A Game-Theoretic Credit Period and Promotion Model in a Supplier-Retailer Channel

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ABSTRACT

It has been established that trade credit can be influenced by a lot of factors. However, no specific function has been used to neither represent these factors nor consider their effects. This paper considers a supplier-retailer Stackelberg game in which the supplier as the channel leader supplies credit goods to the retailer who in-turn sells to the consumers. It uses a credit function based on credit period, supplier's price margin and product promotion effort to model the players' payoffs. The work considers two game scenarios: a situation involving the provision of trade credit and a situation without trade credit. The work obtains a closed-form solution for the credit period for the credit provision scenario, and the promotion efforts and payoffs for both scenarios, and shows that credit period prolongation may not be in favour of the retailer, and that the retailer can attain a larger payoff than the supplier. It also shows that the retailer's margin is very crucial for both channel scenarios, and observes that the players are better-off with trade credit.

Keywords: Trade Credit, Supply Chain, Stackelberg Game, Credit Function

2010 Mathematics Subject Classification: 91A65, 91A10.

1. INTRODUCTION

Trade credit is a transaction between a supplier and a customer in which goods are not paid for immediately, but rather, the supplier provides the goods to the customer on credit, and allows payment to be delayed till a future date [1]. It is a short term financial strategy aimed at incentivising the customer to purchase the product, thereby aiding the sale of supplier's product [2, 3]. The importance of trade credit for small firms and starts-ups in the midst of scarce resources cannot be over emphasized [4, 5]. For instance, according to [6], the existence of taxes has been observed as one of the motivations for the employment of trade credit since the parties involved in a deal can be shielded from taxes by adopting trade credit. Further a survey by [7] shows that in most countries, apart from financing through bank, trade credit is the next most employed external financing source. Findings show that trade credit provision is influenced by a lot of market and environmental factors; however, no function has been clearly deployed to study the effect of these factors. In this work we will consider trade credit using a credit function that incorporates some of these market factors

Works on trade credit can be grouped into empirical data based and mathematical models. Mathematical trade credit models can further be grouped into general non-game theoretic models and game theoretic models. Some of these models use simulation to make helpful predictions. For instance [8] developed a model to compute the effect of trade credit default by firms. The paper used data from firms to investigate bankruptcy, and showed that it is possible to predict a great percentage of possible default. Another related consideration was done by [9]. They examined the role played by trade credit in reduction of information asymmetries which exists between credit providing firms and financial banks. Using a switching regression method and incorporating the simultaneous decisions taken by the financial banks to provide credit and decision of the firms to implement same they arrived at helpful managerial decisions. Wan et al. [10] considered a dual channel in which a manufacturer can sell his products through retail supply channel and at the same time directly to consumers through an online product supply channel. They developed a model on competitive and supplementary product pricing strategies, and observed that instead of unconditional provision of trade credit, the manufacturer's provision of trade credit should be strategic. In a consideration of product's price-dependent demand and constant deterioration rate, Das et al [11] developed a model that combined product reliability and trade credit. They obtained a number of non-linear optimization problem, and the model was validated using seven numerical examples. Another paper that factor in product deterioration was considered by [12]. They considered a dynamic problem in which a retailer sells a deteriorating product with a demand rate varying with the level of inventory and credit period length given to the consumers. They used a mathematical model to obtain trade credit strategy as well as replenishment strategy that can maximize the retailer's profit in a planning horizon.

Game-theoretic models are very useful tools in supply chain studies and analyses [13-15]. There are relatively very few game-theoretic trade credit models. Shi [16] modelled the determination of trade credit from a supplier to a retailer as a Nash bargaining problem between the two channel members. The paper showed that the relationship between both players' financing cost influences the credit offered to the retailer. Using a Stackelberg model [17] studied a supply chain in which a product that has a stable demand is sold by the supplier through a retailer. They determined two trade credit possibilities: a conditional trade credit situation which was found to be beneficial to both parties, and an unconditional trade credit situation which they found to be of benefit to the retailer, but detrimental to the supplier. They showed that with good design, the supplier's trade credit decision can lead to a win-win outcome for the players. Considering capital and replenishment cycle as the buyer's decision variables, and the trade credit financing and shipments as the seller's decision variables [18] obtained a Nash solution based on non-cooperative relationship and an integrated solution which is based on cooperation. In a study of replenishment plan where channel players are uncooperative, Wu and Zhao [19] used a Stackelberg game theory to X-ray conditions surrounding the adoption of trade credit by the retailer and supplier. Jaggi et al. [20] studied a supplier-retailer channel in which the product demand depends on the stock displayed. They considered optimal decision using three policies which include centralized equilibrium solution, a Stackelberg equilibrium solution and a Nash solution. Their work considered the influence of trade credit, replenishment and integration on deteriorating items

This paper considers trade credit involving a supplier and a retailer in a supply channel. It uses game theory to consider a credit transfer and non-credit provision in a supplier-retailer supply channel in which the supplier is the channel leader and the retailer is the follower. The work will address the effect of price margin and credit period on the promotion effort and payoffs for a situation where credit is provided and a situation where credit is not provided.

2. THE MODEL

This paper considers a bilateral monopoly in which the retailer is assumed to sell only the supplier's product brand in a class of similar products. Further we assume that to increase the demand for the supplier's product, the retailer engages in promotion campaign. On the other hand, to encourage the retailer, the supplier engages in the provision of trade credit to him. Thus the retailer and the supplier's decision variables are the promotion effort ρ and the credit period T respectively.

2.1 List of Notations

To aid the understanding of the work we use the following notations:

S_m Supplier's price margin

r_m Retailer's price margin

r_p	Retailer's promotion effort
S_t	Supplier's credit period to the retailer
c	Retailer's promotion effectiveness parameter
R_p	Retailer's payoff
S_p	Supplier's payoff

2.2 Promotion-Demand Function

Considering the close relationship between advertising and promotion with the exception that while advertising is sometimes considered a long term strategy, trade credit is a short term strategy we represent the effect of promotion on demand by adopting the demand function

$$h(r_p) = c\sqrt{r_p} \tag{1}$$

employed by [15, 21, 22], where c is the promotion effectiveness parameter.

Observe that (1) is an increasing concave function of the promotion effort r_p. This representation is in consonance with the saturation effect observed in advertising where an additional spending on advertising results in diminishing returns [23-26].

2.3 Trade Credit Function

We note that a large value of supplier's price margin implies much revenue through the retailer. To reciprocate for this, the supplier can provide large trade credit to the retailer. As such, it would be appropriate to assume a proportional relationship between trade credit to the supplier's price margin S_m.

The promotion effort is an expenditure which can naturally lead to a strain on the retailer's finance. The effect of such strain can be cushioned by the availability of credit. Such a gesture can motivate the retailer to engage more in promoting the supplier's product. Thus we assume that the credit trade T_C is proportional to the promotion expenditure which exhibits diminishing returns.

Further, the credit period is very important to the supplier. We note that it is quite natural for the supplier to give large credit to a retailer if the payment time is short, and will reduce the credit with increase (long) payment time. Thus we let trade credit to be inversely proportional to the time S_t.

Thus we have a credit function of the form

$$T_C = \frac{KS_m\sqrt{r_p}}{S_t}, \tag{2}$$

where K is the constant of proportionality.

2.4 The Game Decision Trend

We consider a supply channel in which the supplier is the Stackelberg leader and the retailer is the follower. The supplier's decision variable is his allowable credit period T which is the same as the credit payment time, while the retailer's decision variable is his promotion effort ρ. The decision sequence is that the supplier informs the retailer of his credit to him (the retailer). This is a function of his allowable credit period T as can be seen in (2) above. Based on this information, the retailer decides on his promotion effort ρ. We will establish the Stackelberg equilibrium through backward induction approach [26]. Thus, based on the supplier's decision we have that the retailer's problem is to

$$\max_{r_p>0} R_p = r_m c\sqrt{r_p} - r_p + \frac{KS_m\sqrt{r_p}}{S_t}. \tag{3}$$

The supplier incorporates the retailer's anticipated response to

$$\max_{S_t>0} S_p = S_m c\sqrt{r_p} - \frac{KS_m\sqrt{r_p}}{S_t}. \tag{4}$$

We will consider two scenarios: a situation where the supplier provides credit support to the retailer, and a situation where he does not. We note that a similar approach was adopted by [27] which is the first to consider a Stackelberg cooperative advertising model with two followers using differential game. Another such approach was adopted by [28] where four channel structures were considered.

3. CREDIT PROVISION SCENARIO

Rearranging (3) we have

$$R_p = \left[cr_m + \frac{KS_m}{S_t}\right]\sqrt{r_p} - r_p \tag{5}$$

which is clearly concave in r_p.

Maximizing (5) wrt r_p we have

$$\frac{\partial R_p}{\partial r_p} = \left[cr_m + \frac{KS_m}{S_t}\right]\frac{1}{2\sqrt{r_p}} - 1 = 0$$

$$\Longrightarrow \quad r_p = \left[\frac{cr_m S_t + KS_m}{2S_t}\right]^2. \tag{6}$$

Using (6) in (4) we have

$$S_p = S_m c\left[\frac{cr_m S_t + KS_m}{2S_t}\right] - \frac{KS_m\sqrt{r_m}}{S_t}\left[\frac{cr_m S_t + KS_m}{2S_t}\right]$$

$$= \left[cS_m - \frac{KS_m}{S_t}\right]\left[\frac{cr_m}{2} + \frac{KS_m}{2S_t}\right]. \tag{7}$$

Now

$$\frac{\partial M_p}{\partial T} = \left[cS_m - \frac{KS_m}{S_t}\right]\left[\frac{KS_m}{2}\right]\left[-\frac{1}{S_t^2}\right] + KS_m\left[\frac{1}{S_t^2}\right]\left[\frac{cr_m}{2} + \frac{KS_m}{2S_t}\right] = 0$$

$$\Longrightarrow \quad cS_m - \frac{KS_m}{S_t} = cr_m + \frac{KS_m}{S_t}$$

$$\Longrightarrow \quad S_t = \frac{2KS_m}{c(S_t - r_m)}. \tag{8}$$

From (6) and (8)

$$r_m = \left[\frac{cr_m\left(\frac{2KS_m}{c(S_t - r_m)}\right) + KS_m}{2\left(\frac{2KS_m}{c(S_t - r_m)}\right)}\right]^2$$

$$= \left[\frac{c(r_m + S_m)}{4}\right]^2.$$

From (5) and (6) we have that

$$R_p = \left[cr_m + \frac{KS_m}{S_t}\right]\left[\frac{cr_m S_t + KS_m}{2S_t}\right] - \left[\frac{cr_m S_t + KS_m}{2S_t}\right]^2, \tag{9}$$

and from (8) we have that

$$R_p = \left[cr_m + \frac{KS_m}{\frac{2KS_m}{c(S_t - r_m)}}\right]\left[\frac{cr_m\left(\frac{2KS_m}{c(S_t - r_m)}\right) + KS_m}{2\left(\frac{2KS_m}{c(S_t - r_m)}\right)}\right] - \left[\frac{cr_m\left(\frac{2KS_m}{c(S_t - r_m)}\right) + KS_m}{2\left(\frac{2KS_m}{c(S_t - r_m)}\right)}\right]^2$$

$$= \left[\frac{c(r_m + S_m)}{4}\right]^2. \tag{10}$$

From (7) and (8) we have

$$S_p = \left[cS_m + \frac{KS_m}{\frac{2KS_m}{c(S_t - r_m)}}\right]\left[\frac{cr_m}{2} + \frac{KS_m}{2\left(\frac{2KS_m}{c(S_t - r_m)}\right)}\right]$$
$$= \frac{1}{8}[c(S_m + r_m)]^2.$$

4. NO-CREDIT PROVISION SCENARIO

Suppose that the supplier does not provide trade credit to the retailer, then the retailer's control problem will be given by

$$\max_{r_p>0} R_p = r_m c\sqrt{r_p} - r_p. \qquad (11)$$

Maximizing (11) wrt r_p we have

$$\frac{\partial R_p}{\partial r_p} = \frac{1}{2} r_m c \frac{1}{\sqrt{r_p}} - 1 = 0$$
$$\Rightarrow \quad \frac{r_m c}{2\sqrt{r_p}} = 1$$
$$\Rightarrow \quad r_p = \left[\frac{r_m c}{2}\right]^2. \qquad (12)$$

Since the provision of trade credit is inversely proportional to the period, it follows that no-credit implies very large S_t. Thus from (6)

$$r_p = \left[\frac{r_m c}{2} + \frac{KS_m}{2S_t}\right]^2.$$

As $S_t \longrightarrow \infty$, $\frac{KS_m}{2S_t} \longrightarrow 0$ so that $r_p \longrightarrow \left[\frac{r_m c}{2}\right]^2$, which is the same as (12).

Using (12) in (11) we have

$$R_p = r_m c\left[\frac{r_m c}{2}\right] - \left[\frac{r_m c}{2}\right]^2$$
$$= \frac{c^2 r_m^2}{4}.$$

Since no credit is give we note that

$$S_p = S_m c\sqrt{r_p}. \qquad (13)$$

From (12) and (13) we have

$$S_p = S_m c\left[\frac{r_m c}{2}\right]$$
$$= \frac{c^2 r_m S_m}{2}.$$

5. DISCUSSION

In this section we use numerical values to discuss the results. To achieve this, we let $r_m = 2$, $S_m = 4$, $K = 0.2$ and $c = 0.3$. Further we let the subscript $T_C \neq 0$ and $T_C = 0$ to represent a situation where credit is given and when it is not given respectively.

5.1 The Effect of Credit Payment Time on Promotion Effort

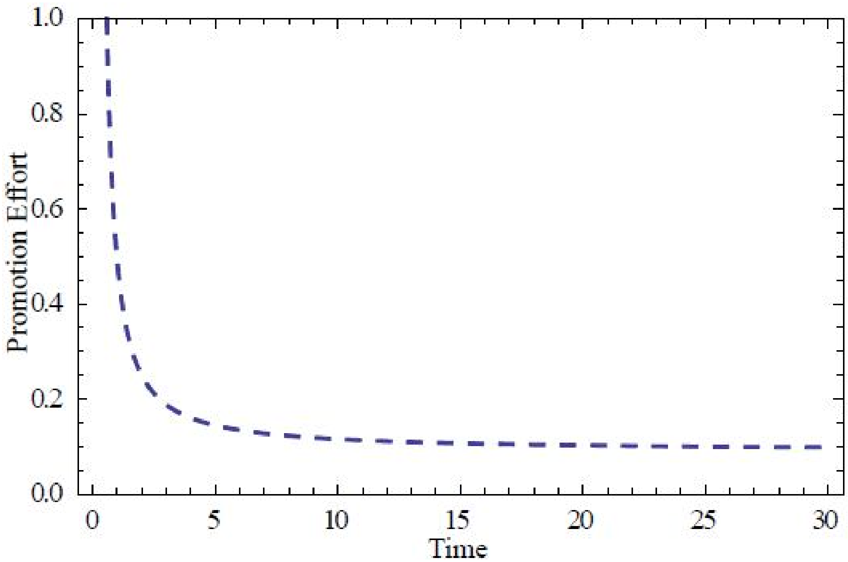

Fig. 5.1. An Illustration of the Reduction Associated with Payment Time

From Fig. 5.1 we note that the promotion effort reduces with time, and as payment time prolongs (that is as $S_t \longrightarrow \infty$), the promotion effort approaches a constant value. Clearly, this constant value cannot be exceeded irrespective of the prolongation of the time. Clearly, the promotion effort diminishes marginally. Initially, this is very rapid. The rapidity reduces over time, and eventually becomes 0, thus stabilizing over time. The reduction in the promotion effort can be seen as a result of the fact that the retailer does not need to engage much in promotion spending since he has enough payment time.

5.2 The Effect of Credit Payment Time on Players' Payoff

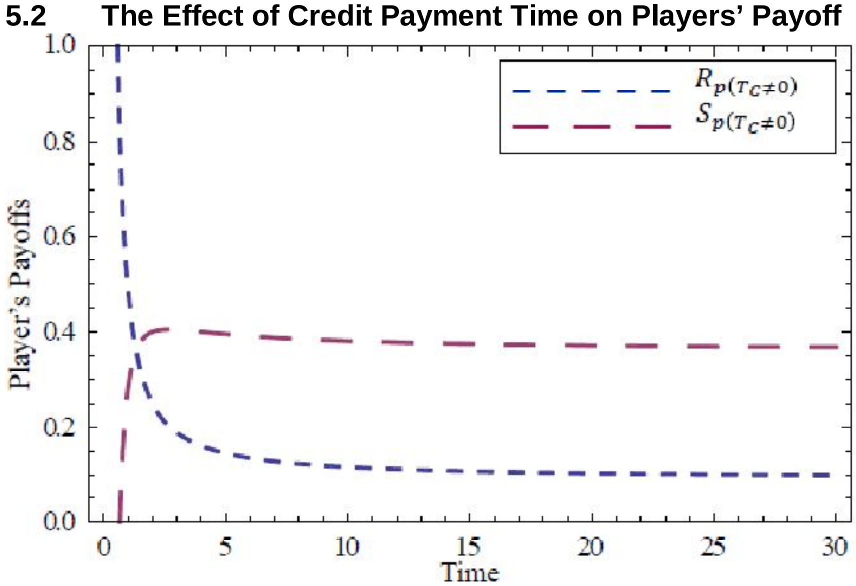

Fig. 5.2. Illustration of the Effect of Prolonged Payment Time on the Payoffs

Quite unfortunately the reduction in the promotion effort as shown Fig. 5.1 above translates to affect the retailer's payoff which reduces with time as can be seen in Fig. 5.2. On the other hand, we observe that the supplier's payoff increases with time, though, exhibiting diminishing marginal return up till the attainment of a maximum. This suggests that allowing some time for credit payment helps the supplier to achieve large payoff as can be seen from the maximum in the graph. However, over prolongation leads to reduction in his payoff, and attainment of a stable value in the long-run. Thus over prolonging the credit payment time has some limiting, or even negative effect on the supplier's payoff. This is because over-prolongation of time does not add value to his payoff, but rather it leads to a reduction due to reduction in the time value of money resulting from inflation and the likes. Thus the optimal time should be adopted.

Considering Fig. 5.2 we observe that while the supplier's payoff S_p increases, the retailer's payoff R_p reduces. Thus at a certain time both payoffs can become equal. From (7) and (9) we have that equality of payoffs would imply

$$\left(cr_m + \frac{KS_m}{S_t}\right)\left(\frac{cr_m S_t + KS_m}{2S_t}\right) - \left(\frac{cr_m S_t + KS_m}{2S_t}\right)^2 = \left(cS_m + \frac{KS_m}{S_t}\right)\left(\frac{cr_m}{2} + \frac{KS_m}{2S_t}\right)$$

$$\Longrightarrow \quad S_t = \frac{3KS_m}{c(2S_M - r_m)}. \tag{14}$$

This shows that the retailer can achieve a large payoff by ensuring that his payment period does not linger up to the time S_t (as given in (14)).

From (14) we observe that as the retailer's margin increases, the quantity $c(2S_M - r_m)$ reduces so that S_t increases. That is, the payment equilibrium time S_t increases with the retailer's margin r_m. This is because increase in r_m will lead to low patronage from consumers which impedes sales. Thus the retailer would need more time for the sale.

Now, (14) implies

$$\frac{1}{S_t} = \frac{c(2S_M - r_m)}{3KS_t} = \frac{2cS_m}{3KS_m} - \frac{cr_m}{3KS_m} = \frac{c}{2K}\left(2 - \frac{r_m}{S_m}\right).$$

Clearly, as S_m increases, $\frac{1}{S_t}$ also increases, which implies that S_t reduces. That is, increase in S_m implies increase in S_t. To further see this more clearly, we observe that

$$\frac{\partial S_t}{\partial S_m} = \frac{c(2S_M - r_m)(3K) - 3KS_m(2c)}{(2cS_m - cr_m)^2} = -\frac{3cKS_m r_m}{(2cS_m - cr_m)^2} < 0$$

Thus, as S_m increases, the supplier would want to allow less credit period so that he can provide much credit. In the nutshell, this means that a switch between the supplier's price margin and credit period is possible and can be used to coordinate the channel.

5.3 The Effect of the Players' Margins on the Promotion Effort and Payoffs

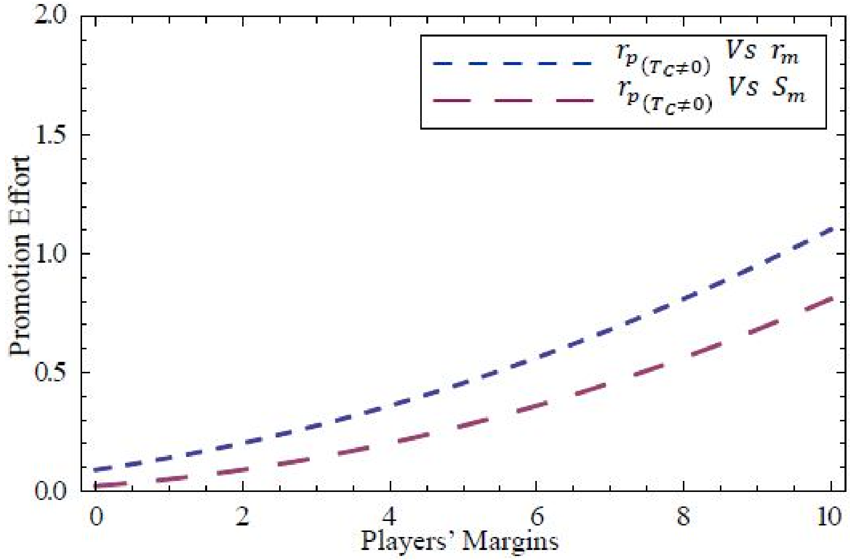

Fig. 5.3. **The Effect of the Players' Margins on the Promotion Effort**

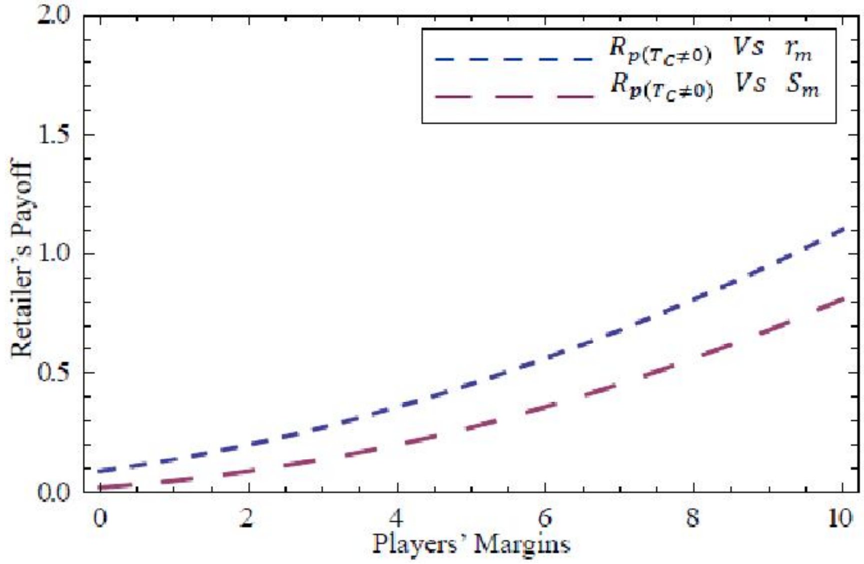

Fig. 5.4. **The Effect of the Players' Margins on the Retailer's Payoff**

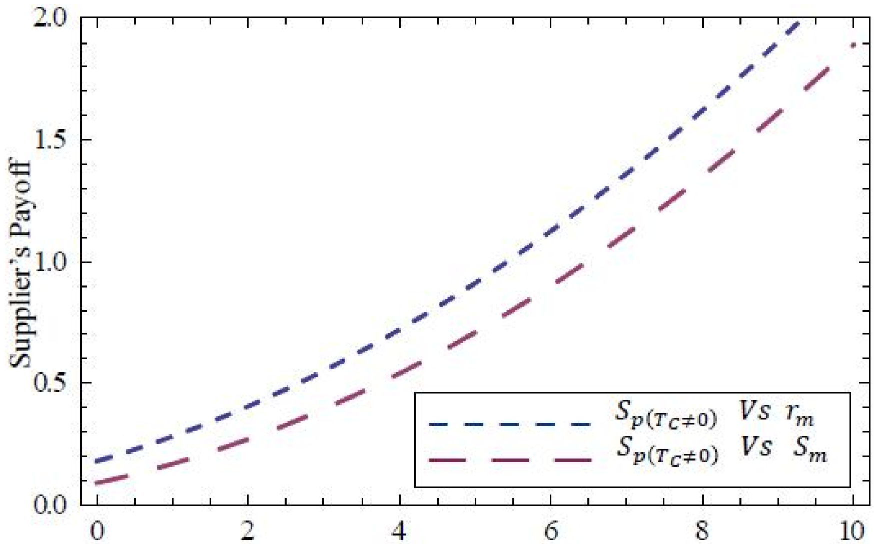

Fig. 5.5. The Effect of the Players' Margins on the Supplier's Payoff

From Fig. 5.3 we observe that the retailer is much more motivated to increase promotion with his margin than with the supplier's margin. A further look at Fig. 5.4 and Fig. 5.5 show that the retailer and the supplier are better-off with the retailer's margin than with the supplier's margin. In essence an increase in the retailer's margin translates to an increase in the promotion effort which translates to increase in the payoffs of both players. This shows that the retailer's margin is very crucial to both players.

5.4 The Players' Optimal Payoffs

Table 5.1 A Comparison of the Players' Optimal Payoffs for both Game Scenarios

	No Credit Provision	Credit Provision
R_p	0.0900	0.2025
S_p	0.3600	0.4050

Clearly, Table 5.1 shows that both players perform better with the adoption of trade credit than with non-provision since the provision of credit benefits both of them.

6. CONCLUSION

This paper studied trade credit in a supplier-retailer setting in which the supplier is the Stackelberg channel leader, while the retailer is considered to be the follower. The work considered two game scenarios which include a situation where the supplier provides trade credit and a situation where credit is not provided. The paper determined the optimal promotion effort and credit period, and hence the channel members' payoffs for both scenarios. The work shows that increase in credit period reduces the retailer's promotion effort and payoff, but increases the supplier's payoff. Further, we observe that the players are better-off with the retailer's increasing margin. Both players and entire channel are better-off with trade credit.

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