

Real-time offshore supply vessel routing problem with random service requests

ABSTRACT

The routing of supply vessels to offshore platforms is one of the major tasks in upstream logistics planning in the oil and gas industry. This task involves the formulation and solution of a real-time variant of the stochastic dynamic vehicle routing problem, which reflects the operational environment of offshore platforms in practice. In this paper, a novel mixed integer linear programming-based heuristic is presented with a view to determining the optimal route of a single supply vessel allocated to provide service to a group of offshore platforms through the supply of spare parts, liquid bulk liquids, and other materials, allowing for random service requests by the offshore platform group. The proposed heuristic is applied to a real-world case study of vessel routing to offshore platforms in the South Atlantic Ocean Shelf. The computational performance of the heuristic, as assessed by the degree of dynamism, is found to be very good.

Keywords: Supply vessel routing; Offshore platforms; Random service requests; Mixed integer linear programming heuristic; Degree of dynamism; Oil and gas industry.

1. Introduction

Logistics supply chains in the oil and gas industry extend from the oil and gas onshore and offshore wells to the final consumer, thus incurring substantial costs and lead times. In view of this feature, it is normal practice to divide the logistics oil and gas supply chain into two major parts, referred to as the upstream logistics chain extending from production wells to petroleum refineries, and the downstream logistics chain which starts at the petroleum refinery and ends at the final consumer. The high costs and lead times that are inherent in oil and gas logistics chains has resulted in extensive work on the planning and optimisation of the oil and gas logistics chain as a whole; cf., [1-7] for recent work in this connection.

In the work presented in this paper, focus is on one type of operation in the offshore upstream logistics chain; i.e., supply vessel real-time routing to petroleum production offshore platforms. An offshore platform for oil and gas production normally consists of a subsea drilling system and a topside processing plant, along with living quarters for operating personnel. As expensive assets, offshore platforms need uninterrupted operation, which in turn requires regular transportation of personnel and material supplies between offshore platforms and onshore base ports. In so far as personnel transportation is concerned, it is normal practice to employ helicopters; cf., [8] for the helicopter routing problem. On the other hand, material supplies are transported to and from platforms, using special supply vessels; cf., [9] for an overview of the selection of offshore supply vessels to offshore platforms. Supplies carried in these vessels include equipment (pumps, compressors, etc.) and materials (oil,

chemicals, spare parts, solid and liquid waste materials, food, medicines, etc.). In [10], risks in offshore platform logistics of both personnel and materials have been studied, these arising due to severe environmental conditions of stormy weather and low visibility.

For a group of offshore platforms, a fleet of supply vessels is allocated to attend the demands of the offshore platforms constituting the group. The determination of the number of supply vessels in the fleet in the long term, is known as the fleet sizing problem. The scope of the fleet sizing problem may include a number of considerations; e.g.,

- uncertainties in weather conditions, offshore platform demands, and delays in supply vessel departure from base port.
- determination of supply vessel schedules (a sequence of supply vessel voyages) and routes (the order in which supply vessels visit offshore platforms in each voyage).

The formulation and solution of the fleet sizing problem, in all its variants, normally employ optimisation and simulation-based optimisation; c.f., [11-16].

The solution of the offshore supply vessel fleet sizing problem, by its very nature, is a strategic long-term plan, typically in a time horizon of one or more years, and as such needs to be implemented by a medium-term tactical plan, typically in a time horizon of several weeks, in order to take into account inevitable changes in offshore platform demand, supply vessel maintenance, weather conditions, and other factors, which could not have been foreseen in the strategic plan of supply vessel fleet sizing; c.f., [17-20].

The solution of the offshore tactical scheduling problem provides the basis for the voyage routing of each supply vessel in the fleet to the group of offshore platforms to which each supply vessel is allocated. The voyage routing of a supply vessel to the offshore platform group to which it is allocated is an operational real-time problem, is a complex problem in view of its dual dynamic and stochastic nature. This is because once a supply vessel departs from its base port with a route plan, offshore platform demands may be altered *during* the voyage tour of the supply vessel; furthermore, these demands are altered in an unforeseen manner as a function of offshore platform necessities which give rise to these demands. As such, this problem is a variant of the stochastic dynamic vehicle routing problem (SDVRP) for which a wide overview may be found in [21]. To the authors' knowledge, based on an extensive literature search, there does not exist any work reported on the SDVRP variant which is applicable to real-time routing of supply vessel to offshore platforms. It is the objective of the work presented in this paper to contribute towards filling this gap.

The rest of the paper is organised as follows. The problem under consideration in this paper is described in Section 2, along with a mathematical formulation of the problem, which is based on that of Ulmer and coworkers, whereby the problem is modelled as a Markov decision process; cf., [22,21,23-27], as well as a solution methodology based on a proposed two-step procedure and mixed

integer linear programming (MILP) heuristic is presented. In Section 3, the application of the aforesaid solution methodology is applied to a real-world problem, arising in the South Atlantic Ocean Shelf, in supply vessel routing in offshore platform groups. A number of problem instances are solved, with a view to assessing the computational performance of the two-step MILP heuristic. The paper is concluded in Section 4 with an assessment of the computational performance of the proposed heuristic, along with recommendations for future research.

2. Materials and methods

2.1. Problem description

The general problem of the routing of supply vessels from onshore base port and back through a group of offshore platforms belongs to the classical vehicle routing problem (VRP) of operational research, whereby a supply vessel corresponds to a vehicle and an offshore platform corresponds to a customer; c.f., [28] for a recent taxonomy and review of the vehicle routing problem. The problem variant that is considered in this paper is focused on supply vessel real-time routing, which may be described as follows. Having allocated a supply vessel to a group of offshore platforms, a single supply vessel is to serve the aforesaid group of offshore platforms by a voyage tour starting at an onshore port base, visiting all offshore platforms in the group to which it is allocated, and returning to the onshore port base. There are two types of service requests that can be placed by an offshore platform: planned service requests (PSRs) and additional random service requests (RSRs). On the one hand, PSRs are made prior to vessel departure from onshore base port, and are obligatory; i.e., the supply vessel must attend all PSRs during its tour. The routing of the supply vessel *prior* to departure from onshore base port is based on PSRs, and is a static (i.e.; time-independent) deterministic vehicle routing problem; cf., [29] for this problem variant in supply vessel routing to offshore platforms. On the other hand, RSRs are optional, in that an offshore platform may or may not place such a request during the supply vessel voyage tour; however, once an RSR is placed, it becomes obligatory for the supply vessel to pay a visit to the offshore platform which placed the aforesaid RSR. Furthermore, an RSR may be one of two types: priority random service request (PRSR) and non-priority random service request (NRSR). A PRSR means that the offshore platform making such a request must be placed at the top of the list of offshore platforms to be visited when the next rerouting of the supply vessel is determined. The motive in placing a PRSR by an offshore platform is that the PSR placed by that offshore platform prior to supply vessel tour start has assumed an *emergency* role at the time instant when the PRSR is placed. Consequently, the PRSR *substitutes* the PSR of that offshore platform, unless that platform had already been visited to attend its PSR *prior* to placing its PRSR. This is in contradistinction to the motive in placing an NRSR by an offshore platform, which is that during the supply vessel tour, an *additional* (to the PSR placed by that platform) non-emergency service has arisen, and that implies that the offshore platform concerned needs to be visited *twice* by the supply vessel during its tour. At each time instant that an RSR (whether of the PRSR or the NPSR) is placed, the supply vessel needs to be *rerouted*, and at each such rerouting, the

remaining PSRs and RSRs that have not yet been served need to be considered in the rerouting of the supply vessel. Rerouting is determined such that the distance covered by the supply vessel to the base port is a minimum. As described, the problem under consideration in this paper is a variant of the stochastic dynamic vehicle routing problem (SDVRP) for which a wide overview may be found in [21].

2.2. Mathematical formulation

The mathematical modelling approach to the SDVRP that is adopted in this paper follows that of Ulmer and coworkers, which has been successfully applied to a number of its variants in environments other than real-time routing of supply vessels to offshore platforms; c.f., [22-29]. It is worth noting that the words customer and platform are equivalent in this paper.

2.2.1. Route-based MDP model

The route-based MDP model for the SDVRP variant under consideration in this paper is characterised by the following elements.

- Decision epoch, which is a point in time at which a decision is made; i.e., when the vehicle is located at a platform.
- State, which is a tuple containing all information necessary to define the feasible actions at a particular decision epoch; i.e., time epoch (and thus vehicle-platform location pair), and lists of platforms yet-to-be served for PSRs and RSRs.
- Action, which is a feasible decision that is available at a particular cost for choosing an action, and the resulting transition state; i.e., a route plan which is a sequence of platforms, each of which has requested a PSR and/or an RSR but is yet to be served.
- Cost which is the quantity (distance that the vehicle traverses) incurred by choosing a particular action at a particular state.
- Transition, which is a function describing how the system evolves given a chosen action in a particular state, and which involves exogenous information; i.e., RSRs.
- Objective, which is a function of the costs, which typically involves the minimisation of the expected sum of costs across all decision epochs.

2.2.2. Illustrative examples

With a view to gaining a good understanding of the way the MDP models the SDVRP variant under consideration in this work, two illustrative examples are presented as follows. In the first example, suppose that the vehicle leaves the depot, denoted by node 0 in Fig.1, with the initial optimal plan (with PSRs only) represented by the list 0-4-2-5-1-3-0, as shown in the left panel of Fig.1. Furthermore, suppose that after vehicle arrival at platform 4 and serving its PSR, a PRSR is placed by platform 3, recalling that the PSRs of platforms of 2, 5, 1, and 3 have not been as yet served. A decision x_k is taken, and the system moves to a post-decision state S_k^x in which the new route is determined, as shown in the right panel of Fig.1, and the cost is added to the total tour cost. Having done that, a stochastic transition w_{k+1} moves the system from post-decision state S_k^x to a new pre-decision state S_{k+1} , which is shown in the right panel of Fig.1. This two-step procedure is repeated until the vehicle returns to the depot node, which is depicted by 0. Having done that, a stochastic transition w_{k+1} moves

the system from post-decision state S_k^x to a new pre-decision state S_{k+1} . This two-step procedure is repeated until the vehicle returns to the depot.

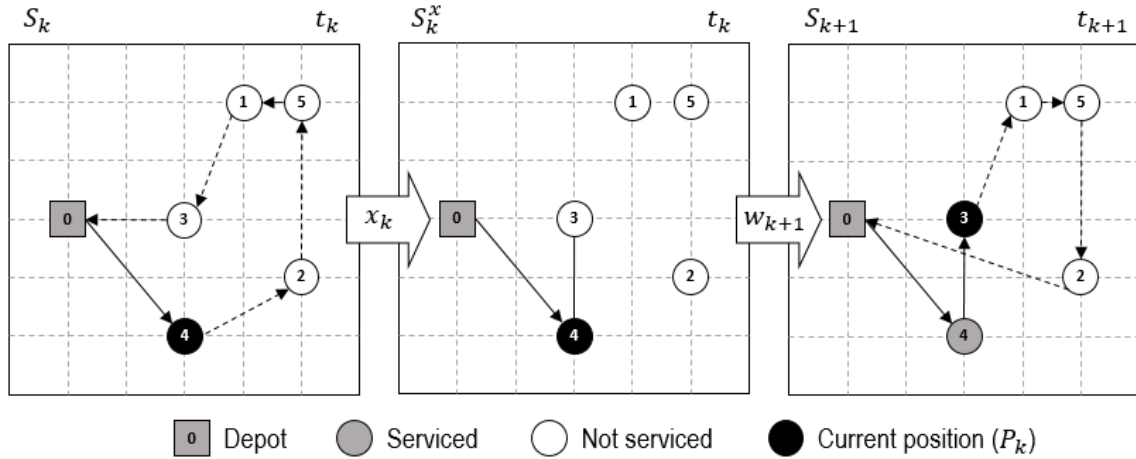


Fig.1. PRSR example.

Following the same objective of the example given above, another illustrative example is presented below for a non-priority random service request (NRSR) which is demonstrated through Fig. 2 and 3 according to the same policy previously described.

In the second example, suppose that the vehicle leaves the depot, denoted by node 0, with the initial optimal plan (with PSRs only) represented by the list 0-4-2-5-1-3-0, as shown in the left panel of Fig.2. The system is at pre-decision state S_k at time epoch t_k , as shown in the left panel of Fig.2. As no RSR is placed by any platform, a decision x_k is taken and the system moves to a post-decision state S_k^x in which the vehicle is moved to platform 2 as represented in the middle panel of Fig.2, ending in a new pre-decision state S_{k+1} , as shown in the right panel of Fig.2.

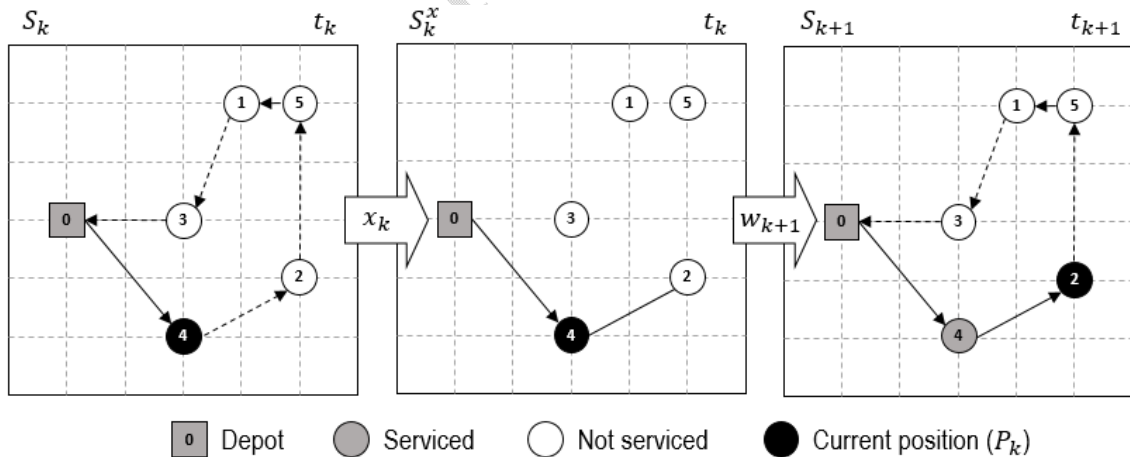


Fig. 2. NRSR example-first part

Now the system is at pre-decision state S_{k+1} at time epoch t_{k+1} , platform 4 have already been served and vehicle has been moved to platform 2 to serve its PSR as shown in the left panel of Fig.3. At this moment, a NRSR is placed by platform 1. Recall that the PSRs of platforms of 1 and 4 have not been

as yet served. Then, a new decision x_{k+1} is taken, and the system moves to a post-decision state S_{k+1}^x in which the new route is determined by adding the NRSR for platform 1, and the cost is added to the total tour cost. Now, a stochastic transition w_{k+2} moves the system from post-decision state S_{k+1}^x to a new pre-decision state S_{k+2} , which is shown in the right panel of Fig.3. From now onwards, this two-step procedure is repeated until the vehicle returns to the depot.

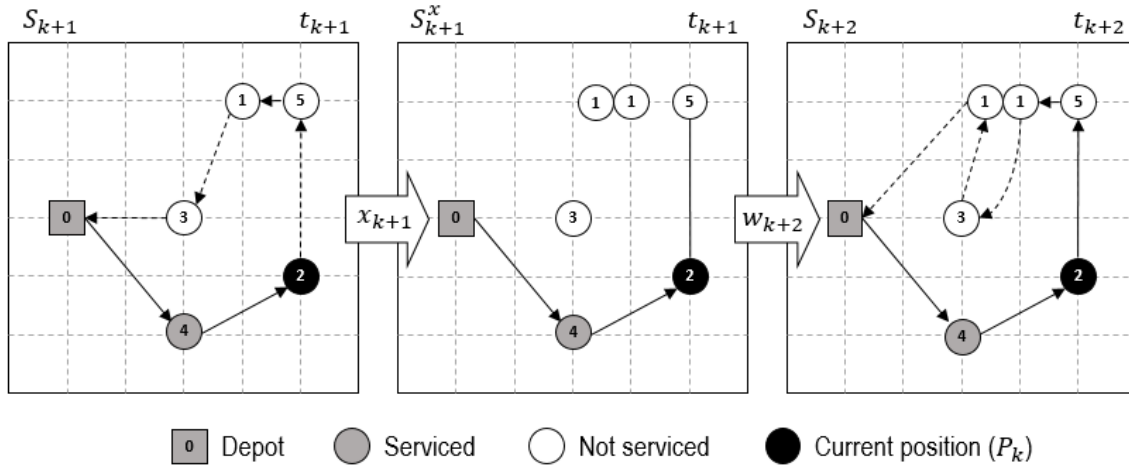


Fig. 3. NRSR example-second part.

2.2.4. Notation

a – PSR

b - RSR

k – index for time epoch; $t=0,1,\dots,K$

A – set of PSRs

B – set of RSRs

C_k - set of not-yet-served customer requests at time epoch t_k

C_k^{new} – set of new RSRs at time epoch t_k

t_k – time epoch

P_k – vehicle position at time epoch t_k

$R(S_k, x)$ – reward (cost) of decision x in state S_k

x_k – decision

w_k – decision

S_k – decision state at time epoch t

T – planning time horizon

θ_k – tour plan at time epoch t

θ_k^x – updated tour plan at time epoch t_k

2.2.5. Optimal policy

The objective of the problem under consideration in this paper is the determination of an optimal policy $\pi^* \in \Pi$, where a policy π is a sequence of decision rules $(X_0^\pi, \dots, X_K^\pi)$ that are formulated in order to

assign every state S_k to a decision $X_k^\pi(S_k)$. Decision rule $X_k^\pi(S_k)$ depends on state S_k and is induced by policy π in time epoch t_k . An optimal policy π^* is defined as the policy that minimises the expected costs over all decision (time) epochs $k=0,1,\dots,K$; i.e.,

$$\pi^* = \operatorname{argmin}_{(\pi \in \Pi)} E [\sum_{k=0,\dots,K} R(S_k, X_k^\pi(S_k)) | S^0]. \quad (1)$$

For every state S_k , π^* satisfies the Bellman equation

$$X_k^{\pi^*}(S_k) = \operatorname{argmin}_{(x \in X)} \{(S_k) + E [\sum_{j=k+1,\dots,K} R(S_j, X_j^{\pi^*}(S_j)) | S^0]\}. \quad (2)$$

In every decision time epoch $k=0,1,\dots,K$, the optimal policy π^* selects the decision that minimises the sum of immediate cost and the expected future costs.

2.2.6. Solution methodology

The solution of the Bellman equation constitutes a serious challenge in view of the curse of dimensionality in state, decision, and transition space; cf. [30]. Consequently, for practical problems and real world case studies, resort to heuristics is mandatory. The heuristic that is developed in this work makes use of the following features of the problem under consideration, as described in Section 2.

- 1) The number of customers is a known finite time-invariant integer.
- 2) In a given vehicle tour, each customer may place one PSR.
- 3) In a given vehicle tour, each customer may place at most one optional RSR.

The heuristic that is adopted in this paper is a two-step MILP procedure, whereby, the vehicle routing problem is solved every time instant at which a new RSR is placed by a customer; cf., [31,32].

2.2.6.1. Two-step MILP computational algorithm

In order to implement the two-step MILP computational algorithm, two lists are created, named A and B. List A corresponds to PSRs, and it consists of elements a_i , $i=1,\dots,l$, where l is the number of offshore platforms. The element $a_i = 0$ if the PSR of platform i has not been served yet, and $a_i = 1$ if the PSR of platform i has been served. List B corresponds to information revelation with regard to RSRs, and it consists of elements b_i , $i=0,1,\dots,l$. The element $b_i = 0$ corresponds to no information being revealed when the supply vessel is at offshore platform i . For $i \neq 0$, then $a_i = 0$ if the RSR of platform i has not been served yet, and $a_i = 1$ if the RSR of platform i has been served. The CPLEX IBM LOG solver is used in the computational implementation of the heuristic algorithm.

2.2.6.2. Pseudo code

The computational algorithm may be transcribed as a pseudo code as follows.

Main algorithm.

- 1: vehicle is at depot;

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2:   set  $a_i, a_j$  (list A) parameters in Excel spreadsheet equal to 1 if customers require services;
3:   if priority service is requested for the first stop then
4:     set arch  $x[0][j]$  equal to 1, where  $j$  is the customer that request priority;
5:   end if
6:   run CPLEX model in order to obtain optimised route and write in Excel sheet the destination  $j$ ;
7:   move vehicle to customer  $j$  and perform the service;
8:   while vehicle do not return to depot ( $j \neq 0$ ) do
9:     if customer request to split or interrupt the service then
10:      if customer has already been visited then
11:        no additional visits are allowed on list B;
12:        call function Interaction
13:      else
14:        if customer requires two visits then
15:          set  $b_i, b_j$  (list B) parameter equal to 1: do it manually in Excel
            spreadsheet;
16:          call function Interaction;
17:        else
18:          call function Interaction;
19:        end if
20:      end if
21:    else
22:      call function Interaction
23:    end if

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Sub-algorithm.

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1:   function Interaction
2:     if some random request is placed by customers, then
3:       set  $b_i, b_j$  (list B) parameter equal to 1: do it manually in Excel spreadsheet;
4:     end if
5:     if some priority request is placed by customers, then
6:       set arch  $x[i][j]$  equal to 1: do it manually in Excel spreadsheet (only one request per
            time allowed);
7:     end if
8:     run CPLEX model in order to obtain optimised route and write in Excel sheet the destination  $j$ ;
9:     move vehicle to customer  $j$  and perform the service;

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2.2.6.3. Flowchart

For visualisation purposes, the solution procedure is presented below in flowchart form.

2.2.6.4. VRP model

The formulation is based on the modelling introduced by [9] which allows each customer be visited once or twice, notwithstanding the difference in purpose of the possibility of two visits between [9] and the present work; i.e., in [9], allowance for two visits to a customer is considered necessary whenever pickup and delivery are made on separate visits, whilst in the present work, two visits may be necessary to service PSRs and RSRs at a customer.

UNDER PEER REVIEW

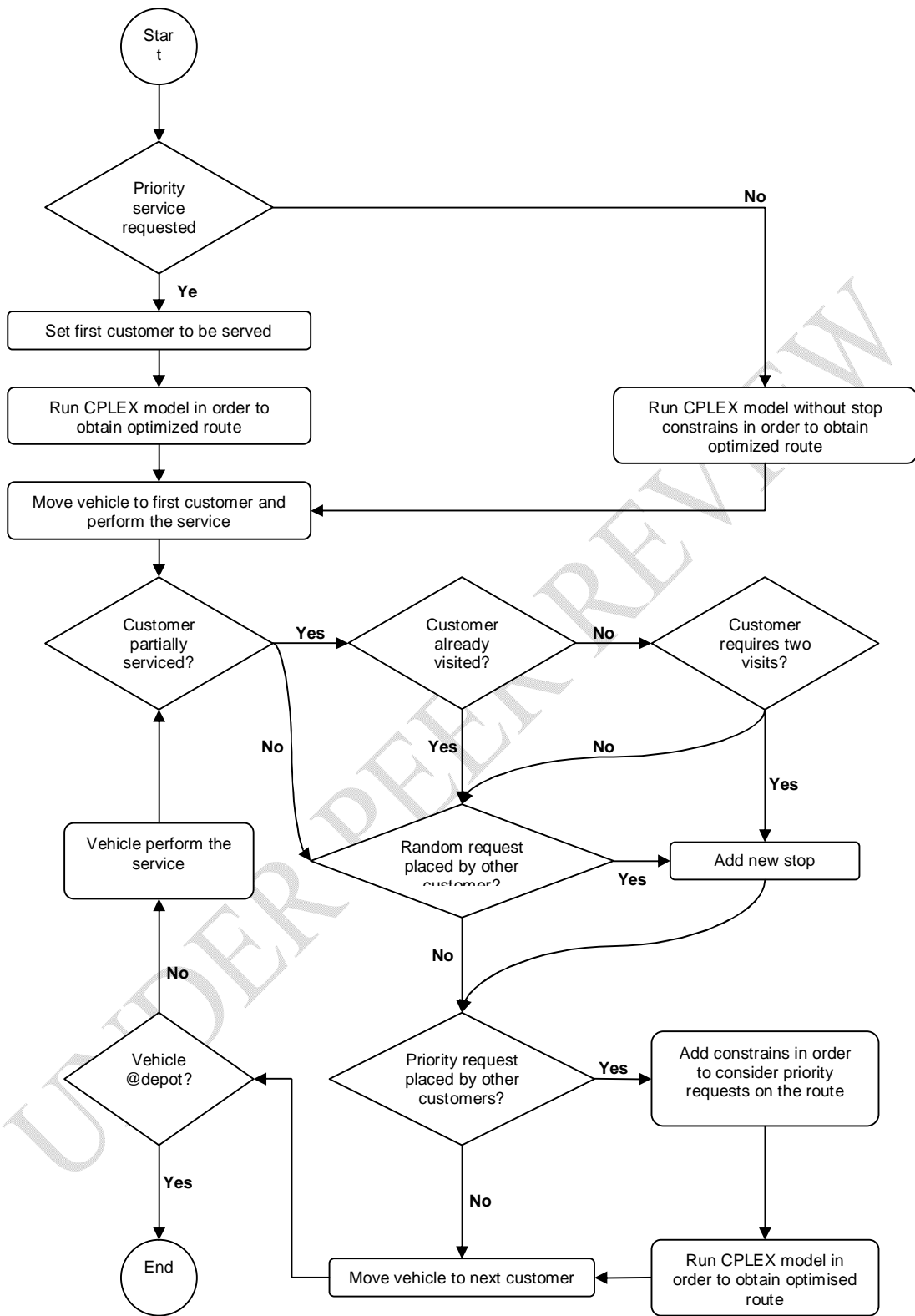


Fig. 4. Solution procedure flow chart.

2.2.6.5. Notation

Parameters

n Number of customers to be serviced;

d_{ij} Distance between customers. Node i and node j ;

a_i Integer parameter which takes value 1 if customer i demands a visit, otherwise takes value 0;

b_i Integer parameter which takes value 1 if customer i demands a second visit, otherwise takes value 0;

Decision variables

x_{ij} Boolean variable which takes values 1, if vehicle travels directly from i to j , and 0, otherwise;

u_i, u_j Sub-route elimination auxiliary variables. Both of them represents the identity of each node itself.

The VRP model may be stated as follows.

(0) Minimise $\sum_i \sum_j d_{ij} x_{ij}$ $i, j = 1, \dots, 2n; i \neq j; j \neq i + n$ if $1 \leq i \leq n; j \neq i - n$ if $n + 1 \leq i \leq 2n$

subject to:

$$(1) \sum x_{ij} = a_j \quad \forall i = 1, 2, \dots, 2n; j = 1, \dots, n; i \neq j;$$

$$(2) \sum x_{ij} = b_j \quad \forall i = 1, 2, \dots, n; j = 1, \dots, 2n; i \neq j;$$

$$(3) \sum x_{ij} = b_{j-n} \quad \forall i = 1, 2, \dots, 2n; j = n + 1, n + 2, \dots, 2n;$$

$$(4) \sum x_{ij} = b_{i-n} \quad \forall i = n + 1, n + 2, \dots, 2n; j = 1, 2, \dots, 2n;$$

$$(5) u_i - u_j + n x_{ij} \leq n - 1 \quad \forall i = 1, 2, \dots, 2n; j = 1, \dots, 2n; i \neq j;$$

$$(6) x_{ij} \in \{0, 1\} \quad \forall i = 1, 2, \dots, 2n; j = 1, \dots, 2n; i \neq j;$$

$$(7) a_i, b_i \in \{0, 1\} \quad \forall i = 1, 2, \dots, n;$$

$$(8) u_i \geq 0 \quad \forall i = 1, 2, \dots, n;$$

2.2.6.6. Description of constraints

Objective function (0) minimises the distance travelled by the supply vessel in its tour in the offshore platforms. Constraints (1) ensure that vehicle arrives just once at the first node of each customer. In order to allow two visits for each customer (node), two nodes are associated for each customer. The second node ($i+n$) is identical to the first (i). For this reason the summation is performed for the interval between $i=1$ and $2n$. constraints (2) ensure that the vehicle leaves just once the first node of each customer. Constraints (3) ensure that vehicle arrives just once at the second node of each customer (in case of a second visit). Constraints (4) ensure that the vehicle leaves just once the second node of each customer (in case of a second visit). Constraints (5) ensure sub-tour elimination, whereby the Miller-Tucker-Zemlin formulation is used. Constraints (6) Domain constraints (6) define the domain of the binary decision variables, which do not allow same indices for i and j in view of the fact that the vehicle must not travel to the same place from where it is coming from. Constraints (7) and (8) define the domain of the parameters.

3. Results and Discussion

Problem instances have been solved using AMD[®] A10 with 2.10 GHz CPU and 8GB of RAM, and the TSP model has been implemented using IBM ILOG CPLEX, which produces precise and logical decisions for planning and resource allocation problems using powerful algorithms, taking advantage of a distributed parallel algorithm for mixed integer programming and flexible, high-performance mathematical programming solvers for linear programming and mixed integer programming.

The supply vessels employed are of the following types: Driving Support Vessels (DSV), Pipe Laying Support Vessels (PLSV), Well Stimulation Vessels (WSV), and Research Vessels (RV).

A hypothetical distance matrix has been created for this purpose. It has been found that large instances (with $n > 48$) can be solved to optimality within a reasonable time as shown in Fig. 3. Moreover, instances with additional visits to all customers do not make much difference to computing time.

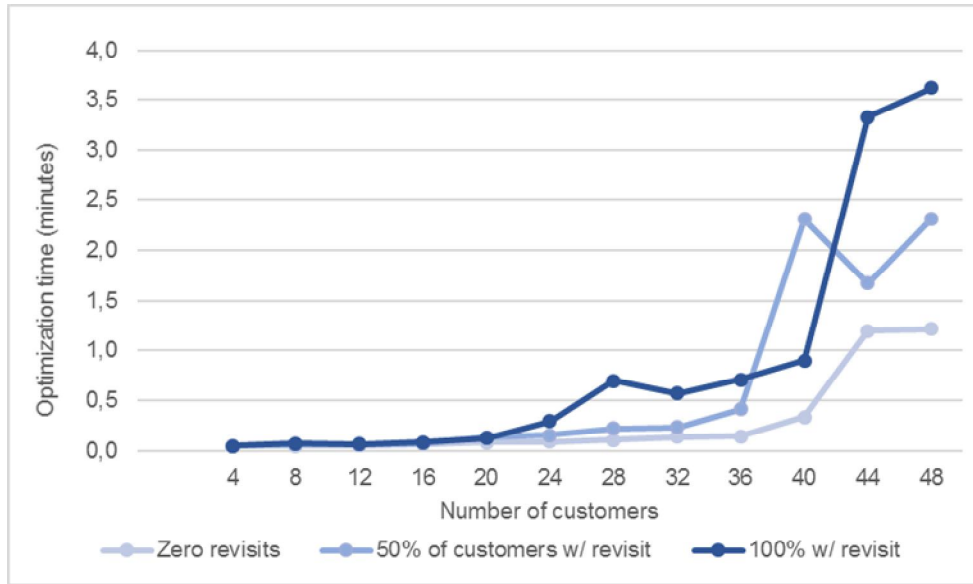


Fig. 5. Computing time vs. number of customers.

As shown in Fig. 5, when the number of customers is greater than 28, the computing time needed to find the optimal solution starts to increase exponentially. That withstanding, for instances with 48 customers in which all customers place requests for additional visits, computing time does not exceed 4 minutes. This time is acceptable for real world cases, as they are unlikely to involve such a large number of platforms.

3.1. Problem instances

In order to evaluate the two-step MILP heuristic, the following parameters will be compared:

- Static route which represents the route that is based solely on PSRs; i.e., the vehicle services for customers in only one visit for each customer as planned, and the route is optimal in that the distance travelled in its tour is minimised.
- Offline route which represents the optimal solution to the solution in which all relevant information (PSRs and RSRs) are assumed to be *hypothetically* known before vehicle departure from base depot.
- Online route which represents the objective function value of the solution generated by the use of the two-step MILP heuristic; i.e., the total time travelled by the vehicle to serve all customer requests (PSRs and RSRs).

The comparisons between these parameters will be made through the two competitive ratios which are defined as follows.

CR1: competitive ratio of the online route distance to the static distance.

CR2: competitive ratio of the online route distance to the offline route distance;

In addition to the CR1 and CR2 parameters, the degree of dynamism (DOD) may be defined as the ratio of the number of RSRs to the sum of the numbers of PSRs and RSRs.

Two sets of problem instances have been studied. The first set is a group of four offshore platforms, to each of which a single supply vessel is allocated. The aforesaid platforms are located in Santos Basin (South Atlantic Ocean Shelf) in Brazil, whose distance matrix (in nautical miles) is shown in Table 1.

Table 1

Distance matrix of of offshore platform 1 (X,Y,Z,W denote platforms)

	Depot	X	Y	Z	W
Depot	0 5	159.89 5	155.53 6	151.93 3	157.30 5
X	159.89 5	0	6.848	9.678	4.062
Y	155.53 6	6.848	0	3.614	2.787
Z	151.93 3	9.678	3.614	0	5.887
W	157.30 5	4.062	2.787	5.887	0

The frequency of random customer service requests for a four platform group is shown in Figure 6 for the period June/2017 - January/2018, from which it may be observed that the number of random service requests decreases rapidly with the number of vehicle tours.

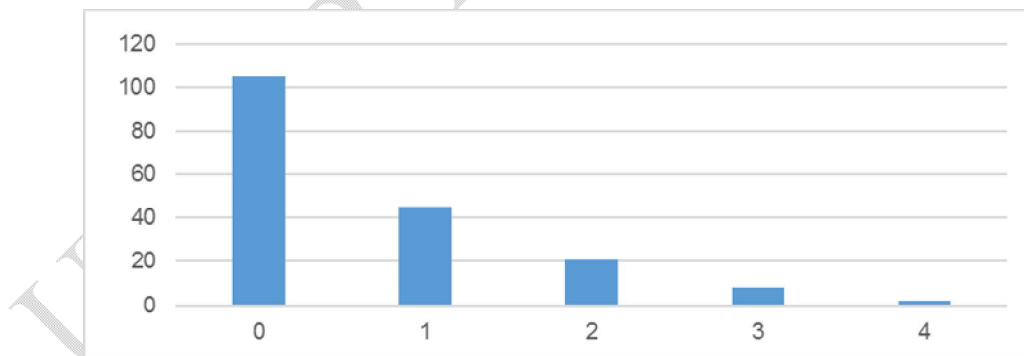


Fig. 6. Number of vehicle tours vs. number of random customer service request.

In Figure7, a typical route of a vessel which is serving these platforms may be visualised. The offshore platform group is located in the Santos Basin in the South Atlantic Ocean Shelf.



Fig.7.Route of a vessel through a four-platforms group.

The second group of offshore platforms that have been studied consists of twelve platforms which are also located in the Santos Basin, and whose distance matrix is summarised in Table 2.

Table 2

Distance matrix of offshore platform group 2 (A,...,L are platforms).

	Depot	A	B	C	D	E	F	G	H	I	J	K	L
Depot	0.00	118.52	159.90	158.00	166.63	135.54	139.72	155.54	151.93	118.79	157.31	175.25	163.92
A	118.52	0.00	43.77	58.54	57.76	23.01	24.78	38.23	34.65	27.93	40.45	67.21	47.54
B	159.90	43.77	0.00	33.92	21.36	24.51	20.19	6.85	9.68	62.12	4.06	28.98	4.04
C	158.00	58.54	33.92	0.00	16.86	36.27	37.06	38.70	38.92	84.46	36.69	20.50	35.20
D	166.63	57.76	21.36	16.86	0.00	34.91	33.26	27.69	29.31	80.44	25.09	9.45	21.11
E	135.54	23.01	24.51	36.27	34.91	0.00	5.20	21.38	18.13	48.21	22.48	44.37	28.56
F	139.72	24.78	20.19	37.06	33.26	5.20	0.00	16.41	13.05	47.85	17.75	42.65	24.20
G	155.54	38.23	6.85	38.70	27.69	21.38	16.41	0.00	3.61	55.38	2.79	35.69	9.73
H	151.93	34.65	9.68	38.92	29.31	18.13	13.05	3.61	0.00	52.50	5.89	37.76	13.05
I	118.79	27.93	62.12	84.46	80.44	48.21	47.85	55.38	52.50	0.00	58.12	89.55	65.01
J	157.31	40.45	4.06	36.69	25.09	22.48	17.75	2.79	5.89	58.12	0.00	32.96	7.16
K	175.25	67.21	28.98	20.50	9.45	44.37	42.65	35.69	37.76	89.55	32.96	0.00	27.74
L	163.92	47.54	4.04	35.20	21.11	28.56	24.20	9.73	13.05	65.01	7.16	27.74	0.00

In Fig. 8, the geographical location of the twelve-platform group is shown.



Fig. 8. Group of twelve platforms.

3.2. Results

The performance of the two-step MILP heuristic proposed in this work is assessed by studying, for each platform group, two aspects: time epoch (platform) at which a random service request is made by a platform, and priority/non-priority random requests. For the four-platform and twelve-platform groups, results of instances selected with these criteria in mind, results are presented in Tables 3-4 and Tables 5-13, respectively.

For the four-platform group, Tables 3 and 4, respectively, show computational results for the case of non-priority requests and priority requests that are placed when the supply vessel is located at platform 2. It may be observed for the maximum value of DOD (0.250), the performance of the two-step MILP heuristic is excellent, as indicated by the fact that CR2 does not exceed 1.0154.

Table 3

Four-platform case - Instances representing additional non-priority visit requests for each platform when vehicle is at the second platform of the online route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	322.291	322.291	322.291	1.0000	1.0000	0.0000	No random requests.
1	322.291	330.414	330.414	1.0252	1.0000	0.250	Non-priority random request placed by platform 1 when vehicle is at 2nd stop.
2	322.291	324.780	324.780	1.0077	1.0000	0.250	Non-priority random request placed by platform 2 when vehicle is at 2nd stop.
3	322.291	324.007	324.007	1.0053	1.0000	0.250	Non-priority random request placed by platform 3 when vehicle is at 2nd stop.
4	322.291	323.763	323.763	1.0046	1.0000	0.250	Non-priority random request placed by platform 4 when vehicle is at 2nd stop.

Table 4.

Four-platform case - Instances representing priority visit requests for each platform when vehicle is at second platform of the online route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	322.291	322.291	322.291	1.0000	1.0000	0.0000	No priority requests
1	322.291	322.291	323.762	1.0046	1.0046	0.0000	Priority random request placed by platform 1 when vehicle is at 2nd stop.
2	322.291	-	-	-	-	-	No priority request is available for platform 2 when the vehicle is on platform 2 itself.
3	322.291	324.007	329.005	1.0208	1.0154	0.250	Priority random request placed by platform 3 when vehicle is at 2nd stop. Additional visit was required to platform 3 since it had already been visited on the 1st stop of the vehicle.
4	322.291	322.291	322.291	1.0000	1.0000	0.0000	Priority random request placed by platform 4 when vehicle is at 2nd stop. But platform 4 would already be the next stop according to the static route.

For the twelve-platform group, Tables 5-13 exhibit computational results, with a view to assessing the effect of the time instant when a RSR is placed and how many such requests are placed. It may be seen that up to the maximum value of DOD (1.000), the value of CR2 does not exceed 1.363. The results show that the performance of the two-step MILP heuristic is excellent, notwithstanding the large number of platforms considered.

Table 5.

Twelve-platform case - Instances representing additional visit (non-priority) requests for each platform when vehicle is at 1st stop of the online route.

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	411.546	411.546	411.546	1.0000	1.0000	0.0000	No random requests.
1	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 1 when vehicle is at 1st stop.
2	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 2 when vehicle is at 1st stop.
3	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 3 when vehicle is at 1st stop.
4	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 4 when vehicle is at 1st stop.
5	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 5 when vehicle is at 1st stop.
6	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 6 when vehicle is at 1st stop.
7	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 7 when vehicle is at 1st stop.
8	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 8 when vehicle is at 1st stop.
9	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 9 when vehicle is at 1st stop.
10	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 10 when vehicle is at 1st stop.

11	411.546	415.832	417.358	1.0141	1.0037	0.0833	Non-priority random request placed by platform 11 when vehicle is at 1st stop.
12	411.546	415.832	437.041	1.0619	1.0510	0.0833	Non-priority random request placed by platform 12 when vehicle is at 1st stop.

Table 6.

Twelve-platform case - Instances representing priority visit requests for each platform when vehicle is at 1st stop of the online route

Instance	Static route (NM)	Offline route (MN)	Online route (NM)	CR1	CR2	DOD	Tour description
0	411.546	411.546	411.546	1.0000	1.0000	0.0000	No random requests.
1	411.546	418.260	418.260	1.0678	1.0000	0.0833	Priority random request placed by platform 1 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
2	411.546	418.260	418.260	1.0678	1.0000	0.0833	Priority random request placed by platform 2 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
3	411.546	418.260	418.260	1.0752	1.0069	0.0833	Priority random request placed by platform 3 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
4	411.546	418.260	418.260	1.1529	1.0797	0.0833	Priority random request placed by platform 4 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
5	411.546	418.260	418.260	1.1655	1.0915	0.0833	Priority random request placed by platform 5 when vehicle is at 12th stop. Two required visits since the platform had

							already been visited when the request occurred.
6	411.546	418.260	418.260	1.1744	1.0999	0.0833	Priority random request placed by platform 6 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
7	411.546	418.260	432.795	1.2796	1.1983	0.0833	Priority random request placed by platform 7 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
8	411.546	418.260	430.233	1.3005	1.2179	0.0833	Priority random request placed by platform 8 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
9	411.546	-	-	-	-	-	No priority request is available for platform 9 when the vehicle is on platform 9 itself.
10	411.546	418.260	444.403	1.0454	1.0286	0.0833	Priority random request placed by platform 10 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
11	411.546	418.260	444.403	1.0516	1.0348	0.0833	Priority random request placed by platform 11 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.
12	411.546	418.260	444.403	1.0798	1.0625	0.0833	Priority random request placed by platform 12 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.

Table 7.

Twelve-platform case - Instances representing additional visit (non-priority) requests for each platform when vehicle is at 6th stop of the online route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	411.546	411.546	411.546	1.0000	1.0000	0.0000	No random requests.
1	411.546	427.910	430.601	1.0463	1.0063	0.0833	Non-priority random request placed by platform 1 when vehicle is at 6th stop.
2	411.546	415.832	415.832	1.0104	1.0000	0.0833	Non-priority random request placed by platform 2 when vehicle is at 6th stop.
3	411.546	439.450	439.450	1.0678	1.0000	0.0833	Non-priority random request placed by platform 3 when vehicle is at 6th stop.
4	411.546	417.358	417.358	1.0141	1.0000	0.0833	Non-priority random request placed by platform 4 when vehicle is at 6th stop.
5	411.546	421.830	425.353	1.0335	1.0084	0.0833	Non-priority random request placed by platform 5 when vehicle is at 6th stop.
6	411.546	418.516	430.324	1.0456	1.0282	0.0833	Non-priority random request placed by platform 6 when vehicle is at 6th stop.
7	411.546	417.119	417.119	1.0135	1.0000	0.0833	Non-priority random request placed by platform 7 when vehicle is at 6th stop.
8	411.546	418.260	418.260	1.0163	1.0000	0.0833	Non-priority random request placed by platform 8 when vehicle is at 6th stop.

9	411.546	448.854	456.798	1.1100	1.0177	0.0833	Non-priority random request placed by platform 9 when vehicle is at 6th stop.
10	411.546	416.606	418.730	1.0175	1.0051	0.0833	Non-priority random request placed by platform 10 when vehicle is at 6th stop.
11	411.546	427.625	427.625	1.0391	1.0000	0.0833	Non-priority random request placed by platform 11 when vehicle is at 6th stop.
12	411.546	418.690	418.690	1.0174	1.0000	0.0833	Non-priority random request placed by platform 12 when vehicle is at 6th stop.

Table 8.

Twelve-platform case - Instances representing priority visit requests for each platform when vehicle is at 6th stop of the online route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	411.546	411.546	411.546	1.0000	1.0000	0.0000	No random requests.
1	411.546	427.910	487.429	1.1844	1.1391	0.0833	Priority random request placed by platform 1 when vehicle is at 6th stop. Two required visits since the platform had already been visited when the request occurred.
2	411.546	411.546	418.729	1.0175	1.0175	0.0000	Priority random request placed by platform 2 when vehicle is at 6th stop. Only one visit was required.
3	411.546	411.546	446.763	1.0856	1.0856	0.0000	Priority random request placed by platform 3 when vehicle is at 6th stop. Only one visit was required.
4	411.546	411.546	449.782	1.0929	1.0929	0.0000	Priority random request placed by platform 4 when vehicle is at 6th stop. Only one visit was required.

5	411.546	421.830	452.617	1.0998	1.0730	0.0833	Priority random request placed by platform 5 when vehicle is at 6th stop. Two required visits since the platform had already been visited when the request occurred.
6	411.546	418.516	442.916	1.0762	1.0583	0.0833	Priority random request placed by platform 6 when vehicle is at 6th stop. Two required visits since the platform had already been visited when the request occurred.
7	411.546	-	-	-	-	-	No priority request is available for platform 7 when the vehicle is on platform 7 itself.
8	411.546	418.260	418.260	1.0163	1.0000	0.0833	Priority random request placed by platform 8 when vehicle is at 6th stop. Two required visits since the platform had already been visited when the request occurred.
9	411.546	448.854	522.265	1.2690	1.1636	0.0833	Priority random request placed by platform 9 when vehicle is at 6th stop. Two required visits since the platform had already been visited when the request occurred.
10	411.546	411.546	411.546	1.0000	1.0000	0.0000	Priority random request placed by platform 10 when vehicle is at 6th stop. Only one visit was required.
11	411.546	411.546	451.163	1.0963	1.0963	0.0000	Priority random request placed by platform 11 when vehicle is at 6th stop. Only one visit was required.
12	411.546	411.546	421.854	1.0250	1.0250	0.0000	Priority random request placed by platform 12 when vehicle is at 6th stop. Only one visit was required.

Table 9.

Twelve-platform case - Instances representing additional visit (non-priority) requests for each platform when vehicle is at 12th stop of the online route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	411.546	411.546	411.546	1.0000	1.0000	0.0000	No random requests.
1	411.546	418.260	418.260	1.0163	1.0000	0.0833	Non-priority random request placed by platform 12 when vehicle is at 1st stop.
2	411.546	418.260	418.260	1.0163	1.0000	0.0833	Non-priority random request placed by platform 12 when vehicle is at 2nd stop.
3	411.546	418.260	418.260	1.0163	1.0000	0.0833	Non-priority random request placed by platform 12 when vehicle is at 3rd stop.
4	411.546	418.260	418.260	1.0163	1.0000	0.0833	Non-priority random request placed by platform 12 when vehicle is at 4th stop.
5	411.546	418.260	418.260	1.0163	1.0000	0.0833	Non-priority random request placed by platform 12 when vehicle is at 5th stop.
6	411.546	418.260	418.260	1.0163	1.0000	0.0833	Non-priority random request placed by platform 12 when vehicle is at 6th stop.
7	411.546	418.260	423.049	1.0280	1.0114	0.0833	Non-priority random request placed by platform 12 when vehicle is at 7th stop.
8	411.546	418.260	430.233	1.0454	1.0286	0.0833	Non-priority random request placed by platform 12 when vehicle is at 8th stop.
9	411.546	418.260	432.795	1.0516	1.0348	0.0833	Non-priority random request placed by platform 12 when vehicle is at 9th stop.
10	411.546	418.260	444.403	1.0798	1.0625	0.0833	Non-priority random request placed by platform 12 when vehicle is at 10th stop.
11	411.546	418.260	444.403	1.0798	1.0625	0.0833	Non-priority random request placed by platform 12 when vehicle is at 11h stop.
12	411.546	418.260	444.403	1.0798	1.0625	0.0833	Non-priority random request placed by platform 12 when vehicle is at 12th stop.

Table 10.

Twelve-platform case - Instances representing priority visit requests for each platform when vehicle is at 12th stop of the online route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	411.546	411.546	411.546	1.0000	1.0000	0.0000	No random requests.
1	411.546	411.546	420.648	1.0221	1.0221	0.0000	Priority random request placed by platform 12 when vehicle is at 1st stop. Only one visit was required.
2	411.546	411.546	424.742	1.0321	1.0321	0.0000	Priority random request placed by platform 12 when vehicle is at 2nd stop. Only one visit was required.
3	411.546	411.546	430.206	1.0453	1.0453	0.0000	Priority random request placed by platform 12 when vehicle is at 3rd stop. Only one visit was required.
4	411.546	411.546	411.546	1.0000	1.0000	0.0000	Priority random request placed by platform 12 when vehicle is at 4th stop. Only one visit was required.
5	411.546	-	-	-	-	-	No priority request is available for platform 12 when vehicle is at 5th stop because the vehicle is on platform 12 itself.
6	411.546	418.260	418.260	1.0163	1.0000	0.0833	Priority random request placed by platform 12 when vehicle is at 6th stop. Two required visits since the platform had already been visited when the request occurred.
7	411.546	418.260	423.049	1.0280	1.0114	0.0833	Priority random request placed by platform 12 when vehicle is at 7th stop. Two required visits since the platform had already been visited when the request occurred.
8	411.546	418.260	430.233	1.0454	1.0286	0.0833	Priority random request placed by platform 12 when vehicle is at 8th stop. Two required visits since the platform had already been visited when the request occurred.
9	411.546	418.260	432.795	1.0516	1.0348	0.0833	Priority random request placed by platform 12 when vehicle is at 9th stop. Two required visits since the platform had already been visited

							when the request occurred.
10	411.546	418.260	469.161	1.1400	1.1217	0.0833	Priority random request placed by platform 12 when vehicle is at 10th stop. Two required visits since the platform had already been visited when the request occurred.
11	411.546	418.260	467.727	1.1365	1.1183	0.0833	Priority random request placed by platform 12 when vehicle is at 11th stop. Two required visits since the platform had already been visited when the request occurred.
12	411.546	418.260	444.403	1.0798	1.0625	0.0833	Priority random request placed by platform 12 when vehicle is at 12th stop. Two required visits since the platform had already been visited when the request occurred.

Table 11.

Twelve-platform case - Instances representing additional visit (non-priority) plus priority visit requests being made when vehicle is at 1st stop of the route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	322.291	322.291	322.291	1.000	1.000	0.000	No random requests
1	322.291	427.910	439.198	1.363	1.026	0.083	Non-priority request: 1; Priority request: N/A;
2	322.291	322.291	500.992	1.554	1.554	0.000	Non-priority request: N/A; Priority request: 1;
3	322.291	475.338	487.602	1.513	1.026	0.500	Non-priority request: 1, 2, 3, 4, 5, 6; Priority request: N/A;
4	322.291	459.950	539.165	1.673	1.172	0.417	Non-priority request: 2, 3, 4, 5, 6; Priority request: 1;
5	322.291	471.052	520.793	1.616	1.106	0.417	Non-priority request: 1, 3, 4, 5, 6; Priority request: 2;
6	322.291	-	-	-	-	-	Non-priority request: 1, 2, 4, 5, 6; Priority request: 3;
7	322.291	469.526	483.309	1.500	1.029	0.417	Non-priority request: 1, 2, 3, 5, 6;

							Priority request: 4;
8	322.291	471.906	524.392	1.627	1.111	0.417	Non-priority request: 1, 2, 3, 4, 6; Priority request: 5;
9	322.291	469.344	527.706	1.637	1.124	0.417	Non-priority request: 1, 2, 3, 4, 5; Priority request: 6;
10	322.291	527.706	544.990	1.691	1.033	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: N/A;
11	322.291	542.256	620.327	1.925	1.144	0.917	Non-priority request: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 1;
12	322.291	531.786	578.029	1.794	1.087	0.917	Non-priority request: 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 2;
13	322.291	-	-	-	-	-	Non-priority request: 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 3;
14	322.291	526.914	542.161	1.682	1.029	0.917	Non-priority request: 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 4;
15	322.291	529.294	581.780	1.805	1.099	0.917	Non-priority request: 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12; Priority request: 5;
16	322.291	526.732	585.094	1.815	1.111	0.917	Non-priority request: 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12; Priority request: 6;
17	322.291	532.212	582.382	1.807	1.094	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12; Priority request: 7;
18	322.291	526.012	573.307	1.779	1.090	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12; Priority request: 8;
19	322.291	504.520	600.276	1.863	1.190	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12; Priority request: 9;
20	322.291	527.666	578.345	1.794	1.096	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12; Priority request: 10;
21	322.291	519.630	531.894	1.650	1.024	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12; Priority request: 11;
22	322.291	527.706	573.346	1.779	1.086	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11; Priority request: 12;

Comment: for the instances 6 and 13 no results are presented because the vehicle is on the requesting platform.

Table 12.

Twelve-platform case - Instances representing additional visit (non-priority) plus priority visit requests being made when vehicle is at 6th stop of the route.

Instance	Static route (NM)	Offline route (Nm)	Online route (NM)	CR1	CR2	DOD	Tour description
0	322.291	322.291	322.291	1.000	1.000	0.000	No random requests
1	322.291	427.910	439.198	1.363	1.026	0.083	Non-priority request: 1; Priority request: N/A;
2	322.291	322.291	476.658	1.479	1.479	0.000	Non-priority request: N/A; Priority request: 1;
3	322.291	475.338	521.505	1.618	1.097	0.500	Non-priority request: 1, 2, 3, 4, 5, 6; Priority request: N/A;
4	322.291	459.950	559.536	1.736	1.217	0.417	Non-priority request: 2, 3, 4, 5, 6; Priority request: 1;
5	322.291	471.052	527.784	1.638	1.120	0.500	Non-priority request: 1, 3, 4, 5, 6; Priority request: 2;
6	322.291	447.434	528.563	1.640	1.181	0.500	Non-priority request: 1, 2, 4, 5, 6; Priority request: 3;
7	322.291	469.526	529.524	1.643	1.128	0.500	Non-priority request: 1, 2, 3, 5, 6; Priority request: 4;
8	322.291	471.906	542.777	1.684	1.150	0.417	Non-priority request: 1, 2, 3, 4, 6; Priority request: 5;
9	322.291	469.344	540.568	1.677	1.152	0.417	Non-priority request: 1, 2, 3, 4, 5; Priority request: 6;
10	322.291	527.706	574.104	1.781	1.088	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: N/A;
11	322.291	542.256	629.353	1.953	1.161	0.917	Non-priority request: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 1;
12	322.291	531.786	580.892	1.802	1.092	1.000	Non-priority request: 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 2;
13	322.291	504.822	580.892	1.802	1.151	1.000	Non-priority request: 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 3;
14	322.291	526.914	583.374	1.810	1.107	1.000	Non-priority request: 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 4;
15	322.291	529.294	590.252	1.831	1.115	0.917	Non-priority request: 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12; Priority request: 5;
16	322.291	526.732	590.163	1.831	1.120	0.917	Non-priority request: 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12; Priority request: 6;
17	322.291	532.212	574.501	1.783	1.079	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12; Priority request: 7;
18	322.291	526.012	568.571	1.764	1.081	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12; Priority request: 8;
19	322.291	504.520	585.672	1.817	1.161	0.917	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12; Priority request: 9;

20	322.291	527.666	-	-	-	-	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12; Priority request: 10;
21	322.291	519.630	581.831	1.805	1.120	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12; Priority request: 11;
22	322.291	527.706	578.511	1.795	1.096	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11; Priority request: 12;

Comment: for the instance 20 no results are presented because the vehicle is on the requesting platform.

Table 13.

Twelve-platform case - Instances representing additional visit (non-priority) plus priority visit requests being made when vehicle is at 12th stop of the route

Instance	Static route (NM)	Offline route (NM)	Online route (NM)	CR1	CR2	DOD	Tour description
0	322.291	322.291	322.291	1.000	1.000	0.000	No random requests
1	322.291	427.910	439.198	1.363	1.026	0.083	Non-priority request: 1; Priority request: N/A;
2	322.291	322.291	439.198	1.363	1.363	0.083	Non-priority request: N/A; Priority request: 1;
3	322.291	475.338	565.292	1.754	1.189	0.500	Non-priority request: 1, 2, 3, 4, 5, 6; Priority request: N/A;
4	322.291	459.950	565.292	1.754	1.229	0.500	Non-priority request: 2, 3, 4, 5, 6; Priority request: 1;
5	322.291	471.052	576.869	1.790	1.225	0.500	Non-priority request: 1, 3, 4, 5, 6; Priority request: 2;
6	322.291	447.434	582.338	1.807	1.302	0.500	Non-priority request: 1, 2, 4, 5, 6; Priority request: 3;
7	322.291	469.526	590.888	1.833	1.258	0.500	Non-priority request: 1, 2, 3, 5, 6; Priority request: 4;
8	322.291	471.906	578.922	1.796	1.227	0.500	Non-priority request: 1, 2, 3, 4, 6; Priority request: 5;
9	322.291	469.344	576.802	1.790	1.229	0.500	Non-priority request: 1, 2, 3, 4, 5; Priority request: 6;
10	322.291	527.706	622.814	1.932	1.180	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: N/A;
11	322.291	542.256	622.814	1.932	1.149	1.000	Non-priority request: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 1;
12	322.291	531.786	641.045	1.989	1.205	1.000	Non-priority request: 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 2;

13	322.291	504.822	630.758	1.957	1.249	1.000	Non-priority request: 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 3;
14	322.291	526.914	640.772	1.988	1.216	1.000	Non-priority request: 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12; Priority request: 4;
15	322.291	529.294	627.342	1.947	1.185	1.000	Non-priority request: 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12; Priority request: 5;
16	322.291	526.732	625.222	1.940	1.187	1.000	Non-priority request: 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12; Priority request: 6;
17	322.291	532.212	628.799	1.951	1.181	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12; Priority request: 7;
18	322.291	526.012	622.814	1.932	1.184	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12; Priority request: 8;
19	322.291	-	-	-	-	-	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12; Priority request: 9;
20	322.291	527.666	634.055	1.967	1.202	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12; Priority request: 10;
21	322.291	519.630	643.255	1.996	1.238	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12; Priority request: 11;
22	322.291	527.706	641.194	1.989	1.215	1.000	Non-priority request: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11; Priority request: 12;

Comment: for the instance 19 no results are presented because the vehicle is on the requesting platform.

The performance of the two-step MILP heuristic may be seen to be excellent in that for DOD values as high as 1.000, CR2 values do not exceed 1.216, as shown in Tables 10 and 11. In other words, even for high values of DOD, which are rarely encountered in industrial practice as shown in Fig. 4, CR2 values are not far from unity. In this manner, the two-step MILP heuristic ensures a high service level for customers under highly dynamic conditions entailed by a large number of random service requests in comparison with the number of planned service requests.

4. Conclusion

The problem variant of the single supply vessel serving a group of offshore platforms, an important task in the upstream logistics chain in the oil and gas industry, has been formulated within the framework of the SDVRP. The aforesaid problem variant has been solved employing a two-step MILP heuristic for a real-world case study based on industrial practice in the Atlantic Ocean South Shelf.

There are several extensions and generalisations of the problem variant problem studied in this work, which merit consideration in future research work. These include the following.

1) Finite storage capacity limitations in supply vessels as well as offshore platforms, in addition to the incorporation of time window requests, thus extending the work reported in [9] to allow for random service requests.

- 2) Multiple supply vessel allocation to groups of offshore platforms, with a view to extending the work reported in [17] to allow for random service requests in real time.
- 3) Supply vessel breakdown during its tour in the offshore platform group to which it is allocated, with a view to incorporating fault tolerance in the SDVRP, and thus extending the work reported in [33] to allow for random service requests.

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COMPETING INTERESTS

The authors declare that there are no financial and personal relationships with other people or organizations that could inappropriately influence (bias) their work.

AUTHORS' CONTRIBUTIONS

The author RYQ contributed to problem definition and mathematical model development. The author SK contributed to mathematical model development and computational implementation. Both authors RYQ and SK worked on the conclusions of the work.

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