

A Comparative Study on Mathematical Problems in Chinese and Japanese Textbooks using the Synthesized Bloom Taxonomy

Abstract: In order to study the characteristics of the problem design of mathematics textbooks in junior high school, this paper focuses on the comparative study on linear function sections of Chinese textbooks of people's edition (P1) and Japanese textbooks named New Mathematics 2 (J1). Using the Synthesized Bloom Taxonomy(SBT), this paper compares the types of mathematical problems in two editions from two dimensions. In the cognitive process dimension, P1 and J1 both put emphasis on grasping basics and the competence to solve simple practical problems. In the knowledge dimension, P1 inclines toward processes of problem solving, requiring a step-by-step approach according to the clues provided and J1 better focus on the diversity and synthesis of the solutions. These results may provide references for mathematics teaching and learning in junior high schools.

Keywords: the Synthesized Bloom Taxonomy; Comparative Study; Linear function; Mathematical Problems

1 Introduction

Textbooks are the basis of all educational and teaching activities and the fundamental carrier of education. In order to study the characteristics of the problem design of junior high school mathematics textbooks in China and Japan, Chinese textbooks of people's edition (P1) and Japanese textbooks named New Mathematics 2 (J1) . In the evaluation framework of TIMSS2019, the content areas of the eighth grade evaluation are numbers and algebra, which account for 60%.

In recent years, the cross-national comparison of mathematics textbooks is hotspot of research on mathematics education. China and Japan belong to Asia, and are influenced by Confucianism, and have a certain similarity in culture and education background. Therefore, comparative study of Chinese and Japanese textbooks is helpful to understand their advantages and disadvantages, and promote cultural exchanges and learn from each other [1]. Given similarities between China and Japan in terms of cultural backgrounds and education systems, we chose problems in Chapter 19 , Linear Functions, of People's Education Press-Mathematics in China (P1) and Chapter 3, Linear Functions, of New Mathematics 2 in Japan (J1) as the objects of our study and comparison. The Synthesized Bloom Taxonomy (SBT) [2] is used to assess cognitive process and knowledge dimensions of problems. It gives an opportunity to assess cognition not just in terms of knowledge but simultaneously in terms of processes [3]. This study hopes to collect information on the characteristics of the mathematical problems in the primary function section of the two editions of the textbook through the two dimensional framework of the SBT model, so as to provide reference for the design of mathematics exercises and teachers' teaching in China.

Searches of the knowledge network shows that the current literature on comparative studies of mathematics materials focuses on text-based studies, which includes comparative studies of mathematical problems. Lorin W and his colleagues developed a two-dimensional framework focusing on knowledge and cognitive processes to provide teachers with a framework for understanding and organizing objectives. Literature [4] built on the framework of literature [3] to give learning objectives corresponding to each level of the knowledge and cognitive processes dimensions. In particular, Rizvi pointed out that the stages of cognitive processes in existing studies [5-8] overlap and identify different levels, developing a new taxonomy called Synthetic Bloom's Taxonomy. Literature [2] presented a comprehensive classification and comparative study of mathematical problems in Turkish and Canadian school mathematics textbooks with the help of the SBT provided in literature [9]. Based on the above study, the SBT can be improved and extended to offer some reference value for a comparative study of the type of mathematics problems in the functions section of junior secondary school mathematics textbooks in various countries.

The research problems which guided this study were: What are the similarities and differences in the design of problems on linear functions in the two textbooks and suggestions for further studies, as well as the room for improvement.

2 Theoretical framework

2.1 Research Model

In this study, the SBT from reference [2] was chosen to code and classified all mathematics problems in the chapter on linear functions. As shown in Table 1 below, the SBT contains two dimensions: cognitive process and knowledge.

Table 1 Synthesized Bloom taxonomy

Knowledge dimension	Cognitive Process		
	1 .Assimilation	2 . Transformation	3 . Creative and Validation
A. Factual			
B. Conceptual			
C. Procedural			
D. Meta- cognitive			

On the horizontal axis, the cognitive process dimension is divided into levels 1, 2 and 3. Where 1 denotes assimilation, i. e., examining students' memory of concepts and 2 denotes transformation, i. e., converting real- life problems into mathematical problems or performing simple calculations, and 3 denotes creation and validation, i. e., choosing the most appropriate solution by analyzing and creating.

On the vertical axis, the knowledge dimension is divided into A, B, C and D levels. Where A denotes facts, i. e., containing general knowledge of mathematics, concepts, and formulae that students have mastered or facts that already exist objectively; B denotes concepts, i. e., solving problems through concepts; C denotes procedural, i. e., having a fixed method of solving problems; and D denotes meta-cognitive, i. e., multiple methods of solving problems.

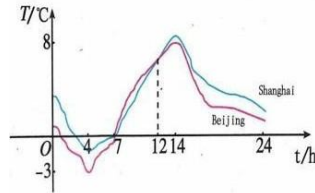
When coding the problems from P1 and J1, we specified these problems into the following parts: examples, exercises in this chapter and each sections taken from P1; examples, exercises in this chapter and each sections, supplementary problems taken from J1. As counting the total number of problems, only the main problems were considered. If several sub-problems appeared in a problem, these were considered as one problem. In particular, it should be noted that each mathematical problem may be evaluated as more than one unit, but in this study, we assumed that each problem was evaluated as only one unit.

Some problem samples related to coding of problems in middle school mathematics

Code Chinese Japanese

The graph below shows the temperature in Beijing and Shanghai on one day as a function of time.

- (1) When is the temperature in Shanghai the same as that in Beijing during this day?
- (2) At which time is the weather in Shanghai warmer than that in Beijing? And at which time was it cooler?



x	...	4	...	6	...
y	...	40	...	50	...

x	...	-3	...	1	...
y	...	5	...	25	...

B1

Find variables and constants in the question.

The water price cities is 4 yuan per tonne.

The researchers samples water usage data of some families. Write a function to show the total cost y per month for water for x tonnes in a home.

Find the slope and y-intercept of each function:

(1) $y = 3x + 1$ (2) $y = -\frac{1}{2}x - 3$

Find the equation of the line with the given slope and passing through the given point.

slope=-2; (3,1)

For example, the main activities in A involved recalling about definitions of linear function or extracting the objective facts from the problem. The problem in Chinese textbook has given the temperature changes in Beijing and Shanghai, which is a fact, therefore it is in the category of A. And students should consider the relation of two functions in terms of the function expressions, thus it is in the category of 2. Next example, the problem in J1 is expected that students should be able to remember and understand concepts of the intercept and slope accurately, therefore it is in the category of 1 and B. Another mathematics problem in Chinese textbook has been categorized as C2 since it is a procedural knowledge for students to determine whether the given point is on the image of a function and it examines their understanding of the concept of graph of functions. The sample problem, taken from the P1 and assessed as 3, expects students to have the ability to apply the concepts of function expressions and graphs to select the optimal decisions. And since there are clear methods to draw the graph of the function, it is in the category of C. Lastly, according to information in the D1 problem from J1, in the given case, students are supposed to understand the concept of images. And there are two ways to draw the image of functions, Point-by-Point method and the migration method of graphing linear functions.

2 . 2 Validity and Reliability of Data

For assessing the credibility of the coding results of the model, we selected 20 mathematics problems from the chapter on linear functions in P1 and 20 problems from J1 as a sample for the test. Two group members acted as raters to check the coding reliability. According to the coding model, the raters independently coded each of the 40 sample problems from two dimensions, and the coding results are shown in Table 3 below.

Table 3 Coding test results

No.	Coder	Coder	No.	Coder	Coder	No.	Coder	Coder	No.	Coder	Coder
	A	B		A	B		A	B		A	B
1	A2	B2	11	D1	C1	21	B2	B2	31	C2	C2
2	B1	B1	12	C2	C2	22	B1	B1	32	B2	B2
3	B2	B1	13	D1	D1	23	C1	C1	33	C1	C1
4	A2	A2	14	B1	C1	24	C2	C2	34	A1	B1
5	B2	B2	15	D2	C2	25	B3	B1	35	B2	B2
6	B2	B2	16	D2	D2	26	A1	A1	36	B2	B2
7	B2	B2	17	B2	D2	27	B1	B1	37	B3	B3
8	A2	A2	18	B2	B2	28	B2	B2	38	C1	C1
9	B2	B2	19	D1	D2	29	C1	C1	39	C2	D2
10	D1	D1	20	D1	D1	30	C1	C1	40	C3	C3

The above results were analyzed in SPSS[9] using Kendall's Coefficient [10] to obtain the credibility of the model. The four levels of the knowledge dimension in the SBT were assigned values of 1, 2, 3 and 4, and the three levels of the cognitive process dimension were assigned values of 1, 2 and 3. After iterative testing and adjustment by the panelists, the following results were obtained as shown in Figures 1 and 2.

Correlations

			Coder A	Coder B
Kendall's tau_b	Coder A	Correlation Coefficient	1.000	.822**
		Sig.(2-tailed)	.	.000
		N	40	40
	Coder B	Correlation Coefficient	.822**	1.000
		Sig.(2-tailed)	.000	.
		N	40	40

**Correlation is significant at the 0.01 level (2-tailed).

Figure 1 Relevance test for knowledge dimension

Correlations

			Coder A	Coder B
Kendall's tau_b	Coder A	Correlation Coefficient	1.000	.800**
		Sig.(2-tailed)	.	.000
		N	40	40
	Coder B	Correlation Coefficient	.800**	1.000
		Sig.(2-tailed)	.000	.
		N	40	40

**Correlation is significant at the 0.01 level (2-tailed).

Figure 2 Relevance test for the cognitive process dimension

The correlation coefficient in the knowledge and cognitive dimension were 0.822 and 0.8, respectively, which indicated a high level of reliability.

3 Results

In the present study, 165 mathematics problems in a chapter of linear function in the Japanese and Chinese textbooks are investigated according to the SBT.

The mathematics problems in the linear function are investigated in detail in terms of the knowledge dimension and the cognitive processes dimension and the obtained coding data are presented in Table 4, Table 5 and Table 6. Statistics show that the number of the mathematics problems in J1 is higher than that in P1 (Japan: n = 90, China: n = 75).

Table 4 Distribution of problem in P1 according to SBT

Knowledge dimension	Cognitive Process		
	1 .Assimilation	2 . Transformation	3 . Creative and Validation
A. Factual	8	6	0
B. Conceptual	6	12	5
C. Procedural	9	22	2
D. Meta- cognitive	4	1	0

Table 5 Distribution of problem in J1 according to SBT

Knowledge dimension	Cognitive Process		
	1 .Assimilation	2 . Transformation	3 . Creative and Validation
A. Factual	3	8	0
B. Conceptual	10	12	1
C. Procedural	11	12	0
D. Meta- cognitive	22	11	0

Table 6 Summary of the type of mathematics problems coding results of linear function in two versions of textbooks

Dimension Level	Chinese textbook		Japanese textbook	
	Volume	Percentage of problems	Volume	Percentage of problems
Cognitive Process	1 . Assimilation	27 36.00%	46 51.11%	
	2 . Transformation	41 54.67%	43 47.78%	
	3 . Creative and Validation	7 9.33%	1 1.11%	
Knowledge dimension	A. Factual	14 18.67%	11 12.22%	
	B. Conceptual	23 30.67%	23 25.55%	
	C. Procedural	33 44.00%	23 25.55%	
	D. Meta- cognitive	5 6.67%	33 36.67%	

Additional, the types of mathematics problems in the chapter on linear functions in both editions of the textbook are similar with no mathematics problems observed that could be assessed within the A3 and D3 categories.

A z-test [11] was conducted on the data obtained from Table 6. The number of the types of mathematics problems in the chapter on linear functions in the two editions of the textbook was

There are some differences in cognitive process dimension. At the assimilation and transformation stage of the cognitive process dimension, there is no significant difference between the two versions of the textbook in terms of the type of mathematics problems in the chapter on linear functions ($z = 1.9$, $p = 0.0534 > 0.05$; $z = 0.9$, $p = 0.3704 > 0.05$). In addition, at the creation and validation level, there is a significant difference between the two editions in terms of the types of mathematics problems in the chapter on linear functions ($z = 2.4$, $p = 0.015 < 0.05$).

There are some differences in cognitive process dimension. There is no significant difference between the two editions in terms of the types of mathematics problems in the chapter on linear functions at the factual and conceptual stage of the knowledge dimension ($z = 1.2$, $p = 0.2121 > 0.05$; $z = 0.7$, $p = 0.4776 > 0.05$). Additionally, at the procedural and Meta-cognitive levels, there are significant differences between the two editions in the types of mathematics problems used in the chapter on linear functions ($z = 2.4$, $p = 0.0152 < 0.05$; $z = 4.5$, $p < 0.0001$).

4 Discussion and conclusion

This paper compares and analyzed the differences in the knowledge and cognitive dimensions of mathematical problems in the linear function part of two different versions of mathematics textbooks: People's Education Press- Mathematics in China(P1) and New Mathematics 2 in Japan(J1), for the purpose of investigating their own characteristics, and gaining understandings and implications in their differences.

Mathematics Curriculum Standard [12] requires students to be able to list the expressions of linear function in practical problems, and solve simple practical problems by combining the images of linear function and the properties of expressions. Initially, the discoveries of the research illustrate that, in the cognitive dimension, both of two textbooks attach a considerable importance to transformation knowledge, and the proportion of each one is approximately half of their respective problem sets, which is in line with the requirements of Standard and is consistent with Yiming Cao's view in the literature [13]. Meanwhile, both of them pay attention to assimilation knowledge, especially J1. And it is obvious that, based on the data, the sum of the proportions of assimilation and transformation problems in each textbook is more than 90% respectively (J1 even exceeds 98%), which indicates that both take students' mastery of basic knowledge and the solution of simple practical problems into account. Despite the fact that the research reveals many close parallels in these two textbooks' sets on linear function, it still has distinctions. Both of P1 and J1 apparently account for relatively small proportions of the creative and validation problems, requiring students to analyze and synthesize information and give the appropriate solution, however, compared with the J1 (1.11%), the proportion of P1 is significantly larger, with 9.33%, reflecting the requirements of the Standards.

Considering the knowledge dimension, proportions on factual and conceptual are comparison consistent on the basis of and Z-test results, whereas that on procedural and meta-cognitive do not. Procedural knowledge of P1 takes up the biggest part while J1 is quite the opposite, manifesting P1 trends to the process of solving problems which enquire students to solve problems step by step according to the given clues and J1 better focus on the diversity and synthesis of the solutions, which is corresponding with results reported by LiJun Ye (2014) in his research about the comparison of China and Japanese mathematics textbooks in junior high school[14].

The results may provide references for mathematics teaching and learning in junior high schools. These may help teachers to consider textbooks under different perspective. Nevertheless, this study focused only on the mathematics problems of linear function sections in P1 and J1. Therefore, future researchers may adopt the analytical framework and research model offered to do relevant research. Moreover, supporting the research model with quantitative data is enlightening so as to make the findings may become more significant.

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