

# The wealth effects on demand for insurance under ambiguity aversion

## Abstract:

Under which condition does the optimal insurance demand decrease in wealth? In the expected utility model a decreasing concavity condition is necessary and sufficient for this result. However in general, the result does not hold under ambiguity aversion. By introducing several types of controlling relation about random loss variables, we constrained the structure of the ambiguity and obtained several unambiguous results. The results show that under these constraints the demand for insurance against ambiguous losses also decreases in wealth.

**JEL Classification:** D81, G22

**Key words:** Decreasing aversion; Smooth ambiguity aversion; Decreasing concavity; Optimal insurance demand

## 1. Introduction

Insurance provides risk-averse consumers with protection against many kinds of risks such as damages to property, liability exposures, health care costs, and crop income loss etc. A large number of documents have studied the factors influencing insurance demand (e.g. Schlesinger, [15]; Lee [12]; Richard Peter and Jie Ying [14]). The classical model considers a risk-averse agent that aims to ensure wealth against a possible loss. These analyses focus mainly on the optimal coinsurance rate that can maximize the expected utility of wealth. In economics, one of the prevailing assumptions is that wealthier people are less risk-averse. This property is called decreasing aversion (Cherbonnier and Gollier [5]).

There are many definitions of decreasing aversion in documents. For instance, an agent is called to be of decreasing aversion type if any risk is not preferred at a specific level of wealth, then it is also not preferred at any lower level of wealth. The second definition is that the demand for risky assets increases with the initial wealth in the portfolio selection problem of risk-free and risky assets. A similar definition of decreasing aversion in the insurance demand problem is that demand for insurance is a decreasing function of initial wealth.

Since Arrow [1] assumes decreasing absolute risk aversion, many studies have verified this supposition by means of experimental (Levy [13]; Guiso and Paiella [10]) and econometric methods (Bar-Shira et al. [2]). As Gollier [9] shows, this generally accepted characteristic of individual risk preference plays a vital role in many applications of expected utility theory. Risk aversion is capital to capture the mechanism of individual choices about insurance, portfolio accumulation. Many studies extend these models by considering labor income (such as Viceira [16], Coco

et al. [7]) or housing (Coco [6]), or modifying preferences with persistent habits (Brunnermeier and Nagel [3]) or ambiguity aversion (Campanale [4]; Cherbonnier and Gollier [5]). However, to some extent, how risk aversion shapes individual insurance demand decisions is still worth considering.

For the KMM decision-making criteria under fuzzy conditions, we determine the condition that the rich have low aversion to uncertain losses.

In most cases, the exact distribution of random loss is not completely known, and is called ambiguous. In this study, we investigate the property of decreasing aversion in the context of ambiguity aversion in the case of insurance demand. For the decision criteria of KMM under ambiguity, this study discusses the condition under which the wealthier agent demands less insurance.

## 2 The Model

We now introduce the decision model for insurance demand. We consider a decision maker with initial wealth  $\varpi$ , who is subject to a loss of  $L \in (0, \varpi)$ , and can purchase coinsurance against the risk of loss. The insurance premium is  $p$ , where  $p > EL = l$ . Let  $\alpha$  be the co-insurance rate. In the expected utility model, this decision problem can be represented as

$$\max_{\alpha} Eu(\varpi - (1 - \alpha)L - \alpha p). \quad (1)$$

For the convenience of the discussion, we state the following two definitions.

**Definition 1** We say that an agent is Decreasingly Averse if a reduction in wealth can never lead to more insurance demand against unendurable random loss.

**Definition 2.** A function  $f : R \rightarrow R$  satisfies the decreasing concavity (DC) if  $-f'' / f'$  is non-increasing.

It is obvious that the  $f$  DC means that there exists a concave function  $g$  such that  $-f' = g \circ f$ .

**Definition 3.** We say that the uncertain loss  $L_i$  dominates  $L_j$  in the sense of Jewitt if the following condition is satisfied: if  $L_i$  is weakly more endurable than  $L_j$ , for all increasing and concave  $u$ , then  $L_i$  is weakly more endurable than  $L_j$  for all agents more risk averse than  $u$ . This is denoted by  $L_j \prec_j L_i$ .

Suppose that the accurate distribution function of the random loss is known to the insurer and consumer. In Model (1), it is easy to prove that insurance demand

decreases with initial wealth if and only if  $u$  satisfies decreasing concavity. Because the objective function is concave in  $\alpha$ , this requires proving that the cross-derivative of the objective function in (1) with respect to  $\alpha$  and  $\varpi$  evaluated at the optimal  $\alpha$  is negative. The first-order condition of the above program yields

$$E(L-p)u'(\varpi-(1-\alpha)L-\alpha p)=0.$$

Because  $u$  DC implies that  $-u'$  is a concave function  $g$  of  $u$ , we can have

$$\begin{aligned} & E(L-p)u''(\varpi-(1-\alpha)L-\alpha p) \\ & = -E(L-p)g'(u(\varpi-(1-\alpha)L-\alpha p))u'(\varpi-(1-\alpha)L-\alpha p) \\ & \leq -g'(u(\varpi-p))E(L-p)u'(\varpi-(1-\alpha)L-\alpha p)=0. \end{aligned}$$

The inequality above holds because that for all  $p-l > 0$ ,

$$(p-l)g'(u(\varpi-(1-\alpha)l-\alpha p)) < (p-l)g'(u(\varpi-p)).$$

Thus, in the expected utility model, if and only if  $u$  is decreasing concavity, then the demand for insurance decreases with initial wealth.

Now, suppose that the distribution of loss  $L$  is ambiguous. This ambiguity is characterized by  $n$  possible random variables  $(L_1, L_2, \dots, L_n)$ . Under the KMM smooth ambiguity framework the decision problem becomes

$$\alpha^*(\varpi) = \arg \max_{\alpha} E\phi(Eu(\varpi-(1-\alpha)L-\alpha p)). \quad (2)$$

So, the first order condition can be expressed by

$$E[\phi'(Eu(\varpi-(1-\alpha)L-\alpha p))E(L-P)u'(\varpi-(1-\alpha)L-\alpha p)]=0. \quad (3)$$

Next, we investigate the condition under which the coinsurance demand decreases in initial wealth. It is true that the objective function in (3) is concave in  $\alpha$ , so decreasing aversion property is satisfied if and only if

$$\begin{aligned} & E[\phi''(Eu(\varpi-(1-\alpha^*)L-\alpha^*p))Eu'(\varpi-(1-\alpha^*)L-\alpha^*p)E(L-P)u'(\varpi-(1-\alpha^*)L-\alpha^*p)] \\ & + E[\phi'(Eu(\varpi-(1-\alpha^*)L-\alpha^*p))E(L-P)u''(\varpi-(1-\alpha^*)L-\alpha^*p)] < 0. \end{aligned} \quad (4)$$

The asterisk denotes the optimal level of the endogenous variable obtained from the first-order conditions. For notation convenience, we omit the asterisk throughout the rest of this article.

### 3. The results

In this section, we derive a necessary and sufficient condition for a set of priors to satisfy the decreasing aversion for any ambiguity aversion  $\phi$ , which is DC.

**Lemma 1.** Consider the KMM insurance demand problem (2), and assume that  $u$  is DC. For a set of priors  $L=(L_1, L_2, \dots; L_n)$ , then the demand for co-insurance against the ambiguous loss described by  $\{L_i\}$  decreases in initial wealth for any decreasing concavity ambiguity aversion function  $\phi$  if and only if for the  $\alpha$  and any  $i \neq j$ ,

$$E(L_i - p)u'(\varpi - (1-\alpha)L_i - \alpha p)] > 0 > E(L_j - p)u'(\varpi - (1-\alpha)L_j - \alpha p)]$$

$$\Rightarrow \begin{cases} Eu(\varpi - (1-\alpha)L_i - \alpha p) \leq Eu(\varpi - (1-\alpha)L_j - \alpha p) \\ Eu'(\varpi - (1-\alpha)L_i - \alpha p)] \geq Eu'(\varpi - (1-\alpha)L_j - \alpha p)] \end{cases} \quad (5)$$

**Proof.** See Appendix A.

That is to say, the decreasing aversion property is satisfied whenever an agent with utility  $u$  prefers to buy the insurance contract more than  $\alpha$  against  $L_i$  and less than  $\alpha$  against  $L_j$ , then this agent and the agent with utility  $-u'$  both do in this way. Before using Lemma 1 to get two propositions, we introduce the following lemma which is to be used later.

**Lemma 2**

Define function  $\psi$  such that  $\psi(l) = (l - p)u'(w - (1-\alpha)l - \alpha p)$  for all  $l < p$ . Then the function  $\psi$  is more concaved than  $u$  if the relation

$$(p - l)(P[\varpi - (1-\alpha)l - \alpha p] - A[\varpi - (1-\alpha)l - \alpha p])] \geq \frac{1}{1-\alpha} \quad (6)$$

holds for the optimal coinsurance rate  $\alpha$ , where  $A$  and  $P$  are indices of absolute risk aversion and absolute prudence of  $u$ , respectively.

**Proof.**

Now, we define the function  $K$  such that  $\psi(l) = K(u(l))$  in the joint support of  $(L_1, L_2, \dots; L_n)$ . According to the definition of  $\psi$ , fully differentiates this equality twice, we have

$$\begin{aligned} & -(l - p)u''(\varpi - (1-\alpha)l - \alpha p)(1-\alpha) + u'(\varpi - (1-\alpha)l - \alpha p) \\ & = -K'(u)u'(\varpi - (1-\alpha)l - \alpha p)(1-\alpha), \end{aligned}$$

and

$$\begin{aligned} & -2u''(\varpi - (1-\alpha)l - \alpha p)(1-\alpha) + (l - p)u'''(\varpi - (1-\alpha)l - \alpha p)(1-\alpha)^2 = \\ & K''(u)(u'(\varpi - (1-\alpha)l - \alpha p))^2(1-\alpha)^2 + K'(u)u''(\varpi - (1-\alpha)l - \alpha p)(1-\alpha)^2. \end{aligned}$$

Eliminating  $K'$  from the two equations above yields

$$K''(u)(u'(\varpi - (1-\alpha)l - \alpha p))^2 = (l-p)[u'''(\varpi - (1-\alpha)l - \alpha p) - \frac{(u''(\varpi - (1-\alpha)l - \alpha p))^2}{u'(\varpi - (1-\alpha)l - \alpha p)}] - \frac{u''(\varpi - (1-\alpha)l - \alpha p)}{(1-\alpha)}$$

This means that if condition (6) is satisfied, the concavity of  $K$  is proved.

Next, we use this lemma to obtain the following propositions.

**Proposition 1.** Consider the KMM insurance demand problem in (2). Assuming that  $u$  and  $\phi$  are DC, and the optimal coinsurance rate at wealth level  $\varpi$  is  $\alpha$ , if  $Eu(\varpi - (1-\alpha)L_1 - \alpha p) \leq Eu(\varpi - (1-\alpha)L_2 - \alpha p) \leq L \leq Eu(\varpi - (1-\alpha)L_n - \alpha p)$  and  $L_1 \leq L_2 \leq \dots \leq L_n$ , then the demand for coinsurance decreases in wealth around  $\varpi$ , provided (6) is satisfied for all  $l$  in the joint support of  $(L_1, L_2, \dots, L_n)$ .

**Proof.** Because  $u$  is decreasing concave, we know that  $-u'$  is more concave than  $u$ . Because  $L_{i+1}$  is more endurable than  $L_i$  for the agent with utility function  $u$ , then  $L_{i+1}$  is also more endurable than  $L_i$  for the agent with utility function  $-u'$ . Therefore,

$$E[-u'(\varpi - (1-\alpha)L_i - \alpha p)] \leq E[-u'(\varpi - (1-\alpha)L_{i+1} - \alpha p)].$$

Moreover, condition (6) implies that  $\psi(l) = (p-l)u'(w - (1-\alpha)l - p)$  is more concave in  $l$  than  $u$ , so  $E[(p-L_i)u'(\varpi - (1-\alpha)L_i - \alpha p)]$  is increasing in  $i$ , hence  $E[(L_i - p)u'(\varpi - (1-\alpha)L_i - \alpha p)]$  is decreasing in  $i$ , then, according to **Lemma 1, Proposition 1** is proved.

Next, we investigated another constraint on the set of priors based on the stochastic dominance order defined by Gollier[8]. Now we define the location weighted-probability function  $T(\theta)$  as:

$$T(\theta) = \int_L t dF_\theta(t),$$

where  $F_\theta$  is the cumulative distribution function of uncertain loss  $L_\theta$ . Like Gollier[8], we argue that  $L_\theta$  is dominated by  $L_{\theta+1}$  in the sense of central dominance (CD) if there is a non-negative scalar  $m$  such that  $T_\theta(x) \geq mT_{\theta+1}(x)$  for all  $x$  in the joint support of  $L_\theta$  and  $L_{\theta+1}$ . According to Gollier[8], SSD-dominance is neither necessary nor sufficient for CD-dominance. In Proposition 2, we suppose that this set of priors of the loss can be symmetrically ranked by the two orders.

**Proposition 2.** Assume that  $u$  and  $\phi$  have decreasing concavity. In the insurance demand problem (2), the demand for insurance against the ambiguous loss decreases in wealth, if

$$L_1 \underset{SSD}{\succ} L_2 \underset{SSD}{\succ} L_n \text{ and } L_1 \underset{CD}{\succ} L_2 \underset{CD}{\succ} L_n.$$

**Proof.** Because  $u$  and  $-u'$  are increasing and concave, the right properties in (5) hold for any pair  $(i, j)$  such that  $i < j$ , for SSD-ordering. According to Gollier[8], because  $L_1 \underset{CD}{\succ} L_2 \underset{CD}{\succ} L_n$ , it is true that the left condition in (5) means that  $i < j$ . So, the right condition in (5) holds. Thus, the proposition is proved.

#### 4. Conclusion

The hypothesis that decision makers are averse to and decreasingly averse to uncertainty becomes increasingly important in decision theory, especially in the case of portfolio choice and insurance demand. In this study we define the decreasing aversion by the property that optimal insurance demand decreases when wealth increases. We investigate the effect of initial wealth on insurance demand using a smooth ambiguity aversion model. In fact, we have shown that in the smooth ambiguity aversion model of insurance demand with a decreasingly concave ambiguity-related function  $\phi$ , the unambiguous comparative static result requires some assumptions on the structure of ambiguity.

#### Appendix A

**Proof of Lemma 1.** we prove sufficiency firstly.

Because the utility  $u$  is DC, it is true that the second term in the left hand side of inequality (4) is negative.

It is known that  $u$  DC implies that  $A(c) = -u''(c)/u'(c)$  is decreasing. So,

$$\begin{aligned} & E[\phi'(Eu(\varpi - (1-\alpha)L - \alpha p))E[(L-P)u''(\varpi - (1-\alpha)L - \alpha p)]] \\ &= -E[\phi'(Eu(\varpi - (1-\alpha)L - \alpha p))E[A(\varpi - (1-\alpha)L - \alpha p)(L-P)u'(\varpi - (1-\alpha)L - \alpha p)]] \\ &\leq -A(\varpi - p)E[\phi'(Eu(\varpi - (1-\alpha)L - \alpha p))E[(L-P)u'(\varpi - (1-\alpha)L - \alpha p)]] = 0. \end{aligned}$$

As above, we rewrite the first term of inequality (3) as:

$$\begin{aligned} & E[\phi''(Eu(\varpi - (1-\alpha)L_\theta - \alpha p))Eu'(\varpi - (1-\alpha)L_\theta - \alpha p)E(L_\theta - P)u'(\varpi - (1-\alpha)L_\theta - \alpha p)] \\ &= -E[\tau(\theta)\phi'(Eu(\varpi - (1-\alpha)L_\theta - \alpha p))E(L_\theta - P)u'(\varpi - (1-\alpha)L_\theta - \alpha p)], \end{aligned}$$

with  $\tau(\theta) = A_\theta(Eu(\varpi - (1-\alpha)L_\theta - \alpha p)Eu'(\varpi - (1-\alpha)L_\theta - \alpha p))$

where  $A_\phi(u) = -\phi''(u)/\phi'(u)$  is the absolute measure of ambiguity aversion. Now we rank  $\theta$ s such that  $E(L_\theta - P)u'(\varpi - (1-\alpha)L_\theta - \alpha p)$  is negative if and only if  $\theta \geq \hat{\theta}$  for some  $\hat{\theta}$ . Condition (5) implies  $\tau(\theta) \leq \tau(\hat{\theta})$  for all  $\theta \geq \hat{\theta}$ , and  $\tau(\theta) \geq \tau(\hat{\theta})$  for all  $\theta \leq \hat{\theta}$ . Then, we have

$$E[\phi''(Eu(\varpi - (1-\alpha)L_\theta - \alpha p))Eu'(\varpi - (1-\alpha)L_\theta - \alpha p)E(L_\theta - P)u'(\varpi - (1-\alpha)L_\theta - \alpha p)] \leq -\tau(\theta)E[\phi'(Eu(\varpi - (1-\alpha)L_\theta - \alpha p))E(L_\theta - P)u'(\varpi - (1-\alpha)L_\theta - \alpha p)] = 0.$$

Thus, the two terms in (4) are negative, and sufficiency is proved.

On the other hand, when property (5) does not hold, that is when one of the two inequalities on the right is not satisfied for a pair  $(i, j)$  such that  $E(L_i - p)u'(\varpi - (1-\alpha)L_i - \alpha p)] > 0 > E(L_j - p)u'(\varpi - (1-\alpha)L_j - \alpha p)]$ , then there exists a distribution  $\{p_i, p_j\}$  of those priors and a function  $\phi$  such that there is an increasing aversion. In fact, it is easy to find  $\{p_i, p_j\}$  for a given function  $\phi$  such that condition (3) does hold. Then the only thing we need to do is to choose  $\phi$  such that relation (4) is false. It is easy to prove that relation (4) equals to  $E[\gamma(\theta)\phi'(Eu(\varpi - (1-\alpha)L_\theta - \alpha p))E(L_\theta - P)u'(\varpi - (1-\alpha)L_\theta - \alpha p)] \geq 0$  where

$$\gamma(\theta) = \frac{\phi''(Eu(\varpi - (1-\alpha)L_\theta - \alpha p))Eu'(\varpi - (1-\alpha)L_\theta - \alpha p)}{\phi'(Eu(\varpi - (1-\alpha)L_\theta - \alpha p))} - \frac{E(L_\theta - P)u''(\varpi - (1-\alpha)L_\theta - \alpha p)}{E(L_\theta - P)u'(\varpi - (1-\alpha)L_\theta - \alpha p)}.$$

If the first condition does not hold, that is

$$Eu(\varpi - (1-\alpha)L_i - \alpha p)] > Eu(\varpi - (1-\alpha)L_j - \alpha p)]$$

then we pick  $\phi(z) = \int e^{\lambda/z^n} dz$  with

$\lambda$  and  $n$  large enough such that  $\gamma_i < \gamma_j$ . If the second condition does not hold, that is

$$Eu'(\varpi - (1-\alpha)L_i - \alpha p)] < Eu'(\varpi - (1-\alpha)L_j - \alpha p)]$$

then we pick  $\phi(z) = -e^{-\lambda z}$  with  $\lambda$

large enough, so we have  $\gamma_i < \gamma_j$ . In both cases, this condition and condition (3) imply that relation (4) does not hold. Thus the lemma is proved.

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