Original Research Article

The wealth effects on demand for insurance under ambiguity aversion

Abstract:

Under which condition does the optimal insurance demand decrease in wealth? In the expected utility model the decreasing concavity condition is necessary and sufficient for this result. But in general, under ambiguity aversion, the result does not hold. We constrain the structure of ambiguity and obtain unambiguous results.

Comment [u1]: It would be good if there is a small introduction of 2-3 lines followed by methodology and the results and conclusion.

Key words: Decreasing aversion; Smooth ambiguity aversion; Decreasing concavity; Optimal insurance demand

1. Introduction

Insurance provides risk-averse consumers with protection against many kinds of risks such as damages to property, liability exposures, health care costs, crop income loss etc. A large number of documents have studied the factors influencing insurance demand (e.g. Schlesinger, [15]; Lee[12] ; Richard Peter and Jie Ying [14]). The classical model considers a risk-averse agent aiming to insure her wealth against a possible loss. These analyses mainly concern the optimal coinsurance rate that can maximize her expected utility of wealth. In economics, one of the prevailing assumption is that when people are wealthier they are less risk-averse. This property can be called decreasing aversion.

There are many definitions for the decreasing aversion in the documents. For instance, an agent is called to be of decreasing aversion kind if any risk that is not preferred at some specific level of wealth then it is also not preferred at any lower level of wealth. The second definition is that the demand for risky assets increases with initial wealth in the portfolio selection problem of risk free and risky assets. A similar definition of decreasing aversion in insurance demand problem is that the demand for insurance is a decreasing function of initial wealth.

Since Arrow[1] assume a decreasing absolute risk aversion, much literature has verify this supposition by the means of experimental (Levy[13]; Guiso and Paiella [10])and econometric methods (Bar-Shira et al.[2]). As Gollier [9] shows, this generally accepted characteristic of individual risk preference plays a vital role in many applications of expected utility theory. Risk aversion is capital to capture the mechanism of individual choices about insurance, portfolio accumulation. Many studies extend these models by considering labor income (such as Viceira [16], Coco et al. [7]) or housing (Coco [6]), or modifying preferences with persistent habits (Brunnermeier and Nagel [3]) or ambiguity aversion (Campanale [4]; Cherbonnier and Gollier [5]). However, to some extent, how risk aversion shapes individual insurance demand decision is still worth to be considered.

For KMM decision-making criteria under fuzzy conditions, we determine the condition that the rich have low aversion to uncertain loss.

In most cases, the exact distribution of the random loss is not completely known, which is called ambiguous. In this paper, we investigate the decreasing aversion property in the context of ambiguity aversion in the case of insurance demand. For the decision criteria of KMM under ambiguity, this paper discuss the condition under which the wealthier agent demand less insurance .

2 The Model

Now, we introduce a decision model of insurance demand. We consider a decision maker with initial wealth ϖ , who is subject to a loss of $L \in (0, \varpi)$.The decision maker can purchase coinsurance against the risk of loss. The insurance premium is p, let α be the coinsurance rate. In the expected utility model, this decision problem can be represented by

$$\max_{\alpha} Eu(\varpi - (1-\alpha)L - \alpha p). \tag{1}$$

For the convenience of the discussion, we state the following two definitions.

Definition 1. A function $f: R \to R$ satisfies Decreasing Concavity (DC) if $-f''/f'$ is non- increasing.

It is obvious that f DC means that there exists a concave function g such that $-f' = g \circ f$.

Definition 2. We say that the uncertain loss L_i dominates L_j in the sense of Jewitt if the following condition is satisfied: if L_i is weakly more endurable than L_j, for all increasing and concave u, then L_i is weakly more endurable than L_j for all agents more risk averse than u .

Suppose the accurate distribution function of the random loss is known to the insurer and the consumer. In model (1), it is easy to prove that the insurance demand decreases with the initial wealth if and only if u satisfies decreasing concavity. To see that, because the objective function is concave in α, this requires to prove that the cross derivative of the objective function in (1) with respect to α and ϖ evaluated at the optimal α is negative. The first order condition of the above program yields

$$E(L-p)u'(\varpi-(1-\alpha)L-\alpha p)=0.$$

Since u DC implies that $-u'$ is a concave function g of u, we can have

$$E(L-p)u''(\varpi-(1-\alpha)L-\alpha p)=-E(L-p)g'(\varpi-(1-\alpha)L-\alpha p)u'(\varpi-(1-\alpha)L-\alpha p)$$
$$\leq -g'(u(\varpi-p))E(L-p)u'(\varpi-(1-\alpha)L-\alpha p)=0.$$

The inequality above holds because that for all $p-l>0$,

$$(p-l)g'(u(\varpi-(1-\alpha)l-\alpha p))<(p-l)g'(u(\varpi-l))<(p-l)g'(u(\varpi-p)).$$

So, in the expected utility model, if and only if u is decreasing concavity, then the demand for insurance decreases with initial wealth.

Now, suppose that the distribution of the loss L is ambiguous. This ambiguity is characterized by n possible random variables $(L_1, L_2, \cdots; L_n)$. Under the KMM smooth ambiguity framework the decision problem becomes

$$\alpha^*(\varpi)=\arg\max_{\alpha} E\phi(Eu(\varpi-(1-\alpha)L-\alpha p)). \quad (2)$$

So, the first order condition can be expressed by

$$E[\phi'(Eu(\varpi-(1-\alpha)L-\alpha p))E(L-P)u'(\varpi-(1-\alpha)L-\alpha p)]=0. \quad (3)$$

Next, we investigate the condition under which the coinsurance demand is decreasing in initial wealth. It is true that the objective function in (3) is concave in α, so decreasing aversion property is satisfied if and only if

$$E[\phi''(Eu(\varpi-(1-\alpha^*)L-\alpha^* p))Eu'(\varpi-(1-\alpha^*)L-\alpha^* p)E(L-P)u'(\varpi-(1-\alpha^*)L-\alpha^* p)]$$
$$+E[\phi'(Eu(\varpi-(1-\alpha^*)L-\alpha^* p))E(L-P)u''(\varpi-(1-\alpha^*)L-\alpha^* p)]<0.$$
(4)

The asterisk denotes the optimal level of the endogenous variable obtained from the first order conditions. For notation convenience we will omit the asterisk through the rest of this article.

3.The results

In this section, we will derive a necessary and sufficient condition for a set of priors to satisfy decreasing aversion for any ambiguity aversion ϕ that is DC.

Lemma 1. Consider the KMM insurance demand problem (2), and assume that u is DC. For a set of priors $L=(L_1, L_2, \cdots; L_n)$, then the demand for the coinsurance against the ambiguous loss described by $\{L_i\}$ is decreasing in initial wealth for any decreasing concavity ambiguity aversion function ϕ if and only if for the α and any $i \neq j$,

$$E(L_i - P)u'(\varpi - (1-\alpha)L_i - \alpha p)] > 0 > E(L_j - P)u'(\varpi - (1-\alpha)L_j - \alpha p)]$$

$$\Rightarrow \begin{cases} Eu(\varpi - (1-\alpha)L_i - \alpha p) \leq Eu(\varpi - (1-\alpha)L_j - \alpha p) \\ Eu'(\varpi - (1-\alpha)L_i - \alpha p)] \geq Eu'(\varpi - (1-\alpha)L_j - \alpha p)] \end{cases}. \quad (5)$$

Poof. See **Appendix** .

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That is to say, the decreasing aversion property is satisfied whenever an agent with utility u prefers to buy the insurance contract more than α against L_i and less than α against L_j, then this agent and the agent with utility $-u'$ both do in this way. Before using **Lemma 1** to get two propositions, we introduce the following lemma which is to be used later.

Lemma 2

Define function ψ such that $\psi(l) = (l-p)u'(w-(1-\alpha)l - \alpha p)$ for all $l < p$. Then the function ψ is more concaved than u if the relation

$$(p-l)(P[\varpi - (1-\alpha)l - \alpha p) - A[\varpi - (1-\alpha)l - \alpha p])]) \leq \frac{1}{1-\alpha} \quad (6)$$

holds for the optimal coinsurance rate α. Where A and P are the indices of absolute risk aversion and of absolute prudence of u, respectively.

Proof.

Now we define function K such that $\psi(l)=K(u(l))$ in the joint support of $(L_1, L_2, \cdots; L_n)$. According to the definition of ψ, fully differentiating this equality twice , we have

$$\begin{aligned} &-(l-p)u''(\varpi - (1-\alpha)l - \alpha p)(1-\alpha) + u'(\varpi - (1-\alpha)l - \alpha p) \\ &= -K'(u)u'(\varpi - (1-\alpha)l - \alpha pl)(1-\alpha), \end{aligned}$$

and

$$\begin{aligned} &-2u''(\varpi - (1-\alpha)l - \alpha p)(1-\alpha) + (l-p)u'''(\varpi - (1-\alpha)x - \alpha p)(1-\alpha)^2 = \\ &K''(u)(u'(\varpi - (1-\alpha)l - \alpha p))^2(1-\alpha)^2 + K'(u)u''(\varpi - (1-\alpha)l - \alpha p)(1-\alpha)^2. \end{aligned}$$

Eliminating K' from the two equations above yields

$$K''(u)(u'(\varpi-(1-\alpha)l-\alpha p))^2=(l-p)[u'''(\varpi-(1-\alpha)l-\alpha p)-\frac{(u''(\varpi-(1-\alpha)l-\alpha p))^2}{u'(\varpi-(1-\alpha)l-\alpha p)}]$$
$$-\frac{u''(\varpi-(1-\alpha)l-\alpha p)}{(1-\alpha)}$$

This means that if condition (6) is satisfied, then the concavity of K is proved.

Next, we use this lemma to obtain the following propositions.

Proposition1. Consider the KMM insurance demand problem (2). Assume that u and ϕ are DC, and the optimal coinsurance rate at wealth level ϖ is α. If $Eu(\varpi-(1-\alpha)L_1-\alpha p)\le Eu(\varpi-(1-\alpha)L_2-\alpha p)\le \cdots \le Eu(\varpi-(1-\alpha)L_n-\alpha p)$ and $L_1 \preceq_J L_2 \preceq_J \cdots \preceq_J L_n$, then the demand for coinsurance decreases in wealth around ϖ provides (6) is satisfied for all l in the joint support of $(L_1, L_2, \cdots; L_n)$.

Proof. Because u is decreasing concave, we know that $-u'$ is more concave than u. Because L_{i+1} is more endurable than L_i, then L_{i+1} is also more endurable than L_i for the agent with utility function $-u'$. So,

$$E[-u'(\varpi-(1-\alpha)L_i-\alpha p)]\le E[-u'(\varpi-(1-\alpha)L_{i+1}-\alpha p)].$$

Moreover, condition (6) implies that $\psi(l)=(p-l)u'(w-(1-\alpha)l-p)$ is more concave in l than u, so $E(p-L_i)u'(\varpi-(1-\alpha)L_i-\alpha p)]$ is increasing in i, hence $E(L_i-p)u'(\varpi-(1-\alpha)L_i-\alpha p)]$ is decreasing in i, then, according to **Lemma 1**, **Proposition 1** is proved.

Next, we investigate another constraint on the set of priors based on the stochastic dominance order defined by Gollier[8]. Now we define the location weighted-probability function $T(\theta)$ by:

$$T(\theta)=\int_L t dF_\theta(t),$$

where F_θ is the cumulative distribution function of uncertain loss L_θ. Like Gollier [8], we argue that L_θ is dominated by $L_{\theta+1}$ in the sense of Central Dominance (CD) if there is a non-negative scalar m such that $T_\theta(x)\ge mT_{\theta+1}(x)$ for all x in the in the joint support of L_θ and $L_{\theta+1}$. According to Gollier [8], SSD-dominance is neither necessary nor sufficient for CD-dominance. In the following proposition 2, we suppose that the set of priors of the loss can be ranked by the two orders symmetrically.

Proposition 2. Assume that u and ϕ are decreasing concave. In the insurance demand problem (2), the demand for the insurance against the ambiguous loss decreases in weath, if

$$L_1 \precsim_{SSD} L_2 \precsim_{SSD} \cdots \precsim_{SSD} L_n \text{ and } L_1 \precsim_{CD} L_2 \precsim_{CD} \cdots \precsim_{CD} L_n.$$

Proof. Since u and $-u'$ are increasing and concave, then the right properties in (5) hold for any pair (i, j) such that $i < j$, for the SSD ordering. According to Gollier[8], because $L_1 \precsim_{CD} L_2 \precsim_{CD} \cdots \precsim_{CD} L_n$, it is true that the left condition in (5) means that $i < j$. So, the right condition in (5) holds. Then the proposition is proved.

4. Conclusion

The hypotheses that decision makers are averse to and decreasingly averse to uncertainty becomes more and more important in decision theory, especially in the case of portfolio choice and insurance demand. In this paper we define the decreasing aversion by the property that optimal insurance demand decreases when wealth increases. We mainly investigate the effect of initial wealth on insurance demand through the smooth ambiguity aversion model. In fact, we have shown that in the smooth ambiguity aversion model of insurance demand with a decreasingly concave ambiguity-related function ϕ, the unambiguous comparative static result requires some assumptions on the structure of ambiguity.

Appendix A

Proof of Lemma 1. we prove sufficiency firstly.

Because the utility u is DC, it is true that the second term in the left hand side of the inequality (4) is negative.

It is known that u DC implies that $A(c) = -u''(c)/u'(c)$ is decreasing. So,

$$\begin{aligned}
&E[\phi'(Eu(\varpi-(1-\alpha)L-\alpha p))E[(L-P)u''(\varpi-(1-\alpha)L-\alpha p)]] \\
&= -E[\phi'(Eu(\varpi-(1-\alpha)L-\alpha p))E[A(\varpi-(1-\alpha)L-\alpha p)(L-P)u'(\varpi-(1-\alpha)L-\alpha p)]] \\
&\le -A(\varpi-p)E[\phi'(Eu(\varpi-(1-\alpha)L-\alpha p))E[(L-P)u'(\varpi-(1-\alpha)L-\alpha p)]]=0.
\end{aligned}$$

As above, we rewrite the first term of inequality (3) as:

$$\begin{aligned}
&E[\phi''(Eu(\varpi-(1-\alpha)L_\theta-\alpha p))Eu'(\varpi-(1-\alpha)L_\theta-\alpha p)E(L_\theta-P)u'(\varpi-(1-\alpha)L_\theta-\alpha p)] \\
&= -E[\tau(\theta)\phi'(Eu(\varpi-(1-\alpha)L_\theta-\alpha p))E(L_\theta-P)u'(\varpi-(1-\alpha)L_\theta-\alpha p)]
\end{aligned}$$

with $\tau(\theta)=A_\phi(Eu(\varpi-(1-\alpha)L_\theta-\alpha p)Eu'(\varpi-(1-\alpha)L_\theta-\alpha p)$ where

$A_\phi(u)$=-$\phi''(u))/\phi'(u)$ is the absolute measure of ambiguity aversion. Now we rank θs such that $E(L_\theta - P)u'(\varpi-(1-\alpha)L_\theta-\alpha p)$ is negative if and only if $\theta \geq \acute{\theta}$ for some $\acute{\theta}$. Condition (5) implies that $\tau(\theta) \leq \tau(\acute{\theta})$ for all $\theta \geq \acute{\theta}$, and $\tau(\theta) \geq \tau(\acute{\theta})$ for all $\theta \leq \acute{\theta}$, Then, we have

$$E[\phi''(Eu(\varpi-(1-\alpha)L_\theta-\alpha p))Eu'(\varpi-(1-\alpha)L_\theta-\alpha p)E(L_\theta-P)u'(\varpi-(1-\alpha)L_\theta-\alpha p)]$$
$$\leq -\tau(\theta)E[\phi'(Eu(\varpi-(1-\alpha)L_\theta-\alpha p))E(L_\theta-P)u'(\varpi-(1-\alpha)L_\theta-\alpha p)]=0.$$

Thus, the two terms in (4) is negative, the sufficiency is proved.

In other hand, when property (5) does not hold, i.e. when one of the two inequalities on the right is not satisfied for a pair (i, j) such that $E(L_i-P)u'(\varpi-(1-\alpha)L_i-\alpha p)]>0>E(L_j-P)u'(\varpi-(1-\alpha)L_j-\alpha p)]$, then there exist a distribution $\{p_i, p_j\}$ of those priors and a function ϕ such that there is an increasing aversion. In fact, it is easy to find $\{p_i, p_j\}$ for a given function ϕ such that the condition (3) does hold. Then the only thing we need to do is to choose ϕ such that the relation (4) is false. It is easy to prove that the relation (4) equals to $E[\Gamma(\theta)\phi'(Eu(\varpi-(1-\alpha)L_\theta-\alpha p))E(L_\theta-P)u'(\varpi-(1-\alpha)L_\theta-\alpha p)]\geq 0$ where

$$\gamma(\theta)=-\frac{\phi''(Eu(\varpi-(1-\alpha)L_\theta-\alpha p)}{\phi'(Eu(\varpi-(1-\alpha)L_\theta-\alpha p)}Eu'(\varpi-(1-\alpha)L_\theta-\alpha p)$$
$$-\frac{E(L_\theta-P)u''(\varpi-(1-\alpha)L_\theta-\alpha p)}{E(L_\theta-P)u'(\varpi-(1-\alpha)L_\theta-\alpha p)}.$$

If the first condition does not hold, i.e. $Eu(\varpi-(1-\alpha)L_i-\alpha p)]>Eu(\varpi-(1-\alpha)L_j-\alpha p)$ then we pick $\phi(z)=\int e^{\lambda/z^n}dz$ with λ and n large enough such that $\gamma_i<\gamma_j$. If the second condition does not hold, i.e. $Eu'(\varpi-(1-\alpha)L_i-\alpha p)]<Eu'(\varpi-(1-\alpha)L_j-\alpha p)$ then we pick $\phi(z)=-e^{-\lambda Z}$ with λ large enough, so we have $\gamma_i<\gamma_j$. In both case, this condition and the condition (3) implies the relation (4) does not hold. Then the lemma is proved.

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