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# MATHEMATICAL MODELING OF ALCOHOLISM INCORPORATING MEDIA AWARENESS AS AN INTERVENTION STRATEGY

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Original Research article

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## Abstract

Alcohol addiction is a phenomenon that has attracted the attention of numerous researchers and academics in a variety of professions due to its serious repercussion on all spheres of human life. Alcoholism is a common addiction in adults throughout the globe. Hence there is need to target efficient preventative and therapeutic measures. The impact of media awareness and treatment on the drinking behavior of various drinker classes is discussed in several mathematical models. Nevertheless, the impact of the exposed class of alcoholics on light and heavy drinkers in the presence of media awareness, has not been addressed. The model was formulated based on a system of differential equations. To perform stability analysis of the model at each equilibrium point, Jacobian matrix method was employed. By use of MATLAB, numerical simulations on the impact of awareness on alcoholism were performed. Secondary data obtained from NACADA and data from rehabilitation centers in Kenya was used to validate the analytical results of the impact of media awareness on the drinking population. Analysis of the model indicated that the Alcohol Free Equilibrium (AFE) point is locally asymptotically stable whenever  $R_0 < 1$  and unstable whenever  $R_0 > 1$ . The Alcohol Endemic Point (AEP) exists and is locally asymptotically stable when  $R_0 > 1$ . Further increase in media awareness programs reduces alcohol prevalence in the community. The study concluded that maximum media awareness is an ideal measure in curbing alcohol abuse in the community. The findings of this study will provide useful insight to the government and policy makers in targeting suitable media awareness programs in combating alcoholism.

Keywords: Alcohol Free Equilibrium; Endemic equilibrium point; Reproduction number; Media awareness; Jacobian matrix; Next generation matrix

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# 1 Introduction

Alcoholism has become a serious global challenge due to its social and economic ramifications on different strata of the community. Alcoholism is the misuse and binge drinking of alcohol, which can have negative effects on everyone in the society on a physical, social, and moral level. There have been reports of disastrous repercussions on people, families, and societies as a result of rising alcohol and other drug usage. In addition to various other physical repercussions like road accidents, chronic diseases alcohol abuse can have a psychological, social, and economic burden on society(16). Therefore, various prevention and treatment strategies such as rehabilitation, have been targeted to address the problems of the affected individuals worldwide. Furthermore, awareness has been emphasized to curb the spread and occurrence of alcoholism (9; 12; 15) .

The World Health Organization has calculated that alcohol caused nearly 3 million deaths worldwide in the past year. According to current estimates, 7.9 million people in the United Kingdom consume alcohol, up from 6.5 million the year before, or a 22 percent increase (15). Rwanda alone in Africa has a 17.4% addiction prevalence, this corresponds to 76% of the entire population(7). According to a NACADA report in (12), alcohol abuse is most prevalent in Kenya, with a prevalence rate of roughly 31%. About 60% of Kenyans have used alcohol, and nearly half of them have experienced negative consequences from consuming alcohol. Every year, alcohol misuse claims the lives of four out of every 100 Kenyans (12).

Based on this troubling statistics of alcohol abuse, teenagers and adults need alcohol use prevention mass media programs because they raise knowledge of alcoholism's effects. Evidence from research such as (2; 11) shows that media awareness campaigns are a sensible strategy to educate the public about alcohol abuse. Any method of information transmission to a large audience at once is referred to as mass media. Digital media, print media, broadcast media, and outdoor media may all fall under this category. Music, videos, television tutorials, and print media are all examples of broadcast media. Social media platforms like Instagram, Twitter, Whatsapp, and MySpace are examples of digital media. Print media is internet-based communication that includes newspapers, periodicals and journals. Outdoor media consists of placards, billboards in cities, roadsides, and augmented reality commercials. When effective campaigns are launched against alcohol abuse, such media outlets serve as important informational resources and not only change people's behavior but also raise the government's engagement in health care.

According to the studies such as (11; 9; 15), alcohol consumption is still a serious public health issue throughout the globe and is by no means under control. There is evidence that alcoholism spreads like an infectious disease, according to Misra in (8), therefore it can be represented mathematically. The main objective of this study is to formulate and analyze a mathematical model of alcoholism incorporating media awareness as an intervention strategy and the influence of the exposed class on light and heavy drinkers. A number of mathematical models on alcoholism incorporating treatment have been formulated. The models have addressed the impact of media awareness on various drinkers' classes and have assumed that after successful treatment individuals quit alcohol, rather than considering the fact that some may become susceptible again (4; 6; 11). Recent mathematical models have examined alcoholism and have incorporated media awareness (11). However, the influence of the exposed class of alcoholics on light and heavy drinkers in the presence of media awareness throughout the alcoholism process, has not been addressed. The exposed class is critical as it would aid in understanding the spread, nature and extent of alcoholism at the population level. Thus, a mathematical model of alcoholism that incorporated the influence of the exposed class on light and heavy drinkers in presence of media awareness was formulated and analyzed.

The paper is organized as follows; in section 2, the model is formulated and the dynamics of drinking described. The invariant region, positivity and boundedness of the model solutions have also been examined in section 3. In section 4, the model has been analyzed by performing local stability analysis at the Alcohol-Free Equilibrium and at Endemic Equilibrium point. Numerical simulation of the model has been performed in section 5, where graphical representation of the impact of media

awareness on alcoholism has been done. To conclude, the study has discussed the main results and future directions implicated by findings of this research.

## 2 Model

The model was formulated based on a system of differential equation. The model has five compartments considering the entire population. These classes are; the susceptible individuals who either have or have never consumed alcohol in their lifetime and can also result from media awareness individuals and denoted by  $S$ . The exposed class denoted by  $E$ , comprise of individuals who are at risk of becoming alcoholics as a result of contact with light and heavy drinkers. The light drinkers denoted by  $V_1$ , are individuals who drink occasionally and can do with or without alcohol. The heavy drinkers denoted by  $V_2$ , are individuals who are highly addicted to alcohol or rather dependent on alcohol. The media awareness class denoted by  $A$ , are Individuals undergoing treatment. Individuals are brought into the model at a rate of  $\Lambda$ . The rate of progression from  $S$  to  $E$  is given by  $\omega$ , the progression rate to  $V_1$  from  $E$  is  $\alpha_1$ , the rate of progression from  $V_1$  to  $V_2$  is given by  $\alpha_2$  and the progression rate from  $V_2$  to  $V_1$  is  $\kappa$ . Fatalities from other causes occur at the rate of  $\mu$  and alcohol-related causes at a rate of  $\sigma$ .  $\tau_1$  and  $\tau_2$  represent the rates at which light and heavy drinkers transit to the dangers of alcoholism, respectively. The rate at which an individual becomes susceptible again after undergoing awareness is given by  $\beta$ .

### 2.1 Model Assumptions

The study was based on the following assumptions.

- (i) Alcohol addicts must be exposed to media awareness if they want to stop drinking because they cannot recover on their own by self-control.
- (ii) Not all individuals will quit alcohol completely.
- (iii) After undergoing media awareness, an individual can get back to being susceptible.
- (iv) The exposed group results from contact with both light and heavy drinkers.
- (v) Individuals in the light drinking class drink occasionally and can do with or without alcohol.

### 2.2 Model Flow Chart and Equations

The model below summarizes the variables and parameters described in section 2.

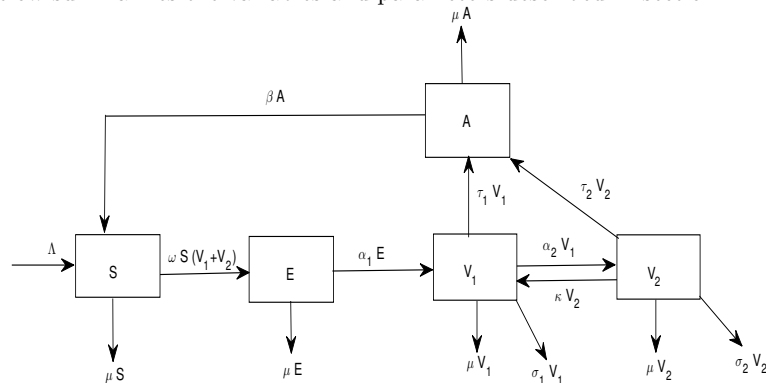


Fig 2.2: Schematic Diagram of the Proposed Model

The following sets of equations governed the model:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda + \beta A - \mu S - \omega S(V_1 + V_2) \\
\frac{dE}{dt} &= \omega S(V_1 + V_2) - \mu E - \alpha_1 E \\
\frac{dV_1}{dt} &= \alpha_1 E + \kappa V_2 - (\alpha_2 + \tau_1 + \mu + \sigma_1)V_1 \\
\frac{dV_2}{dt} &= \alpha_2 V_1 - (\kappa + \tau_2 + \mu + \sigma_2)V_2 \\
\frac{dA}{dt} &= \tau_1 V_1 + \tau_2 V_2 - (\mu + \beta)A
\end{aligned} \tag{2.1}$$

By comparison theorem, let;  $k_1 = \mu + \beta, k_2 = \alpha_2 + \tau_1 + \mu + \sigma_1, k_3 = \kappa + \tau_2 + \mu + \sigma_2$ .

Table 2.1 summarizes the variables and parameters that have been used in model (2.1).

Table 2.1: Description of Variables and Parameters

Variable or Parameter	Description
$S$	Number of susceptible individuals at location $l$ and time $t$ .
$E$	Number of individuals exposed to alcohol at time $t$ .
$V_1$	Number of light drinkers at location $l$ and time $t$ .
$V_2$	Number of heavy drinkers at location $l$ and time $t$ .
$A$	Number of alcohol users at location $l$ and time $t$ and undergoing awareness.
$\Lambda$	Recruitment rate of the population.
$\mu$	Natural death rate.
$\omega$	Addition rate from $S$ to the exposed individuals.
$\sigma_1$	The alcohol-related death rate of $V_1$ .
$\sigma_2$	The alcohol-related death rate of $V_2$ .
$\alpha_1$	The progression rate to $V_1$ from $E$ .
$\kappa$	Progression rate to $V_1$ from $V_2$ .
$\alpha_2$	The progression rate to $V_2$ from $V_1$ .
$\tau_1$	The proportion of light drinkers undergoing awareness programs.
$\tau_2$	The proportion of heavy drinkers undergoing awareness programs.
$\beta$	The permanent withdrawal rate from $A$ back to susceptible.

### 3 Positivity and boundedness of the Model

Lemma 3.1 and lemma 3.2 have been used to study the invariant region, boundedness and positivity of model solutions.

#### 3.1 Invariant Region

**Lemma 3.1.** All feasible solutions of the system in equation (2.1) are bounded and enter into the following region;  $\Omega = S(t), E(t), V_1(t), V_2(t), A(t) \in R_+^5$

**Proof.** If  $(S, E, V_1, V_2, A)$  is a solution to the system in equation (2.1) with non-negative initial conditions, summing the five equations i.e  $N = S + E + V_1 + V_2 + A$  the following solution is obtained;

$$\frac{dN}{dt} = \Lambda - \mu N \tag{3.1}$$

Integrating equation (3.1) with respect to time using the integrating factor;  $\exp^{\mu t}$  the following is obtained:  $N \exp^{\mu t} \leq \frac{\Lambda}{\mu} \exp^{\mu t} + C \exp^{-\mu t}$ .

This can be simplified as;  $N \leq \frac{\Lambda}{\mu} + C \exp^{-\mu t}$ , where  $C = \exp^C$  is a constant of integration. As  $t$  tends to infinity, the limit of  $N(t)$  becomes;

$$\lim_{t \rightarrow \infty} N(t) \leq \frac{\Lambda}{\mu} \quad (3.2)$$

From equation (3.2) it is clear that  $N(t)$  is bounded and  $0 \leq N(t) \leq \frac{\Lambda}{\mu}$ . That is,  $N(t)$  is bounded above by  $\frac{\Lambda}{\mu}$  and below by 0. Hence,  $0 \leq N(t) \leq \frac{\Lambda}{\mu} + N(0) \exp^{\mu t}$

Where  $N(0)$  serves as the initial value of the total population. If  $N(0) > \frac{\Lambda}{\mu}$ , then the solution enters  $\Omega$  in finite time or  $N(t)$  approaches  $\frac{\Lambda}{\mu}$  asymptotically. As the  $\lim_{t \rightarrow \infty}, 0 \leq N(t) \leq \frac{\Lambda}{\mu} + N(0) \exp^{\mu t}$  reduces to  $0 \leq N(t) \leq \frac{\Lambda}{\mu}$ . Thus, the investigation of the study shows that the feasible solutions set of the system equations enters and remain in the region  $\Omega$  for all future time, where;

$$\Omega = (S, E, V_1, V_2, A) \in R_+^5, \quad 0 \leq N(t) \leq \frac{\Lambda}{\mu} \text{ as } t \rightarrow \infty. \quad \square$$

As a result, the model is well posed from equation (3.2), and the dynamics of alcohol abuse in the model may be examined in  $\Omega$ . That is the population growth is always bounded by  $\frac{\Lambda}{\mu}$ .

### 3.2 Positivity of the model solutions

**Lemma 3.2.** If the initial values  $S(0), E(0), V_1(0), V_2(0)$  and  $A(0)$  are positive, then the system in equation (2.1) has positive solutions of  $S(t), E(t), V_1(t), V_2(t), A(t)$  for all  $t > 0$ .

Proof. Assuming the initial conditions are as follows;  $S(0) > 0, E(0) > 0, V_1(0) > 0, V_2(0) > 0, A(0) > 0$ . Then from the first equation of the system in equation (2.1);

$$\frac{dS}{dt} \geq -\mu S - \omega S(V_1 + V_2) \quad (3.3)$$

The following answer is obtained by integrating equation (3.3). with respect to time  $t$ ;

$$S(t) \geq S(0) \exp^{-(\mu + \omega(V_1 + V_2))t} > 0 \quad (3.4)$$

As a result, equation (3.3) is positive regardless of time  $t$ . The same procedure applies to differential equations involving  $E, V_1, V_2$  and  $A$  and the solutions obtained are;

$$E(t) \geq E(0) \exp^{-(\mu + \alpha_1)t} > 0 \quad (3.5)$$

$$V_1(t) \geq V_1(0) \exp^{-k_2 t} > 0 \quad (3.6)$$

$$V_2(t) \geq V_2(0) \exp^{-k_3 t} > 0 \quad (3.7)$$

$$A(t) \geq A(0) \exp^{-(\beta + \mu)t} > 0 \quad (3.8)$$

Equations (3.5), (3.6), (3.7) and (3.8) are always positive for all time  $t$ . This shows that any instant  $t < 0$ , the population is positive (there is population growth). Therefore, the system of equations in model (2.1) are all positive for future time  $t$  and thus the system is biologically and mathematically well posed.  $\square$

## 4 Model Analysis

This section examines the, alcohol reproduction number, local stability of the alcohol free equilibrium and the local stability of the endemic equilibrium.

#### 4.1 The Alcohol Reproduction Number, $R_0$

It is the average number of secondary infections that an infectious person causes in a community that is susceptible. For this case, it is the average number of secondary cases produced by one alcohol user throughout the alcoholism period. The alcohol reproduction number,  $R_0$ , is determined by use of the next generation matrix approach, which was employed by Catillo-Chavez et. al, 2002 (1). In most models, the initial infection is denoted by the letter F, and the transfer of infection with the letter V, observing that  $S^0 = \frac{\Lambda}{\mu}$ . In this model the infection compartments are E,  $V_1$  and  $V_2$ .

$$F = \begin{bmatrix} \omega S(V_1 + V_2) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V = V_i^- - V_i^+ = \begin{bmatrix} (\mu + \alpha_1)E \\ (\mu + \tau_1 + \alpha_2 + \sigma_1)V_1 - \kappa V_2 - \alpha_1 E \\ (\mu + \sigma_2 + \kappa + \tau_2)V_2 - \alpha_2 V_1 \\ (\mu + \omega E)S - \beta A - \Lambda \\ (\mu + \beta)A - \tau_1 V_1 + \tau_2 V_2 \end{bmatrix}$$

Differentiating  $F_i$  and  $V_i$  with respect to  $E, V_1$  and  $V_2$  gives;

$$F = \begin{bmatrix} \omega S^* & 0 & 0 \\ \omega S^* & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \mu + \alpha_1 & 0 & 0 \\ -\alpha_1 & k_2 & -k \\ 0 & -\alpha_2 & k_3 \end{bmatrix}$$

So that;

$$V^{-1} = \frac{1}{(\mu + \alpha_1)(k_3 k_4 - \kappa \alpha_2)} \begin{bmatrix} -(\mu + \alpha_1) & \alpha_1 & 0 \\ 0 & -k_2 & \alpha_2 \\ 0 & \kappa & -k_3 \end{bmatrix}$$

Therefore,

$$FV^{-1} = \begin{bmatrix} \frac{-\omega S^*}{k_2 k_3 - \kappa \alpha_2} & \frac{\omega S^* \alpha_1}{(\mu + \alpha_1)(k_2 k_3 - \kappa \alpha_2)} & 0 \\ \frac{-\omega S^*}{k_2 k_3 - \kappa \alpha_2} & \frac{\omega S^* \alpha_1}{(\mu + \alpha_1)(k_2 k_3 - \kappa \alpha_2)} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The eigenvalues are finally computed from  $|FV^{-1} - \lambda I| = 0$ . The results obtained are;  $\lambda_1 = \frac{-\omega S^*}{k_2 k_3 - \kappa \alpha_2}$ ,  $\lambda_2 = 0$  and  $\lambda_3 = \frac{\omega S^* \alpha_1}{(\mu + \alpha_1)(k_2 k_3 - \kappa \alpha_2)}$ . Since the Alcohol-Free equilibrium of the model was given by  $E^0 = [\frac{\Lambda}{\mu}, 0, 0, 0, 0]$ , the Jacobian matrix is evaluated at the AFE to obtain  $\lambda_1 = \frac{-\omega \Lambda}{\mu(k_2 k_3 - \kappa \alpha_2)}$ ,  $\lambda_2 = 0$  and  $\lambda_3 = \frac{\Lambda \omega \alpha_1}{\mu(\mu + \alpha_1)(k_2 k_3 - \kappa \alpha_2)}$ . The maximum modulus or dominant eigenvalue therefore defines the alcohol reproduction number,  $R_0$ . That is the spectral radius of the Jacobian matrix,  $\rho(FV^{-1})$ . From  $\rho(FV^{-1})$ , the alcohol reproduction number  $R_0$  is therefore given by;

$$R_0 = \max |FV^{-1} - \lambda I| = \max[|\lambda_1|, |\lambda_2|, |\lambda_3|] = \frac{\Lambda \omega \alpha_1}{\mu(\mu + \alpha_1)(k_2 k_3 - \kappa \alpha_2)} \quad (4.1)$$

where;  $\frac{\Lambda\omega}{\mu(\mu+\alpha_1)}$  is the average secondary infections arising from the light drinking class while  $\frac{\alpha_1}{(k_2k_3-\kappa\alpha_2)}$  is the average secondary infections arising from the heavy drinking class.

## 4.2 Local Stability Analysis of the Alcohol Free Equilibrium (AFE)

All drinking classes and awareness levels are set to zero to derive the AFE of the system in equation 2.1, that is  $E = V_1 = V_2 = A = 0$  and  $S \neq 0$ . Hence, the result obtained is  $S^0 = \frac{\Lambda}{\mu}$ . Therefore, the model developed in Equation (2.1) has an alcohol free equilibrium (AFE) given by;

$$E^0 = [S^0, E^0, V_1^0, V_2^0, A^0] = \left[\frac{\Lambda}{\mu}, 0, 0, 0, 0\right] \quad (4.2)$$

**Theorem 4.1.** The AFE point  $E^0$  is locally asymptotically stable if  $R_0 < 1$  and unstable if  $R_0 > 1$

**Proof.** To perform local stability analysis of the AFE, the Jacobi matrix, J is computed by differentiating each equation in system (2.1) with respect to S, E,  $V_1, V_2$  and A;

$$J = \begin{bmatrix} -\mu - \omega(V_1 + V_2) & 0 & -\omega S & -\omega S & \beta \\ \omega(V_1 + V_2) & -\mu - \alpha_1 & \omega S & -\omega S & 0 \\ 0 & \alpha_1 & -k_2 & \kappa & 0 \\ 0 & 0 & \alpha_2 & -k_3 & 0 \\ 0 & 0 & \tau_1 & \tau_2 & -k_1 \end{bmatrix} \quad (4.3)$$

By evaluating the Jacobi matrix in equation (4.3) at the AFE,  $E^0$  the result obtained is;

$$J_{E^0} = \begin{bmatrix} -\mu & 0 & -\frac{\Lambda\omega}{\mu} & -\frac{\Lambda\omega}{\mu} & \beta \\ 0 & -\mu - \alpha_1 & \frac{\Lambda\omega}{\mu} & \frac{\Lambda\omega}{\mu} & 0 \\ 0 & \alpha_1 & -k_2 & \kappa & 0 \\ 0 & 0 & \alpha_2 & -k_3 & 0 \\ 0 & 0 & \tau_1 & \tau_2 & -k_1 \end{bmatrix} \quad (4.4)$$

To investigate the stability of the AFE, eigenvalues are computed from equation (4.4) as follows;

$$J_{E^0} - \lambda I = \begin{bmatrix} -\mu - \lambda & 0 & -\frac{\Lambda\omega}{\mu} & -\frac{\Lambda\omega}{\mu} & \beta \\ 0 & -\mu - \alpha_1 - \lambda & \frac{\Lambda\omega}{\mu} & \frac{\Lambda\omega}{\mu} & 0 \\ 0 & \alpha_1 & -k_2 - \lambda & \kappa & 0 \\ 0 & 0 & \alpha_2 & -k_3 - \lambda & 0 \\ 0 & 0 & \tau_1 & \tau_2 & -k_1 - \lambda \end{bmatrix} \quad (4.5)$$

Using the Routh Hurwitz criterion as used in (11), the eigenvalues obtained by matrix (4.5) are negative since the trace at AFE is given by;  $Tr(E^0) = -[\mu + \alpha_1 + k_1 + k_2 + k_3] < 0$  and the determinant at DFE is given by;  $k_1[-\mu(\mu + \alpha_1)(k_2k_3 + \kappa\alpha_2) + \Lambda\omega\alpha_1(k_3 + \alpha_2)] > 0$  and on substitution with  $R_0$  the following is obtained;  $k_1\mu(\mu + \alpha_1)(k_2k_3 + \alpha_2)[-1 + (k_3 + \alpha_2)R_0]$ .

Factoring out  $(k_3 + \alpha_2)$ , the result obtained is;  $k_1\mu(\mu + \alpha_1)(k_2k_3 + \alpha_2)(k_3 + \alpha_2)\left(\frac{-1}{k_3 + \alpha_2} + R_0\right) > 0$  and if  $\frac{1}{k_3 + \alpha_2} \rightarrow 1$ , that is  $\alpha_2 = 1 - k_3$  then; the determinant becomes,  $k_1\mu(\mu + \alpha_1)(k_2k_3 + \alpha_2)(k_3 + \alpha_2)(R_0 - 1) > 0$  when  $R_0 < 1$ , thus the AFE is locally asymptotically stable.  $\square$

In light of this, the analysis comes to the conclusion that the AFE is locally asymptotically stable whenever  $R_0 < 1$ . That is, given a small alcoholic population, each alcoholic in the entire time frame of alcoholism, will produce on average less than one drinker when  $R_0 < 1$ . This implies that alcohol abuse vanishes in the population when  $R_0 < 1$ . This is because media awareness might have been well implemented hence further minimization of alcohol abuse cases.

### 4.3 Existence of Endemic Equilibrium Point (EEP)

When alcoholism persists in the community, the endemic point of the model is reached. The system of equation (2.1) is solved in terms of the force of infection at the steady state  $\lambda^*$  to determine the prerequisites for the presence of an equilibrium where alcohol misuse is pervasive in the population. When the right side of equation (2.1) is set to zero and it is noted that at equilibrium,  $\lambda = \lambda^*$ , the following result is obtained:

$$\begin{aligned}
 S^* &= \frac{\Lambda\beta\omega\alpha_1\xi(V_1 + V_2) - \Lambda k_1(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)}{\beta\omega\alpha_1\xi(V_1 + V_2)(\mu + \omega(V_1 + V_2))} \\
 E^* &= \frac{\Lambda\beta\omega\alpha_1\xi(V_1 + V_2) - \Lambda k_1(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)}{\beta\alpha_1\xi(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))} \\
 V_1^* &= \frac{\Lambda\beta\omega\alpha_1\xi k_4(V_1 + V_2) - \Lambda k_1 k_3(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)}{\beta\xi(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)} \\
 V_2^* &= \frac{\Lambda\beta\omega\alpha_1\alpha_2\xi(V_1 + V_2) - \Lambda k_1\alpha_2(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)}{\beta\xi(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)} \\
 A^* &= \frac{\Lambda\beta\omega\alpha_1\xi(V_1 + V_2) - \Lambda k_1(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)}{\beta k_1(\mu + \alpha_1)(\mu + \omega(V_1 + V_2))(k_2k_3 - k\alpha_2)} \quad (4.6)
 \end{aligned}$$

### 4.4 Endemic Equilibrium in Terms of $R_0$

The endemic equilibrium point is expressed in terms of the alcohol reproduction number by substituting some of the constants that make up the reproduction number in place of those in equation (4.5). Clearly, from  $R_0$  in equation 4.1,  $\frac{\Lambda\omega\alpha_1}{\mu + \alpha_1} = \mu R_0(k_2k_3 - k\alpha_2)$  and substituting in all equations of system (4.5) the following is obtained:

$$\begin{aligned}
 S^* &= \frac{\Lambda\beta\xi(V_1 + V_2)\mu R_0 - \Lambda^2 k_1(\mu + \omega(V_1 + V_2))}{\beta(V_1 + V_2)\xi\mu R_0(\mu + \omega(V_1 + V_2))} \\
 E^* &= \frac{\Lambda\beta\omega\mu R_0\xi(V_1 + V_2) - \Lambda^2\omega k_1(\mu + \omega(V_1 + V_2))}{\beta\mu R_0\xi(\mu + \omega(V_1 + V_2))(\mu + \alpha_1)} \\
 V_1^* &= \frac{\beta(V_1 + V_2)k_3\mu R_0\xi - \Lambda k_1 k_2(\mu + \omega(V_1 + V_2))}{\beta\xi(\mu + \omega(V_1 + V_2))} \\
 V_2^* &= \frac{\beta\alpha_2(V_1 + V_2)\mu R_0\xi - \Lambda k_1\alpha_2(\mu + \omega(V_1 + V_2))}{\beta\xi(\mu + \omega(V_1 + V_2))} \\
 A^* &= \frac{\beta(V_1 + V_2)\xi\mu R_0 - \Lambda k_1(\mu + \omega(V_1 + V_2))}{\beta k_1(\mu + \omega(V_1 + V_2))} \quad (4.7)
 \end{aligned}$$

**Theorem 4.2.** Endemic equilibrium exist and is locally asymptotically stable if  $R_0 > 1$ .

**Proof.** To investigate stability of the endemic equilibrium point, the steady states,  $E^* = (S^*, E^*, V_1^*, V_2^*, A^*)$  in equation (4.6) are substituted in the Jacobi in equation (4.3) and the solution obtained is;

$$J_E = \begin{bmatrix} -\mu - \omega(V_1 + V_2) & 0 & -\omega S & -\omega S & \beta \\ \omega(V_1 + V_2) & -\mu - \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & -k_2 & \kappa & 0 \\ 0 & 0 & \alpha_2 & -k_3 & 0 \\ 0 & 0 & \tau_1 & \tau_2 & -k_1 \end{bmatrix} \quad (4.8)$$

The eigenvalues of the system (4.8) are calculated in order to determine the necessary and sufficient prerequisites for the occurrence of the endemic equilibrium point. That is;

$$\left\{ \begin{array}{ccccc} -\mu - \omega(V_1 + V_2) - \lambda & 0 & -\omega S & -\omega S & \beta \\ \omega(V_1 + V_2) & -(\mu - \alpha_1) - \lambda & 0 & 0 & 0 \\ 0 & \alpha_1 & -k_2 - \lambda & \kappa & 0 \\ 0 & 0 & \alpha_2 & -k_3 - \lambda & 0 \\ 0 & 0 & \tau_1 & \tau_2 & -k_1 - \lambda \end{array} \right\} = 0 \quad (4.9)$$

The following characteristic equation is obtained from Jacobi in equation (4.9);

$$P(\lambda) = \lambda^5 + A_1\lambda^4 + A_2\lambda^3 + A_4\lambda^2 + A_5 = 0 \quad (4.10)$$

where;

$$\begin{aligned} A_1 &= k_1 + k_2 + \mu + \alpha_1 + \frac{\mu(k_2k_3 + \kappa\alpha_2)(\mu + \alpha_1)(V_1 + V_2)}{\Lambda\alpha_1} (R_0 - 1) \\ A_2 &= k_1k_3 + k_1k_2 + k_2k_3 + 2k_3 + k_1 + k_2 + \frac{\mu(k_1 + k_2 + k_3 + \mu + \alpha_1)(k_2k_3 + \kappa\alpha_2)(V_1 + V_2)(\mu + \alpha_1)}{\Lambda\alpha_1} (R_0 - 1) \\ A_3 &= \frac{(k_1k_3 + k_1k_2 + k_2k_3)(\mu + \alpha_1)^2(k_2k_3 + \kappa\alpha_2)(V_1 + V_2)\mu}{\Lambda\alpha_1} (R_0 - 1) + k_1k_2k_3 \\ A_4 &= \frac{\mu(k_2k_3 + \kappa\alpha_2)(V_1 + V_2)(\mu + \alpha_1)}{\Lambda\alpha_1} (R_0 - 1)(k_1k_2k_3(1 + \mu + \alpha_1) + k_1k_2 + k_2k_3) + \mu + \alpha_1 \\ A_5 &= \frac{\mu k_1 k_2 k_3 (\mu + \alpha_1)^2 (V_1 + V_2) (k_2 k_3 + \kappa \alpha_2)}{\Lambda \alpha_1} (R_0 - 1) \end{aligned}$$

Therefore, the number of negative real roots of equation (4.10) is dependent on signs of  $A_1, A_2, A_3, A_4$  &  $A_5$ . This analysis can be done using the Descartes Rules of Signs of the polynomial given by equation (3).

$$P(\lambda) = A\lambda^4 + B\lambda^3 + D\lambda^2 + E\lambda + G \quad (4.11)$$

The Descartes Rule of Signs (3) states that, the number of negative real zeros of P is either equal to the number of variations in signs of  $P(-\lambda)$  or less than this by an even number. Therefore, the maximum number of variations of signs in  $P(-\lambda)$  is four, thus the characteristic polynomial in (4.10) has four negative roots. Thus,

$$P(-\lambda) = -\lambda^5 + A_1\lambda^4 - A_2\lambda^3 + A_4\lambda^2 - A_5 = 0 \quad (4.12)$$

has negative roots. Thus if the average infections arising from both light and heavy drinkers;  $\frac{1}{(\mu + \alpha)(k_2k_3 + \kappa\alpha_2)} \rightarrow 1$ , and for  $R_0 - 1 > 0$  then the model (2.1) is locally asymptotically stable if  $R_0 > 1$ .  $\square$

## 4.5 Summary

If  $R_0 < 1$ , then  $E_0$  is an alcohol free equilibrium of system (2.1) and it is locally asymptotically stable. This implies that alcoholism cases disappear in the community. This may be as a result of maximum utilization of media programs in creating awareness on the effects of alcoholism. Furthermore, there exists an endemic equilibrium if  $R_0 > 1$  and is locally asymptotically stable. The existence of the endemic equilibrium indicates that, given a small alcoholic population, each alcoholic will produce on average less than one drinker when  $R_0 > 1$ . This implies that alcoholism persists in the community. That is the number of light and heavy drinkers rises further as a result of poor implementation of media programs thus the need for the government and other stakeholders such as NACADA to formulate suitable policies in targeting media awareness as an intervention measure in combating the rampant alcoholism cases in the community.

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## 5 Numerical Simulations

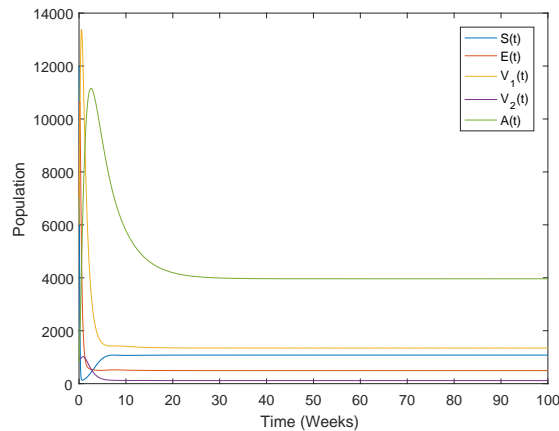
### 5.1 Parameter Estimation

Utilizing MATLAB software, the system of equation (3.1) are studied. The original population estimates for  $S$ ,  $E$ ,  $V_1$  and  $V_2$  and  $A$  come from the 2019 reports from the Kenya Bureau of Statistics (KNBS) and the United Nations and Social Affairs (5; 13). The initial population of the media awareness class was determined using secondary data from rehabilitation facilities in Kenya and information from the National Authority for the Campaign against Alcohol and Drug Abuse report (12). The population of Kenya, which the United Nations estimates to be 52 million people in 2019, was used to calculate the initial conditions of the steady states (13). Alcohol use is predicted to be prevalent in 31 percent of people, with alcohol addiction occurring in 13.3 percent of these people (12). This amounts to around 2.028 million people who are alcohol dependent and 15.6 million people in the classifications  $V_1$ ,  $V_2$ , and  $A$ . Thus, the initial conditions for the variables are taken to be:  $S=24000000$ ,  $E=14000000$ ,  $V_1 = 12000000$ ,  $V_2 = 2000000$ ,  $A=1600000$ . The parameters, their corresponding values, and the source from which they were received are all listed in Table 4.1. The parameter is directly related to the fundamental reproduction number, as indicated by the positive index. The following table summarizes the parameters of alcoholism and the respective sources obtained from.

Table 4.1: Model Parameters and their Respective Sources

Parameter	Value	Source
$\Lambda$	1674000	(11)
$\mu$	0.5	(6; 14)
$\beta$	0.03	Assumed
$\omega$	0.001	Assumed
$\alpha_1$	3	Assumed
$\alpha_2$	0.1	Assumed
$\kappa$	0.01	Assumed
$\sigma_1$	0.05	(10)
$\sigma_2$	0.2	(11)
$\tau_1$	0.05/0.6	Assumed
$\tau_2$	0.2/ 0.9	Assumed

## 6 Numerical Results



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Figure 4.1: All Alcohol Classes of the Model.

The relationship between each population class in the model is shown in Figure 4.1. Increasing  $\tau$  implies that enormous number of light and heavy drinkers who enter the awareness class and that is why the awareness class graph starts out very high before dipping somewhat and stabilizing when some people return to the susceptible class. As more people joined the exposed population, the susceptible individuals became fewer over time. As some awareness individuals return to the susceptible class, the susceptible class rises somewhat before stabilizing. The exposed population decreases due to individuals joining the light drinking group. The light drinkers decreases in the first few days then stabilizes since most individuals join the awareness class. The heavy drinkers decreases further then stabilize for the remaining time. In comparison to the other classes, the awareness class has a very big enrollment. This is a result of the high rate of alcohol-using people recruited into media awareness efforts. Information can be transmitted more quickly and efficiently by being cognizant of the media. This means that most people become aware of the risks associated with alcoholism and immediately take action.

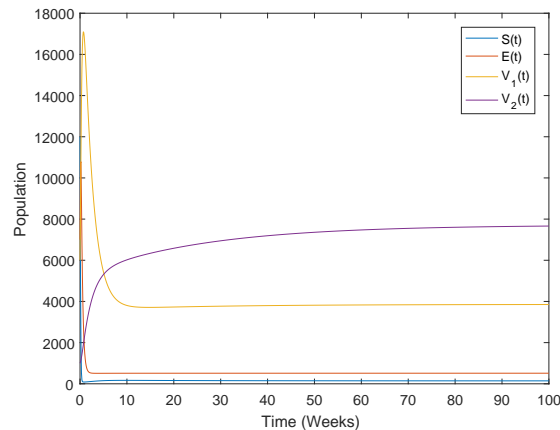


Figure 4.2: Alcoholism Model with No Media Awareness.

Figure 4.2 represents a case with no awareness, the heavy drinking class increases rapidly due to many light drinkers advancing their drinking habits. This implies that individuals in the light drinking class become heavily dependent on alcohol thus becoming addicts. This further causes the number of heavy drinkers in the neighborhood to rise. Low economic output, a rise in social crime, and even an increase in mortality could follow from this. The exposed and light drinkers reduce and then stabilize while the susceptible population reduces further as a result of transition into subsequent classes.

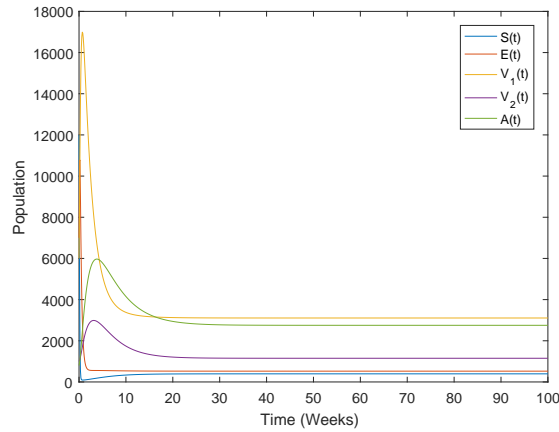


Figure 4.3: Alcoholism Model with Minimal Media Awareness.

Figure 4.3 implies minimal media awareness rate. The treatment rate rises slightly, the number of people in the light drinking class rises a little more, and then it starts to decline a slightly with minimal awareness efforts. This is because more individuals are joining the light drinking class from the exposed group and the heavy drinking class with a small number being recruited into awareness class. The increase in the number of light drinkers implies that individuals are averagely enlightened on the dangers of alcohol abuse thus they are less dependent on alcohol and become addicts at a relatively slower rate. The heavy drinkers slightly increase and then stabilizes. This indicates that suitable media awareness programs have not been utilized in efforts to minimize the abuse of alcohol.

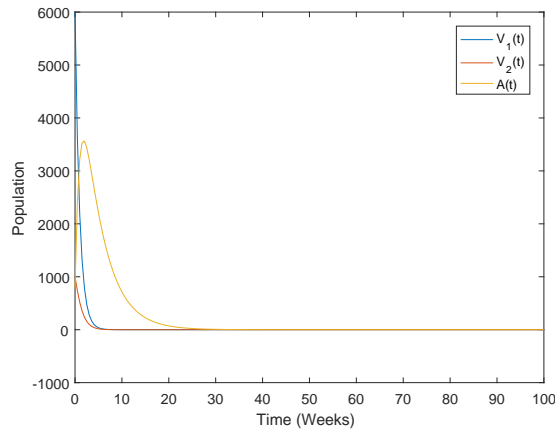


Figure 4.4: Alcohol Prevalence of the Model

Figure 4.4 represents the alcohol prevalence classes plotted in the same axes. Alcohol prevalence simply means the fraction of the population that is infected with alcoholism. The light drinkers decrease in the first few days because some individuals move to the heavy drinking class while many others move to awareness class after which they may quit drinking since they are not highly addicted to alcohol. The awareness class population increase in the first few days then decrease but the number in the class is less than the number in the heavy drinking class and also in the light drinking class. This situation indicates the relationship between drinking classes and treatment (media awareness). This implies that, with adequate media awareness, alcohol abuse stabilizes in the community. This is because, as the cases of alcoholism rises, they are immediately and

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effectively controlled.

## 7 Conclusions and Recommendations

### 7.1 Conclusion

- (a). The model was formulated based on a system of differential equations. The study examined the invariant region and positivity of the model and proved that the model is biologically well-posed.
- (b). The conditions for the existence of the AFE point were investigated and it was shown that to be locally asymptotically stable whenever  $R_0 < 1$ . This suggests that alcoholism becomes extinct in the community. additionally, the endemic equilibrium of the model exists and is positive if  $R_0 > 1$ , hence the prerequisites for its existence were satisfied. This demonstrates how alcoholism spreads through a community, highlighting the importance of making the most of media awareness resources.
- (c). Numerical analysis of the model indicated that increase in media awareness programs reduces alcohol prevalence in the community. The awareness degree is varied as in Figure 4.1, 4.2 and 4.3 and with high rate of media awareness, alcohol prevalence decrease. With minimal media awareness, there is an increase in the number of light drinkers. With no awareness then more individuals remain in heavy drinking class thus increased alcohol dependency and addiction. Thus, increase in media awareness programs plays a key role in further reduction of the light and heavy drinkers. Thus, the study concluded that the exposed class has a greater impact in minimizing the numbers of light and heavy drinkers in the presence of media awareness. Further, the study concluded that media awareness is the best intervention strategy in curbing alcoholism in a community but it cannot alter its spread.

### 7.2 Recommendations

- The study recommends that the Kenya government should encourage awareness programs on alcohol abuse by strengthening mass media campaigns against alcohol consumption.
- More importantly the Kenyan government should endeavor setting up more rehabilitation centers where addicts can be sensitized on dangers of alcohol abuse and rehabilitated either through detoxification process or other forms of treatment.
- The study suggests additional research be done on the effects of media awareness campaigns on other substances that are often abused in Kenya, with a focus on the influence of the exposed class.
- The study advocates that the future works should target other effective treatment and prevention strategies to combat drug abuse.
- Future research should work together with narcotic bodies such as NACADA, so as suitable policies against alcoholism can be emphasized.

## Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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