Temperature dependence on transienthydrodynamicgravitydrivenflowdownaninclinedplane

Abstract

Transient hydrodynamic gravity driven flow of a viscous incompressible fluid down an inclined plane occupying a semi-infinite region of space bounded by an infinite plate is studied. The effect of modified Froude number plays an important role in determinng the flow situation. Exact solution is obtained by employing Laplace transform method. This problem is concerned with traffic flow with a decisive importance to a variation of depth subjected to a stop leading to a flood wave. A reducible influence of a frictional drag has an active influence to the flow reversal characteristics with time variation satisfying the separation due to the existance of a modified Froude number. The stability of a roll wave is determined by the critical Froude number in a time varying flow. Numerical results of velocity distributions and frictional shear stress are depicted graphically for different values of flow parameters. Numerical results of a critical Froude number are presented in tables. Temperature analysis has been presented analytically with a view to point out the expression of Nusselt number at the plate.

Keywords

Gravity driven flow, Modified Froude number, Traffic flow, Flood wave, Frictional drag, Laplace transform method.

1. Introduction

Resonance exerts its influence of a controlled thermonuclear fusion reaction of the Sun subjected to a periodic driving force which oscillates harmonically with time with reference to a resonant condition $\omega > \frac{1}{2}\, Cos\omega T\,(16K^4 - M^4Sin^4\theta)^{\frac{1}{2}}\omega > \frac{1}{2}\, cos\omega T\,(16K^4 - M^4sin^4\theta)^{\frac{1}{2}}$where M^2 is the Hartmann number, K^2 is the rotation parameter which is the reciprocal of Ekman number, ω is the excitation frequency, ωT is the phase angle and θ is the angle of inclination of a magnetic field with the positive direction of the axis of rotation. A magnetic mirror with the Sun corresponds to an inclined magnetic field to exhibit strong ionizing radiation inside the solar atmosphere. Plasma fusion in a controllable region in the presence of a strong magnetic field communicateshottest electron spiraling in a magnetic field. A controlled thermonuclear fusion reaction of the Sun interacts with radio emission in the presence of a magnetic mirror so that the hottest charged particle is liberated from the Sun with the formation of cloud particle and the fusion reaction starts with the charged particle to begin artificial rain fall initiated by the Sun. This implicates the situation of a rotating environment that inscribed by a strong ionizing radiation in the atmosphere. This has been studied by Ghosh [1], Ghosh et al. [2] and Ghosh [3 -5].A controlled thermonuclear fusion reaction of the Sun at the resonant level appears to an oceanic circulation with the swollen of sea water in a periodic balance and the movement of sea water in a haphazard direction due to a gravitational pull attracted by the Sun as the Earth is put into a vacuum so that the gravity of the Moon becomes unpredictable as soon as fusion reaction starts. In such a case, the movement of sea water due to periodicity of the Sun is subjected to a charged particle at the resonant level that becomes a strong evidence of the existence of magnetic field in the deep ocean. A periodic behaviour of a river at the resonant level of the Sun corresponds to a river bed rolling like an oceanic circulation. This concept plays an important role on the behaviour of a traffic flow. Thus, the roll wave is propagated in a river with a periodic behaviour that depends on time. A representation of a Froude number appears to be a significant manner as the gravitational force is balanced by the viscous force to determine the stability on roll wave. The traffic flow is subjected to a flood wave on a river bed to ensure a pioneer book which was written by Whitham [6].The importance of a flood wave is represented in a manner down an inclined plane subjected to a slope with depth variation. Nevertheless, a few of references has been highlighted by the authors to describe the structure of a flow behaviour through an inclined plane by mentioning their works of Balmforth [7], Chamkha et al [8] Chen etal. [9-10], Fan etal [11],Fujii and Imura [12], Gudhe et al. [13], Ghosh and Pop [14] and Beg et al [15].

An attempt has been made to a study of car flowing theory to incorporate a traffic flow with a view to analyze a gravity driven flow with reference to a pressure gradient down an inclined plane where the effect of Froude number becomes relevant to the depth variation. A time dependent motion down an inclined plane with a decisive importance to gravity does over the flow medium which is balanced by the viscous force to initiate thermal radiation. The equation of state is balanced by the energy equation subject to a modified pressure gradient to find out the modified Froude number. A stability criterion of roll wave takes place in a critical Froude number which depends on time. The frictional shear stress on the plane exerts its influence of Froude number so that the rolling friction down an inclined plane with the angle of inclination reveals to a gravity driven flow in the presence of a viscous force. This problem is solved to find out the solutions of the velocity distributions, frictional shear stresses and the critical Froude number by employing Laplace transform method.An analytical approach on the energy equation satisfying the temperature boundary condition takes place of the rate of heat transfer at the surface of the plane subject to the angle of inclination of a plane which makes an angle $\alpha$to the horizon. The importance of a study of this problem lies in its application of a geophysical flow such as oceanic circulation, atmospheric signal observation, stability criteria of roll wave and flood wave observation.

2.Formulation of the problem and its solution

Consider the unsteady flow of a viscous incompressible fluid down an inclined plane at an angle α to the horizon occupying a semi infinite region of space bounded by infinite plate moving with a constant velocity U in its own plane. The x'-axis is oriented along the plate and y' - axis is perpendicular to the plate. Since a constant pressure gradient is applied along x'-direction, all physical quantities except pressure will be functions of y' and t only. The flow is considered fully developed into the semi-infinite region of space so that all inertia terms vanish. There arise a slope with a decisive importance to the variation of depth subjected to a time variation to represent a traffic flow problem. This situation reveals to a flood wave with reference to the variation of depth.

The momentum equation under Boussinesq approximation,

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty)\sin\alpha \quad (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y'} - g\beta(T' - T'_\infty)sos\ \alpha\ . \quad (2)$$

Comments:

1- "Authors can apply this table for all mathematical equations"
2- I think no need they use first letter of "sin" and "cos" as capital letter.
3- I think using u' and x' and etc. will be confusing. Better they replace u_1 and x_1 and … instead of them.

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty)Sin\ \alpha \quad (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y'} - g\beta(T' - T'_\infty)Cos\ \alpha \quad (2)$$

The energy equation reads

$$\frac{\partial T'}{\partial t} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}. \quad (3)$$

The equation of state becomes

$$\rho = \rho_0[1 - \beta(T' - T'_\infty)]. \quad (4)$$

The initial and the boundary conditions for velocity become,

$$u' = 0 \quad for\ all\ y' \geq 0\ and\ t \leq 0,$$

$$u' = U \quad \text{at } y' = 0\ for\ t > 0, \quad (5)$$

$$u' \to 0 \quad as\ y' \to \infty\ for\ t > 0$$

Combining Equationsequations(3) and (4)with reference to differentiation with respect to$t$'t', the following condition is satisfied.

$$T' - T'_\infty = \frac{1}{\beta} \quad (6)$$

Using Equation (2), we have

$$p = (h - y')\rho g\ Cos\alpha \quad (7)$$

From Equation (7), we get

$$\frac{dp}{dx'} = \rho g \frac{dh}{dx'} \, Cos \propto \qquad (8)$$

Let is assume that the depth variation along$x' - \, direction \; i.e \frac{dh}{dx'} = F \; (say)$.

Comments:

1- All "Equation" and "Equations" in the text need to change "equation" and "equation". (No need you use Capital letter for equation.)

2- When your new line starting with Capital letter don't forget to use stop point "." before it.

Finally, Equationequation(8) becomes,

$$\frac{dp}{dx'} = \rho g F Cos \propto \qquad (9)$$

Using Equations (6) and (9), Equation (1) reads,

$$\frac{\partial u'}{\partial t} = \nu \frac{\partial^2 u'}{\partial y'^2} + g(Sin\alpha - FCos\alpha) \qquad (10)$$

u', ρ, $\nu, g, \beta, h, p, t, K, C_p, T'$, T'_w, T'_∞ and α are respectively, the velocitycomponent, fluid density, kinematic coefficient of viscosity, gravitational acceleration, coefficient of thermal expansion, depth, pressure, time, thermal conductivity, specific heat of constant pressure, temperature of the fluid, plate temperature, the temperature far away from the plate and the angle of inclination of the plate.

Introducing dimensionless quantities

u $=\frac{u'}{U}$ (Velocity), y $=\frac{y'U}{\nu}$ (distance), τ $=\frac{tU^2}{\nu}$ (time) and T$=\frac{T'-T'_\infty}{T'_w-T'_\infty}$ (temperature). (11)

Equation (1) can be represented in a dimensionless form reads

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial y^2} + \frac{1}{F_r^2}(Sin\alpha - F \; Cos \; \alpha) \qquad (12)$$

where $F_r = \frac{U^{3/2}}{\sqrt{g\nu}}$ is the modified Froude number.

The corresponding dimensionless velocity boundary conditions become

$$u = 0 \qquad for \; all \; y \geq 0 \; and \; \tau \leq 0,$$

$$u = 1 \qquad \text{at } y = 0 \; for \; \tau > 0, \qquad (13)$$

$$u \to 0 \qquad as \; y \to \infty \; for \; \tau > 0$$

Using Laplace transform method, the Equation 12 takes the form

$$su^* = \frac{\partial^2 u^*}{\partial y^2} + \frac{1}{F_r^2}(Sin \propto \ -F\ Cos \propto)\frac{1}{s} \qquad (14)$$

The corresponding velocity boundary conditions are

$$u^* = 0 \qquad for\ all\ y \geq 0\ and\ \tau \leq 0,$$

$$u^* = \frac{1}{s} \qquad \text{at } y = 0\ for\ \tau > 0, \qquad (15)$$

$$u^* \to 0 \qquad as\ y \to \infty\ for\ \tau > 0$$

Equation (14) together with the boundary conditions(15) can be solved and the solution becomes

$$u^* = \frac{1}{s}e^{-y\sqrt{s}} - \frac{1}{F_r^2}(\text{Sin } \alpha - \text{F Cos } \alpha)\frac{1}{s^2}\left[e^{-y\sqrt{s}} - e^{-2y\sqrt{s}}\right] \quad (16)$$

Equation (16) subject to the boundary condition (15), the solution for the velocity distributions can be obtained by applying inverse Laplace transform method such as

$$u(y, \tau) =$$

$$erfc\left(\frac{y}{2\sqrt{\tau}}\right) + \frac{1}{F_r^2}(Sin\alpha - FCose\alpha)\left[(\tau + 2y^2)\ erfc\left(\frac{y}{\sqrt{\tau}}\right) - 2y\sqrt{\frac{\tau}{\pi}}\ e^{-\frac{y^2}{\tau}} - \left(\tau + \frac{y^2}{2}\right)\ erfc\ \left(\frac{y}{2\sqrt{\tau}}\right) + y\sqrt{\frac{\tau}{\pi}}\, e^{\frac{y^2}{4\tau}}\right] \qquad (17)$$

Frictional shear stress τ_x at the plate y = 0 $y = 0$ becomes

$$\tau_x = \frac{du}{dy}\Big|_{y=0} = -\frac{1}{\sqrt{\pi\tau}} - 2\sqrt{\frac{\tau}{\pi}}\frac{1}{F_r^2}(Sin \propto -F\ Cos \propto) \qquad (18)$$

The point of separation is determined by the condition

$$\frac{du}{dy}\Big|_{y=0} = 0 \qquad (19)$$

Hence, from Equations (18 – 19), we get the critical Froude number with a decisive importance to the stability of roll wave.

$$F_r^2\big|_{critical} = 2\tau\ (F\ Cos\ \alpha - Sin\ \alpha) \qquad (20)$$

Temperature analysis:

The temperature profile in the fluid region

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty} = \left(1 - \frac{y}{1+y}\right)^2 \qquad (21)$$

Equation (21) can be written in the form

$$\psi = \psi_w \left(1 - \frac{y}{1+y}\right)^2 \qquad (22)$$

where $\psi = T' - T'_\infty$ and $\psi_w = T'_w - T'_\infty$

The temperature boundary conditions become

$$\psi = \psi_w \quad at\ y = 0, \qquad (23)$$

$$\psi = 0\ at\ y \to \infty$$

The heat transfer coefficient is determined by the equation

$$\propto = -\frac{K}{T_w - T_\infty}\left(\frac{dT}{dy}\right)_{y=0} \qquad (24)$$

From Equation (22), it follows that

$$\frac{dT}{dy} = -\frac{2}{(1+y)^3}\psi_w \qquad (25)$$

and $\frac{dT}{dy}|_{y=0} = -2\psi_w$ (26)

Substituting Equation (26) to Equation (24), we have

$$\alpha = 2K \qquad (27)$$

Therefore, a relation can be obtained by the Equation (27) to satisfy the heat transfer coefficient at the plate.

From Equation (3) subject to dimensionless quantities (11) reads

$$\frac{\partial T}{\partial \tau} = \frac{1}{P_r}\frac{6}{(1+y)^4} \qquad (28)$$

where P_r *is the Prandtl number*

Integrating Equation (28), we get

$$T = \frac{1}{P_r}\frac{6\tau}{(1+y)^4} + C \qquad (29)$$

where C is arbitrary constant

Boundary condition at the plate

T=1 at y=0 (30)

From Equations (29) and (30), we have

$$C = 1 - \frac{1}{P_r}6\tau \qquad (31)$$

Using Equation (31), the Equation (29) takes the form

$$T(y,\tau) = 1 - \frac{6\tau}{P_r}\left[1 - \frac{1}{(1+y)^4}\right] \qquad (32)$$

Therefore, Nusselt number at the plate

$$-\frac{dT}{dy}\Big|_{y=0} = 24\frac{\tau}{P_r} \qquad (33)$$

3 DISCUSSIONS OF RESULTS:

3. Discussions of Results

A physical interpretation takes place of a gravity driven flow down an inclined plane leading to the variation of depth with a decisive importance to a traffic flow followed by a roll wave that exerted by the river bed with a significant effect of circulation of a river in a periodic in nature to determine a stability criteria down an inclined plane. Since Froude number is the measure of gravity, critical Froude number determines the stability criteria that depends on time with the angle of inclination of a plane subjected to the variation of depth. Numerical results are depicted graphically for velocity distributions and frictional shear stresses by using flow parameters in taking into account of various values of F_r(Froude number), α (angle of inclination of a plane), τ (time) and F (variation of depth) in Figures 1 to 7. Numerical results of a critical Froude number (F_r) are presented in Table I and II for different values of α, τ and F. It is evident from Figures 1 to 5 that the profiles are skewed in nature and all the profiles converge quickly near the plate. This happens due to a gravity driven flow so that Froude number plays an important role on the flow behaviour with the skewness of all the profiles. Figures 1 and 2 show that the velocities decrease with the increase in small as well as large values of F_r (Froude number). Since gravitational force is balanced by viscous force, the effect of Froude number (F_r) leads to fall the velocity down an inclined plane to deaccelerate the flow particle and dies out near the plate. Since viscosity offers a resistance to its motion, the modified Froude number (F_r) has an active influence to the motion of a river flow. It is convenient to cast that the stability structure of a roll wave is determined by the modified Froude number. It is seen from Figure 3 that the velocity increases with the increase in depth variation (F). since depth varies linearly along the leading edge of the plane, a gravity driven flow tends to increase the velocity down an inclined plane and it converges near the plate. There is a tendency to fluctuate the flow particle on increasing depth variation (F). It is observed that the flow circulation provides a flood wave to represent fluctuation on increasing depth variation (F). Figure 4 demonstrates that the velocity decreases with the increase in angle of inclination (α) of the plane of flow. An occurrence of fluctuation reveals to a gravity driven flow with a decisive importance to the Froude number on increasing α. An interesting situation appears to exert its influence of steady of steady motion where no fluctuation is observed when α = $\frac{\pi}{2}$. It is evident from Figure 5 that the velocity increases with the increase in time (τ). This happens in the case of flow field that a car flowing theory is subjected to a traffic flow where the time variation represents to speed up the flow velocity and the fluctuation occurs due to time variation with reference to a slope so that velocity increases with time. Figure 6 reveals that the frictional shear stress (τ_x) down an inclined plane decreases in magnitude with increase in Froude number (F_r) while it increases in magnitude with increase in time (τ) whereas there exists a separation close to the plane of flow. In a realistic situation, a reducible influence on frictional drag communicates with the Froude number (F_r) to determine separation of flow. It is noticed from Figure 7 that all the profiles are almost linear in nature. The frictional shear stress (τ_x) decreases in magnitude with the angle of inclination (α) whereas the frictional shear stress increases in magnitude with the increase in time (τ) in the range $0 \leq \propto \leq {}^{\pi}/_{4}$ but the total separation of flow occurs with the increase in α in the range ${}^{\pi}/_{3} \leq \propto \leq {}^{\pi}/_{2}$. Here frictional drag is reduced with the angle of inclination (α) so that flow separation becomes predominant. Table 1 shows that the critical Froude number (F_r^2) increases with increase in depth variation (F) for any value of time (τ) whereas it decreases with increase in time (τ) for any value of depth variation (F). This indicates that the stabilization of flow in a time varying motion is characterized by the Critical Froude number (F_r^2) so that a gravity driven flow has become a critical influence to a Froude number. It is notice from Table 2 that the critical Froude number (F_r^2) increases with increase in time (τ) while it decreases with increase in angle of inclination (α) in the range $0 \leq \propto \leq {}^{\pi}/_{3}$. The stabilization of flow behaviour is determined by the Critical Froude Number (F_r^2) in a time varying flow. On the other hand, the Critical Froude Number (F_r^2) does not exist due to flow reversal when the angle of inclination $\alpha = \frac{\pi}{2}$ because negative value for $\alpha = {}^{\pi}/_{2}$ at different values of time (τ) should not appear in the case of the critical influence of a Froude number (F_r^2). Indeed, the importance of a stability on roll wave plays a significant role in determining the effect of Froude number (F_r^2).

4.Conclusion

A gravity driven flow of a viscous incompressible fluid down an inclined plane with the variation of depth subjected to a slope leading to a traffic flow with time variation emerges the backbone of a flood wave. A modified Froude number has an active influence to the flow field with an occurrence of a fluctuation that leads to an instability pattern with the time variation of a flood wave. This problem is solved by employing Laplace transform method. A reducible influence on frictional drag introduces flow reversal criteria with time variation satisfying with separation due to the existence of a modified Froude number. The stability on roll wave is characterized by the critical Froude number in a time varying flow.

Comments:

1- Quality of "Figures" are very low. They can replace high quality Figures.

Table -1Critical Froude Number(F_r^2) for $\alpha = {}^{\pi}/_{4}$

F \ τ	0.1	0.5	0.7	1.0
2.0	0.14116	0.70583	0.98816	1.41166

3.0	0.28250	1.41251	1.97751	2.82502
5.0	0.56517	2.82587	3.95622	5.65174
7.0	0.84784	4.23923	5.93492	8.47847

Table -2Critical Froude number(F_r^2) for F=2.0

α \ τ	0.1	0.5	0.7	1.0
0	0.400	2.0	2.800	4.00
$\pi/6$	0.24626	1.23130	1.72382	2.46260
$\pi/4$	0.14116	0.70583	0.98816	1.41166
$\pi/3$	0.02643	0.13218	0.18505	0.26436
$\pi/2$	- 0.200	- 1.00	- 1.40057	- 2.00082

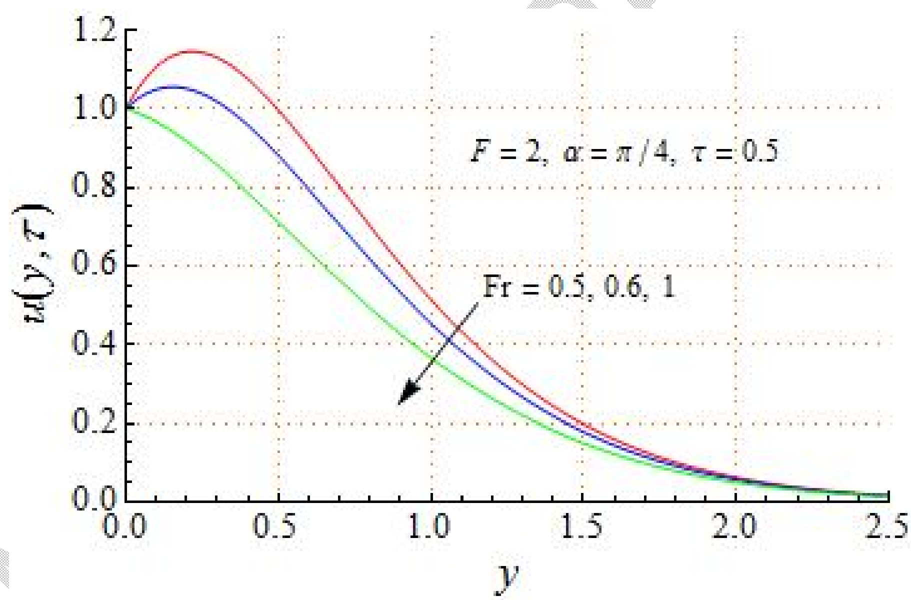

Figure 1. Velocity profile on varying F_r (small)

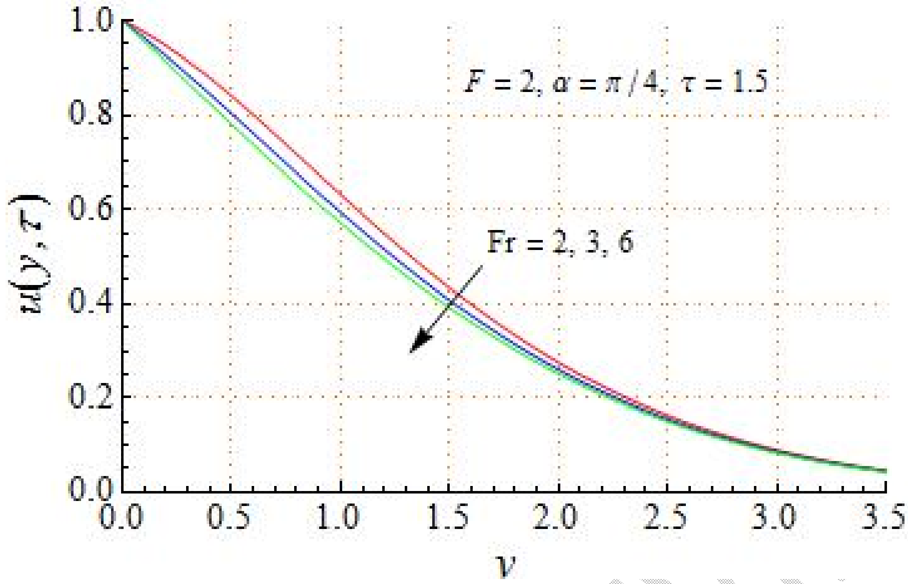

Figure 2. Velocity profile on varying F_r (large)

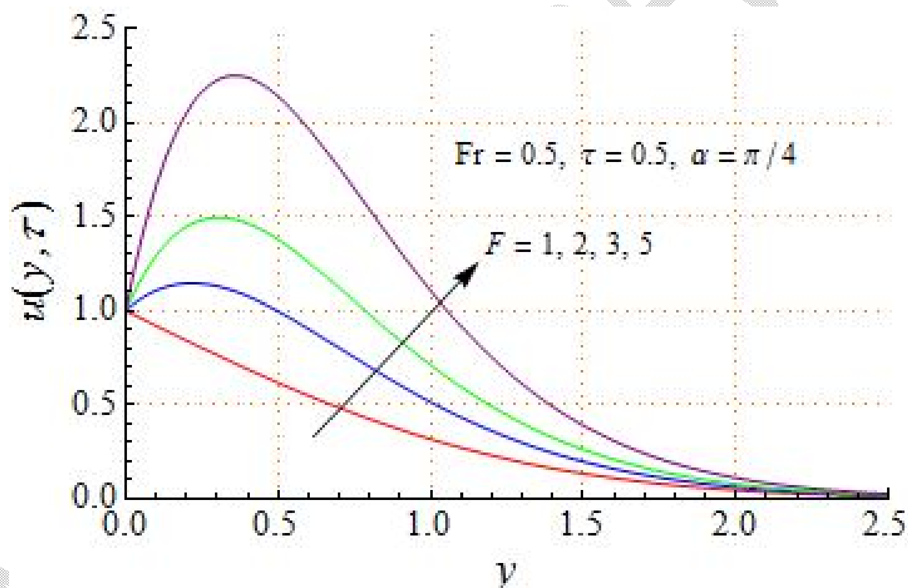

Figure 3. Velocity profile on varying F

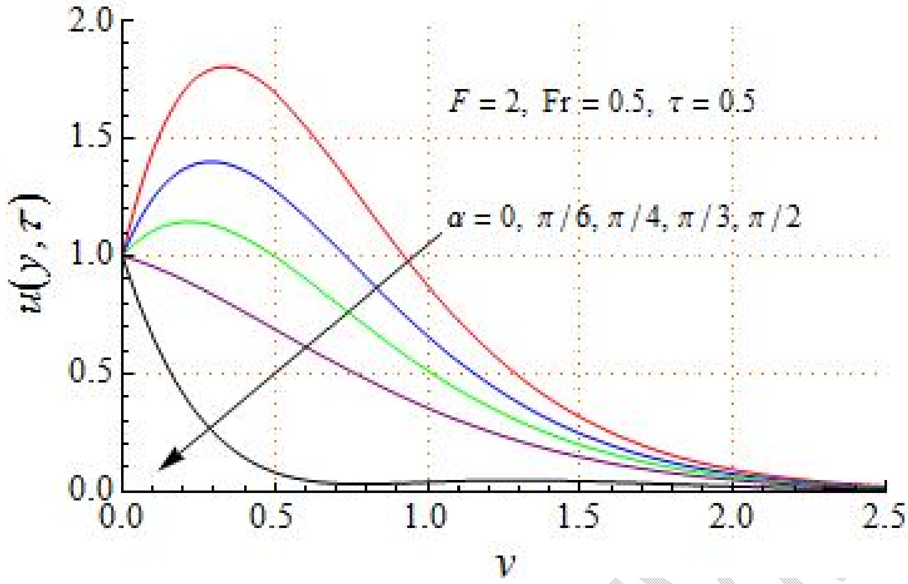

Figure 4. Velocity profile on varying α

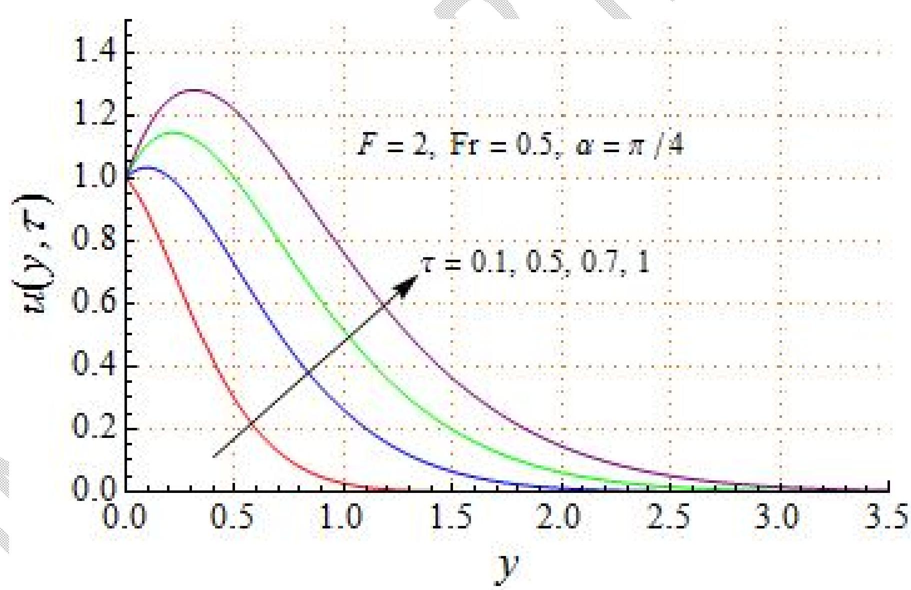

Figure 5. Velocity profile on varying time τ

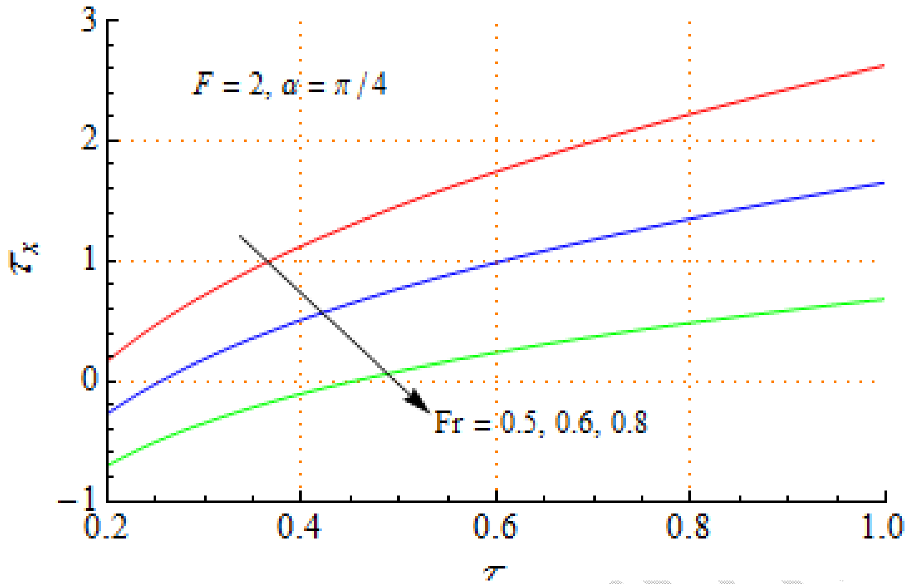

Figure 6. Variation of shear stress on varying F_r and time τ

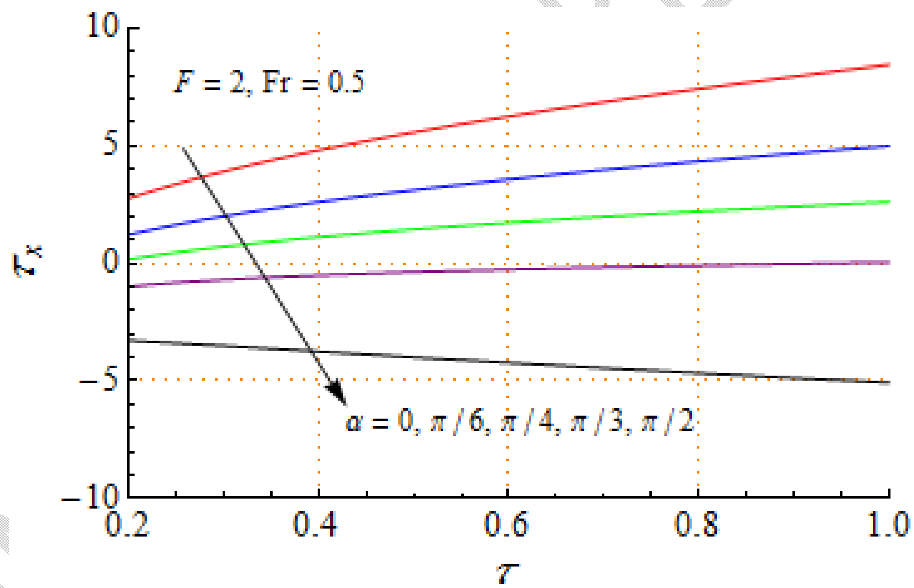

Figure 7. Variation of shear stress on varyingα and time τ

REFERENCES:

Comments:

1- I cannot see any newly published paper inside them references list! Just I can see one paper from 2018! Authors need to update the Reference list.

References

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inclined magnetic field in the presence of an oscillator, Czech. J.Phys., 2001, 51, 799 – 804.

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