

# A Comparative Study on Mathematical Problems in Chinese and Japanese Textbooks using the Synthesized Bloom Taxonomy

**Abstract:** In order to study the characteristics of the problem design of mathematics textbooks in junior high school, this paper focuses on the comparative study on linear function sections of Chinese textbooks of people's edition (P1) and Japanese textbooks named New Mathematics 2 (J1). Using the Synthesized Bloom Taxonomy(SBT), this paper compares the types of mathematical problems in two editions from two dimensions. In the cognitive process dimension, P1 and J1 both put emphasis on grasping basics and the competence to solve simple practical problems. In the knowledge dimension, P1 inclines toward processes of problem solving, requiring a step-by-step approach according to the clues provided and J1 better focus on the diversity and synthesis of the solutions. These results may provide references for mathematics teaching and learning in junior high schools.

**Keywords:**the Synthesized Bloom Taxonomy;Comparative Study; Linear function; Mathematical Problems

## 1 Introduction

Textbooks are the basis of all educational and teaching activities and the fundamental carrier of education. In order to study the characteristics of the problem design of junior high school mathematics textbooks in China and Japan, Chinese textbooks of people's edition (P1) and Japanese textbooks named New Mathematics 2 (J1). In the evaluation framework of TIMSS2019, the content areas of the eighth grade evaluation are numbers and algebra, which account for 60%. Given similarities between China and Japan in terms of cultural backgrounds and education systems, we chose problems in Chapter 19, Linear Functions, of People's Education Press- Mathematics in China(P1) and Chapter 3, Linear Functions, of New Mathematics 2 in Japan(J1) as the objects of our study and comparison.The Synthesized Bloom Taxonomy (SBT) [1] is used to assess cognitive process and knowledge dimensions of problems. Referring to the SBT, this paper will compare the differences in types of problems between the two editions in terms of the cognitive process dimension and the knowledge dimension, so as to provide reference for the design of mathematics exercises and teachers' teaching in China.

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Searches of the knowledge network shows that the current literature on comparative studies of mathematics materials focuses on text-based studies, which includes comparative studies of mathematical problems. Lorin W and his colleagues developed a two-dimensional framework focusing on knowledge and cognitive processes to provide teachers with a framework for understanding and organizing objectives. Literature[2] built on the framework of literature[3] to give learning objectives corresponding to each level of the knowledge and cognitive processes dimensions. In particular, Rizvi pointed out that the stages of cognitive processes in existing studies[4-7] overlap and identify different levels, developing a new taxonomy called Synthetic Bloom's Taxonomy. Literature[1] presented a comprehensive classification and comparative study of mathematical problems in Turkish and Canadian school mathematics textbooks with the help of the SBT provided in literature[8]. Based on the above study, the SBT can be improved and extended to offer some reference value for a comparative study of the type of mathematics problems in the functions section of junior secondary school mathematics textbooks in various countries.

The research problems which guided this study were: What are the similarities and differences in the design of problems on linear functions in the two textbooks and suggestions for further studies, as well as the room for improvement.

## 2 Theoretical framework

### 2.1 Research Model

In this study, the SBT from reference[1] was chosen to code and classified all mathematics problems in the chapter on linear functions. As shown in Table 1 below, the SBT contains two dimensions: cognitive process and knowledge.

**Table 1 Synthesized Bloom taxonomy**

| Knowledge dimension | Cognitive Process |                  |                           |
|---------------------|-------------------|------------------|---------------------------|
|                     | 1.Assimilation    | 2.Transformation | 3.Creative and Validation |
| A.Factual           |                   |                  |                           |
| B.Conceptual        |                   |                  |                           |
| C.Procedural        |                   |                  |                           |
| D.Meta-cognitive    |                   |                  |                           |

On the horizontal axis, the cognitive process dimension is divided into levels 1, 2 and 3. Where 1 denotes assimilation, i.e., examining students' memory of concepts and 2 denotes transformation, i.e., converting real-life problems into mathematical problems or performing simple calculations, and 3 denotes creation and validation, i.e., choosing the most appropriate solution by analyzing and creating.

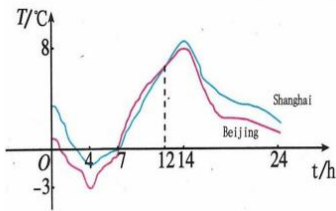
On the vertical axis, the knowledge dimension is divided into A, B, C and D levels. Where A denotes facts, i.e., containing general knowledge of mathematics, concepts, and formulae that students have mastered or facts that already exist objectively; B denotes concepts, i.e., solving problems through concepts; C denotes procedural, i.e., having a fixed method of solving problems; and D denotes meta-cognitive, i.e., multiple methods of solving problems.

When coding the problems from P1 and J1, we specified these problems into the

following parts: examples, exercises in this chapter and each sections taken from P1; examples, exercises in this chapter and each sections, supplementary problems taken from J1. As counting the total number of problems, only the main problems were considered. If several sub-problems appeared in a problem, these were considered as one problem. In particular, it should be noted that each mathematical problem may be evaluated as more than one unit, but in this study, we assumed that each problem was evaluated as only one unit.

Some problem samples related to coding of problems in middle school mathematics textbooks in both countries were given in Table 2.

**Table 2 Coding examples of problems in P1 and J1**

| Code | Chinese   | Japanese   |     |     |     |     |   |     |   |     |    |     |    |     |   |     |    |     |   |     |   |     |   |     |    |     |
|------|---|--|-----|-----|-----|-----|---|-----|---|-----|----|-----|----|-----|---|-----|----|-----|---|-----|---|-----|---|-----|----|-----|
| A2   | <p>The graph below shows the temperature in Beijing and Shanghai on one day as a function of time.</p> <p>(1) When is the temperature in Shanghai the same as that in Beijing during this day?</p> <p>(2) At which time is the weather in Shanghai warmer than that in Beijing? And at which time was it cooler?</p>  | <p>Given a function <math>y = 5x + 20</math>, try to investigate the change of the y values with the change of the x values.</p> <p>(1) When the x values increases from 4 to 6, increase in the x values: <math>6 - 4 = 2</math><br/>increase in the y values: <math>(5 \times 6 + 20) - \{5 \times (-3) + 20\} = 10</math></p> <p>(2) When the x values increases from -3 to 1, increase in the x values: <math>1 - (-3) = 4</math><br/>increase in the y values: <math>(5 \times 1 + 20) - \{5 \times (-3) + 20\} = 10</math></p> <p>In both (1) and (2), the increase in the y values is 5 times the increase in the x values.</p> <p>That is, <math>\frac{\text{(the increase in the y values)}}{\text{(the increase in the x values)}} = 5</math></p> <table border="1" data-bbox="874 1115 1045 1167"> <tr> <td>x</td> <td>...</td> <td>4</td> <td>...</td> <td>6</td> <td>...</td> </tr> <tr> <td>y</td> <td>...</td> <td>40</td> <td>...</td> <td>50</td> <td>...</td> </tr> </table><br><table border="1" data-bbox="874 1182 1045 1234"> <tr> <td>x</td> <td>...</td> <td>-3</td> <td>...</td> <td>1</td> <td>...</td> </tr> <tr> <td>y</td> <td>...</td> <td>5</td> <td>...</td> <td>25</td> <td>...</td> </tr> </table> | x   | ... | 4   | ... | 6 | ... | y | ... | 40 | ... | 50 | ... | x | ... | -3 | ... | 1 | ... | y | ... | 5 | ... | 25 | ... |
| x    | ...   | 4  | ... | 6   | ... |     |   |     |   |     |    |     |    |     |   |     |    |     |   |     |   |     |   |     |    |     |
| y    | ...   | 40   | ... | 50  | ... |     |   |     |   |     |    |     |    |     |   |     |    |     |   |     |   |     |   |     |    |     |
| x    | ...   | -3   | ... | 1   | ... |     |   |     |   |     |    |     |    |     |   |     |    |     |   |     |   |     |   |     |    |     |
| y    | ...   | 5  | ... | 25  | ... |     |   |     |   |     |    |     |    |     |   |     |    |     |   |     |   |     |   |     |    |     |
| B1   | <p>Find variables and constants in the question.</p> <p>The water price cities is 4 yuan per tonne.</p> <p>The researchers samples water usage data of some families. Write a function to show the total cost y per month for water for x tonnes in a home.</p>   | <p>Find the slope and y-intercept of each function:</p> <p>(1) <math>y = 3x + 1</math>      (2) <math>y = -\frac{1}{2}x - 3</math></p>   |     |     |     |     |   |     |   |     |    |     |    |     |   |     |    |     |   |     |   |     |   |     |    |     |
| C2   | <p>1. (1) Sketch the graph of the function <math>y = 2x - 1</math>;</p> <p>(2) Judge whether the points A <math>(-2.5, -4)</math>, B <math>(1, 3)</math> and C <math>(2.5, 4)</math> are on the graph of the function <math>y = 2x - 1</math>.</p>  | <p>Find the equation of the line with the given slope and passing through the given point.</p> <p>slope=-2; (3,1)</p>  |     |     |     |     |   |     |   |     |    |     |    |     |   |     |    |     |   |     |   |     |   |     |    |     |

|    |  |  |
|----|--|--|
| C3 | <p>A,B two stores usually sell the same goods at the same price.During the Spring Festival, A store sells all goods at a 20% discount, B store offers a 30% discount on one-off purchases priced over 200 yuan.</p> <p>(1) Let <math>x</math> (yuan) represent the original price of goods,and <math>y</math> (yuan)represent spending amount.Write function expressions according to the discounts of the two stores.</p> <p>(2) Draw graphs of two functions in (1) on the same axes.</p> <p>(3) How to choose the two stores to go shopping during the Spring Festival to save money?</p> | <p>Draw graphs of <math>y = -6</math> and <math>y = -6x + 5</math> on the same axes.</p> |
| D1 | <p>Draw each linear function:</p> <p>(1) <math>x - 2y + 4 = 0</math></p> <p>(2) <math>y = -3</math></p> <p>(3) <math>4x - 6 = 0</math></p>   | <p>Draw graphs of <math>y = -6</math> and <math>y = -6x + 5</math> on the same axes.</p> |

For example,the main activities in A involved recalling about definitions of linear function or extracting the objective facts from the problem. The problem in Chinese textbook has given the temperature changes in Beijing and Shanghai, which is a fact,therefore it is in the category of A. And students should consider the relation of two functions in terms of the function expressions, thus it is in the category of 2. Next example, the problem in J1 is expected that students should be able to remember and understand concepts of the intercept and slope accurately, therefore it is in the category of 1 and B. Another mathematics problem in Chinese textbook has been categorized as C2 since it is a procedural knowledge for students to determine whether the given point is on the image of a function and it examines their understanding of the concept of graph of functions. The sample problem, taken from the P1 and assessed as 3, expects students to have the ability to apply the concepts of function expressions and graphs to select the optimal decisions. And since there are clear methods to draw the graph of the function, it is in the category of C. Lastly, according to information in the D1 problem from J1, in the given case, students are supposed to understand the concept of images. And there are two ways to draw the image of functions, Point-by-Point method and the migration method of graphing linear functions.

## 2.2 Validity and Reliability of Data

For assessing the credibility of the coding results of the model, we selected 20 mathematics problems from the chapter on linear functions in P1 and 20 problems from J1 as a sample for the test. Two group members acted as raters to check the coding reliability. According to the coding model, the raters independently coded each of the 40 sample problems from two dimensions, and the coding results are shown in Table 3 below.

**Table 3 Coding test results**

| No. | Coder | Coder | No. | Coder | Coder | No. | Coder | Coder | No. | Coder | Coder |
|-----|-------|-------|-----|-------|-------|-----|-------|-------|-----|-------|-------|
|     | A     | B     |     | A     | B     |     | A     | B     |     | A     | B     |
| 1   | A2    | B2    | 11  | D1    | C1    | 21  | B2    | B2    | 31  | C2    | C2    |
| 2   | B1    | B1    | 12  | C2    | C2    | 22  | B1    | B1    | 32  | B2    | B2    |
| 3   | B2    | B1    | 13  | D1    | D1    | 23  | C1    | C1    | 33  | C1    | C1    |
| 4   | A2    | A2    | 14  | B1    | C1    | 24  | C2    | C2    | 34  | A1    | B1    |
| 5   | B2    | B2    | 15  | D2    | C2    | 25  | B3    | B1    | 35  | B2    | B2    |
| 6   | B2    | B2    | 16  | D2    | D2    | 26  | A1    | A1    | 36  | B2    | B2    |
| 7   | B2    | B2    | 17  | B2    | D2    | 27  | B1    | B1    | 37  | B3    | B3    |
| 8   | A2    | A2    | 18  | B2    | B2    | 28  | B2    | B2    | 38  | C1    | C1    |
| 9   | B2    | B2    | 19  | D1    | D2    | 29  | C1    | C1    | 39  | C2    | D2    |
| 10  | D1    | D1    | 20  | D1    | D1    | 30  | C1    | C1    | 40  | C3    | C3    |

The above results were analyzed in SPSS[8] using Kendall's Coefficient[9] to obtain the credibility of the model. The four levels of the knowledge dimension in the SBT were assigned values of 1, 2, 3 and 4, and the three levels of the cognitive process dimension were assigned values of 1, 2 and 3. After iterative testing and adjustment by the panelists, the following results were obtained as shown in Figures 1 and 2.

**Correlations**

|                 |         |                         | Coder A | Coder B |
|-----------------|---------|-------------------------|---------|---------|
| Kendall's tau_b | Coder A | Correlation Coefficient | 1.000   | .822**  |
|                 |         | Sig.(2-tailed)          | .       | .000    |
|                 |         | N                       | 40      | 40      |
|                 | Coder B | Correlation Coefficient | .822**  | 1.000   |
|                 |         | Sig.(2-tailed)          | .000    | .       |
|                 |         | N                       | 40      | 40      |

\*\* .Correlation is significant at the 0.01 level (2-tailed).

**Figure 1 Relevance test for knowledge dimension**

**Correlations**

|                 |         |                         | Coder A | Coder B |
|-----------------|---------|-------------------------|---------|---------|
| Kendall's tau_b | Coder A | Correlation Coefficient | 1.000   | .800**  |
|                 |         | Sig.(2-tailed)          | .       | .000    |
|                 |         | N                       | 40      | 40      |
|                 | Coder B | Correlation Coefficient | .800**  | 1.000   |
|                 |         | Sig.(2-tailed)          | .000    | .       |
|                 |         | N                       | 40      | 40      |

\*\* .Correlation is significant at the 0.01 level (2-tailed).

**Figure 2 Relevance test for the cognitive process dimension**

The correlation coefficient in the knowledge and cognitive dimension were 0.822 and 0.8, respectively, which indicated a high level of reliability.

### 3 Main results

In the present study, 165 mathematics problems in a chapter of linear function in the Japanese and Chinese textbooks are investigated according to the SBT.

The mathematics problems in the linear function are investigated in detail in terms of the knowledge dimension and the cognitive processes dimension and the obtained coding data are presented in Table 4, Table 5 and Table 6. Statistics show that the number of the mathematics problems in J1 is higher than that in P1 ( Japan : n = 90, China : n = 75 ).

**Table 4 Distribution of problem in P1 according to SBT**

| Knowledge dimension | Cognitive Process |                  |                           |
|---------------------|-------------------|------------------|---------------------------|
|                     | 1.Assimilation    | 2.Transformation | 3.Creative and Validation |
| A.Factual           | 8                 | 6                | 0                         |
| B.Conceptual        | 6                 | 12               | 5                         |
| C.Procedural        | 9                 | 22               | 2                         |
| D.Meta-cognitive    | 4                 | 1                | 0                         |

**Table 5 Distribution of problem in J1 according to SBT**

| Knowledge dimension | Cognitive Process |                  |                           |
|---------------------|-------------------|------------------|---------------------------|
|                     | 1.Assimilation    | 2.Transformation | 3.Creative and Validation |
| A.Factual           | 3                 | 8                | 0                         |
| B.Conceptual        | 10                | 12               | 1                         |
| C.Procedural        | 11                | 12               | 0                         |
| D.Meta-cognitive    | 22                | 11               | 0                         |

**Table 6 Summary of the type of mathematics problems coding results of linear function in two versions of textbooks**

| Dimension           | Level                     | Chinese textbook   |            | Japanese textbook  |            |
|---------------------|---------------------------|--------------------|------------|--------------------|------------|
|                     |                           | Volume of problems | Percentage | Volume of problems | Percentage |
| Cognitive Process   | 1.Assimilation            | 27                 | 36.00%     | 46                 | 51.11%     |
|                     | 2.Transformation          | 41                 | 54.67%     | 43                 | 47.78%     |
|                     | 3.Creative and Validation | 7                  | 9.33%      | 1                  | 1.11%      |
| Knowledge dimension | A.Factual                 | 14                 | 18.67%     | 11                 | 12.22%     |
|                     | B.Conceptual              | 23                 | 30.67%     | 23                 | 25.55%     |
|                     | C.Procedural              | 33                 | 44.00%     | 23                 | 25.55%     |
|                     | D.Meta-cognitive          | 5                  | 6.67%      | 33                 | 36.67%     |

Additionally, the types of mathematics problems in the chapter on linear functions in

both editions of the textbook are similar with no mathematics problems observed that could be assessed within the A3 and D3 categories.

A z-test[9] was conducted on the data obtained from Table 6. The number of the types of mathematics problems in the chapter on linear functions in the two editions of the textbook was compared and analyzed according to the results of the test.

There are some differences in cognitive process dimension. At the assimilation and transformation stage of the cognitive process dimension, there is no significant difference between the two versions of the textbook in terms of the type of mathematics problems in the chapter on linear functions ( $z = 1.9$ ,  $p = 0.0534 > 0.05$ ;  $z = 0.9$ ,  $p = 0.3704 > 0.05$ ). In addition, at the creation and validation level, there is a significant difference between the two editions in terms of the types of mathematics problems in the chapter on linear functions ( $z = 2.4$ ,  $p = 0.015 < 0.05$ ).

There are some differences in cognitive process dimension. There is no significant difference between the two editions in terms of the types of mathematics problems in the chapter on linear functions at the factual and conceptual stage of the knowledge dimension ( $z = 1.2$ ,  $p = 0.2121 > 0.05$ ;  $z = 0.7$ ,  $p = 0.4776 > 0.05$ ). Additionally, at the procedural and Meta-cognitive levels, there are significant differences between the two editions in the types of mathematics problems used in the chapter on linear functions ( $z = 2.4$ ,  $p = 0.0152 < 0.05$ ;  $z = 4.5$ ,  $p < 0.0001$ ).

#### 4 Summary and discussion

This paper compares and analyzed the differences in the knowledge and cognitive dimensions of mathematical problems in the linear function part of two different versions of mathematics textbooks: People's Education Press-Mathematics in China(P1) and New Mathematics 2 in Japan(J1), for the purpose of investigating their own characteristics, and gaining understandings and implications in their differences.

Initially, the discoveries of the research illustrate that, in the cognitive dimension, both of two textbooks attach a considerable importance to transformation knowledge, and the proportion of each one is approximately half of their respective problem sets. Meanwhile, both of them pay attention to assimilation knowledge, especially J1. And it is obvious that, based on the data, the sum of the proportions of assimilation and transformation problems in each textbook is more than 90% respectively(J1 even exceeds 98%), which indicates that both take students' mastery of basic knowledge and the solution of simple practical problems into account. Despite the fact that the research reveals many close parallels in these two textbooks' sets on linear function, it still has distinctions. Both of P1 and J1 apparently pay little attention to the creative and validation problems, requiring students to analyze and synthesize information and give the appropriate solution, however, compared with the P1 (9.33%), J1 is much more rare, accounting for only 1.11%.

Considering the knowledge dimension, proportions on factual and conceptual are comparison consistent on the basis of and Z-test results, whereas that on procedural and meta-cognitive do not. Procedural knowledge of P1 takes up the biggest part while J1 is quite the opposite, manifesting P1 trends to the process of solving problems which enquire students to solve problems step by step according to the given clues and J1 better focus on the diversity and synthesis of the solutions.

The results may provide references for mathematics teaching and learning in junior high schools. These may help teachers to consider textbooks under different perspective. Nevertheless, this study focused only on the mathematics problems of linear function sections in P1 and J1. Therefore, future researchers may adopt the analytical framework and research model offered to do relevant research. Moreover, supporting the research model with quantitative data is enlightening so as to make the findings may become more significant.

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