Establishing equivariant class $[\mathcal{O}]$ for hyperbolic groups

Abstract
This paper aims to create a class $[\mathcal{O}]$ concerning the groups associated with Gromov hyperbolic groups over correspondence and equivalence through Fuchsian, Kleinian, and Schottky when subject to Laplace – Beltrami in the Teichmüller space where for the hyperbolic 3-manifold when the fundamental groups of Dehn extended to Gromov – any occurrence of Švarc-Milnor lemma satisfies the same class $[\mathcal{O}]$ for quotient space and Jørgensen inequality. Thus the relation (and class) extended to Mostow – Prasad Rigidity Theorem in a finite degree isometry concerning the $\text{Quasi} - \text{Isomorphic}$ structure of the commensurator in higher order generalizations suffice through $\text{CAT}(k)$ space. The map of the established class $[\mathcal{O}]$ is shown at the end of the paper.

Keywords– Teichmüller Space; Dehn; Švarc-Milnor; Jørgensen inequality; Laplace – Beltrami; Lickorish – Wallace; Haken Space.

I Introduction

Any non-Euclidean geometry having a saddle or negative curvature where both the omega and Kretschmann scaler is less than 1 being defined for Riemann curvature tensor R_{mnop} there lies a vanishing Ricci R_{mn} through[1],

$$K = R_{mnop}R^{mnop} \text{ for } deg[L]^{-4}$$

Every Gauss-Bolyai-Lobachevsky Space is a Riemann space provided it is a symmetric non-compact type where the Gaussian Curvature G being negative gives the inverse root where there exists a limit in the Geodesic curvature for 2-such cases as identified[2],

$$\frac{1}{\sqrt{-G}} = \begin{cases} 1, & Horocycle \\ [0,1], & Hypercycle \end{cases}$$

Riemann and Hyperbolic space are equivalent over actions on PGL groups for any isomorphism over 3-manifold as the Dehn surgery segregates the Flat (Euclidean) from hyperbolic provided any 3-manifold can take on different geometries for the same structures where the conformal boundary associated with the Riemann sphere can be distinguished by group[3],

$$\text{PGL}(2, \mathbf{C})$$

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I Gauss-Bolyai-Lobachevsky Theorem with Dehn Surgery

Being associated with the homology classes and established over simplicial norm there exists the Dehn Link and Dehn Twists satisfying the properties[4],

[1] $Lickorish - Wallace\ Theorem$ – Where Dehn Twist D can be defined as an automorphism for roundabout channel C such that the mapping factor φ gives,

$$\varphi: C \longrightarrow C \text{ over time } T\ giving\ D \cong Ce^{i2\pi T} \exists\ T \in [0,1]$$

[2] $Manifold\ Type$ – Closed and orientable.

II Fuchsian Group

By taking the Gauss-Bolyai-Lobachevsky over the associated and irreducible prescriptions of Dehn Twist the PGL$(2, \mathbf{C})$ preserves the isometry for any Hyperbolic 3-manifold – a category where the necessary functors of orientations suffice the Möbius Transform group we get a Kleinian group representing the Riemann sphere over conformal transformations in PSL$(2, \mathbf{C})$. Thus by considering 2 factors a realization can be made for connections of Fuschain to Kleinian groups[5],

[1] Categorizing Kleinian as the discrete subgroup ω and the hyperbolic 3-manifold as M_H^3 then two cases can be established over the equation,

a. Co-compact being finitely co-volume
b. Co-volume being finitely generated
   i. $Over - M_H^3/\omega$

[2] For the real domain R of PSL$(2, \mathbf{C})$ then over any finite generations being an isometry group taking on the upper half of the unit disc throughout any conformal transformations – it can be shown that[6],

a. M_H^3 is discontinuous over PSL$(2, \mathbf{R})$ when Kleinian takeover the Fuchsian for the discrete subgroup ω.
b. For any polynomial P there exists a non-repetitive root for the equation

$$x^3 + ax + b$$

$\exists$ there exists a deg_3 Potential of P in x for every y^2 establishing the above equation

- This again gives another subgroup ω^0 satisfying the moduli space of y^2 via PSL$(2, \mathbf{Z})$

- Fuchsain norms for non-abelian ω.

- PSL$(2, \mathbf{C})$ is then satisfied for the hyperbolic 3-fold over functors M_H^3 of Möbius group where a 2-sphere ${}^{\infty}_{B^3}S^2$ is taken at infinity being associated with the conformal homeomorphism where there exists an associated hyperbolic isometry on 3-ball B^3. Thus the discontinuity as mentioned in [a] of point [2] gives an extension to 2 categories[7],

A. Schottky group over any Hausdorff measure $h < 2$ for the discrete subgroup ω for $2g$ weight for a total convergence over the bundle norm $b \geq 1$ over linear fractional transformations $\partial \equiv \frac{\partial_1}{\partial_2}$,

$\sum_{\partial \in \omega} {\partial_2}^{-2g} h\partial$ ∃ as mentioned earlier H is Hausdroff < 2 ≅ Poincarė series $b \geq 1$

B. Denoting area of discontinuity D^0 we again get 2 relations,

1. $Orbifold\ Riemann$ − for the discrete subgroup ω there is $D^0(\omega)/\omega$

2. $Fuchsian \equiv Schottky$ − For area inequality D^0/ω.

III Teichmüller Space

Any Teichmüller space $T(M)$ can be defined over a Riemann manifold M endowed with a hyperbolic structure where there exists identity homeomorphism. For any universal covering making an identification for genus $g \geq 2$ - The compact structure or the surface that the topology establishes prescribed 5 interrelated parameters[8],

[1] Non-compact space
[2] Teichmüller space $T(M)$
[3] Fuchsian group
[4] Isotopy norm over Riemann M for metric g_M suffice smoothness s,

$$s \sum_{i=1}^{n} dx_i^2$$

[5] Uniformization theorem for Hyperbolic 3-folds taking Thurston's 8-geometries taking over[9],

- Point [4] noting $s \sum_{i=1}^{n} dx_i^2$ as metric $d\ell^2$ giving,
    1. Hodge star ⋆ for isothermal coordinates equipped with above metric $d\ell^2$ such that for smoothness s there exists $Jacobean \neq 0$,
    2. s is equipped with the differential $\partial\alpha^2 + \partial\beta^2$ (Let α, β be the coordinates where ⋆ generates 3 peculiar forms,
        - $\star \partial \star \partial$
        - $\star \partial\alpha = \partial\beta$
        - $\star \partial\beta = \partial\alpha$

- $Laplace - Beltrami$ − For the exterior derivative (being non-trivially hidden in sub-points 1 and 2 under [5]) we would be getting a scalar potential ϕ such that this can be differentiated for the Riemann metric g_M over[10],

$$\partial_m |g_M|^{-1/2}(\partial_m \phi g_M^{mn} |g_M|)$$

Giving area inequality D^0/ω for discrete subgroup ω with a discontinuity $D^0 - Schottky \cong Fuchsian$ provided Poincarė series stands at $b \geq 1$. While all this occupying genus g Teichmüller is identified with the Fuchsian over 3 norms[11],

1. Genus $g \geq 2$
2. Ball B having deg_{6g-6}
3. For scalar potential ϕ with Riemann $\partial_m |g_M|^{-1/2}(\partial_m \phi g_M^{mn} |g_M|)$ there exists[12],
    a. $T(M) \xrightarrow{bijection} (M, \phi)$
    b. Equivalence class $[\mathcal{O}]$ for the isotopy, diffeomorphism, and holomorphism such that for closed interval $[0,1]$ Teichmüller gives[13],
        1. $T(M) \Longrightarrow (g_M, M)$
        2. Diffeomorphism to ϕ for contractable $\mathbb{R}^2$ $\forall$ $6g - 6$ making an equivalence as,

$$[\mathcal{O}] \equiv T(M) \cong \mathbb{R}^2 \; \forall \; 6g - 6 \; \exists \; Fuchsian \equiv Schottky \equiv \text{Klenia} \Longrightarrow \text{Lie Group } \mathrm{PSL}_2(\mathbb{R})$$

IV Gromov Hyperbolic Groups

No 3-dimensional manifold can occupy a single class of geometry. Where either there exists a certain curvature on the outside while a certain curvature is inside. In some cases – the structure can be equipped with the surface geometry taking different curvatures on different parts to say the frequency concerned with that manifold. There have been groups equipped with structures for the homotopy invariant spaces but the segregation can be possible by the Dehn surgery for making a distinct classification from Euclidean to Hyperbolic geometries. Any isomorphisms can be satisfied by this homotopy invariant spaces M_X that are Riemann with a negative curvature. Thus taking X as the topological space for the negative Riemann M - one can deduce the fundamental groups for a completely connected path along the surface being parameterized by P along X[14],

$$\rho_1(X_P)$$

If we take another topological space Y for hyperbolic manifold M_Y then we can deduce 3 relations making a symmetric equivalence for class$[\mathcal{O}]$[15],

1. For any hyperbolic 3-manifold $\rho_1(X_P)$ satisfies a quasi-isometric form such that for any definite deg_{finite} Lie Group $\mathcal{L}$ one can find the Riemann metric g_M where there exists closed–connectedness between the topological manifold M_X or M_Y satisfying quotients via[16],

$$\check{S}varc - Milnor\ lemma = \begin{cases} X/\mathcal{L} \\ Y/\mathcal{L} \end{cases} \forall\ \rho_1(X_P) \simeq \rho_1(Y_P)\ \exists\ \hat{}: M_X \longrightarrow M_Y$$

2. The quotient topology norm with $\check{S}varc - Milnor\ lemma$ is indeed equivalent to the same fundamental group denoted as $G_{\mathcal{L}}$ suffice[17],

$$\begin{cases} \rho_1(X_P) \longrightarrow \rho_1(X_P/G_{\mathcal{L}}) \\ . \\ \rho_1(Y_P) \longrightarrow \rho_1(Y_P/G_{\mathcal{L}}) \end{cases} \cong G_{\mathcal{L}}\ ^{\text{Note 1}}$$

3. For establishing equivariant hyperbolic groups for class $[O] -$ The hyperbolic 3-manifold M_H^3 suffices an inequality relation over the same class correspondence – $[O] -$[18]

$$[O] \equiv \begin{cases} \text{Quotient Topological Space } M_H^3/G_{\mathcal{L}} \\ . \\ \text{Jørgensen inequality} \end{cases} \cong \text{Kleinian Groups}$$

If we take any complex plane $\mathbb{C}$ and suffice that any element existing on that complex plane gives the trace μ^1 and μ^1 of a 2×2 matrix then any parabolic function gives the cusp of a Riemann 3-fold that iff generates through the Kleinian Groups then – This suffice a correlation for any suitable parameters belonging to those traces μ^1 and μ^1 norms,

$$\cong Fuchsian\ Group \text{ for } \cup^{\frac{1}{2}}/G_{\mathcal{L}}\ \exists\ \gamma \coloneqq \cup^{\frac{1}{2}}/G_{\mathcal{L}}\ ^{\text{Note 2}}$$

This again suffices the $[O] -$ class in a more concrete structure.

IV.I Mostow – Prasad Rigidity Theorem

If we take the same fundamental group of Dehn which Gromov extended – for the hyperbolic 3-fold then we will again find ourselves embedded in the Teichmüller space for every deg_{finite} closed manifolds one would surprisingly find the $[O] -$ class equivariant as stated in [Page 5] for manifold M suffice Teichmüller$T(M)$ in contractable$\mathbb{R}^2$ for all that norm $6g - 6$ taking the Lie Group $\text{PSL}_2(\mathbb{R})$[19].

Note 1 – Higher order generalizations for the fundamental group can be achieved by making $\rho_n(X_P)$ and $\rho_n(Y_P)$ where in the case of complex plane $\mathbb{Z}$ – There are higher-order homotopy groups of $n -$ spheres S^n in the ρ_n order making as $\rho_n(S^n)$

Note 2 – $\cup^{\frac{1}{2}}$ denotes the upper half-plane where for any projections parameterized by γ there exists a difference in the relational subset for Group – $G_{\mathcal{L}}$ from its mean $\langle G_{\mathcal{L}} \rangle$

For assessment of clarifications when the connected path P along topological space X for the concerned manifold M_X – as we have seen in $\check{S}varc - Milnor\ lemma$ where the established equation of path connectivity $\rho_i(X_P)$ takes[20],

$$\sum_{i=1}^{n} \rho_i \bigvee_{p \in p^n} X_P$$

The map $\hat{}: M_X \longrightarrow M_Y$ goes prominent in the same deg_{finite} isometry where the fundamental group $\rho_i \ \exists\ i$ takes $[1, n]$ – the same map can be re-written through this group structure,

$$\hat{}: \sum_{i=1}^{n} \rho_i\,(M_X) \longrightarrow \sum_{i=1}^{n} \rho_i\,(M_Y)$$

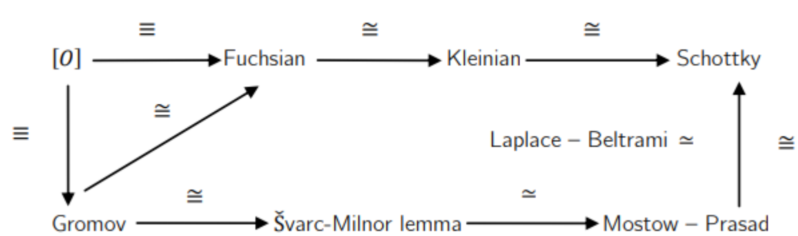
The same class can be satisfied for $n -$ sphere via any higher degree generalizations – where $M_H^3 \longrightarrow M_H^n$ via,

$$\rho_n(S^n)$$

The large–scale structures are highlighted for any established deg_{finite} connectivity where a peculiar functor **Comm** takes over the generators of the group $G_{\mathcal{L}}$ where there exists the path–connectivity for some arbitrary commensurator Π over any intersections of sub-group ${G_{\mathcal{L}}}^1$ and ${G_{\mathcal{L}}}^2$ such that the **Comm** over group $G_{\mathcal{L}}$ establishes[21],

$$\textbf{Comm}(\Pi) \text{ justifies } {G_{\mathcal{L}}}^1, {G_{\mathcal{L}}}^2 \in G_{\mathcal{L}} \text{ iff } \Pi \equiv \text{Quasi} - \text{Isomorphic}$$

Hence a $[O] -$ class equivariant relation,



This indeed suffices for an argument if we consider the fundamental group considering $n -$ sphere $\rho_n(S^n)$ where there exists a $\text{CAT}(k)$ space for every M_H^n for manifold M_X equipped with metric g such that this suffice for every $\leq k$ in $\text{CAT}(k)$[22,23],

$$
(M_X, g) \begin{cases} M_H^n \begin{cases} \text{CAT}(k) \text{ for } k\ k = -1 \\ . \\ \text{Hadamard space for } k = 0 \end{cases} \\ . \\ . \\ S^n \begin{cases} \text{Fundamental group } \longrightarrow \rho_n(S^n) \\ . \\ \text{CAT}(k) \text{ for } k = 1 \end{cases} \end{cases}
$$

Thus we get $[\mathcal{O}] \equiv T(M) \cong \mathbb{R}^2\ \forall\ 6g - 6\ \exists\ Fuchsian\ \equiv Schottky \equiv$ Klenia $\Longrightarrow$ Lie Group $\text{PSL}_2(\mathbb{R})$ where there can be $\text{CAT}(k)$ space for every M_H^n as (M_X, g) with g being the equipped metric. [Note 3]

Results

Equivariance is satisfied for class $[\mathcal{O}]$ with the necessary hyperbolic groups – that being considered for this paper. The extended relation that is shown is the large-scale and also higher-degree generalization when subject to specific Lemma and Theorem mentioned throughout this paper. Concerning the Gauss-Bolyai-Lobachevsky space, the equivariant being satisfies over equivalence among $Fuchsian\ \equiv Schottky \equiv$ Klenia all being subject to Gromov and Thurston's 8-geometries to suffice the related extension from M_H^3 to M_H^n all being justified through the fundamental group, $\text{CAT}(k)$ space and Hadamard space for the associated metric concerned with the hyperbolic manifold (M_X, g).

Note 3 – The Lie Group will not be the same as $\text{PSL}_2(\mathbb{R})$ for casewise alterations.

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