of non-uniform absolute intrinsic static flow speed  $\varnothing\hat{V}_m$  along the initial straight line absolute intrinsic metric space  $\varnothing\hat{\rho}$  and along the initial straight line absolute intrinsic metric time ‘dimension’  $\varnothing\hat{c}_s\hat{\varnothing}\hat{t}$  in Fig. 2, causes the straight line  $\varnothing\hat{\rho}$  to evolve into curved absolute intrinsic metric space  $\varnothing\hat{\rho}$ , where  $\varnothing\hat{\rho}$  will have largest curvature at the point  $(S, S^0)$  where  $\varnothing\hat{V}_m$  is largest and virtually zero curvature at point O, which is far removed from point  $(S, S^0)$ , where  $\varnothing\hat{V}_m$  vanishes virtually.

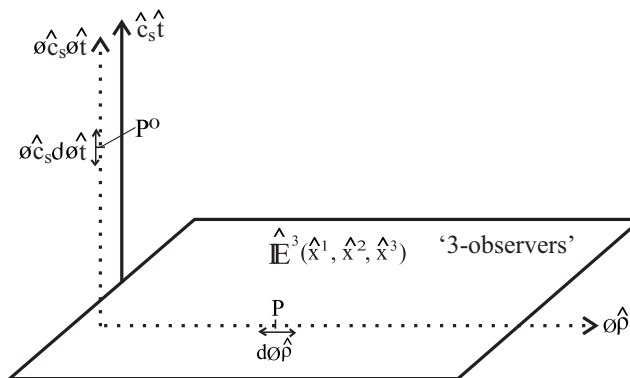


Figure 2: Flat ‘four-dimensional’ absolute metric spacetime and its underlying flat ‘two-dimensional’ absolute intrinsic metric spacetime with assumed absence of a long-range metric force field (or absence of absolute intrinsic Riemannian spacetime geometry); (Fig. 6 of [3]).

On the other hand, the absolute intrinsic metric time ‘dimension’  $\varnothing\hat{c}_s\hat{\varnothing}\hat{t}$  and absolute metric time dimension  $\hat{c}_s\hat{t}$  along the vertical are invariant (or remain unaffected) in the context of the absolute intrinsic metric phenomenon that causes the curvature of the absolute intrinsic metric spacetime ‘dimensions’, with respect to ‘3-observers’ in the absolute space  $\hat{\mathbb{E}}^3$  in Fig. 2. Graphically, this implies that the straight line absolute intrinsic metric time ‘dimension’  $\varnothing\hat{c}_s\hat{\varnothing}\hat{t}$  along the vertical in that figure will remain not curved from its vertical position, thereby yielding the half-geometry of Fig. 1 of [3], reproduced as Fig. 3 of this article, which is valid with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  in that figure.

The initial straight line absolute intrinsic metric time ‘dimension’  $\varnothing\hat{c}_s\hat{\varnothing}\hat{t}$  in Fig. 2 remains not curved from the vertical, while the initial straight line absolute intrinsic metric space  $\varnothing\hat{\rho}$  in that figure becomes curved absolute intrinsic metric space  $\varnothing\hat{\rho}$  in Fig. 3, because the absolute metric time ‘dimension’  $\hat{c}_s\hat{t}$  and the absolute intrinsic metric time ‘dimension’  $\varnothing\hat{c}_s\hat{\varnothing}\hat{t}$  are invariant (or unaffected), that is, do not evolve into the absolute proper metric time dimensions  $c_{sab}t'_{ab}$  and absolute proper intrinsic metric time dimension  $\varnothing c_{sab}\varnothing t'_{ab}$ , with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  along the horizontal in that figure, in the contexts of absolute





with respect to 3-observers in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  and 1-observers in the newly formed relativistic metric time dimension  $c_s t$ , in the contexts of the theory of relativity and intrinsic theory of relativity associated with the presence of a long-range relative proper metric force field in relative proper metric spacetime and long-range relative proper intrinsic metric force-field in relative proper intrinsic metric spacetime.

As mentioned in section 4 of [5], affine spacetime coordinates and intrinsic affine spacetime coordinates that appear in SR/ $\emptyset$ SR shall have over-head tilde label as,  $\tilde{x}, \tilde{y}, \tilde{z}, c_d \tilde{t}, \emptyset \tilde{x}$  and  $\emptyset c_d \emptyset \tilde{t}$ , while the metric spacetime coordinates and intrinsic metric spacetime coordinates that appear in the theory of relativity and intrinsic theory of relativity associated with the presence of metric force field in metric spacetime and intrinsic metric force field in intrinsic metric spacetime, shall have no over-head tilde label, appearing as,  $x^0, x^1, x^2, x^3, \emptyset x$  and  $\emptyset c_s \emptyset t$ .

An implication of the penultimate paragraph is that the introduction of non-uniform relative proper intrinsic static flow speeds  $\emptyset V'_m$  identically along the straight line relative proper intrinsic metric space  $\emptyset \rho'$  and along the straight line relative proper intrinsic metric time dimension  $\emptyset c_s \emptyset t'$  in Fig. 1 of this article, will cause both  $\emptyset \rho'$  and  $\emptyset c_s \emptyset t'$  to be identically curved anticlockwise simultaneously relative to the horizontal and vertical respectively, such that the curved  $\emptyset \rho'$  lying in the first quadrant and the curved  $\emptyset c_s \emptyset t'$  lying in the second quadrant on the larger spacetime hyperplane of combined positive and negative universes, constitute pseudo-orthogonal curvilinear intrinsic metric dimensions, with respect to 3-observers in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  in our (or positive) universe. It is to be remembered, as mentioned earlier that, the projective non-uniform absolute proper intrinsic static flow speed  $\emptyset V'_{mab}$  along  $\emptyset \rho'$  and  $\emptyset c_s \emptyset t'$  in Fig. 1 at the first stage of evolutions of spacetime and intrinsic spacetime in a long-range metric force field cannot cause the curvature of  $\emptyset \rho'$  and  $\emptyset c_s \emptyset t'$ .

In symmetry, the relative proper intrinsic metric space  $-\emptyset \rho'^*$  and the relative proper intrinsic metric time dimension  $-\emptyset c_s \emptyset t'^*$  of the negative universe are identically curved anticlockwise simultaneously relative to the horizontal and vertical respectively, such that the curved  $-\emptyset \rho'^*$  lying in the third quadrant and the curved  $-\emptyset c_s \emptyset t'^*$  lying in the fourth quadrant, constitute pseudo-orthogonal curvilinear intrinsic metric dimensions with respect to 3-observers\* in the relativistic metric Euclidean 3-space  $-\mathbb{E}^{*3}$  in the negative universe. These curvatures of relative proper intrinsic metric spacetime dimensions and those of the preceding paragraph will take place simultaneously within symmetry-partner long-range relativistic metric force fields and their underlying long-range relativistic intrinsic metric force fields in the positive (or our) universe and the negative universe, at the second stage of evolutions of spacetime and intrinsic spacetime within symmetry-partner long-range relativistic metric force fields in the positive and negative universes.

A consequence of the foregoing is that the geometry of Fig. 5 will evolve with respect to 3-observers in the relativistic metric Euclidean 3-spaces,  $\mathbb{E}^3$  and  $-\mathbb{E}^{*3}$ , of

the positive and negative universes, as indicated, at the second stage of evolutions of spacetime/intrinsic spacetime within symmetry-partner long-range relativistic metric force fields in the positive and negative universes. The non-uniform relative proper intrinsic static flow speed  $\varnothing V'_m$  introduced along the straight line relative proper intrinsic metric space  $\varnothing\rho'$  and straight line relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  in Fig. 1, have largest magnitude at a point  $(S, S^0)$  on  $(\varnothing\rho', \varnothing c_s \varnothing t')$ , due to the sources of relative proper intrinsic metric force field located at that point (not shown) in Fig. 5, and decrease in magnitude continuously until it vanishes virtually at point O in  $(\varnothing\rho', \varnothing c_s \varnothing t')$ , which is far removed from the point  $(S, S^0)$  in that figure.

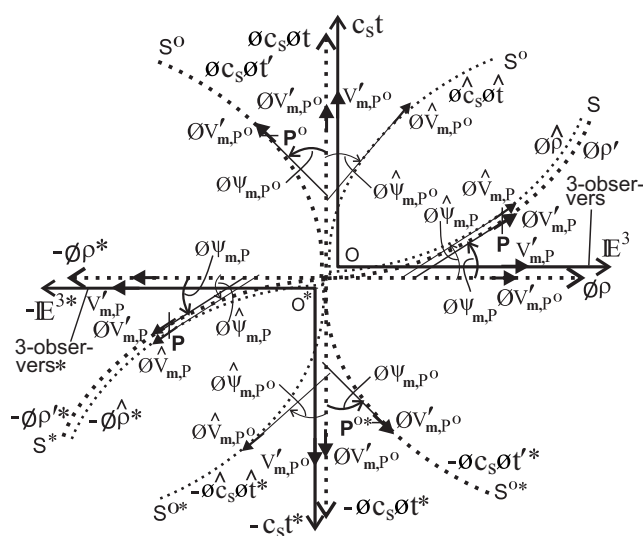


Figure 5: The extended curved two-dimensional relative proper intrinsic metric spacetimes with pseudo-orthogonal curvilinear intrinsic dimensions, project flat two-dimensional relativistic intrinsic metric spacetimes that underlie their outward manifestations namely, the flat four-dimensional relativistic metric spacetimes, with respect to 3-observers in the relativistic metric Euclidean 3-spaces on the positive and negative universes, at the second stage of evolutions of spacetimes and intrinsic spacetimes within symmetry-partner long-range relativistic metric force fields in the two universes, shown along with the curved ‘two-dimensional’ absolute intrinsic metric spacetimes with absolute intrinsic sub-Riemannian metric tensors in the two universes brought forward from the first stage: The global metric spacetime/intrinsic metric spacetime diagram with respect to 3-observers in the relativistic metric Euclidean spaces in our universe and the negative universe.

Figure 5 has evolved from Fig. 1 upon introducing non-uniform relative proper intrinsic static flow speed along the straight line relative proper intrinsic metric space  $\varnothing\rho'$  and straight line relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  in our

universe in that figure, and their counterparts,  $-\varnothing\rho'^*$  and  $-\varnothing c_s\varnothing t'^*$ , in the negative universe (not shown in Fig. 1). Hence the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$  in our universe in that figure and the corresponding curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(-\varnothing\hat{\rho}^*, -\varnothing\hat{c}_s\varnothing\hat{t}^*)$  in the negative universe (not shown in that figure), have remained in Fig. 5.

A relative proper static flow speed  $V'_m$  is relativistic for,  $V'_m/c_m > 0$  relative to all observers, since  $V'_m$  is the same relative to all observers. A relativistic metric force field is one within which  $V'_m/c_m > 0$ . Thus relativistic as being used to qualify metric force fields and metric spacetime within a relativistic metric force field, does not connote the presence of the special theory of relativity (SR), in the context of which “relativistic” has usually appeared.

It is to be noted that the straight line absolute proper intrinsic metric spacetime ‘dimensions’,  $\varnothing c_{sab}\varnothing t'_{ab}$  and  $\varnothing\rho'_{ab}$ , which are embedded in the straight line relative proper intrinsic metric spacetime dimensions,  $\varnothing\rho'$  and  $\varnothing c_s\varnothing t'$ , in Fig. 5, are curved along with  $\varnothing\rho'$  and  $\varnothing c_s\varnothing t'$  in that figure. They project ‘absolute relativistic’ intrinsic metric spacetime ‘dimensions’,  $\varnothing\rho_{ab}$  and  $\varnothing c_{sab}\varnothing t_{ab}$ , (absolute proper intrinsic dimensions without prime label) (not shown in Fig. 5), which are imperceptibly embedded in the projective straight line relativistic intrinsic metric spacetime dimensions,  $\varnothing\rho$  and  $\varnothing c_s\varnothing t$ , in the first quadrant (or in our universe) in Fig. 5. The invariance,  $\varnothing\rho_{ab} = \varnothing\rho'_{ab}$  and  $\varnothing c_{sab}\varnothing t_{ab} = \varnothing c_{sab}\varnothing t'_{ab}$ , obtain, as shall be shown elsewhere.

The projective  $\varnothing\rho_{ab}$  and  $\varnothing c_{sab}\varnothing t_{ab}$  embedded in  $\varnothing\rho$  and  $\varnothing c_s\varnothing t$  are then made manifested outwardly in ‘absolute relativistic’ metric spacetime ‘dimensions’,  $\rho_{ab}$  and  $c_{sab}t_{ab}$  (not shown in Fig. 5), which are imperceptibly embedded in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  and relativistic metric time dimension  $c_s t$  respectively in our (or positive) universe in Fig. 5. All of these occur simultaneously and in perfect symmetry in the negative universe in that figure.

All of the curved relative proper intrinsic metric spacetimes,  $(\varnothing\rho', \varnothing c_s\varnothing t')$  and  $(-\varnothing\rho'^*, -\varnothing c_s\varnothing t'^*)$ , the projective flat relativistic intrinsic metric spacetimes,  $(\varnothing\rho, \varnothing c_s\varnothing t)$  and  $(-\varnothing\rho^*, -\varnothing c_s\varnothing t^*)$ , and the flat four-dimensional relativistic metric spacetimes,  $(\mathbb{E}^3, c_s t)$  and  $(-\mathbb{E}^{*3}, -c_s t^*)$ , along with the curved absolute proper intrinsic metric spacetimes,  $(\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab})$  and  $(-\varnothing\rho'^*_{ab}, -\varnothing c_{sab}\varnothing t'^*_{ab})$ , the projective flat ‘absolute relativistic’ intrinsic metric spacetimes,  $(\varnothing\rho_{ab}, \varnothing c_{sab}\varnothing t_{ab})$  and  $(-\varnothing\rho^*_{ab}, -\varnothing c_{sab}\varnothing t^*_{ab})$ , and the flat ‘absolute relativistic’ metric spacetimes,  $(\rho_{ab}, c_{sab}t_{ab})$  and  $(-\rho^*_{ab}, -c_{sab}t^*_{ab})$ , imperceptibly embedded in them, shall be found of important relevance in determining the hierarchy of intrinsic theories and theories of a metric force field and intrinsic metric force field later in this article and elsewhere.

However the main interest in this article is in the formulation of the intrinsic theory of relativity and theory of relativity associated with the presence of a long-range metric force field in metric spacetime and its underlying long-range intrinsic metric force field in intrinsic metric spacetime. It is the curved relative proper intrinsic metric spacetimes  $(\varnothing\rho', \varnothing c_s\varnothing t')$  and  $(-\varnothing\rho'^*, -\varnothing c_s\varnothing t'^*)$ , their projective flat relativistic intrinsic metric spacetimes  $(\varnothing\rho, \varnothing c_s\varnothing t)$  and  $(-\varnothing\rho^*, -\varnothing c_s\varnothing t^*)$  and

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the outward manifestations flat relativistic metric spacetimes  $(\mathbb{E}^3, c_s t)$  and  $(-\mathbb{E}^{3*}, -c_s t^*)$  that are relevant for doing this.

Now the curved relative proper intrinsic metric space  $\mathcal{O}\rho'$  in the first quadrant and the curved relative proper intrinsic metric time dimension  $\mathcal{O}c_s\mathcal{O}t'$  in the second quadrant in Fig. 5, evolve simultaneously with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  in the first quadrant (or in the positive universe), and the curved relative proper intrinsic metric space  $-\mathcal{O}\rho'^*$  in the third quadrant and the curved relative proper intrinsic metric time dimension  $-\mathcal{O}c_s\mathcal{O}t'^*$  in the fourth quadrant, evolve simultaneously with respect to 3-observers\* in the relativistic metric Euclidean 3-space  $-\mathbb{E}^{3*}$  in the third quadrant (or in the negative universe) in that figure.

The curved relative proper intrinsic metric space  $\mathcal{O}\rho'$  in the first quadrant projects a straight line relativistic intrinsic metric space  $\mathcal{O}\rho$  along the horizontal, which is made manifested outwardly in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  in which 3-observers are now located, as indicated. The curved relative proper intrinsic metric time dimension  $\mathcal{O}c_s\mathcal{O}t'$  in the second quadrant, likewise projects straight line relativistic intrinsic metric time dimension  $\mathcal{O}c_s\mathcal{O}t$  along the vertical, which is made manifested outwardly in the relativistic metric time dimension  $c_s$ , in which 1-observers in time dimension are now located in our universe.

The curved relative proper intrinsic metric space  $-\mathcal{O}\rho'^*$  in the third quadrant projects relativistic intrinsic metric space  $-\mathcal{O}\rho^*$  along the horizontal, which is made manifested outwardly in the relativistic Euclidean 3-space  $-\mathbb{E}^{3*}$  in which 3-observers\* are now located in the negative universe, as indicated, and the curved relative proper intrinsic metric time dimension  $-\mathcal{O}c_s\mathcal{O}t'^*$  in the fourth quadrant projects relativistic intrinsic metric time dimension  $-\mathcal{O}c_s\mathcal{O}t^*$  along the vertical, which is made manifested outwardly in the relativistic metric time dimension  $-c_s t^*$  in which 1-observers\* in time dimension are now located in the negative universe.

However 1-observers are not indicated to exist in the relativistic time dimensions,  $c_s t$  and  $-c_s t^*$ , in Fig. 5, because the geometry of Fig. 5 is valid with respect to 3-observers in the Euclidean 3-spaces,  $\mathbb{E}^3$  and  $-\mathbb{E}^{3*}$ , solely, as indicated. Recall from section 4 of [7] that the anti-clockwise sense of inclination (or rotation) by positive angle of the curved relative proper intrinsic metric spacetimes dimensions relative to their projective flat relativistic intrinsic metric spacetimes by varying positive intrinsic angles, is valid with respect to 3-observers in the relativistic metric Euclidean 3-spaces in Fig. 5. It is in the complementary diagram to Fig. 5, to be developed shortly, which is valid with respect to 1-observers in the relativistic time dimensions that 1-observers in  $c_s t$  and  $-c_s t^*$ , in which the 1-observers will be indicated along  $c_s t$  and  $-c_s t^*$ .

Thus the flat four-dimensional relative proper metric spacetime  $(\mathbb{E}'^3, c_s t')$  and its underlying flat two-dimensional relative proper intrinsic metric spacetime  $(\mathcal{O}\rho', \mathcal{O}c_s\mathcal{O}t')$ , which appear within a long-range metric force field at the first stage of evolution of spacetime/intrinsic spacetime in our universe in Fig. 11 of [3], reproduced as Fig. 1 of this article, evolve into flat four-dimensional relativistic metric spacetime  $(\mathbb{E}^3, c_s t)$

and its underlying flat two-dimensional relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s \varnothing t)$  in Fig. 5, at the second stage of evolution of spacetime/intrinsic spacetime in the long-range metric force-field.

The flat 4-dimensional relative proper metric spacetime  $(-\mathbb{E}'^{3*}, -c_s t'^*)$  and its underlying flat relative proper intrinsic metric spacetime  $(-\varnothing\rho'^*, -\varnothing c_s \varnothing t'^*)$ , which appear within the symmetry-partner long-range metric force field in the negative universe (not shown in Fig. 1), at the first stage of evolutions of spacetime/intrinsic spacetime, likewise evolve into flat four-dimensional relativistic metric spacetime  $(-\mathbb{E}^{3*}, -c_s t^*)$  and its underlying flat two-dimensional relativistic intrinsic metric spacetime  $(-\varnothing\rho^*, -\varnothing c \varnothing t^*)$  in Fig. 5, at the second stage of evolution of spacetime/intrinsic spacetime within the symmetry-partner long-range metric force-field.

There are some other features of Fig. 5 that are important for remark. First, the absolute intrinsic metric space  $\varnothing\hat{\rho}$  and the relative proper intrinsic metric space  $\varnothing\rho'$ , are shown to be identically curved relative to the relativistic intrinsic metric space  $\varnothing\rho$  along the horizontal. Indeed the curved  $\varnothing\rho'$  should fall along the curved  $\varnothing\hat{\rho}$  in Fig. 5. This means that the point P along the curved  $\varnothing\hat{\rho}$  in Fig. 1 is the same as point P along the curved  $\varnothing\rho'$  in Fig. 5. Consequently the absolute intrinsic angle  $\varnothing\hat{\psi}_{m,P}$  of inclination of the curved  $\varnothing\hat{\rho}$  to the horizontal at point P along  $\varnothing\hat{\rho}$  in Fig. 1 and the relative proper intrinsic angle  $\varnothing\psi_{m,P}$  of inclination of the curved  $\varnothing\rho'$  to the horizontal at point P along the curved  $\varnothing\rho'$  in Fig. 5, are equal in magnitude. It then follows that the absolute intrinsic static flow speed  $\varnothing\hat{V}_{m,P}$  at point P along the curved  $\varnothing\hat{\rho}$  in Fig. 1 and the relative proper intrinsic static flow speed  $\varnothing V'_{m,P}$ , at point P along  $\varnothing\rho'$  in Fig. 5 are equal in magnitude. That is,

$$\sin |\varnothing\hat{\psi}_{m,P}| = \sin |\varnothing\psi_{m,P}| \tag{1a}$$

or

$$\left| \frac{\varnothing\hat{V}_{m,P}}{\varnothing\hat{c}_m} \right| = \left| \frac{\varnothing V'_{m,P}}{\varnothing c_m} \right| \tag{1b}$$

In order for relations (1a) and (1b) to hold, it must be that the source of absolute intrinsic metric force field located at the point S along the curved absolute intrinsic metric space  $\varnothing\hat{\rho}$  in Fig. 1, which establishes non-uniform absolute intrinsic static flow speed  $\varnothing\hat{V}_{m,P}$  between points S and O along the curved  $\varnothing\hat{\rho}$  in that figure, is ‘projected’ as a source of absolute proper intrinsic metric force field of identical magnitude, into the corresponding point S along the projective absolute proper intrinsic metric space  $\varnothing\rho'_{ab}$ .

A source of relative proper intrinsic metric force field of identical magnitude then appears automatically at the corresponding point S along the straight line relative proper intrinsic metric space  $\varnothing\rho'$ , which appears along the horizontal automatically along with the projection of  $\varnothing\rho'_{ab}$  along the horizontal in Fig. 1. The source of relative proper intrinsic metric force field that appears automatically thereby establishes non-uniform relative proper intrinsic static flow speed  $\varnothing V'_m$  (of identical magnitudes as

the projective non-uniform absolute proper intrinsic static flow speed  $\emptyset V'_{m,ab}$ ) along the straight line  $\emptyset\rho'$  in Fig. 1 and, consequently, along the curved  $\emptyset\rho'$  in Fig. 5.

The point  $P^0$  along the curved relative proper intrinsic metric time dimension  $\emptyset c_s \emptyset t'$  in the second quadrant is the symmetry-partner to point P along the curved relative proper intrinsic metric space  $\emptyset\rho'$  in the first quadrant in Fig. 2. Consequently the relative intrinsic angle  $\emptyset\psi_{P^0}$  of inclination of the curved  $\emptyset c_s \emptyset t'$  to the vertical at point  $P^0$  along the curved  $\emptyset c_s \emptyset t'$  and the relative intrinsic angle  $\emptyset\psi_P$  of inclination of the curved  $\emptyset\rho'$  to the horizontal at point P along the curved  $\emptyset\rho'$ , are equal in magnitude. It then follows that the non-uniform relative proper intrinsic static flow speeds,  $\emptyset V'_{m,P^0}$  and  $\emptyset V'_{m,P}$ , are equal in magnitude. That is,

$$\sin \emptyset\psi_{m,P^0} = \sin \emptyset\psi_{m,P} \tag{2a}$$

or

$$\frac{\emptyset V'_{m,P^0}}{\emptyset c_m} = \frac{\emptyset V'_{m,P}}{\emptyset c_m} \tag{2b}$$

Finally, the relative proper intrinsic static flow speed  $\emptyset V'_{m,P}$  of point P along the curved relative proper intrinsic metric space  $\emptyset\rho'$  is shown to be invariantly projected as relative proper intrinsic static flow speed  $\emptyset V'_{m,P}$  into the relativistic intrinsic metric space  $\emptyset\rho$  along the horizontal, and this is made manifested in relative proper static flow speed  $V'_{m,P}$  in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  in Fig. 5. The relative proper intrinsic static flow speed  $\emptyset V'_{m,P^0}$  of point  $P^0$  along the curved relative proper intrinsic metric time dimension  $\emptyset c_s \emptyset t'$  is likewise shown to be invariantly projected as relative proper intrinsic static flow speed  $\emptyset V'_{m,P^0}$  into the relativistic intrinsic metric time dimension  $\emptyset c_s \emptyset t$ , which is made manifested in relative proper static flow speed  $V'_{m,P^0}$  along the relativistic metric time dimension  $c_s t$  along the vertical in Fig. 5.

On the other hand, one expects that the non-uniform relative proper intrinsic static flow speed  $\emptyset V'_m$  along the curved relative proper intrinsic metric space  $\emptyset\rho'$  should be projected as non-uniform relativistic intrinsic static flow speed  $\emptyset V_m$  (without prime label) into the projective relativistic intrinsic metric space  $\emptyset\rho$  along the horizontal and that the non-uniform relative proper intrinsic static flow speed  $\emptyset V'_m$  along the curved relative proper intrinsic metric time dimension  $\emptyset c_s \emptyset t'$  should be projected as non-uniform relativistic intrinsic static flow speed  $\emptyset V_m$  into the projective relativistic intrinsic metric time dimension  $\emptyset c_s \emptyset t$  along the vertical in Fig. 2.

The fact that the non-uniform relative proper intrinsic static flow speed  $\emptyset V'_m$  along the curved  $\emptyset\rho'$  and  $\emptyset V'_m$  along the curved  $\emptyset c_s \emptyset t'$  are invariantly projected as relative proper intrinsic static flow speed  $\emptyset V'_m$  into  $\emptyset\rho$  along the horizontal and  $\emptyset c_s \emptyset t$  along the vertical respectively in Fig. 5, is a graphical interpretation of the invariance of intrinsic static flow speed in the context of the intrinsic theory of

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relativity associated with the presence of a long-range relativistic intrinsic metric force field in intrinsic metric space. This invariance is stated as

$$\emptyset V_m = \emptyset V'_m, \tag{3a}$$

hence,

$$V_m = V'_m, \tag{3b}$$

where Eqs. (3a) and (3b) have been written at an arbitrary pair of symmetry-partner points along the curved  $\emptyset\rho'$  and  $\emptyset c_s \emptyset t'$ .

The invariance of relative proper intrinsic static flow speed and relative proper static flow speed (3a) and (3b), in the context of the theory of relativity and theory of intrinsic relativity associated with the presence of a relative proper metric force field in relative proper metric spacetime and relative proper intrinsic metric force field in relative proper intrinsic metric spacetime, which involve relative proper static flow speed and relative proper intrinsic static flow speed respectively, established in spacetime and intrinsic spacetime by the source of a long-range relative proper metric force field, at the second stage of evolutions of spacetimes and intrinsic spacetimes within the metric force field, shall be given formal proof elsewhere, upon particularizing to the gravitational field.

The corresponding invariance of absolute intrinsic static flow speed and absolute static flow speed,

$$\emptyset V'_{mab} = \emptyset \hat{V}_m \text{ and } V'_{mab} = \hat{V}_m,$$

deduced and presented as Eqs. (83a) and (83b) of [3], in the context of absolute intrinsic metric theory of physics and absolute metric theory of physics, involving absolute intrinsic static flow speeds and absolute static flow speed respectively, established in absolute spacetime and absolute intrinsic spacetime by the source of a long-range absolute metric force-field, at the first stage of evolution of spacetime/intrinsic spacetime within the metric force field, shall likewise be given formal proofs elsewhere upon particularizing to the gravitational field.

Now the perfect symmetry of state among the four universes namely, positive (or our) universe, negative universe, positive time-universe and negative time-universe, isolated in [5–8], implies that, as the geometry of Fig. 5 evolves with respect to 3-observers in the relativistic Euclidean 3-spaces,  $\mathbb{E}^3$  and  $-\mathbb{E}^{3*}$ , in our universe and the negative universe, at the second stage of evolutions of spacetime/intrinsic spacetime, within symmetry-partner long-range metric force fields in our universe and the negative universe, the geometry of Fig. 3 evolves simultaneously with respect to 3-observers in the relativistic Euclidean 3-spaces  $\mathbb{E}^{03}$  and  $-\mathbb{E}^{03*}$  in the positive time-universe and the negative time-universe, at the second stage of evolution of spacetime/intrinsic spacetime within the symmetry-partner long-range metric force fields in the positive time-universe and the negative time-universe.

Figure 6 in the positive time-universe and the negative time-universe co-exists with Fig. 5 in the positive (or our) universe and the negative universe. It should serve

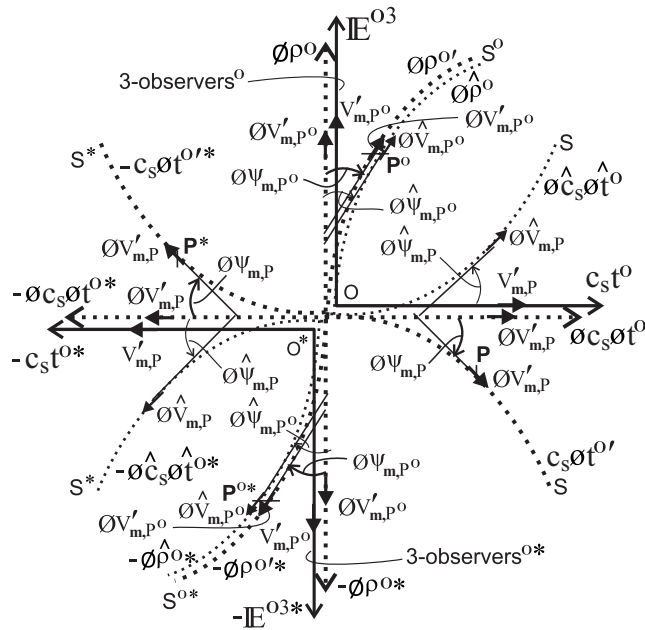


Figure 6: The symmetrical global metric spacetime/intrinsic metric spacetime diagram in the positive time-universe and the negative time-universe, which evolve simultaneously with Fig. 5 in our universe and the negative universe, at the second stage of evolutions of spacetimes/intrinsic spacetimes within symmetry-partner long-range metric force fields in the positive time-universe and the negative time-universe, with respect to 3-observers<sup>0</sup> in the relativistic metric Euclidean 3-spaces in those universes.

as a complementary diagram to Fig. 5 toward formulating the theory of relativity associated with the presence of symmetry-partner relative proper metric force fields in the relative proper spacetimes in our universe and the negative universe. However Fig. 6 in its present form cannot serve as a complementary diagram to Fig. 5. This is so, because the spacetime and intrinsic spacetime dimensions of the positive time-universe and the negative time-universe in Fig. 6 are elusive to observers in our universe and the negative universe and cannot appear in physics in our universe and the negative universe.

In order to make Fig. 6 a valid complementary diagram to Fig. 5, the spacetime and intrinsic spacetime dimensions of the positive time-universe and the negative time-universe in it must be transformed to those of our universe and the negative universe, as developed in [8] (see system (15) of that article). This means that the following transformations of spacetime/intrinsic spacetime dimensions must be

performed on Fig. 6, thereby transforming Fig. 6 to Fig. 7.

$$\begin{aligned}
 \mathbb{E}^{03} &\rightarrow c_s t; -\mathbb{E}^{03*} \rightarrow -c_s t^*; c_s t^0 \rightarrow \mathbb{E}^3; -c_s t^{0*} \rightarrow -\mathbb{E}^{3*}; \\
 \varnothing \rho^0 &\rightarrow \varnothing c_s \varnothing t; -\varnothing \rho^{0*} \rightarrow -\varnothing c_s \varnothing t^*; \varnothing c_s \varnothing t^0 \rightarrow \varnothing \rho; -\varnothing c_s \varnothing t^{0*} \rightarrow -\varnothing \rho^*; \\
 \varnothing \rho^{0'} &\rightarrow \varnothing c_s \varnothing t'; -\varnothing \rho^{0'*} \rightarrow -\varnothing c_s \varnothing t'^*; \varnothing c_s \varnothing t^{0'} \rightarrow \varnothing \rho'; -\varnothing c_s \varnothing t^{0'*} \rightarrow -\varnothing \rho'^*; \\
 \varnothing \hat{\rho}^0 &\rightarrow \varnothing \hat{c}_s \varnothing \hat{t} - \varnothing \hat{\rho}^{0*} \rightarrow -\varnothing \hat{c}_s \varnothing \hat{t}^*; \varnothing \hat{c}_s \varnothing \hat{t}^0 \rightarrow \varnothing \hat{\rho}; -\varnothing \hat{c}_s \varnothing \hat{t}^{0*} \rightarrow -\varnothing \hat{\rho}^*
 \end{aligned} \tag{4}$$

Implementation of system (4) on Fig. 6 yields Fig. 7.

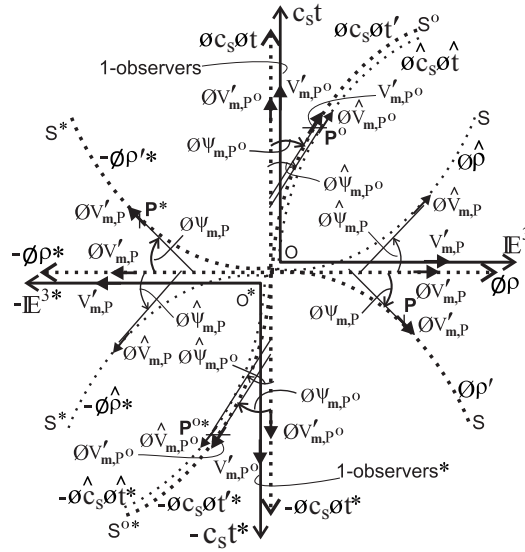


Figure 7: The global metric spacetime/intrinsic metric spacetime diagram obtained by transforming the spacetimes and intrinsic spacetimes of the positive time-universe and the negative time-universe in Fig. 6 to the spacetimes and intrinsic spacetimes of the positive (or our) universe and the negative universe; the complementary diagram to Fig. 5, which is valid with respect to 1-observers in the relativistic metric time dimensions in our universe and the negative universe.

The 3-observers<sup>0</sup> in the relativistic Euclidean 3-spaces,  $\mathbb{E}^{03}$  and  $-\mathbb{E}^{03*}$ , of the positive time-universe and negative time-universe in Fig. 6, become 1-observers in the time dimensions,  $c_s t$  and  $-c_s t^*$ , of our universe and negative universe in Fig. 7, due to the transformations (or contractions) of  $\mathbb{E}^{03}$  and  $-\mathbb{E}^{03*}$  to  $c_s t$  and  $-c_s t^*$  in system (4).

Figure 7 obtained by performing the transformations of system (4) on Fig. 6, is valid with respect to 1-observers in the metric time dimensions,  $c_s t$  and  $-c_s t^*$ , as indicated. Recall from section 2 of [8] that the clockwise inclination (or rotation) of the curved relative proper intrinsic metric spacetimes,  $\varnothing \rho'$  and  $\varnothing c_s \varnothing t'$ , relative to their projective relativistic intrinsic metric spacetimes,  $\varnothing \rho$  and  $\varnothing c_s \varnothing t$ , by varying



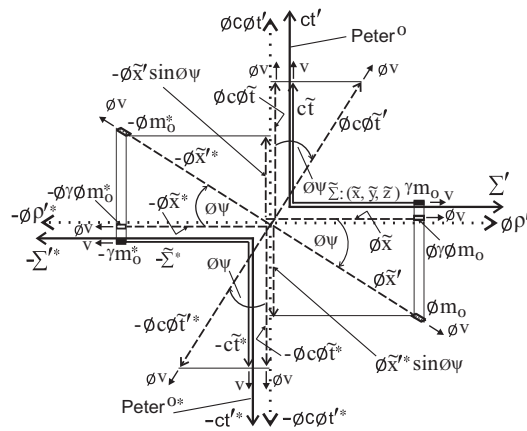


Figure 8: **b** The affine spacetime/intrinsic affine spacetime diagram of the special theory of relativity and intrinsic special theory of relativity with respect to symmetry-partner 1-observers in the metric time dimensions of our universe and the negative universe; the complementary diagram to Fig. 8a; (Fig. 8b of [5]).

of this article, have Fig. 9a and Fig. 9b of that article as their inverses, there are inverses to Fig. 5 and Fig. 7 of this article for the theory of relativity and intrinsic theory of relativity associated with the presence of symmetry-partner long-range metric force fields in our universe and negative universe. Figures 9a and 9b of [5], which are the inverse diagrams to Fig. 8a and Fig. 8b of this article, are reproduced as Fig. 9a and Fig. 9b of this article, for convenience of reading. The derivations of the inverses to Fig. 5 and Fig. 7 of this article are done below.

Now the extended curved relative proper intrinsic metric space  $\varnothing\rho'$  and the extended curved relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  possess varying positive relative proper intrinsic static flow speed  $\varnothing V'_m$  along their lengths, relative to all 3-observers in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  in Fig. 5. Consequently  $\varnothing\rho'$  is curved anticlockwise at varying positive relative intrinsic angles  $\varnothing\psi$  relative to the straight line  $\varnothing\rho$  along the horizontal and  $\varnothing c_s \varnothing t'$  is identically curved anticlockwise (into the second quadrant) at varying positive relative intrinsic angle  $\varnothing\psi$  relative to the straight line  $\varnothing c_s \varnothing t$  along the vertical in Fig. 5.

In obtaining the inverse of Fig. 5, one could, at first thought, consider the projective extended relativistic intrinsic metric space  $\varnothing\rho$  and extended projective straight line relativistic intrinsic metric time dimension  $\varnothing c_s \varnothing t$  to possess varying negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  along their lengths, relative to the curved relative proper intrinsic metric space  $\varnothing\rho'$  and curved relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  respectively. One could then further consider the relativistic intrinsic metric space  $\varnothing\rho$  to be curved into the first quadrant at varying negative intrinsic angle  $-\varnothing\psi$  relative to straight line relative proper intrinsic metric space  $\varnothing\rho'$  along the horizontal and the relativistic intrinsic metric time

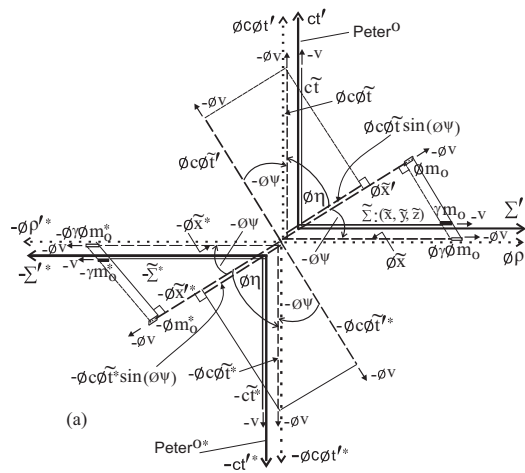


Figure 9: **a** The inverse diagram of Fig. 8a of this article with respect to symmetry-partner 1-observers in the metric time dimensions of our universe and the negative universe; (Fig. 9a of [5]).

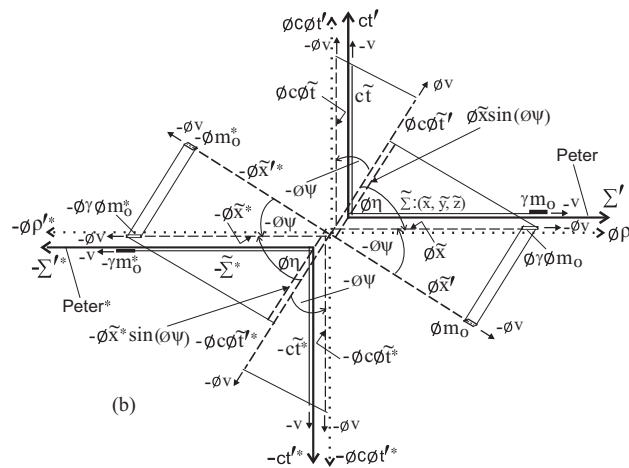


Figure 9: **b** The inverse diagram of Fig. 8b of this article with respect to symmetry-partner 3-observers in the metric Euclidean 3-spaces of our universe and the negative universe; the complementary diagram to Fig. 9a; (Fig. 8b of [5]).

dimension  $\varnothing c_s \varnothing t$  to be curved into the second quadrant at varying negative intrinsic angle  $-\varnothing \psi$  relative to straight line relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  along the vertical. The straight line  $\varnothing \rho'$  along the horizontal and the straight line  $\varnothing c_s \varnothing t'$  along the vertical so formed, will then be made manifested outwardly in three-dimensional relative proper Euclidean space  $\mathbb{I}E'^3$  as hyper-surface along the horizontal and straight line relative proper metric time dimension  $c_s t'$  along the

vertical.

The inverse of Fig. 5 derived as explained in the preceding paragraph is invalid for two major reasons. First, it brings back the flat four-dimensional relative proper metric spacetime  $(\mathbb{E}^3, c_s t')$  and its underlying flat two-dimensional relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s \varnothing t')$  at the second stage of evolutions of metric spacetimes and intrinsic metric spacetimes in a long range metric force field, which cannot be, since the flat relative proper metric spacetime  $(\mathbb{E}^3, c_s t')$  and its underlying flat relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s \varnothing t')$ , have evolved into flat relativistic metric spacetime  $(\mathbb{E}^3, c_s t)$  and its underlying flat relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s \varnothing t)$  permanently at the second stage.

Secondly, projective negative non-uniform relative proper intrinsic static flow speed  $-\varnothing V'_m$  along the length of the relativistic intrinsic metric spacetime dimensions,  $\varnothing\rho$  and  $\varnothing c_s \varnothing t$ , cannot give rise to curvature of  $\varnothing\rho$  and  $\varnothing c_s \varnothing t$ . Rather it is non-uniform negative relativistic intrinsic static flow speed (without prime label)  $-\varnothing V_m$  along the lengths of the relativistic intrinsic metric dimensions,  $\varnothing\rho$  and  $\varnothing c_s \varnothing t$ , that can give rise to their curvature relative to straight line  $\varnothing\rho'$  and  $\varnothing c_s \varnothing t'$ . In other words, contrary to the inverse of Fig. 5 described in the penultimate paragraph, the relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s \varnothing t)$  cannot be curved relative to flat relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s \varnothing t')$  by virtue of non-uniform negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  along the lengths of  $\varnothing\rho$  and  $\varnothing c_s \varnothing t$ .

The curvature of the relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s \varnothing t')$  relative to its projective flat relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s \varnothing t)$  underlying a flat relativistic metric spacetime  $(\mathbb{E}^3, c_s t)$  in Fig. 5, must be retained in the inverse of Fig. 5. However the straight line relativistic intrinsic metric space  $\varnothing\rho$  with varying positive relative proper intrinsic static flow speeds  $\varnothing V'_m$  along its length along the horizontal in Fig. 5, must be considered to possess varying negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  along its length along the horizontal and to be inclined clockwise by varying negative intrinsic angle  $-\varnothing\psi$  relative to the curved  $\varnothing\rho'$  in the inverse diagram, where the curved  $\varnothing\rho'$  also possesses non-uniform relative proper intrinsic static flow speed  $-\varnothing V'_m$  along its length.

The straight line relativistic intrinsic metric time dimension  $\varnothing c_s \varnothing t$  with varying positive relative proper intrinsic static flow  $\varnothing V'_m$  speed along its length along the vertical in Fig. 5, must likewise be considered to possess varying negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  along its length along the vertical and to be inclined clockwise by varying negative relative intrinsic angle  $-\varnothing\psi$  relative to the curved  $\varnothing c_s \varnothing t'$  in the inverse diagram, where the curved  $\varnothing c_s \varnothing t'$  also possesses non-uniform relative proper intrinsic static flow speed  $-\varnothing V'_m$  along its length.

The valid inverse of Fig. 5 that follows from the preceding paragraph is depicted in Fig. 10. It is to be observed that the straight line relativistic intrinsic metric space  $\varnothing\rho$  along the horizontal is rotated clockwise by negative relative intrinsic angle  $-\varnothing\psi_{m,P}$  relative to the tangent to the curved relative proper intrinsic metric

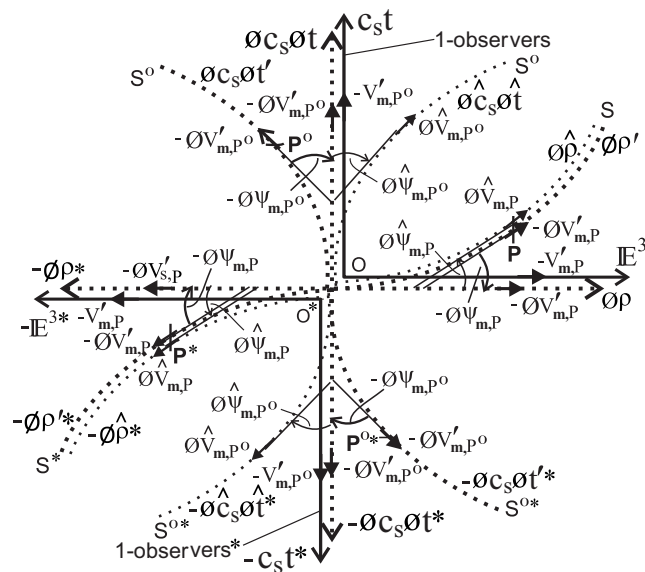


Figure 10: The inverse of the global metric spacetime/intrinsic metric spacetime diagram of Fig. 5; is valid with respect to 1-observers in the relativistic time dimensions  $c_s$  and  $-c_s t^*$  in the positive and negative universes.

space  $\varnothing\rho'$  at the point P along the curved  $\varnothing\rho'$  and the straight line relativistic intrinsic metric time dimension  $\varnothing c_s \varnothing t$  along the vertical is rotated clockwise by equal negative relative intrinsic angle  $-\varnothing\psi_{P^0}$  relative to the tangent to the curved relative proper intrinsic time dimension  $\varnothing c_s \varnothing t'$ , at the symmetry-partner point  $P^0$  along the curved  $\varnothing c_s \varnothing t'$  in Fig. 10. The negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  and negative relative intrinsic angle  $-\varnothing\psi$  vary along the curved  $\varnothing\rho'$  and curved  $\varnothing c_s \varnothing t'$  in Fig. 10.

Now the clockwise sense of inclination of the relativistic intrinsic metric time dimension  $\varnothing c_s \varnothing t$  along the vertical relative to the curved relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  in the second quadrant, by varying negative relative intrinsic angles  $-\varnothing\psi$  along the curved  $\varnothing c_s \varnothing t'$ , due to varying negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  along  $\varnothing c_s \varnothing t$ , is valid with respect to 1-observers in the relativistic time dimensions,  $c_s t$  and  $-c_s t^*$  of the positive and negative universes as indicated. The clockwise sense of inclination of the straight line relativistic intrinsic metric space  $\varnothing\rho$  along the horizontal relative to the curved relative proper intrinsic metric space  $\varnothing\rho'$  in the first quadrant, by varying negative intrinsic angles,  $-\varnothing\psi$  along the curved  $\varnothing\rho'$ , due to varying negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  along  $\varnothing\rho'$  in Fig. 10, is likewise valid with respect to 1-observers in the relativistic time dimensions,  $c_s t$  and  $-c_s t^*$ , of the positive and negative universes as indicated.

The Fig. 10 is valid with respect to 1-observers in the relativistic time dimensions,

$c_s t$  and  $-c_s t^*$ , because the clockwise rotations (or inclinations) of the curved relative proper intrinsic metric dimensions,  $\varnothing c_s \varnothing t'$  and  $\varnothing \rho'$ , relative to their projective straight line relativistic intrinsic metric dimensions,  $\varnothing \rho$  and  $\varnothing c_s \varnothing t$ , respectively, by varying positive intrinsic angle  $\varnothing \psi$  in Fig. 7, are equivalent to clockwise rotations (or inclinations) of the straight line relativistic intrinsic metric dimensions,  $\varnothing \rho$  and  $\varnothing c_s \varnothing t$ , relative to the curved relative proper intrinsic metric dimensions,  $\varnothing \rho'$  and  $\varnothing c_s \varnothing t'$ , respectively, by varying negative intrinsic angle  $-\varnothing \psi$  in Fig. 10. Consequently, Fig. 10, like Fig. 7, is valid with respect to 1-observers in the relativistic time dimensions  $c_s t$  and  $-c_s t^*$ .

The inverse of Fig. 7, which can be derived from that figure by following the derivation of Fig. 10 from Fig. 5 is depicted in Fig. 11.

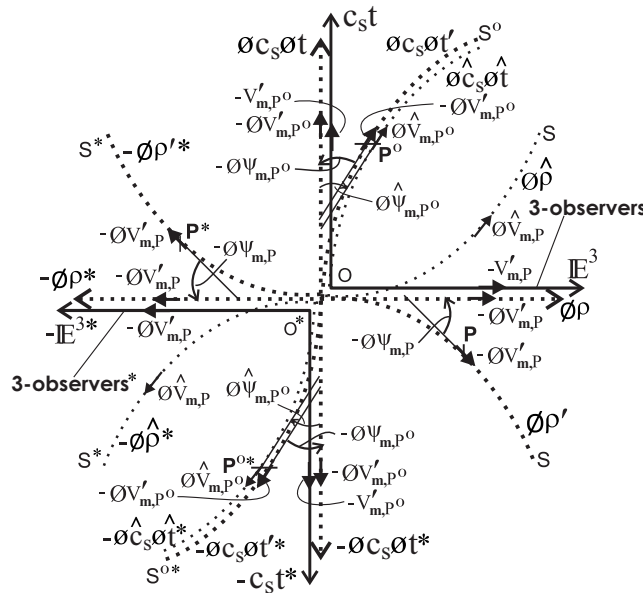


Figure 11: The inverse of the global metric spacetime/intrinsic metric spacetime diagram of Fig. 7; is valid with respect to 3-observers in the relativistic metric Euclidean 3-spaces  $\mathbb{E}^3$  and  $-\mathbb{E}^3$  of the positive and negative universes.

Again the curved relative proper intrinsic metric spacetime ( $\varnothing \rho'$ ,  $\varnothing c_s \varnothing t'$ ) relative to its projective flat relativistic intrinsic metric spacetime ( $\varnothing \rho$ ,  $\varnothing c_s \varnothing t$ ) underlying the flat relativistic metric spacetime ( $\mathbb{E}^3$ ,  $c_s t$ ) in Fig. 7, is retained in the inverse diagram of Fig. 11. However the straight line relativistic intrinsic metric space  $\varnothing \rho$  with varying positive relative proper intrinsic static flow speed  $\varnothing V'_m$  along its length in Fig. 7, is now considered to possess varying negative relative proper intrinsic static flow speed  $-\varnothing V'_m$  along its length along the horizontal relative to the curved relative proper intrinsic metric space  $\varnothing \rho'$  and to be inclined anti-clock-wise by varying negative intrinsic angle  $-\varnothing \psi$  relative to the curved  $\varnothing \rho'$  in Fig. 11. The straight

line relativistic intrinsic metric time dimension  $\mathcal{O}c_s\mathcal{O}t$  with varying positive relative proper intrinsic static flow speed  $\mathcal{O}V'_m$  along its length along the vertical in Fig. 7, is likewise considered to be inclined anti-clockwise by varying negative intrinsic angle  $-\mathcal{O}\psi$ , relative to the curved  $\mathcal{O}c_s\mathcal{O}t'$  in the inverse diagram of Fig. 11.

Figure 11 as the inverse global diagram of Fig. 7, is valid with respect to 3-observers in the relativistic Euclidean 3-spaces,  $\mathbb{E}^3$  and  $-\mathbb{E}^{*3}$ . This is so, because the anti-clockwise rotations (or inclinations) of the curved relative proper intrinsic metric dimensions,  $\mathcal{O}c_s\mathcal{O}t'$  and  $\mathcal{O}\rho'$ , relative to their projective straight line relativistic intrinsic metric dimensions,  $\mathcal{O}c_s\mathcal{O}t$  and  $\mathcal{O}\rho$ , respectively, by varying positive intrinsic angle  $\mathcal{O}\psi$  in Fig. 10, are equivalent to anti-clockwise rotations (or inclinations) of the straight line relativistic intrinsic metric dimensions,  $\mathcal{O}c_s\mathcal{O}t$  and  $\mathcal{O}\rho$ , relative to curved relative proper intrinsic metric dimensions,  $\mathcal{O}c_s\mathcal{O}t'$  and  $\mathcal{O}\rho'$ , respectively, by varying negative relative intrinsic angle  $-\mathcal{O}\psi$  in Fig. 11. Consequently, Fig. 11, like Fig. 5, is valid with respect to 3-observers in  $\mathbb{E}^3$  and  $-\mathbb{E}^{*3}$  as indicated.

Figures 8a and 8b and their inverses, Figs. 9a and 9b, of [5], reproduced as Figs. 8a and 8b and Figs. a and 9b of this article, involve inclined extended straight line pseudo-orthogonal primed (or proper) intrinsic affine spacetime coordinates,  $\mathcal{O}\tilde{x}'$  and  $\mathcal{O}c_s\mathcal{O}\tilde{t}'$ , relative to their projective extended pseudo-orthogonal unprimed (or relativistic) straight line intrinsic affine coordinates,  $\mathcal{O}\tilde{x}$  and  $\mathcal{O}c_s\mathcal{O}\tilde{t}$ . They involve constant relative positive intrinsic dynamical speed and positive dynamical speed,  $\mathcal{O}v$  and  $v$  (in Figs. 8a and 8b) and constant relative negative intrinsic dynamical speed and negative dynamical speed,  $-\mathcal{O}v$  and  $-v$  (in Figs. 9a and 9b), in the context of the intrinsic special theory of relativity and special theory of relativity ( $\mathcal{O}SR/SR$ ), as developed in [5].

On the other hand, Figs. 5 and 7 and their inverses, Figs. 10 and 11, of this article, involve extended inclined curved pseudo-orthogonal curvilinear relative proper intrinsic metric spacetime dimensions,  $\mathcal{O}\rho'$  and  $\mathcal{O}c_s\mathcal{O}t'$ , which are curved relative to their projective extended straight line pseudo-orthogonal relativistic intrinsic metric spacetime dimensions,  $\mathcal{O}\rho$  and  $\mathcal{O}c_s\mathcal{O}t$ . They involve non-uniform positive relative proper intrinsic static flow speed  $\mathcal{O}V'_m$  along the curved and straight line intrinsic metric spacetime dimensions (in Figs. 5 and 7), and non-uniform negative relative proper intrinsic static flow speed  $-\mathcal{O}V'_m$  along the curved and straight line intrinsic metric dimensions (in Figs. 10 and 11), in the context of the intrinsic theory of relativity and theory of relativity associated with the presence of symmetry-partner long-range relative proper metric force fields in metric spacetimes and symmetry-partner long-range relative proper intrinsic metric force fields in the underlying intrinsic metric spacetimes in our universe and the negative universe.

From the point of view of the absolute intrinsic metric theory on the curved 'two-dimensional' absolute intrinsic metric spacetime ( $\mathcal{O}\hat{\rho}, \mathcal{O}\hat{c}_s\mathcal{O}\hat{t}$ ), involving non-uniform absolute intrinsic static flow speed  $\mathcal{O}\hat{V}_m$  along the curved  $\mathcal{O}\hat{\rho}$  and  $\mathcal{O}\hat{c}_s\mathcal{O}\hat{t}$ , developed in the preceding three parts of this paper [1–3], at the first stage of evolutions of spacetime and intrinsic spacetime within a long-range metric force field, on the other

hand, there is only one diagram namely, the curved absolute intrinsic spacetime  $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$  relative to its projective flat absolute proper intrinsic metric spacetime  $(\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab})$  and the relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s\varnothing t')$ , which appears automatically alongside the projection of  $(\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab})$  along the horizontal, as well as the outward manifestation of  $(\varnothing\rho', \varnothing c_s\varnothing t')$  namely, the flat four-dimensional relative proper metric spacetime  $\mathbb{E}^3$  in Fig. 1.

Inverse diagrams and inverse coordinate transformations exist in relativity only and not in the context of the absolute intrinsic metric theory. Consequently the curved  $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$  in the first quadrant and the curved  $(-\varnothing\hat{\rho}^*, -\varnothing\hat{c}_s\varnothing\hat{t}^*)$  in the third quadrant in Figs. 2 and 4 are retained in the inverse diagrams of Fig. 7 and Fig. 11.

## 2.2 Deriving intrinsic local lorentz transformation and local lorentz transformation and their inverses within long-range metric force fields in terms of intrinsic static flow speed and static flow speed

Let us consider an elementary interval  $d\varnothing\rho'$  of the curved relative proper intrinsic metric space  $\varnothing\rho'$  about point P along the curved  $\varnothing\rho'$  in the first quadrant in Fig. 5. The interval  $d\varnothing\rho'$  possesses positive relative proper intrinsic static flow speed  $\varnothing V'_{m,P}$  and is inclined anticlockwise to the horizontal at intrinsic angle  $\varnothing\psi_P$ . It projects relativistic intrinsic metric space interval  $d\varnothing\rho$  along the horizontal that also possesses relative proper intrinsic static speed  $\varnothing V'_{m,P}$ , with respect to all 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  in Fig. 5.

The corresponding elementary interval  $\varnothing c_s d\varnothing t'$  of the curved relative proper intrinsic metric time dimension  $\varnothing c_s\varnothing t'$  about the symmetry-partner point  $P^0$  along the curved  $\varnothing c_s\varnothing t'$  in the second quadrant in Fig. 5, possesses intrinsic static flow speed  $\varnothing V'_{m,P^0}$  and is inclined anticlockwise at intrinsic angle  $\varnothing\psi_{P^0}$  to the vertical. It projects interval  $\varnothing cd\varnothing t$  of relativistic intrinsic metric time dimension along the vertical that also possesses proper intrinsic static flow speed  $\varnothing V'_{m,P^0}$ , with respect to all 3-observers in  $\mathbb{E}^3$  in that figure.

The elementary interval  $-d\varnothing\rho'^*$  of the curved relative proper intrinsic metric space  $-\varnothing\rho'^*$  about the symmetry-partner point  $P^*$  along  $-\varnothing\rho'^*$ , in the third quadrant in Fig. 5, possesses positive relative proper intrinsic static flow speed  $\varnothing V'_{m,P}$  and is inclined anticlockwise at intrinsic angle  $\varnothing\psi_P$  to the horizontal. It projects relativistic intrinsic metric space interval  $-d\varnothing\rho^*$  along the horizontal that also possesses relative proper intrinsic static flow speed  $\varnothing V'_{m,P}$ , with respect to all 3-observers\* in the relativistic Euclidean 3-space  $-\mathbb{E}^{3*}$  in that figure. The corresponding elementary interval  $-\varnothing c_s d\varnothing t'^*$  of the curved relative proper intrinsic metric time dimension  $-\varnothing c_s\varnothing t'^*$ , about the symmetry-partner point  $P^{0*}$  along  $-\varnothing c_s\varnothing t'^*$  in the fourth quadrant in Fig. 5, possesses positive relative intrinsic static flow speed  $\varnothing V'_{m,P^0}$  and it is inclined anticlockwise at intrinsic angle  $\varnothing\psi_{P^0}$  to the vertical. It projects interval  $-\varnothing c_s d\varnothing t^*$  of relativistic intrinsic metric time dimension along the vertical

that also possesses positive relative intrinsic static flow speed  $\emptyset V'_{m,P^0}$ , with respect to all 3-observers\* in  $-\mathbb{E}^{3*}$  in Fig. 5.

The elementary intervals of curved relative proper intrinsic metric spaces and curved relative proper intrinsic metric time dimensions,  $d\emptyset\rho'$ ,  $-d\emptyset\rho'^*$ ,  $\emptyset c_s d\emptyset t'$  and  $-\emptyset c_s d\emptyset t'^*$ , shall be considered to be indefinitely short so that they are short straight line segments within which relative proper intrinsic static flow speed has a constant value. Then since the points,  $P^0$  and  $P^{0*}$ , along the curved  $\emptyset c_s \emptyset t'$  and  $-\emptyset c_s \emptyset t'^*$  and points,  $P$  and  $P^*$ , along the curved  $\emptyset\rho'$  and  $-\emptyset\rho'^*$  are symmetry-partner points in Fig. 5, the intrinsic angle  $\emptyset\psi_{P^0}$  of inclinations of intervals,  $\emptyset c_s d\emptyset t'$  and  $-\emptyset c_s d\emptyset t'^*$ , to the vertical and the intrinsic angle  $\emptyset\psi_{m,P}$  of inclinations of intervals,  $d\emptyset\rho'$  and  $-d\emptyset\rho'^*$ , to the horizontal are equal, that is,  $\emptyset\psi_{P^0} = \emptyset\psi_{m,P}$  in Fig. 5.

By making use of the information in the preceding two paragraphs and drawing the inclined elementary intervals,  $d\emptyset\rho'$ ,  $\emptyset c_s d\emptyset t'$ ,  $-d\emptyset\rho'^*$  and  $-\emptyset c_s d\emptyset t'^*$ , relative to their projections,  $d\emptyset\rho$ ,  $\emptyset c_s d\emptyset t$ ,  $-d\emptyset\rho^*$  and  $-\emptyset c_s d\emptyset t^*$ , respectively, at the symmetry-partner points,  $P$  along the curved  $\emptyset\rho'$ ,  $P^0$  along the curved  $\emptyset c_s \emptyset t'$ ,  $P^*$  along the curved  $-\emptyset\rho'^*$  and  $P^{0*}$  along the curved  $-\emptyset c_s \emptyset t'^*$  in Fig. 5, we have Fig. 12. The local geometry of Fig. 12, derived from the global geometry of Fig. 5, is valid with respect to all 3-observers in the relativistic metric Euclidean 3-spaces,  $\mathbb{E}^3$  and  $-\mathbb{E}^{3*}$ , as is the case with Fig. 5.

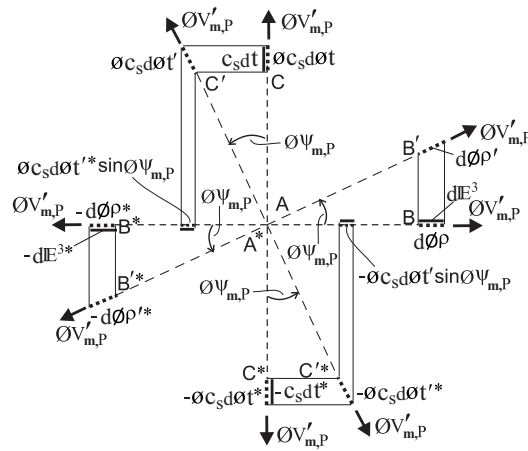


Figure 12: The local metric spacetime/intrinsic metric spacetime diagram drawn at symmetry-partner points in spacetimes/intrinsic spacetimes in the positive (or our) universe and the negative universe obtained from the global diagram of Fig. 5, for deriving partial transformation of elementary intrinsic metric spacetime coordinate intervals in terms of intrinsic static flow speed, with respect to 3-observers in the relativistic Euclidean 3-spaces in the positive and negative universes.

Figure 12 has been drawn at the symmetry-partner points  $P$ ,  $P^0$ ,  $P^*$  and  $P^{0*}$  along the curved relative proper intrinsic metric dimensions  $\emptyset\rho'$ ,  $\emptyset c_s \emptyset t'$ ,  $-\emptyset\rho'^*$

and  $-\varnothing c_s \varnothing t'^*$  respectively in Fig. 5, as mentioned above. Hence the appearance of intrinsic angle of inclination  $\varnothing \psi_{m,P}$  (where  $\varnothing \psi_{m,P} = \varnothing \psi_{P0}$ ) in Fig. 7. The inclined elementary intervals of proper intrinsic metric spacetime  $d\varnothing \rho'$ ,  $\varnothing c_s d\varnothing t'$ ,  $-d\varnothing \rho'^*$  and  $-\varnothing c_s d\varnothing t'^*$ , possess equal positive relative proper intrinsic static flow speed  $\varnothing V'_{m,P}$  and invariantly project same into their projective relativistic components  $d\varnothing \rho$ ,  $\varnothing c_s d\varnothing t$ ,  $-d\varnothing \rho^*$  and  $-\varnothing c_s d\varnothing t^*$ . It is to be noted that the line segments  $AB'$ ,  $AC'$ ,  $A^*B'^*$ , and  $A^*C'^*$  are mere connecting lines and not intrinsic metric coordinates. The line segments  $AB$ ,  $AC$ ,  $A^*B^*$  and  $A^*C^*$  are likewise mere connecting lines.

The component  $d\varnothing \rho$  of interval of relativistic intrinsic metric space projected along the horizontal is made manifested outwardly in an elementary volume  $d\mathbb{I}\mathbb{E}^3$  of the relativistic Euclidean 3-space  $\mathbb{I}\mathbb{E}^3$  in Fig. 12. Likewise the component  $\varnothing c_s d\varnothing t$  of the relativistic intrinsic metric time dimension projected along the vertical is made manifested outwardly in an elementary interval  $c_s dt$  of the relativistic metric time dimension  $c_s t$  along the vertical.

In addition, the inclined negative elementary relative proper intrinsic metric time dimension  $-\varnothing c_s d\varnothing t'^*$  from the negative universe in the fourth quadrant, projects component  $-\varnothing c_s d\varnothing t' \sin \varnothing \psi_{m,P}$  along the horizontal in the first quadrant, which is made manifested outwardly in  $-c_s dt' \sin \psi_{m,P}$  along the horizontal in Fig. 12. The dummy star label has been removed from the projective component  $-\varnothing c_s d\varnothing t'^* \sin \varnothing \psi_{m,P}$  of the inclined  $-\varnothing c_s d\varnothing t'^*$ , because this projective component is now an intrinsic dimension in the positive universe. The star label on the spacetime and intrinsic spacetime and parameters and intrinsic parameters of the negative universe have been consistently used to differentiated from those of our universe in all previous articles, starting from since [5].

Derivation of partial intrinsic local Lorentz transformation from Fig. 12 follows the same procedure used to derive partial intrinsic Lorentz transformation from Fig. 8a of [5] in the context of intrinsic special theory of relativity ( $\varnothing$ SR). The procedure is applied hereunder.

Now  $d\varnothing \rho$  being the projective component along the horizontal of the inclined  $d\varnothing \rho'$ , then  $d\varnothing \rho = d\varnothing \rho' \cos \varnothing \psi_{m,P}$ . Hence we must express the rotated  $d\varnothing \rho'$  in terms of its projection  $d\varnothing \rho$  along the horizontal with respect to all 3-observers in  $\mathbb{I}\mathbb{E}^3$  and write

$$d\varnothing \rho' = d\varnothing \rho \sec \varnothing \psi_{m,P} .$$

This is all the intrinsic metric spacetime interval transformation that should have been possible along the horizontal in the first quadrant, with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{I}\mathbb{E}^3$  in Fig. 12, except that the inclined interval of negative relative proper intrinsic metric time dimension  $-\varnothing c_s d\varnothing t'^*$  in the fourth quadrant also projects interval  $-\varnothing c_s d\varnothing t' \sin \varnothing \psi_{m,P}$  (with the star label removed for the reason given above) along the horizontal, which must be added to the right-hand

side of the last displayed equation to have

$$d\varnothing\rho' = d\varnothing\rho \sec \varnothing\psi_{m,P} - \varnothing c_s d\varnothing t' \sin \varnothing\psi_{m,P} .$$

But the inclined interval  $\varnothing c_s d\varnothing t'$  is related to its projection  $\varnothing c_s d\varnothing t$  along the vertical in the same Fig. 12 as,  $\varnothing c_s d\varnothing t = \varnothing c_s d\varnothing t' \cos \varnothing\psi_{m,P}$ , hence  $\varnothing c_s d\varnothing t' = \varnothing c_s d\varnothing t \sec \varnothing\psi_{m,P}$ . Using this in the last displayed equation gives

$$d\varnothing\rho' = d\varnothing\rho \sec \varnothing\psi_{m,P} - \varnothing c_s d\varnothing t \tan \varnothing\psi_{m,P} ; \tag{5}$$

(with respect to 3 – observers in  $\mathbb{I}\mathbb{E}^3$ ).

Equation (5) is the partial transformation of elementary intrinsic metric spacetime coordinate intervals in terms of the intrinsic angle  $\varnothing\psi_{m,P}$ , which can be derived along the horizontal in the first quadrant, with respect to 3-observers in  $\mathbb{I}\mathbb{E}^3$  in Fig. 12.

The complementary diagram to Fig. 12 that can be drawn at the symmetry-partner points, P, P<sup>0</sup>, P\* and P<sup>0\*</sup>, along the curved relative proper intrinsic metric spacetimes dimensions,  $\varnothing\rho'$ ,  $\varnothing c_s \varnothing t'$ ,  $-\varnothing\rho'^*$  and  $-\varnothing c_s \varnothing t'^*$ , in Fig. 7; Fig. 7 being the complementary diagram to Fig. 5, is depicted in Fig. 13. The local geometry of Fig. 13 derived from the global geometry of Fig. 7 is valid with respect to 1-observers in the relativistic metric time dimensions,  $c_s t$  and  $-c_s t^*$ , as is the case with Fig. 7.

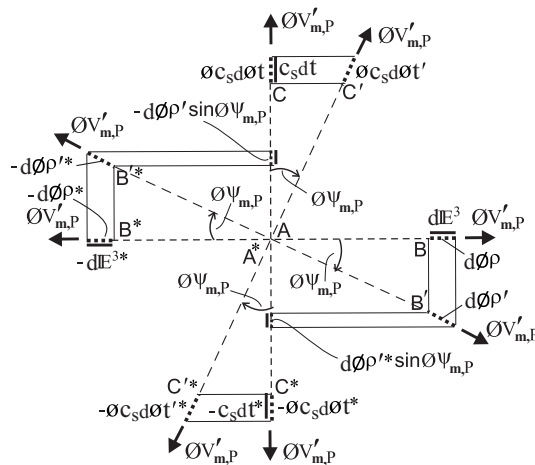


Figure 13: The complementary diagram to Fig. 12 drawn at symmetry-partner points in spacetimes/intrinsic spacetimes in the positive and negative universes, from the global diagram of Fig. 7, for deriving partial transformation of elementary intrinsic metric spacetime coordinate intervals in terms of intrinsic static flow speed with respect to 1-observers in the relativistic metric time dimensions in the positive and negative universes.

Now  $\varnothing c_s d\varnothing t$  being the projective component along the vertical of the inclined  $\varnothing c_s d\varnothing t'$  in the first quadrant in Fig. 13, then  $\varnothing c_s d\varnothing t = \varnothing c_s d\varnothing t' \cos \varnothing \psi_{m,P}$ . We must express the rotated  $\varnothing c_s d\varnothing t'$  in terms of its projection  $\varnothing c_s d\varnothing t$  along the vertical with respect to 1-observers in  $c_s t$  and write,

$$\varnothing c_s d\varnothing t' = \varnothing c_s d\varnothing t \sec \varnothing \psi_{m,P} .$$

This is all the transformation of intrinsic metric spacetime intervals that should have been possible along the vertical in the first quadrant, with respect to 1-observers in the relativistic time dimension  $c_s t$  in Fig. 13, except that the inclined negative relative proper intrinsic metric space interval  $-d\varnothing \rho'^*$  in the second quadrant also projects component  $-d\varnothing \rho' \sin \varnothing \psi_{m,P}$  (its star label has been removed because it is an intrinsic coordinate of our universe) along the vertical, which must be added to the right-hand side of the last displayed equation to have

$$\varnothing c_s d\varnothing t' = \varnothing c_s d\varnothing t \sec \varnothing \psi_{m,P} - d\varnothing \rho' \sin \varnothing \psi_{m,P} .$$

But the inclined interval  $d\varnothing \rho'$  is related to its projection  $d\varnothing \rho$  as,  $d\varnothing \rho = d\varnothing \rho' \cos \varnothing \psi_{m,P}$ , hence,  $d\varnothing \rho' = d\varnothing \rho \times \sec \varnothing \psi_{m,P}$ , along the horizontal in the same Fig. 13. Using this in the last displayed equation gives

$$\begin{aligned} \varnothing c_s d\varnothing t' &= \varnothing c_s d\varnothing t \sec \varnothing \psi_{m,P} - d\varnothing \rho \tan \varnothing \psi_{m,P} ; \\ &\text{(with respect to 1 - observers in } c_s t \text{)} . \end{aligned} \tag{6}$$

Equation (6) is the partial transformation of elementary intrinsic metric spacetime coordinate intervals in terms of the intrinsic angle  $\varnothing \psi_{m,P}$ , which can be derived along the vertical in the first quadrant, with respect to 1-observers in  $c_s t$  in Fig. 13.

Collecting Eqs. (5) and (6) gives the full transformation of elementary intrinsic metric spacetime coordinate intervals from the local geometry of Fig. 12 and its complementary geometry of Fig. 13 as

$$\begin{aligned} \varnothing c_s d\varnothing t' &= \varnothing c_s d\varnothing t \sec \varnothing \psi_{m,P} - d\varnothing \rho \tan \varnothing \psi_{m,P} \\ &\text{(w.r.t 1 - observers in } c_s t \text{)} ; \\ d\varnothing \rho' &= d\varnothing \rho \sec \varnothing \psi_{m,P} - \varnothing c_s d\varnothing t \tan \varnothing \psi_{m,P} ; \\ &\text{(w.r.t 3 - observers in } \mathbb{E}^3 \text{)} . \end{aligned} \tag{7}$$

There is an inverse of system (7), which must be derived from the inverses to Figs. 12 and 13. Now in obtaining the inverse of the local diagrams of Figs. 12 and 13, the inclined position of the primed intrinsic local metric frame ( $d\varnothing \rho', \varnothing c_s d\varnothing t'$ ) and the non-inclined position (or flatness) of the unprimed (or relativistic) intrinsic local metric frame ( $d\varnothing \rho, \varnothing c_s d\varnothing t$ ) in Figs. 12 and 13 must be retained. However the flat (or non-inclined) relativistic (or unprimed) local intrinsic metric frame ( $d\varnothing \rho, \varnothing c_s d\varnothing t$ )

must now be considered to possess negative relative proper intrinsic static flow speed  $-\varnothing V'_{m,P}$  and to be inclined at negative intrinsic angle  $-\varnothing\psi_{m,P}$  relative to the inclined relative proper (or primed) local intrinsic metric frame  $(d\varnothing\rho', \varnothing c_s d\varnothing t')$ . The negative relative proper intrinsic static flow speed of the unprimed local intrinsic metric frame  $(d\varnothing\rho, \varnothing c_s d\varnothing t)$  is still invariantly projected into the inclined primed local intrinsic metric frame  $(d\varnothing\rho', \varnothing c_s d\varnothing t')$ . The resulting inverse diagram to Fig. 12 is depicted in Fig. 14.

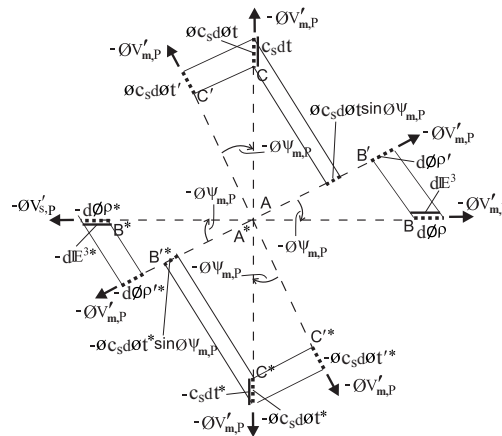


Figure 14: The inverse of the local diagram of Fig. 12 drawn from the global diagram of Fig. 10, for deriving partial inverse transformations of elementary intrinsic metric coordinate intervals in terms of relative proper intrinsic static flow speed with respect to 1-observers in the relativistic time dimensions  $c_s t$  and  $-c_s t^*$  in the positive and negative universes.

The clockwise rotation of the relative proper (or primed) intrinsic local metric frame  $(d\varnothing\rho', \varnothing c_s d\varnothing t')$  relative to its projective relativistic (or unprimed) intrinsic local metric frame  $(d\varnothing\rho, \varnothing c_s d\varnothing t)$ , by a positive intrinsic angle  $\varnothing\psi_{m,P}$  in Fig. 13, is equivalent to the clockwise rotation of the relativistic (or unprimed) intrinsic local metric frame  $(d\varnothing\rho, \varnothing c_s d\varnothing t)$  relative to the relative proper (or primed) intrinsic local metric frame  $(d\varnothing\rho', \varnothing c_s d\varnothing t')$ , by negative intrinsic angle  $-\varnothing\psi_{m,P}$  in Fig. 14. Consequently Fig. 13 and Fig. 14 are both valid with respect to 1-observers in the relativistic metric time dimensions,  $c_s t$  and  $-c_s t^*$ , in our universe and the negative universe.

Figure 14 as the inverse of Fig. 12 in a long-range metric force field and an intrinsic long-range metric force field, corresponds to Fig. 9a as the inverse of Fig. 8a [5], reproduced as Figs. 8a and 9a of the article in the context of SR/ $\varnothing$ SR. Consequently the procedure applied in deriving partial inverse intrinsic affine coordinate transformation from Fig. 9(a) of this article in [5] shall be applied in deriving partial inverse transformation of elementary intrinsic metric spacetime coordinate intervals from

Fig. 14 here.

Now the interval of inclined relative proper intrinsic metric space  $d\varnothing\rho'$  in the first quadrant in Fig. 14 is the projection of the non-inclined interval of relativistic intrinsic metric space  $d\varnothing\rho$  in the first quadrant in that figure. That is,  $d\varnothing\rho' = d\varnothing\rho \cos(-\varnothing\psi_{m,P}) = d\varnothing\rho \cos(\varnothing\psi_{m,P})$ . The non-inclined  $d\varnothing\rho$  must be expressed in terms of its projective  $\varnothing\rho\rho'$  with respect to 1-observers in  $c_s t$  in the inverse diagram of Fig. 14 as

$$d\varnothing\rho = d\varnothing\rho' \sec \varnothing\psi_{m,P} .$$

This is all the elementary intrinsic metric spacetime coordinate interval transformation that should have been possible along the inclined path  $AB'$ , with respect to 1-observers in the relativistic time dimension  $c_s t$  in the first quadrant in Fig. 14, except that the interval  $\varnothing c_s d\varnothing t$  of relativistic intrinsic metric time dimension along the vertical projects a component,  $\varnothing c_s d\varnothing t \cos \varnothing\eta$  along the inclined path  $AB'$ , where  $\varnothing\eta + \varnothing\psi_{m,P} = \varnothing\pi/2$ , or  $\varnothing\eta = \varnothing\pi/2 - \varnothing\psi_{m,P}$ . Hence,  $\varnothing c_s d\varnothing t \cos \varnothing\eta = \varnothing c_s d\varnothing t \sin \varnothing\psi_{m,P}$ . This component must be added to the right-hand side of the last displayed equation to have

$$d\varnothing\rho = d\varnothing\rho' \sec \varnothing\psi_{m,P} + \varnothing c_s d\varnothing t \sin \varnothing\psi_{m,P} ;$$

(w.r.t. 1 – observers in  $c_s t$ ). However,  $\varnothing c_s d\varnothing t' = \varnothing c_s d\varnothing t \cos(-\varnothing\psi_{m,P}) = \varnothing c_s d\varnothing t \cos \varnothing\psi_{m,P}$ , hence,  $\varnothing c_s d\varnothing t = \varnothing c_s d\varnothing t' \sec \varnothing\psi_{m,P}$ , along the vertical in the first quadrant in Fig. 14. Using this in the last displayed equation gives

$$d\varnothing\rho = d\varnothing\rho' \sec \varnothing\psi_{m,P} + \varnothing c_s d\varnothing t' \tan \varnothing\psi_{m,P} ;$$

(w.r.t. 1 – observers in  $c_s t$ ) . (8)

This is the partial inverse transformation of elementary intrinsic metric spacetime coordinate intervals that can be derived along the inclined path  $AB'$  with respect to 1-observers in the relativistic time dimension  $c_s t$  in the first quadrant (or in our universe) in Fig. 14.

Finally the inverse of the local geometry of Fig. 13, which can be derived from the global geometry of Fig. 11 is depicted in Fig. 15. Figure 10 is valid with respect to 3-observers in the relativistic Euclidean 3-spaces,  $\mathbb{E}^3$  and  $-\mathbb{E}^{3*}$ . This is so because anti-clockwise rotation of the relative proper (or primed) intrinsic local metric frame  $(d\varnothing\rho', \varnothing c_s d\varnothing t')$  relative to its projective relativistic (or unprimed) intrinsic local metric frame  $(d\varnothing\rho, \varnothing c_s d\varnothing t)$ , by a positive intrinsic angle  $\varnothing\psi_{m,P}$  in Fig. 12, is equivalent to the anti-clockwise rotation of the relativistic (or unprimed) intrinsic local metric frame  $(d\varnothing\rho, \varnothing c_s d\varnothing t)$  relative to the inclined relative proper intrinsic local metric frame  $(d\varnothing\rho', \varnothing c_s d\varnothing t')$  by negative intrinsic angle  $-\varnothing\psi_{m,P}$  in Fig. 15. Consequently Fig. 15, like Fig. 12, is valid with respect to all 3-observers in the relativistic Euclidean 3-spaces,  $\mathbb{E}^3$  and  $-\mathbb{E}^{3*}$ , in our universe and the negative universe.

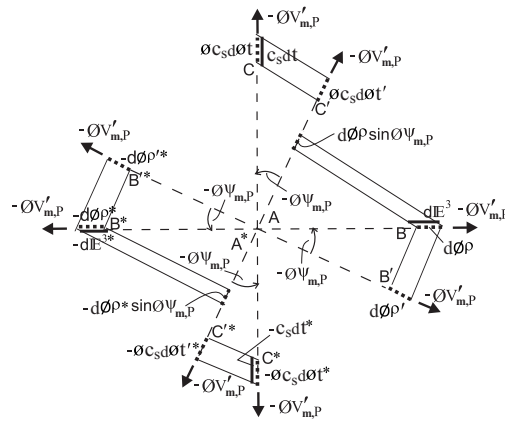


Figure 15: The inverse of the local diagram of Fig. 13, drawn from the global diagram of Fig. 11, for deriving partial inverse transformations of elementary intrinsic metric spacetime coordinate intervals in terms of relative proper intrinsic static flow speed with respect to 3-observers in the relativistic metric Euclidean 3-spaces in the positive and negative universes.

Again Fig. 15 is the inverse of Fig. 13 like Fig. 9(b) is the inverse of Fig. 8(b) in [5], reproduced as Figs.9(b) and 8(b) of this article. Consequently the procedure applied in deriving partial inverse intrinsic affine coordinate transformation from Fig. 9(b) in [5] shall be applied in deriving partial inverse intrinsic metric coordinate interval transformation from Fig. 15 here.

Now the interval  $\varnothing c_s d\varnothing t'$  of the inclined relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  along the inclined path  $AC'$  in the first quadrant in Fig. 15, is the projection of the non-inclined interval  $\varnothing c_s d\varnothing t$  of the relativistic intrinsic metric time dimension  $\varnothing c_s d\varnothing t$  along the horizontal in the first quadrant in that figure. That is,  $\varnothing c_s d\varnothing t' = \varnothing c_s d\varnothing t \cos(-\varnothing \psi_{m,P}) = \varnothing c_s d\varnothing t \cos(\varnothing \psi_{m,P})$ . The interval  $\varnothing c_s d\varnothing t$  must be expressed in terms of its projection  $\varnothing c_s d\varnothing t'$  along the path  $AC'$  with respect of 3-observers in  $\mathbb{E}^3$  in Fig. 15 as

$$\varnothing c_s d\varnothing t = \varnothing c_s d\varnothing t' \sec \varnothing \psi_{m,P} .$$

This is all the elementary intrinsic metric spacetime coordinate interval transformation that should have been possible along the inclined path  $AC'$ , with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  in the first quadrant in Fig.15, except that the interval  $d\varnothing \rho$  of relativistic intrinsic metric space along the horizontal projects a component,  $d\varnothing \rho \cos \varnothing \eta$  along the inclined path  $AC'$ , where  $\varnothing \eta + \varnothing \psi_{m,P} = \varnothing \pi/2$ , or  $\varnothing \eta = \varnothing \pi/2 - \varnothing \psi_{m,P}$ . Hence,  $d\varnothing \rho \cos \varnothing \eta = d\varnothing \rho \sin \varnothing \psi_{m,P}$ . This component must be added to the right-hand side of the last displayed equation

to have

$$\begin{aligned} \varnothing c_s d\varnothing t &= \varnothing c_s d\varnothing t' \sec \varnothing \psi_{m,P} + d\varnothing \rho \sin \varnothing \psi_{m,P} ; \\ &(\text{w.r.t. } 3 - \text{observers in } \mathbb{E}^3) \end{aligned}$$

However,  $d\varnothing \rho' = d\varnothing \rho \cos(-\varnothing \psi_{m,P})$ , hence,  $d\varnothing \rho = d\varnothing \rho' \sec \varnothing \psi_{m,P}$ , along the horizontal in the first quadrant in Fig. 15. Using this in the last displayed equation gives

$$\begin{aligned} \varnothing c_s d\varnothing t &= \varnothing c_s d\varnothing t' \sec \varnothing \psi_{m,P} + d\varnothing \rho' \tan \varnothing \psi_{m,P} ; \\ &(\text{w.r.t. } 3 - \text{observers in } \mathbb{E}^3) . \end{aligned} \tag{9}$$

This is the partial inverse transformation of elementary intrinsic metric spacetime coordinate intervals that can be derived along the inclined path  $AC'$  with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  in the first quadrant (or in our universe) in Fig. 15. Collecting Eqs. (8) and (9) gives

$$\begin{aligned} \varnothing c_s d\varnothing t &= \varnothing c_s d\varnothing t' \sec \varnothing \psi_{m,P} + d\varnothing \rho' \tan \varnothing \psi_{m,P} ; \\ &(\text{w.r.t } 3 - \text{observers in } \mathbb{E}^3) ; \\ d\varnothing \rho &= d\varnothing \rho' \sec \varnothing \psi_{m,P} + \varnothing c_s d\varnothing t' \tan \varnothing \psi_{m,P} ; \\ &(\text{w.r.t } 1 - \text{observers in } c_s t) . \end{aligned} \tag{10}$$

System (10) derived from the local diagrams of Figs. 14 and 15 is the inverse of system (7) derived from the local diagrams of Figs. 12 and 13.

Let us consider an intrinsic event that involves interval  $\varnothing c_s d\varnothing t'$  of relative proper intrinsic metric time dimension but zero interval of relative proper intrinsic metric space ( $d\varnothing \rho' = 0$ ). This reduces system (10) as

$$\begin{aligned} \varnothing c_s d\varnothing t &= \varnothing c_s d\varnothing t' \sec \varnothing \psi_{m,P} ; \\ d\varnothing \rho &= \varnothing c_s d\varnothing t' \tan \varnothing \psi_{m,P} . \end{aligned} \tag{11}$$

Dividing the second into the first equation of system (11) gives

$$\frac{d\varnothing \rho}{\varnothing c_s d\varnothing t} = \sin \varnothing \psi_{m,P} . \tag{12}$$

But  $d\varnothing \rho/d\varnothing t = \varnothing V'_{m,P}$ , is the positive relative proper intrinsic static flow speed of the primed intrinsic local metric frame ( $d\varnothing \rho', \varnothing c_s d\varnothing t'$ ) and its projective unprimed intrinsic local metric frame ( $d\varnothing \rho, \varnothing c_s d\varnothing t$ ), knowing that intrinsic dynamical speed is absent, since no particle is in motion.

Let us also recall the definition of  $\sin \varnothing \hat{\psi}_{m,P}$  in Eq. (80) of the preceding third part of this paper [3] and the discussion following it. The corresponding definition of  $\sin \varnothing \psi_{m,P}$  in the present context is,  $\sin \varnothing \psi_{m,P} = d\varnothing \rho/\varnothing c_s d\varnothing t = d\varnothing \rho'/\varnothing c_s d\varnothing t' =$

$\varnothing V'_{m,P}/c_m$  . As discussed in converting Eq. (80) to Eqs. (81) and (82) of the preceding article, the ratio,  $\sin \varnothing\psi_{m,P} = \varnothing V'_{m,P}/\varnothing c_m$  (where  $\varnothing c_m$ , with magnitude  $3 \times 10^8 \text{ m s}^{-1}$ , is the maximum over all relative proper intrinsic static-flow speeds  $\varnothing V'_m$  that can be established in intrinsic metric spacetime), is the appropriate ratio. The ratio,  $\sin \varnothing\psi_{m,P} = \varnothing V'_{m,P}/\varnothing c_s$ , where  $\varnothing c_s$  is the maximum intrinsic static geodesic flow-speed that appears in the time dimensions,  $\varnothing c_s \varnothing t$  and  $\varnothing c_s \varnothing t'$  (introduced in sub-section... of [7]), is inappropriate. Indeed  $\varnothing c_s$  is equivalent to zero magnitude of  $\varnothing V'_m$ . The speed  $V'_m$  and  $c_m$  shall referred to sa gravitational flow-speed and re-denoted upon particularizing the results of this paper to the gravitational field elsewhere. Then the difference between  $\varnothing c_s$  and  $\varnothing c_m$  shall become clarified. Hence,

$$\sin \varnothing\psi_{m,P} = \varnothing V'_{m,P}/\varnothing c_m \equiv \varnothing\beta_{m,P}(\varnothing V'_{m,P}) \quad (13a)$$

$$\sec \varnothing\psi_{m,P} = \left(1 - \frac{\varnothing V'^2_{m,P}}{\varnothing c^2_m}\right)^{-1/2} \equiv \varnothing\gamma_{m,P}(\varnothing V'_{m,P}) . \quad (13b)$$

Using Eqs. (13a) and (13b) in systems (7) and (10) gives the following respectively

$$d\varnothing t' = \varnothing\gamma_{m,P}(\varnothing V'_{m,P})(d\varnothing t - \frac{\varnothing V'_{m,P}}{\varnothing c^2_m}d\varnothing\rho);$$

(w.r.t. 1 – observers in  $c_s t$ ) ;

(14)

$$d\varnothing\rho' = \varnothing\gamma_{m,P}(\varnothing V'_{m,P})(d\varnothing\rho - \varnothing V'_{m,P}d\varnothing t);$$

(w.r.t. 3 – observers in  $\mathbb{E}^3$ )

and

$$d\varnothing t = \varnothing\gamma_{m,P}(\varnothing V'_{m,P})(d\varnothing t' + \frac{\varnothing V'_{m,P}}{\varnothing c^2_m}d\varnothing\rho');$$

(w.r.t. 3 – observers in  $\mathbb{E}^3$ ) ;

(15)

$$d\varnothing\rho = \varnothing\gamma(\varnothing V'_{m,P})(d\varnothing\rho' + \varnothing V'_{m,P}d\varnothing t');$$

(w.r.t. 1 – observers in  $c_s t$ ) .

Systems (14) and (15) in terms of intrinsic static flow speed  $\varnothing V'_{m,P}$ , take on the forms of intrinsic Lorentz transformation ( $\varnothing\text{LT}$ ) and its inverse respectively in terms of relative intrinsic dynamical speed  $\varnothing v$ , in the context of intrinsic special theory of relativity ( $\varnothing\text{SR}$ ), presented as systems (20) and (21) of [5]. Hence systems (14) and (15) shall be referred to as intrinsic local Lorentz transformation ( $\varnothing\text{LLT}$ ) and its inverse (in terms of intrinsic static flow speed), in the context of the intrinsic theory of relativity associated with the presence of intrinsic metric force field in intrinsic metric spacetime

Either system (10) or its inverse (7), or the explicit form in terms of relative proper intrinsic static flow speed (14) or (15), leads to intrinsic local Lorentz invariance

$$\varnothing c_s^2 d\varnothing t^2 - d\varnothing\rho^2 = \varnothing c_s^2 d\varnothing t'^2 - d\varnothing\rho'^2 . \quad (16)$$

The intrinsic local Lorentz transformation of elementary relative proper intrinsic metric spacetime intervals,  $d\varnothing\rho'$  and  $\varnothing c_s d\varnothing t'$ , into elementary relativistic intrinsic metric spacetime intervals,  $d\varnothing\rho$  and  $\varnothing c_s d\varnothing t$ , of system (7) or (14) and its inverse system (10) or (15), written at symmetry-partner points P and P<sup>0</sup> along the curved relative proper intrinsic metric space  $\varnothing\rho'$  and curved relative proper intrinsic metric time dimension  $\varnothing c_s \varnothing t'$  in Figs. 5 and 7 and their inverses Figs. 10 and 11, can equally be written at another symmetry-partner points Q and Q<sup>0</sup> along those curved relative proper intrinsic metric spaces and curved relative proper intrinsic metric time dimensions, in terms of intrinsic angle  $\varnothing\psi_{m,Q}$  and relative proper intrinsic static flow speed  $\varnothing V'_{m,Q}$  of the new symmetry-partner points, and this can be done at every symmetry-partner points along these curved intrinsic metric spacetime dimensions.

It follows from the preceding paragraph that the intrinsic local Lorentz invariance (16) obtains between every point of the global curved two-dimensional relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s \varnothing t')$  and the corresponding point of the projective relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s \varnothing t)$  in Fig. 5 through Fig. 11. This guarantees that the projective two-dimensional relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s \varnothing t)$  is everywhere flat within every long range metric force field.

Having derived the local diagrams of Figs. 12 and 13 from the global diagrams of Figs. 5 and 7 respectively and the inverse local diagrams of Figs. 14 and 15 from the inverse global diagrams of Figs. 10 and 11 respectively, let us now demonstrate how the global diagrams arise from the respective local diagrams. Now when the inclinations of the primed (or proper) intrinsic local metric frame  $(d\varnothing\rho', \varnothing c_s d\varnothing t')$  relative to its projective flat (or non-inclined) unprimed (or relativistic) intrinsic local metric frame  $(d\varnothing\rho, \varnothing c_s \varnothing t)$  by positive intrinsic angle  $\varnothing\psi_{m,P}$  in Figs. 12 and 13, are drawn at consecutive points away from point O (where  $\varnothing\psi \approx 0$  and  $\varnothing V'_m \approx 0$ ), then one obtains the extended curved relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s \varnothing t')$  relative to its projective extended flat relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s \varnothing t)$  in the positive universe, and the symmetrical extended curved  $(-\varnothing\rho'^*, -\varnothing c_s \varnothing t'^*)$  relative to its projective extended flat  $(-\varnothing\rho^*, -\varnothing c_s \varnothing t^*)$  in the negative universe in Figs. 5 and 7.

Now let us return to the elementary intrinsic metric spacetime interval transformation (14) and its inverse (15) and collect the partial intrinsic transformations that are valid with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  in those systems to have

$$\begin{aligned} d\varnothing\rho' &= \varnothing\gamma_{m,P}(\varnothing V'_{m,P})(d\varnothing\rho - \varnothing V'_{m,P}d\varnothing t) ; \\ d\varnothing t &= \varnothing\gamma_{m,P}(\varnothing V'_{m,P})(d\varnothing t' + \frac{\varnothing V'_{m,P}}{\varnothing c_m^2}d\varnothing\rho') ; \end{aligned} \tag{17}$$

(w.r.t 3-observers in  $\mathbb{E}^3$  .

Now from the point of view of what can be observed and measured as intrinsic

space interval with intrinsic laboratory rod and as intrinsic time interval with intrinsic laboratory clock by ‘intrinsic 1-observers’ in the intrinsic space  $\varnothing\rho$ , the terms  $-\gamma_{m,P}(\varnothing V'_{m,P})\varnothing V'_{m,P}d\varnothing t$  and  $\gamma_{m,P}(\varnothing V'_{m,P})(\varnothing V'_{m,P}\varnothing c_m^2)d\varnothing\rho'$  must be set to zero in system (17), thereby reducing that system as follows from the point of view of what can be measured with intrinsic laboratory rod and clock by hypothetical intrinsic 1-observers in  $\varnothing\rho$

$$d\varnothing\rho = \varnothing\gamma_{m,P}(\varnothing V'_{m,P})^{-1}d\varnothing\rho' = d\varnothing\rho'(1 - \frac{\varnothing V_{m,P}''2}{\varnothing c_m^2})^{1/2} \quad (18)$$

and

$$d\varnothing t = \varnothing\gamma_{m,P}(\varnothing V''_{m,P})d\varnothing t' = d\varnothing t'(1 - \frac{\varnothing V_{m,P}''2}{\varnothing c_m^2})^{-1/2} . \quad (19)$$

Equations (18) and (19) give intrinsic metric space contraction and intrinsic metric time dilation formulae with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$ , explicitly in terms of relative proper intrinsic static flow speed at point P along the curved  $\varnothing\rho'$  and the symmetry-partner point  $O^0$  along the curved  $\varnothing c_s\varnothing t'$  in the global diagrams. These are intrinsic length contraction and intrinsic time dilation formulae in the context of the intrinsic theory of relativity associated with the presence of a long-range intrinsic metric force field in intrinsic metric spacetime.

Now the intrinsic theory of relativity on the flat two-dimensional relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s\varnothing t)$  associated with the presence of a long-range relativistic intrinsic metric force field on  $(\varnothing\rho, \varnothing c_s\varnothing t)$ , will be made manifested outwardly in the theory of relativity on the flat four-dimensional relativistic metric spacetime  $(\mathbb{E}^3, c_s t)$ , due to the presence of a long-range relativistic metric force field in  $(\mathbb{E}^3, c_s t)$ . Consequently the intrinsic local Lorentz transformation ( $\varnothing$ LLT) of system (7) and its inverse of system (10) in the two-dimensional intrinsic metric spacetime, will be made manifested outwardly in local Lorentz transformation (LLT) and its inverse in the four-dimensional metric spacetime respectively as

$$\begin{aligned} c_s dt' &= c_s dt \sec \psi - dx^1 \tan \psi_{m,P}; \\ &\quad (\text{w.r.t. 1 - observers in } c_s t); \\ dx'^1 &= dx^1 \sec \psi_{m,P} - c_s dt \tan \psi_{m,P}; \quad dx'^2 = dx^2; \quad dx'^3 = dx^3; \\ &\quad (\text{w.r.t. 3 - observers in } \mathbb{E}^3) \end{aligned} \quad (20)$$

and

$$\begin{aligned} c_s dt &= c_s dt' \sec \psi_{m,P} + dx'^1 \tan \psi_{m,P}; \\ &\quad (\text{w.r.t. 3 - observers in } \mathbb{E}^3); \\ dx^1 &= dx'^1 \sec \psi_{m,P} + c_s dt' \tan \psi_{m,P}; \quad dx^2 = dx'^2; \quad dx^3 = dx'^3; \\ &\quad (\text{w.r.t. 1 - observers in } c_s t) . \end{aligned} \quad (21)$$

The explicit forms of  $\varnothing$ LLT (14) and its inverse (15) in the two-dimensional intrinsic metric spacetime are likewise made manifested in LLT and its inverse on

the flat four-dimensional metric spacetime respectively as

$$\begin{aligned}
 dt' &= \gamma_{m,P}(V'_{m,P})(dt - \frac{V'_{m,P}}{c_m^2} dx^1); \\
 &\quad \text{(w.r.t. 1 – observers in } c_s t) \\
 dx'^1 &= \gamma_{m,P}(V'_{m,P})(dx^1 - V'_{m,P} dt); \quad dx'^2 = dx^2; \quad dx'^3 = dx^3; \\
 &\quad \text{(w.r.t. 3 – observers in } \mathbb{E}^3)
 \end{aligned} \tag{22}$$

and

$$\begin{aligned}
 dt &= \gamma_{m,P}(V'_{m,P})(dt' + \frac{V'_{m,P}}{c_m^2} dx'^1); \\
 &\quad \text{(w.r.t. 3 – observers in } \mathbb{E}^3); \\
 dx^1 &= \gamma_{m,P}(V'_{m,P})(dx'^1 + V'_{m,P} dt'); \quad dx^2 = dx'^2; \quad dx^3 = dx'^3; \\
 &\quad \text{(w.r.t. 1 – observers in } c_s t);
 \end{aligned} \tag{23}$$

where

$$\gamma_{m,P}(V'_{m,P}) = \sec \psi_{m,P} = (1 - \frac{V'^2_{m,P}}{c_m^2})^{-1/2}. \tag{24}$$

The dimension  $x^1$  of the relativistic Euclidean 3-space  $\mathbb{E}^3$  is considered to be orientated along the isotropic relativistic intrinsic metric space  $\emptyset\rho$ , while the dimensions  $x^2$  and  $x^3$  of  $\mathbb{E}^3$  are orientated along other directions in  $\mathbb{E}^3$ . It then follows that the dimension  $x'^1$  of the relative proper Euclidean 3-space  $\mathbb{E}'^3$  was orientated along the isotropic relative proper intrinsic metric space  $\emptyset\rho'$ , while the dimensions  $x'^2$  and  $x'^3$  of  $\mathbb{E}'^3$  were orientated along other directions in  $\mathbb{E}'^3$  in Fig. 4 or Fig. 11 of [3], reproduced as Fig. 1 of this article, at the first stage of evolutions of spacetimes and intrinsic spacetimes in a long-range metric force field, prior to the evolutions of Figs. 5, 7, 10 and 11 at the second stage.

Now the intrinsic static flow speed  $\emptyset V'_{m,P}$  lies along the isotropic proper intrinsic metric space  $\emptyset\rho'$  underlying  $\mathbb{E}'^3$  (in Fig. 1) at the first stage and along  $\emptyset\rho$  underlying  $\mathbb{E}^3$  at the second stage. Consequently the static flow velocity  $\vec{V}'_{m,P}$  lies along  $x'^1$  in  $\mathbb{E}'^3$  and along  $x^1$  in  $\mathbb{E}^3$ . It has no component along the coordinate  $x'^2$  or  $x'^3$  in  $\mathbb{E}'^3$  and no component along coordinate  $x^2$  or  $x^3$  in  $\mathbb{E}^3$ . These make systems (20) through (23) to take on their forms, in which the intervals  $dx'^2$  and  $dx'^3$  transform into intervals  $dx^2$  and  $dx^3$  trivially as,  $dx'^2 = dx^2$  and  $dx'^3 = dx^3$ . A robust explanation of why systems (20) – (25) take on their forms in all long-ranged metric force-fields—spherically symmetric or not—shall be given when we fully make connection to the gravitational field elsewhere.

Either the LLT (20) or its inverse (21), or the explicit form (22) or (23), leads to local Lorentz invariance (LLI)

$$c_s^2 dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 = c_s^2 dt'^2 - (dx'^1)^2 - (dx'^2)^2 - (dx'^3)^2. \tag{25}$$

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This is the outward manifestation in the four-dimensional metric spacetime of the intrinsic local Lorentz invariance ( $\emptyset$ LLI) (16) in the two-dimensional intrinsic metric spacetime. The local Lorentz invariance (25) is valid at every point on the four-dimensional spacetime, implying flatness everywhere in a long-range metric force field of the four-dimensional relativistic metric spacetime ( $\mathbb{E}^3, c_s t$ ).

The intrinsic length contraction formula (18) and intrinsic time dilation (19) on the flat two-dimensional intrinsic metric spacetime are likewise made manifested outwardly in length contraction and time dilation formulae on the flat four-dimensional metric spacetime as

$$dx^1 = \gamma_{m, (V'_{m,P})^{-1}} dx'^1 = \left(1 - \frac{V'^2_{m,P}}{c_m^2}\right)^{1/2} dx'^1; \quad dx'^2 = dx^2; \quad dx'^3 = dx^3 \quad (26)$$

and

$$dt = \gamma_{m, P} (V'_{m,P}) dt' = \left(1 - \frac{V'^2_{m,P}}{c_m^2}\right)^{-1/2} dt'. \quad (27)$$

As a summary of this section, we have derived the global curved intrinsic metric spacetime/global flat metric spacetime geometries of Figs. 5 – 11 and the associated local spacetime/intrinsic spacetime geometries of Figs. 12 – 15 in the four-world picture. We have derived the intrinsic local Lorentz transformation ( $\emptyset$ LLT) and its inverse of systems (7) and (10), or systems (14) and (15); we have validated intrinsic local Lorentz invariance ( $\emptyset$ LLI) and have derived the intrinsic length contraction and intrinsic time dilation formulas (18) and (19), at an arbitrary point in spacetime within a long range metric force field, with the aid of Figs. 12 – 15 (as must be done at every point in spacetime) in every long-range metric force field. These are results in the context of the intrinsic theory of relativity associated with the presence of an intrinsic metric force field in intrinsic metric spacetime.

The theory of relativity in the metric spacetime due to the presence of a long-range metric force field in the metric spacetime, being mere outward manifestation of the intrinsic theory of relativity in intrinsic metric spacetime, due to the presence of intrinsic metric force field in the intrinsic metric spacetime; the results of the theory of relativity in spacetime have been written directly from the corresponding results of intrinsic theory of relativity in intrinsic metric spacetime. These are the local Lorentz transformation (LLT) and its inverse of system (20) and (21) or system (22) and (23); local Lorentz invariance (25) and the length contraction and time dilation formulae (26) and (27), all of which have been written at an arbitrary point in spacetime in a long-range metric force field.

The central purpose of this article is to develop a new geometrical background for the theory of relativity associated with the presence of a long-range metric force field in the metric spacetime within the four-world picture, in which the four-dimensional metric spacetime is underlay by a hidden two-dimensional intrinsic metric spacetime in each universe, derived in [5–8]. We deem the results derived in this section and

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summarized in the foregoing two paragraphs as adequate for this purpose. It is to be recalled from the derivation of the concept of intrinsic static flow speed and static flow speed in part three of this paper [3] that, the intrinsic static flow speed and static flow speed, which appear in the intrinsic theory of relativity and theory of relativity associated with the presence of a long-range metric force field in spacetime in this article are pure geometrical parameters. Nevertheless, they will not arise in the absence of a source of a metric force field and a source of intrinsic metric force field.

### 2.3 Clarifications of the concepts of relative static flow-speed, relativity associated with static flow-speed and relative metric force fields

It is appropriate to shine some light on the new concepts in the topic of this subsection that are introduced in this article. Let us start with the familiar concept (or parameter) in physics namely, the dynamical velocity  $\vec{v}$  (or speed  $v$ ). It is an observable and measurable property of a particle or object in motion. The dynamical velocity is a relative parameter because its magnitude varies with the observer or frame of reference relative to which the particle is in motion. The relativity of dynamical velocity is the origin of the relativity of motion of material particles and objects described by the special theory of relativity.

On the other hand, the relative proper static flow speed  $V'_m$  is a property of space, established in space by the source of a long-range relative proper (or classical) metric force field, irrespective of whether a particle or object is present in space or not. A particle or object of any mass located at a point P in space where the relative proper static flow speed is  $V'_{m,P}$ , will acquire  $V'_{m,P}$  but will not move relative to any observer or frame of reference at this speed. If it also possesses dynamical velocity  $\vec{v}$  relative to an observer while moving through point P, then it will be observed to move at the velocity  $\vec{v}$  only relative to the observer, despite the static flow speed  $V'_{m,P}$  it has acquired.

The static flow speed established at a point in space cannot be observed or measured. It does not give rise to flow of space and, consequently, it does not give rise to translation in space of a material particle or object that acquired it, as mentioned above. Further more, the static flow speed of a point in space is the same with respect to all observers of frames of reference. It is hence an absolute parameter from the point of view of dynamical relativity (or the special theory of relativity). Then how come the concepts of relative static flow speed and relativity associated with static flow speed?

In order to answer the question ending the preceding paragraph, let us revisit the length contraction and time dilation formulae (26) and (27). Although the relative proper static flow speed  $V'_m$  of a point in space cannot be observed or measured and, although its square  $V'^2_m$  cannot be observed or measured, the quantities  $(1 - V'^2_m/c_m^2)^{\frac{1}{2}} dx'^1$  and  $(1 - V'^2_m/c_m^2)^{-\frac{1}{2}} dt'$  can be observed and measured. This follows

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from the fact to be formally derived upon making connection to gravity elsewhere that  $V'_m$  is related to the classical potential  $\Phi'_m$  of the metric force field that establishes  $V'_m$  in space as,  $\Phi'_m = -\frac{1}{2}V'^2_m$  (for an attractive metric force field). The quantity  $V'^2_m$ , like the potential  $\Phi'_m$  at a point in space, cannot be observed or measured (as is the case with gravitational potential in particular).

Now the quantities,

$$(1 - V'^2_m/c^2_m)^{\frac{1}{2}} dx'^1 = (1/c_m)(c^2_m - V'^2_m)^{\frac{1}{2}} dx'^1$$

and

$$(1 - V'^2_m/c^2_m)^{-\frac{1}{2}} dt' = c_m(c^2_m - V'^2_m)^{-\frac{1}{2}} dt',$$

can be measured, since,  $c^2_m - V'^2_m$ , being equivalent to difference of potentials, can be measured. It then follows that the length contraction and time dilation formulae (26) and (27) can be observed and measured.

Thus by allowing an event that involves proper time interval  $dt'$  and proper space intervals,  $dx'^1$ ,  $dx'^2$  and  $dx'^3$ , to occur at different positions in space within a long-range metric force field, the observed (or relativistic) time interval  $dt$  and the observed (or relativistic) interval  $dx^1$  of the relativistic Euclidean 3-space  $\mathbb{E}^3$  (in Fig. 5) involved in the same event, will vary with position in  $\mathbb{E}^3$ , while the observed intervals,  $dx^2$  and  $dx^2$ , of  $\mathbb{E}^3$  involved in the event will be the same at all positions within the metric force field, according to systems (26) and (27). The variations with the magnitude of the relative proper static flow speed  $V'_m$  and, consequently, with position in space within a long-range metric force field, of the observed (or relativistic) time interval  $dt$  and the observed (or relativistic) interval  $dx^1$  of the Euclidean 3-space  $\mathbb{E}^3$ , of an event, is the concept of relativity associated with relative proper static flow-speed, or with the presence of a long-range metric force field in spacetime.

In brief, the relativity associated with relative proper static flow speed in a long-range metric force field is relativity with position in space within the field (and not relativity with observer or frame of reference). Relativity of relative proper static flow speed likewise refers to variation of magnitude of relative proper static flow speed with position in space within a long-range metric force field. In other words, it refers to the fact that the relative proper static flow speeds,  $V'_{m,P}$  and  $V'_{m,Q}$ , of two positions P and Q of different distances,  $x^1_P$  and  $x^1_Q$ , respectively, from the origin of the long-range metric force field, have different magnitudes. It does not refer to variation of the magnitude of a static flow speed with observers or frames of reference. As mentioned earlier, the relative proper static flow speed at a point in space is the same relative to all observers or frames of reference.

In the light of the foregoing, a relative (or relativistic) metric force field is the one that establishes non-zero relative proper static flow speed in space. That is, one that establishes relative proper static flow speeds of different magnitudes (no matter how small in magnitudes in a strict sense), at different positions in the relative proper

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metric Euclidean 3-space  $\mathbb{E}^3$ , which transforms invariantly as relative proper static flow speeds in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  within the metric force field. The possibility of the relativity of other physical parameters, such as mass, electric and magnetic fields, energy, fluxes, temperature, entropy, potentials, etc, in the sense of the variations of their observed (or relativistic) magnitudes with relative proper static flow speed and, consequently, with position in space within a long-range metric force field, on the flat four-dimensional relativistic metric spacetime ( $\mathbb{E}^3, c_s t$ ) (in Fig. 5), now isolated, shall be investigated upon applying the results of this article to the gravitational field elsewhere.

Expectedly, it will be possible to derive the transformations of physical parameters and physical constants, classical and special-relativistic non-gravitational laws, as well as classical gravitational laws, on flat spacetime within a long-range metric force field, with the aid of the local Lorentz transformation and its inverse in terms of relative proper static flow speed of systems (22) and (23), in the context of the theory of relativity associated with the presence of a long-range metric force field in metric spacetime and, in particular, in the gravitational field. This will be analogous to the Lorentz transformations of parameters and natural laws on flat spacetime in the context of the special theory of relativity.

### 3 Absolute intrinsic Riemann geometry on curved ‘two-dimensional’ absolute intrinsic metric spacetime at the second stage of evolutions of spacetime/intrinsic spacetime in a metric force field

The ‘two-dimensional’ absolute intrinsic metric spacetime  $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$  is curved relative to its projective flat ‘2-dimensional’ absolute proper intrinsic metric spacetime  $(\emptyset\rho'_{ab}, \emptyset c_{sab}\emptyset t'_{ab})$ , which is imperceptibly embedded in the flat relative proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s\emptyset t')$  in Fig. 1, at the first stage of evolutions of spacetimes and intrinsic spacetimes within a long-range metric force field. Consequently the absolute intrinsic Riemann geometry has been formulated on the curved  $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$  with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  that overlies  $\emptyset\rho'$  in [3].

On the other hand, the ‘two-dimensional’ absolute intrinsic metric spacetime  $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$  is curved relative to the flat two-dimensional relativistic intrinsic metric spacetime  $(\emptyset\rho, \emptyset c_s\emptyset t)$  in Fig. 5, at the second stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field. It then follows that absolute intrinsic Riemann geometry must be formulated on the curved  $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$  with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  that overlies  $\emptyset\rho$  in Fig. 5, at the second stage of evolution of spacetime/intrinsic spacetime.

In order to show that absolute intrinsic Riemann geometry on the curved absolute

intrinsic metric spacetime  $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$  takes on the same form with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 4 or Fig. 11 of [3], reproduced as in Fig. 1 of this article, and with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 5 of this article, let us revisit the derivation of the absolute intrinsic metric tensor without star label with the aid of Fig. 7 of [3], reproduced as Fig. 16 of this article, from Eq. (48a-b) through Eq. (64) of [3].

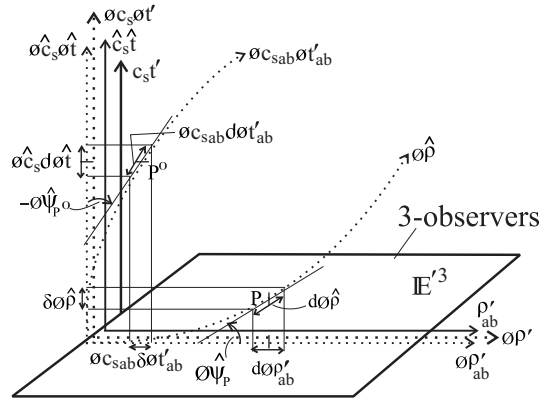


Figure 16: The curved ‘two-dimensional’ absolute intrinsic metric spacetime made valid with respect to 3-observers in the lat relative proper metric 3-space solely; the correct diagram for absolute intrinsic Riemannian metric spacetime geometry in our universe; (Fig. 7 of [3]).

Let us re-write Eq. (53) of that article as follows

$$\begin{aligned} (d\varnothing s'_{ab})^2 &= \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 \left( \cos^2 \varnothing \hat{\psi}_{m,P} + \sin^2 \varnothing \hat{\psi}_{m,P} \right) \\ &\quad - (d\varnothing \rho'_{ab})^2 \left( \sec^2 \varnothing \hat{\psi}_{m,P} - \tan^2 \varnothing \hat{\psi}_{m,P} \right). \end{aligned} \quad (28)$$

Equation (28) gives the absolute intrinsic line element on the curved ‘two-dimensional’ absolute intrinsic spacetime  $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ , written in terms of the intervals of the absolute proper intrinsic metric spacetime  $(\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab})$  in Fig. 16, which is valid with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  in that figure.

Then the established invariance of intrinsic line element (or absolute intrinsic local Lorentz invariance (A\varnothingLLI)) is derived between the curved  $(\varnothing\hat{\rho}, \varnothing c_{sab}\varnothing t'_{ab})$  and its projective flat  $(\varnothing\rho'_{ab}, \varnothing\hat{c}_s\varnothing\hat{t})$  in Fig. 7 of [3], reproduced as Fig. 16 of this article and expressed by Eq. (54) of that article. It shall be reproduced here as

$$\varnothing\hat{c}_s^2 (d\varnothing\hat{t})^2 - (d\varnothing\hat{\rho})^2 = \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 - (d\varnothing\rho'_{ab})^2. \quad (29)$$

The  $(d\varnothing\rho'_{ab})^2$  and  $\varnothing c_{sab}^2(d\varnothing t'_{ab})^2$  in this equation are replaced with  $(d\varnothing\hat{\rho})^2$  and  $\varnothing\hat{c}^2(d\varnothing\hat{t})^2$  respectively, yielding Eq. (56) of [3], reproduced here as

$$(d\varnothing\hat{s})^2 = \varnothing\hat{c}_s^2(d\varnothing\hat{t})^2(\cos^2\varnothing\hat{\psi}_{m,P} + \sin^2\varnothing\hat{\psi}_{m,P}) - (d\varnothing\hat{\rho})^2(\sec^2\varnothing\hat{\psi}_{m,P} - \tan^2\varnothing\hat{\psi}_{m,P}). \quad (30)$$

The absolute intrinsic metric tensor without star label of Eq. (63) or Eq. (64) of [3] and the absolute intrinsic Ricci tensor without star label of Eq. (67) or Eq. (68) of that article, were then derived with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 16, from Eq. (30) above (or Eq. (56) of [3]), between Eqs. (58) and (68) of that article.

Now the absolutism of the absolute intrinsic metric spacetime  $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$  and absolute proper intrinsic metric spacetime  $(\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab})$ , implies that they are invariant with intrinsic local Lorentz transformation (14) and its inverse (15) in the context of the theory of relativity associated with the presence of a long-range metric force field in spacetime. In other words, we can write as follows

$$(d\varnothing\rho_{ab})^2 - \varnothing c_{sab}^2(d\varnothing t_{ab})^2 = (d\varnothing\rho'_{ab})^2 - \varnothing c_{sab}^2(d\varnothing t'_{ab})^2. \quad (31)$$

Just as the flat absolute proper intrinsic metric spacetime  $(\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab})$  is imperceptibly embedded in the flat relative proper intrinsic metric spacetime  $(\varnothing\rho', \varnothing c_s\varnothing t')$  in Fig. 1, at the first stage of evolutions of metric spacetimes and intrinsic metric spacetimes in a long-range metric force field, the flat ‘absolute relativistic’ intrinsic metric spacetime  $(\varnothing\rho_{ab}, \varnothing c_{sab}\varnothing t_{ab})$  is imperceptibly embedded in the flat relativistic intrinsic metric spacetime  $(\varnothing\rho, \varnothing c_s\varnothing t)$  in Fig. 5, at the second stage of evolutions of metric spacetimes and intrinsic metric spacetimes in a long-range metric force field, and the invariance (31) obtains between  $(d\varnothing\rho'_{ab}, \varnothing c_{sab}^2 d\varnothing t'_{ab})$  and its projective  $(d\varnothing\rho_{ab}^2, \varnothing c_{sab}^2 d\varnothing t_{ab}^2)$ . This allows us to replace  $\varnothing c_{sab}^2(d\varnothing t'_{ab})^2$  and  $(d\varnothing\rho'_{ab})^2$  with  $\varnothing c_{sab}^2(d\varnothing t_{ab})^2$  and  $(d\varnothing\rho_{ab})^2$  respectively in Eq. (28) to have

$$(d\varnothing s_{ab})^2 = \varnothing c_{sab}^2(d\varnothing t_{ab})^2 \left( \cos^2\varnothing\hat{\psi}_{m,P} + \sin^2\varnothing\hat{\psi}_{m,P} \right) - (d\varnothing\rho_{ab})^2 \left( \sec^2\varnothing\hat{\psi}_{m,P} - \tan^2\varnothing\hat{\psi}_{m,P} \right). \quad (32)$$

While the primed absolute intrinsic line element  $d\varnothing s'$  in Eq. (28) on relative proper intrinsic metric spacetime  $(\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab})$  is valid with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 1, at the first stage of evolutions of metric spacetimes and intrinsic metric spacetimes in a long-range metric force field, the unprimed absolute intrinsic line element  $d\varnothing s$  in Eq. (32) on the ‘absolute relativistic’ (or absolute unprimed) intrinsic metric spacetime  $(\varnothing\rho_{ab}, \varnothing c_{sab}\varnothing t_{ab})$ , is valid with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 5 of this article at the second stage.

Combining the absolute intrinsic local Lorentz invariance (AØLLI) (29) and (31) we have

$$\begin{aligned} \varnothing c_{sab}^2 (d\varnothing t_{ab})^2 - (d\varnothing \rho_{ab})^2 &= \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 - (d\varnothing \rho'_{ab})^2 = \\ &\varnothing \hat{c}_s^2 (d\varnothing \hat{\rho})^2 - (d\varnothing \hat{\rho})^2 . \end{aligned} \tag{33}$$

Equation (33) allows us to replace  $\varnothing c_{sab}^2 (d\varnothing t_{ab})^2$  and  $(d\varnothing \rho_{ab})^2$  by  $\varnothing \hat{c}_s^2 (d\varnothing \hat{t})^2$  and  $(d\varnothing \hat{\rho})^2$  respectively in Eq. (32) to have Eq. (30) again.

It then follows that the absolute intrinsic metric tensor of Eq. (63) or (64) and absolute intrinsic Ricci tensor of Eq. (67) or (68) of [3], derived from Eq. (30) of this article, with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 4 or Fig. 11 of [3], reproduced as Fig. 1 of this article, but with the aid of Fig. 7 of that article, reproduced as Fig. 16 of this article, at the first stage of evolutions of metric spacetimes and intrinsic metric spacetimes in a long-range metric force field, are equally valid with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 5 of this article at the second stage.

The starred absolute intrinsic line element  $d\varnothing \hat{s}^*$ , the starred absolute intrinsic metric tensor  $\varnothing \hat{g}_{ij}^*$  and the starred absolute intrinsic Ricci tensor  $\varnothing \hat{R}_{ij}^*$  on the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$  in Fig. 1, given by Eqs. (31), (33) and (39) respectively of [3], which are valid partially with respect to 3-observers in the flat relative proper metric 3-space  $\mathbb{E}^3$  and partially with respect to 1-observers in the relative proper metric time dimension  $c_s t'$  in that figure, as explained in that article, are equally valid on the curved  $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$  in Fig. 5 of this article, partially with respect to 3-observers on the flat relativistic metric 3-space  $\mathbb{E}^3$  and partially with respect to 1-observers in the relativistic metric time dimension  $c_s t$  in that figure.

Thus the formulation of absolute intrinsic Riemann geometry on the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$  with respect to 3-observers in the relativistic metric Euclidean 3-space  $\mathbb{E}^3$  solely in Fig. 5 of this article, at the second stage of evolutions of metric spacetimes and intrinsic metric spacetimes within a long-range metric force field, follows the same procedure used to formulate absolute intrinsic Riemann geometry on the curved  $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ , with respect to 3-observers in the relative proper Euclidean 3-space  $\mathbb{E}^3$ , with the aid of Fig. 7 of [3], reproduced as Fig. 16 of this article, in [3] at the first stage.

The preceding paragraph means that just as done at the first stage of evolutions of metric spacetimes/intrinsic metric spacetimes, one must write the pair of absolute intrinsic tensor equations involving starred absolute intrinsic tensors  $\varnothing \hat{g}_{ij}^*$  and  $\varnothing \hat{R}_{ij}^*$ , derived on the curved  $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$  in [3], and presented as Eqs. (34) and (38) of that article. One must then solve those equations algebraically to obtain  $\varnothing \hat{g}_{ij}^*$  and  $\varnothing \hat{R}_{ij}^*$  in terms of the square of the absolute intrinsic curvature parameter  $\varnothing \hat{k}^{*2}$  as Eqs. (64) and (68), or in terms of absolute intrinsic static flow speed as Eqs. (81) and (82) of [3]. The starred absolute intrinsic tensors so derived are valid partially with

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respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  and partially with respect to 1-observers in the relativistic time dimension  $ct$  in Fig. 5 of this article.

Then in order to obtain the absolute intrinsic metric tensor without star label  $\emptyset\hat{g}_{ij}$ , which is valid with respect to 3-observers in the relativistic Euclidean 3-space  $\mathbb{E}^3$  solely, one must use the relations among the components of the starred absolute intrinsic metric tensor  $\emptyset\hat{g}_{ij}^*$  and the components of the absolute intrinsic metric tensor without star label  $\emptyset\hat{g}_{ij}$  in systems (65a) and (65b) of [3]. Once  $\emptyset\hat{g}_{ij}$  has been obtained, one must apply the tensorial statement of intrinsic local Lorentz invariance (66) of [3] to derive the absolute intrinsic Ricci tensor without star label  $\emptyset\hat{R}_{ij}$ , which is valid with respect to 3-observers in  $\mathbb{E}^3$  solely.

The superposition procedure developed in absolute intrinsic Riemann geometry at the first stage of evolutions of spacetimes and intrinsic spacetimes in [3], when two or a larger number of curved absolute intrinsic metric spacetimes co-exist, is equally applicable at the second stage. The clarifications of the concepts of absolute intrinsic static flow speed, absolute proper intrinsic static flow speed, absolute intrinsic metric tensor and absolute intrinsic metric theory of physics associated with them, introduced in [3] and this section, shall be done upon making connection to gravity in the next article and elsewhere.

## 4 Summary, conclusion and direction for further investigation

The summary in brief of the four parts of this paper is that metric spacetime and its underlying intrinsic metric spacetime follow two stages of evolution in the sequence of absolute metric spacetime/absolute intrinsic metric space  $\rightarrow$  proper metric spacetime/proper intrinsic metric spacetime  $\rightarrow$  relativistic metric spacetime/relativistic intrinsic metric spacetime in every long-range metric force field. The proper metric spacetime is comprised of ‘two-dimensional’ absolute proper metric spacetime  $(\rho'_{ab}, c_{sab}t'_{ab})$  and the 4-dimensional relative proper metric spacetime  $(\mathbb{E}'^3, c_s')$ , where  $(\rho'_{ab}, c_{sab}t'_{ab})$  is imperceptibly embedded in  $(\mathbb{E}'^3, c_s t')$ , as illustrated in Fig. 1. The proper intrinsic metric spacetime is likewise comprised of ‘two-dimensional’ of absolute proper intrinsic metric spacetime  $(\emptyset\rho'_{ab}, \emptyset c_{sab}\emptyset t'_{ab})$  and the two-dimensional relative proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s\emptyset t')$ , where  $(\rho'_{ab}, c_{sab}t'_{ab})$  is embedded in  $(\emptyset\rho', \emptyset c_s\emptyset t')$ , as also illustrated in Fig. 1, at the first stage of evolutions of metric spacetimes and intrinsic metric spacetimes in a long-range metric force field.

The relativistic metric spacetime is comprised ‘two-dimensional’ ‘absolute relativistic’ metric spacetime  $(\rho_{ab}, c_{sab}t_{ab})$  and the four-dimensional ‘relative relativistic’ (simply referred to as relativistic) metric spacetime  $(\mathbb{E}^3, c_s t)$ , where  $(\rho_{ab}, c_{sab}t_{ab})$  is imperceptibly embedded in  $(\mathbb{E}^3, c_s t)$  in Fig. 5. The relativistic intrinsic metric spacetime is likewise comprised of ‘two-dimensional’ ‘absolute relativistic’ intrinsic metric spacetime  $(\emptyset\rho_{ab}, \emptyset c_{sab}\emptyset t_{ab})$  and two-dimensional relativistic intrinsic metric spacetime  $(\emptyset\rho, \emptyset c_s\emptyset t)$ , where  $(\rho_{ab}, c_{sab}t_{ab})$  is embedded in  $(\emptyset\rho, \emptyset c_s\emptyset t)$  in Fig. 5, at the second stage of evolutions of metric

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spacetimes and intrinsic metric spacetimes in long-range metric force fields. It shall be shown elsewhere that the second stage of evolutions of metric spacetimes and intrinsic metric spacetimes in a long-range metric force field is the final stage.

The theories and intrinsic theories of a given long-range metric force field encompassed by the geometry of Fig. 1 at the first stage of evolutions of spacetime and intrinsic spacetime in a long-range metric force field and those encompassed by the geometries of Figs. 2 and 7 and their inverses of Figs. 10 and 11, at the second stage, shall be formulated upon particularizing to the gravitational field elsewhere.

The spacetime/intrinsic spacetime geometry of Fig. 1 and the associated theories and intrinsic theories of a given metric force field, which evolve at the first stage of evolutions of spacetime and intrinsic spacetime in the long-range metric force field, endure for no moment before transforming into the enduring spacetime/intrinsic spacetime geometries of Figs. 5, 7, 10 and 11 and the associated theories and intrinsic theories of the given metric force field at the second (and final) stage. Indeed the first and second stages commence simultaneously and progress together, as shall be demonstrated upon particularizing to the gravitational field elsewhere. It is therefore the theories and intrinsic theories encompassed by Figs. 5 and 7 and their inverses of Figs. 10 and 11, at the second stage that exist in every long-range metric force field in the universe.

A crucial conclusion is that the four-dimensional (relativistic) metric spacetime  $(\mathbb{E}^3, c_s t)$  and its underlying two-dimensional (relativistic) intrinsic metric spacetime  $(\emptyset\rho, \emptyset c_s \emptyset t)$  are everywhere flat in long-range metric force fields; the only curved spacetime with Riemannian metric tensor, so to speak, with respect to 3-observers in the flat (or Euclidean) three-dimensional metric space  $\mathbb{E}^3$ , being the ‘two-dimensional’ absolute intrinsic metric spacetime  $(\emptyset\hat{\rho}, \emptyset\hat{c}_s \emptyset\hat{t})$  with absolute intrinsic sub-Riemannian metric tensor  $\emptyset\hat{g}_{ik}$ , isolated progressively in the first three parts of this paper [1–3].

The next natural step is to particularize the newly derived spacetime/intrinsic spacetime geometries of Figs. 5 and 7 and their inverses of Figs. 10 and 11, in long-range metric force fields and the associated flat spacetime theory of metric force field on the flat four-dimensional metric spacetime  $(\mathbb{E}^3, c_s t)$  and hierarchy of theories of intrinsic metric force field on the hierarchy of intrinsic metric spacetimes namely, the flat relativistic intrinsic metric spacetime  $(\emptyset\rho, \emptyset c_s \emptyset t)$ , the curved relative proper intrinsic metric spacetime  $(\emptyset\rho', \emptyset c_s \emptyset t')$  with intrinsic Lorentzian metric tensor and the curved absolute intrinsic metric spacetime  $(\emptyset\hat{\rho}, \emptyset\hat{c}_s \emptyset\hat{t})$ , with absolute intrinsic sub-Riemannian metric tensor, to the gravitational field. This will yield the corresponding flat spacetime theory of gravity and hierarchy of theories of intrinsic gravity in the gravitational field. Particularization to the gravitational field shall be done elsewhere.

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