__Original Research Article__

GENERAL PARAMETERS ESTIMATION IN THE PRESENCE OF NONRESPONSE USING DIFFERENCE-CUM EXPONENTIAL ESTIMATORS

Abstract

Non response is a common problem in a survey process. Therefore, it is necessary to find a way out of handling non response whenever arises. The current study proposed difference-cum exponential ratio-type estimator for estimation of general parameters using auxiliary information which is defined in two situations of non response. A conventional estimator $t^{*}_{(a,b)}$ is used to define population constants including population mean, standard deviation, coefficient of variation and mean square. The expression of bias and mean square error of the proposed estimators are obtained up to the first order of approximation for situation I and situation II. To compare the efficiency of the proposed estimators over the existing ones, an empirical study is carried out using real and simulated data sets. Both the theoretical and empirical study shows that the proposed class of estimators outperformed other existing estimators.

Keywords: Non response; Auxiliary variable; Bias; Efficiency; mean squared error; study variable.

1. Introduction

The information on each unit of the selected sample in statistical procedures is very important to obtain the precise estimate of the population parameters (eg; mean, variance). In practice, it is obvious that the information on some of the selected units may not be available in the first attempt due to some natural occurrence. In such situation, the unavailable selected units are treated as nonresponse by the survey practitioners. The nonresponse group cannot be ignored; thus, Hansen and Hurwitz (1946) developed a procedure of sub-sampling to handle the nonrespondents and used it a more expensive approach introducing second attempt. These attempts include mail questionnaire and personal interview, to obtain the information from the sample as well to provide appropriate inference regard the population parameter. It is quite understandable that estimator with more efficiency can be established by using auxiliary information. Recently, Pal and Singh (2017, 2018), Singh and Usman (2019) have developed an efficient estimator for estimation of population mean to handle sub-sampling of nonrespondents as introduced by Hansen and Hurwitz (1946). Bhat et al. (2018) presented the estimation of population variance in the presence of nonresponse utilizing the linear combination of coefficient of skewness and quartiles as auxiliary information. thus, various parameters like population mean $\bar{X}$, variance S_x^2, coefficient of variation C_x, moment ratio $\beta_2(x)$ (coefficient of kurtosis), and $\beta_2(x)$ (coefficient of skewness) for auxiliary variable $x$can be known easily (see Upadhyaya and Singh (1999)). On advantage of auxiliary information, the nonresponse problem in the study of population mean $\bar{X}$, and variance S_x^2 have been established in literature. Numerous authors including Cochran (1977), Rao (1986, 1987), Khare and Srivastava (1993, 1995, 1997), Singh and Kumar (2008a,b, 2009), Singh, Kumar, and Kozak (2010), Kumar and Bhougal (2011), Kumar (2013), Shabbir and Khan (2013), Pal and Singh (2017, 2018), Rao et. al. (1990), Riaz et al (2014), Singh et al (2016) Shahzad et al (2017), Sinha and Kumar (2017), Bhat et al. (2018) have proposed various estimators for estimation of population mean or variance to handle study variable in the presence of nonresponse.

However, some studies have presented the estimation of general population parameter $t_{(a,b)}$. The work in this direction includes Singh and Karpe (2009) who presented general population parameter for estimation of coefficient of variation in the presence of measurement error. Singh and Pal (2017) and Adichwal et al. (2022) developed estimator for the general parameter of the population when observations are assumed to be free from any type of errors. Also, Gajendra et al. (2022) have considered the general population parameter estimation in the presence of sensitive study variable. To the best our knowledge the estimator for general parameter of the population when observations are subject to nonresponse has not been considered. Hence, in the present study, we present the general difference-cum exponential ratio-type estimators for estimating general population parameters in the presence on nonresponse problem.

2. Notations and Conventional Estimator

Suppose y is the interested variable, and n sample size is randomly selected without replacement, in which the response units is n_1 and non-response units is n_2. From n_2 non-respondents we choose $r = \frac{n_2}{k}, (k \geq 1)$ sample size where k is the rate of inverse sampling at the second phase sample of size n (fixed in advance) and from which necessary information is obtained. Here, it is expected that all the h units are fully respond at this time.

Let N_1 be responding and $N_2 = N - N_1$ be non-responding units from the finite population N having corresponding weights; $W_1 = \frac{N_1}{N}, W_2 = \frac{N_2}{N}$, respectively.

The study considered the modification of Hansen and Hurwitz (1946) unbiased estimator, where general form of the parameters of interest is under consideration and given by

$$t_{(a,b)} = \bar{Y}^a S_y^b \qquad (1.1)$$

where $a \; and \; b$ are suitable chosen scalars and the unbiased estimators of $t_{(a,b)}$ is defined as

$$\hat{t}^*_{(a,b)} = \bar{y}^{*a} s_y^{*b} \qquad (1.2)$$

$$\hat{t}^*_{(1,0)} = \hat{\bar{y}}^* = w_1 \bar{y} + w_2 \bar{y}_{2h} \;, var(\hat{\bar{y}}^*) = \bar{Y}^2 \left[\lambda_1 C_y^2 + \lambda_2 C_{y_{(2)}}^2\right],$$

$$\hat{t}^*_{(0,2)} = \hat{s}_y^{*2} \quad var(\hat{s}_y^{*2}) = \left(S_y^2\right)^2 \left[\lambda_1(\delta_{40} - 1) + \lambda_2\left(\delta_{40(2)} - 1\right)\right]$$

with expression: $\bar{y} = n_1^{-1} \sum_{i=1}^{n_1} y_i$, $\bar{y}_{2h} = h^{-1} \sum_{i=1}^{r} y_{2i}$, $w_1 = \frac{n_1}{n}, w_2 = \frac{n_2}{n}$, $\lambda_1 = \frac{1}{n}$, $\lambda_2 = \frac{N_2(k-1)}{nN}$, $S_y^2 = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})$, $S_{y2}^2 = (N_2 - 1)^{-1} \sum_{i=1}^{N_2} (y_i - \bar{Y}_2)$

The reduce form of general parameter $t_{(a,b)}$ in (1) commonly known population parameters are presented as follows

S/N	Scalars		$t^*_{(a,b)}$	Population Parameters
	a	b		
1	1	0	$t^*_{(1,0)}$	$\bar{Y}^*$ (Population mean of Y)
2	0	1	$t^*_{(0,1)}$	S^*_y (Standard deviation of Y)
3	-1	1	$t^*_{(-1,1)}$	C^*_y (Coefficient of variation of Y)
4	0	2	$t^*_{(0,2)}$	S^{*2}_y (Population variance of Y)

Note that $\mu_{20} = S_y^2,\ C_y^2 = S_y^2/\bar{Y}^2,\ \mu_{02} = S_x^2,\ C_x^2 = S_x^2/\bar{X}^2,\ \mu_{11} = S_{yx},\ \rho_y = \frac{S_{yx}}{S_y}$

Following are the definition of the error terms

$e_0^* = \frac{\bar{y}^*}{\bar{Y}} - 1,\ e_1^* = \frac{s_y^{*2}}{S_y^2} - 1,\ e_2 = \frac{\bar{x}}{\bar{X}} - 1,\ e_3 = \frac{s_x^2}{S_x^2} - 1, e_2^* = \frac{\bar{x}^*}{\bar{X}} - 1,\ e_3^* = \frac{s_x^{*2}}{S_x^2} - 1,$

with $E(e_0^*) = E(e_1^*) = E(e_2) = E(e_3) = (e_2^*) = E(e_3^*) = 0,$

to obtain the bias and MSE of the proposed estimators up to first order approximations, we used the following error terms

$E(e_0^{*2}) = \lambda_1 C_y^2 + \lambda_2 C_{y_2}^2 = V_{20}^*\ ,\ E(e_1^{*2}) = \lambda_1(\delta_{40} - 1) + \lambda_2(\delta_{40(2)} - 1) = V_{40}^*$

$E(e_2^{*2}) = \lambda_1 C_x^2 + \lambda_1 C_{x(2)}^2 = V_{02}^*\ ,\ \ E(e_1^2) = \lambda_1(\delta_{04} - 1) + \lambda_2\left(\delta_{04(2)} - 1\right) = V_{04}^*,$

$E(e_0^* e_1^*) = \lambda_1 \delta_{30} C_y + \lambda_2 \delta_{30} C_{y2} = V_{30}^*, E(e_0^* e_2^*) = \lambda_1 \rho_{yx} C_y C_x + \lambda_2 \rho_{yx} C_y C_{x(2)} = V_{11}^*,$

$E(e_0^* e_3^*) = \lambda_1 \delta_{12} C_y + \lambda_2 \delta_{12} C_{y(2)} = V_{12}^*,\ \ E(e_1^* e_2^*) = \lambda_1 \delta_{21} C_x + \lambda_2 \delta_{21} C_{x(2)} = V_{21}^*$

$E(e_1^* e_3^*) = \lambda_1(\delta_{22} - 1) + \lambda_2\left(\delta_{22(2)} - 1\right) = V_{22}^*, E(e_2^* e_3^*) = \lambda_1 \delta_{03} C_x + \lambda_2 \delta_{03} C_{x(2)} = V_{03}^*$

$E(e_2^2) = \lambda_1 C_x^2 = V_{02},\ \ \ E(e_1^2) = \lambda_1(\delta_{04} - 1) = V_{04},\ E(e_0^* e_2) = \lambda_1 \rho_{yx} C_y C_x = V_{11},\ \ E(e_0^* e_3) = \lambda_1 \delta_{12} C_y = V_{12},$

$E(e_1^* e_2) = \lambda_1 \delta_{21} C_x = V_{21}, E(e_1^* e_3) = (\lambda_1 \delta_{22} - 1) = V_{22},\ E(e_2 e_3) = \lambda_1 \delta_{03} C_x = V_{03}$

where $\delta_{rs} = \frac{\mu_{rs}}{\sqrt{\mu_{20}^{r}\mu_{02}^{s}}}$, $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{Y})^r(x_i - \bar{X})^s$, r and s are non-negative integers. Also $\lambda_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\lambda_2 = \frac{W_2(k-1)}{n}$ with $W_2 = \frac{N_2}{N}$, N_2 is the number of units in the population corresponds to non-response group.

Moreover, the following notation may be found useful

$$M_1^* = a^2V_{20}^* + abV_{30}^* + \frac{b^2}{4}V_{40}^*$$

$$M_2^* = aV_{11}^* + \frac{b}{2}V_{21}^*, \quad M_2 = aV_{11} + \frac{b}{2}V_{21}$$

$$M_3^* = aV_{12}^* + \frac{b}{2}V_{22}^*, \quad M_3 = aV_{12} + \frac{b}{2}V_{22}$$

$$M_4^* = a(a-1)V_{20}^* + abV_{30}^* + \frac{b(b-2)}{4}V_{40}^*$$

To estimate the general population parameters discussed earlier, the conventional estimator of general parameter $t_{(a,b)}$ given in (1.2) is express in terms of errors and we get

$$\hat{t}_{(a,b)}^* = \bar{Y}^a S_y^a (1 + e_0^*)^a (1 + e_1^*)^{b/2} \tag{2.1}$$

$$= t_{(a,b)}^* (1 + e_0^*)^a (1 + e_1^*)^{b/2}$$

Due to the expansion of $(1 + e_0^*)^a$ and $(1 + e_1^*)^{b/2}$, we assume that $|e_0| < 1$ and $|e_1| < 1$. Expanding (2.1) and retain e's up to second degree and subtracting $t_{(a,b)}$ from both side we have

$$\hat{t}_{(a,b)}^* - t_{(a,b)} = t_{(a,b)}\left[ae_0^* + \frac{b}{2}e_1^* + \frac{ab}{2}e_0^*e_1^* + \frac{1}{2}a(a-1)e_0^{*2} + \frac{b(b-2)}{8}e_1^{*2} + \cdots\right] \tag{2.2}$$

Taking the expectation of (2.2), the bias of estimator $\hat{t}_{(a,b)}^*$ given by

$$B\left(\hat{t}_{(a,b)}^*\right) = \frac{1}{2}t_{(a,b)}\left[a(a-1)V_{20}^* + abV_{30}^* + \frac{b(b-2)}{4}V_{40}^*\right] = \frac{1}{2}t_{(a,b)}M_4^*(a,b) \tag{2.3}$$

Squaring both side of (2.2) and retain up to first order of approximation we get

$$\left(\hat{t}_{(a,b)}^* - t_{(a,b)}\right)^2 = t_{(a,b)}^2\left(a^2e_0^{*2} + abe_0^*e_1^* + \frac{b^2}{4}e_1^{*2}\right) \tag{2.4}$$

Taking the expectations of (2.4) we get the Mean square error of $\hat{t}_{(a,b)}$ $\left(MSE\left(\hat{t}_{(a,b)}^*\right)\right)$

The mean squared error (MSE) of estimator $\hat{t}_{(a,b)}^*$ given by

$$MSE\left(\hat{t}_{(a,b)}^*\right) = t_{(a,b)}^2\left[a^2V_{20}^* + abV_{30}^* + \frac{b^2}{4}\delta_{40}^*\right] = t_{(a,b)}^2 M_1^*(a,b) \tag{2.5}$$

3. **The Proposed Estimator**

motivated by Adichwal et al. (2022), employing the information on an auxiliary variable x when population mean $\bar{X}$ and variance S_x^2 are known, a class of difference-cum exponential ratio estimator for population parameters, $\hat{t}^*_{(a,b)}$ have been proposed in two different situations on nonresponse to estimate general parameters of the population

3.1 **Situation I: nonresponse observed only on y and x known**.

Assuming the information is obtained only on $n_1(< n)$ units for study variable while having complete information on sample of auxiliary variable x such that n_2 units of the study variable y are considered as nonrespondents. Therefore, when there is incidence of nonresponse on study variable y, we propose a class of difference-cum exponential ratio estimator, t.

3.1.1. **Proposed Difference-Cum Exponential Ratio Estimator**, $\boldsymbol{t}$.

Assuming that population mean $\bar{X}$ and variance S_x^2 are already known, the estimator t is given by

$$t = \left[\hat{t}^*_{(a,b)} + w(\bar{X} - \bar{x})\right] exp\left[\frac{u(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right] exp\left[\frac{v(S_x^2 - s_x^2)}{S_x^2+(\beta-1)s_x^2}\right] \quad (3.1)$$

where u and v are scalar having real values $(0, -1, 1)$, w, α and β are suitably chosen constants and they can be determined.

Expressing (3.1) in terms of error we obtain

$$t = \left[t^*_{(a,b)}(1 + e_0^*)^a (1 + e_1^*)^{b/2} - w\bar{X}e_2\right] exp[-A_1 e_2\{1 + \theta_1 e_2\}^{-1}] \times exp[-A_2 e_3\{1 + \theta_2 e_3\}^{-1}] \quad (3.2)$$

where $A_1 = \frac{u}{\alpha}$, $A_2 = \frac{v}{\beta}$, $\theta_1 = \left(\frac{\alpha-1}{\alpha}\right)$, $\theta_2 = \left(\frac{\beta-1}{\beta}\right)$

$$= t_{(a,b)}\left[ae_0^* + \frac{b}{2}e_1^* + \frac{ab}{2}e_0^* e_1^* + \frac{a(a-1)}{2}e_0^{*2} + \frac{b(b-2)}{8}e_1^{*2} + \cdots - +w\bar{X}e_2\right]$$

$$\times \left[1 - A_1 e_2 - A_2 e_3 + \frac{1}{2}B_1 e_2^2 + \frac{1}{2}B_2 e_3^2 + A_1 A_2 e_2 e_3 + \cdots\right] \quad (3.3)$$

where $B_1 = A_1(2\theta_1 + A_1) and\ B_2 = A_2(2\theta_2 + A_2)$

Expand (3.3) and retain e's to degree of order two

$$t = t_{(a,b)}\left[1 + ae_0^* + \frac{b}{2}e_1^* - A_1 e_2 - A_2 e_3 + \frac{a(a-1)}{2}e_0^{*2} + \frac{b(b-2)}{8}e_1^{*2} + \frac{ab}{2}e_0^* e_1^* - A_1\left(ae_0^* e_2 + \frac{1}{2}e_1^* e_2\right) - A_2\left(ae_0^* e_3 + \frac{1}{2}e_1^* e_3\right) + A_1 A_2 e_2 e_3 + \frac{1}{2}B_1 e_2^2 + \frac{1}{2}B_2 e_3^2\right] - w\bar{X}e_2 + A_1 w\bar{X}e_2^2 + A_2 w\bar{X}e_2 e_3 \quad (3.4)$$

Subtracting $t_{(a,b)}$ from both side of (3.4) and taking the expectation, we obtain the bias of the proposed estimator up to first order approximations as follow.

$$Bias(t) = E\left(t - t_{(a,b)}\right)$$

$$= \frac{1}{2} t_{(a,b)} \left[a(a-1)V_{20}^{*} + abV_{30}^{*} + \frac{1}{4} b(b-1)V_{40}^{*} + B_1V_{02} + B_2V_{04} - 2A_1\left(aV_{11} + \frac{b}{2}V_{21}\right) - 2A_2\left(aV_{12} + \frac{b}{2}V_{22}\right) + 2A_1A_2V_{03} + 2kR(A_1V_{02} + A_2V_{03})\right] \quad (3.5)$$

$$Bias(t) = \frac{1}{2} t_{(a,b)}[M_4^{*}(a,b) + B_1V_{02} + B_2V_{04} + 2A_1A_2V_{03} - 2A_1M_2(a,b) - 2A_2M_3(a,b) + 2wR\{A_1V_{02} + A_2V_{03}\}] \quad (3.6)$$

where, $R = \frac{\bar{X}}{t_{(a,b)}^{*}}$

Squaring (3.4) we obtain the mean squared error (MSE) of the proposed estimator up to first order approximation as follows.

$$\left(t - t_{(a,b)}^{*}\right)^2 = t_{(a,b)}^{*2}\left[ae_0^{*} + \frac{1}{2}be_1^{*} - A_1e_2 - A_2e_3 - wRe_2\right]^2 \quad (3.7)$$

From (3.7), we get

$$\left(t - t_{(a,b)}^{*}\right)^2 = t_{(a,b)}^{*2}\left[a^2e_0^{*2} + \frac{1}{4}b^2e_1^{*2} + A_1^2e_2^2 + A_2^2e_3^2 + abe_0^{*}e_1^{*} - 2aA_1e_0^{*}e_2 - 2aA_2e_0^{*}e_3 - bA_1e_1^{*}e_2 - bA_2e_1^{*}e_3 + 2A_1A_2e_2e_3 - 2awRe_0^{*}e_2 - bwRe_1^{*}e_2 + 2aA_1wRe_2^2 + 2A_2wRe_2e_3 + w^2R^2e_2^2\right] \quad (3.8)$$

(3.8) can be rewrite as

$$\left(t - t_{(a,b)}^{*}\right)^2 = t_{(a,b)}^{*2}\left[a^2e_0^{*2} + \frac{1}{4}b^2e_1^{*2} + abe_0^{*}e_1^{*} + A_1^2e_2^2 + A_2^2e_3^2 - 2A_1\left\{ae_0^{*}e_2 + \frac{b}{2}e_1^{*}e_2\right\} - 2A_2\left\{ae_0^{*}e_3 + \frac{b}{2}e_1^{*}e_3\right\} + 2A_1A_2e_2e_3 + 2wR\{A_1e_2^2 + A_2e_2e_3\} - 2wR\left\{ae_0^{*}e_2 + \frac{1}{2}be_1^{*}e_2\right\} + w^2R^2e_2^2\right] \quad (3.9)$$

Follow is the expectation of (3.9)

$$MSE(t) = t_{(a,b)}^2\left[a^2V_{20}^{*} + abV_{30}^{*} + \frac{1}{4}b^2V_{40}^{*} + A_1^2V_{02} + A_2^2V_{04} - 2A_1\left\{aV_{11} + \frac{b}{2}v_{21}\right\} - 2A_2\left\{aV_{12} + \frac{b}{2}V_{22}\right\} + 2A_1A_2V_{03} + 2wR\{A_1V_{02} + A_2V_{03}\} - 2wR\left\{aV_{11} + \frac{b}{2}V_{21}\right\} + w^2R^2V_{02}\right] \quad (3.10)$$

$$MSE(t) = t_{(a,b)}^2[M_1^{*} + A_1^2V_{02} + A_2^2V_{04} - 2A_1M_2(a,b) - 2A_2M_3(a,b) + 2A_1A_2V_{03} + 2wR\{A_1V_{02} + A_2V_{03}\} - 2wRM_2(a,b) + w^2R^2V_{02}] \quad (3.11)$$

The optimum values of $A_1$and A_2 which minimizes $MSE(t)$ are obtained by differentiating (3.11) partially with respect to $A_1$and A_2 and given by

$$A_{1(opt)} = \frac{V_{04}M_2(a,b) - V_{03}M_3(a,b)}{V_{02}V_{04} - V_{03}^2} - wR \qquad A_{2(opt)} = \frac{V_{02}M_3(a,b) - V_{03}M_2(a,b)}{V_{02}V_{04} - V_{03}^2} \tag{3.12}$$

Substituting (3.12) into (3.11) the minimum MSE of estimator t up to the first order approximation given by

$$MSE(t) = t_{(a,b)}^2 \left[M_1^* - \frac{\{V_{04}\{M_2(a,b)\}^2 - 2V_{03}M_2(a,b)M_3(a,b) + V_{02}\{M_3(a,b)\}^2\}}{V_{02}V_{04} - V_{03}^2}\right] \tag{3.13}$$

Suppose X is known and $v = 0$, the general population parameter t in (3.1) can be reduced to the following general estimator for estimating population parameters in the presence of nonresponse as

$$t_1 = \left[\hat{t}_{(a,b)}^* + w(\bar{X} - \bar{x})\right] exp\left[\frac{u(\bar{X} - \bar{x})}{\bar{X} + (\alpha - 1)\bar{x}}\right] \tag{3.14}$$

where u is a scalar having real values $(-1, 0, 1)$, w and α is arbitrary constants. The bias and MSE of the estimator t_1 in (3.14) up to first order approximation are given as follows

$$Bias(t_1) = \frac{1}{2} t_{(a,b)}[M_4^*(a,b) + B_1V_{02} - 2A_1M_2(a,b)] \tag{3.15}$$

$$MSE(t_1) = t_{(a,b)}^2[M_1^*(a,b) + A_1^2V_{02} - 2A_1M_2(a,b)] \tag{3.16}$$

Estimating the optimum value of $A_{1(opt)} = M_2(a,b)/V_{02}$, we have the minimum value of $MSE(t_1)$ as

$$MSE_{min}(t_1) = t_{(a,b)}^2 \left[M_1^*(a,b) - \frac{\{M_2(a,b)\}^2}{V_{02}}\right] \tag{3.17}$$

Table 1.1shows the existing estimators obtained from (3.14) considering appropriate values for a, b, w, u and α accordingly.

Considered k and u to be zero while the S_x^2 is known, the population parameter $t_{(a,b)}^*$ in (3.1) reduces to estimator $t_{(1)}$ and is given by

$$t_{(1)} = \hat{t}_{(a,b)}^* exp\left[\frac{v(S_x^2 - s_x^2)}{S_x^2 + (\beta - 1)s_x^2}\right] \tag{3.18}$$

where v is a scalar having real values $(-1, 0, 1)$, and β is a suitably chosen constant. The bias and MSE of the estimator $t_{(1)}$ are given as follows.

$$Bias(t_{(1)}) = \frac{1}{2} t_{(a,b)}[M_4^*(a,b) + B_2V_{04} - 2A_1M_2(a,b)] \tag{3.19}$$

$$MSE\left(t_{(1)}\right) = t_{(a,b)}^2\left[M_1^* + A_2^2 V_{04} - 2A_2 M_3(a,b)\right] \tag{3.20}$$

Estimating the optimum value of $A_{2(opt)} = M_3(a,b)/V_{04}$, we have the minimum value of $MSE(t_{(1)})$ as

$$MSE_{min}\left(t_{(1)}\right) = t_{(a,b)}^2\left[M_1^* - \frac{\{M_3(a,b)\}^2}{V_{04}}\right] \tag{3.21}$$

Table 1.2 shows special cases of the estimator $t_{(1)}$ obtained from (3.18) considering appropriate values for a, b, k, v and β accordingly.

3.1.1.1 **Relative Efficiency of *t***

$$MSE\left(\hat{t}_{(a,b)}^*\right) - MSE_{min}(t) = t_{(a,b)}^2\left[\frac{V_{04}\{M_2(a,b)\}^2 - 2V_{03}M_2(a,b)M_3(a,b) + V_{02}\{M_3(a,b)\}^2}{V_{02}V_{04} - V_{03}^2}\right] \geq 0 \tag{3.22}$$

$$MSE_{min}(t_1) - MSE_{min}(t) = t_{(a,b)}^2\left[\frac{\{V_{03}M_2(a,b) - V_{02}M_3(a,b)\}^2}{V_{02}\left(V_{02}V_{04} - V_{03}^2\right)}\right] \geq 0 \tag{3.23}$$

$$MSE_{min}\left(t_{(1)}\right) - MSE_{min}(t) = t_{(a,b)}^2\left[\frac{\{V_{03}M_3(a,b) - V_{04}M_2(a,b)\}^2}{V_{04}\left(V_{02}V_{04} - V_{03}^2\right)}\right] \geq 0 \tag{3.24}$$

In as much $V_{02}V_{04} - V_{03}^2$ is always greater than zero, as shown in (3.22), (3.23), and (3.24), the proposed estimator t is more efficiently performed than the estimators $\hat{t}_{(a,b)}^*, t_1, t_{(1)}$.

3.1.2. **Estimation of Population Mean of study Variable *Y***

when $(a, b, k, u, v, \alpha, \beta) = (1, 0, k, u, v, \alpha, \beta)$, the estimator t in (3.1) reduces to estimator t_μ and given by

$$t_\mu = [\bar{y}^* + w(\bar{X} - \bar{x})]exp\left[\frac{u(\bar{X} - \bar{x})}{\bar{X} + (\alpha - 1)\bar{x}}\right] exp\left[\frac{v(S_x^2 - s_x^2)}{S_x^2 + (\beta - 1)s_x^2}\right] \tag{3.25}$$

Substituting $(1, 0, w, u, v, \alpha, \beta)$ into (3.6) and (3.11), we obtain the bias and MSE of the estimator t_μ, respectively, as

$$Bias\left(t_\mu\right) = +\frac{1}{2}\bar{Y}[B_1 V_{02} + B_2 V_{04} + 2A_1 A_2 V_{03} - 2A_1 a V_{11} - 2A_2 a V_{12} + 2wR(A_1 V_{02} + A_2 V_{03})] \tag{3.26}$$

$$MSE\left(t_\mu\right) = \bar{Y}^2[V_{20}^* + A_1^2 V_{02} + A_2^2 V_{04} - 2A_1 V_{11} - 2A_2 V_{12} + 2A_1 A_2 V_{03} + 2wR\{A_1 V_{02} + A_2 V_{03}\} - 2wR\, V_{11} + w^2 R^2 V_{02}] \tag{3.27}$$

where, $R = \frac{\bar{X}}{\bar{Y}}$, the MSE of t_μ in (3.27) is minimized for values

$$A_{1(opt)} = \frac{V_{04}V_{11} - V_{03}V_{12}}{V_{02}V_{04} - V_{03}^2} - wR \qquad A_{2(opt)} = \frac{V_{02}V_{12} - V_{03}V_{11}}{V_{02}V_{04} - V_{03}^2} \tag{3.28}$$

Substituting the optimal values in (30), the minimum MSE of estimator t_μ is given by

$$MSE_{min}(t_\mu) = \bar{Y}^2 \left[V_{20}^* - \left\{\frac{V_{04}V_{11}^2 - 2V_{03}V_{11}V_{12} + V_{02}V_{12}^2}{V_{02}V_{04} - V_{03}^2}\right\}\right] \quad (3.29)$$

The special cases of the estimator t_μ are presented with their corresponding minimum MSEs in Table 1.3

3.1.2.1 **Relative Efficiency**

$$V(t_{1.*}) - MSE_{min}(t_\mu) = \bar{Y}^2 \left\{\frac{V_{04}V_{11}^2 - 2V_{03}V_{11}V_{12} + V_{02}V_{12}^2}{V_{02}V_{04} - V_{03}^2}\right\} \geq 0 \quad (3.30)$$

$$MSE_{min}(t_{1.2}) - MSE_{min}(t_\mu) = \bar{Y}^2 \left\{\frac{(V_{03}V_{11} - V_{02}V_2)^2}{V_{02}(V_{02}V_{04} - V_{03}^2)}\right\} \geq 0 \quad (3.31)$$

$$MSE(t_{1.R}) - MSE_{min}(t_\mu) = \bar{Y}^2 \left[\frac{V_{04}V_{11}^2 - 2V_{03}V_{11}V_{12} + V_{02}V_{12}^2}{V_{02}V_{04} - V_{03}^2} - \frac{1}{4}V_{02} + V_{11}\right] \geq 0 \quad (3.32)$$

$$MSE_{min}(t_{1.d}) - MSE_{min}(t_\mu) = \bar{Y}^2 \left\{\frac{(V_{03}V_{11} - V_{02}V_2)^2}{V_{02}(V_{02}V_{04} - V_{03}^2)}\right\} \geq 0 \quad (3.33)$$

3.2 **Situation II: Nonresponse Observed on both y and x when $\bar{X}$ and S_x^2 are known**.

Assuming the information is obtained only on $n_1(< n)$ units for both study variable y and auxiliary variable x and n_2 units are considered as group of nonresponses among the selected sample. Following Hansen and Hurwitz (1946) technique, we propose a class of difference-cum exponential ratio estimator, t^* for general population parameter

3.2.1. **Proposed Difference-Cum Exponential Ratio Estimator**, t^*.

Assuming that population mean $\bar{X}$ and variance S_x^2 are already known, the estimator t^* is given by

$$t^* = \left[\hat{t}_{(a,b)}^* + w(\bar{X} - \bar{x}^*)\right] exp\left[\frac{u(\bar{X} - \bar{x}^*)}{\bar{X} + (\alpha - 1)\bar{x}^*}\right] exp\left[\frac{v(S_x^2 - s_x^{*2})}{S_x^2 + (\beta - 1)s_x^{*2}}\right] \quad (3.34)$$

where u and v are scalar having real values $(0, -1, 1)$, w, α and β are suitably chosen constants and they can be determined.

Expressing (3.34) in terms of error we obtain

$$t^* = \left[t_{(a,b)}(1 + e_0^*)^a (1 + e_1^*)^{b/2} - w\bar{X}e_2^*\right] exp[-A_1^* e_2^* \{1 + \theta_1 e_2^*\}^{-1}]$$
$$\times\ exp[-A_2^* e_3^* \{1 + \theta_2 e_3^*\}^{-1}] \quad (3.35)$$

where $A_1^* = \frac{u}{\alpha}, A_1^* = \frac{v}{\beta},\ \theta_1 = \left(\frac{\alpha - 1}{\alpha}\right),\ \theta_2 = \left(\frac{\beta - 1}{\beta}\right)$

$$= t_{(a,b)} \left[a e_0^* + \frac{b}{2} e_1^* + \frac{ab}{2} e_0^* e_1^* + \frac{a(a-1)}{2} e_0^{*2} + \frac{b(b-2)}{8} e_1^{*2} + \cdots - + w\bar{X}e_2^*\right]$$

$$\times \left[1 - A_1^* e_2^* - A_2^* e_3^* + \frac{1}{2} B_1^* e_2^{*2} + \frac{1}{2} B_2 e_3^{*2} + A_1^* A_2^* e_2^* e_3^* + \cdots\right] \quad (3.36)$$

where $B_1^* = A_1^* (2\theta_1 + A_1^*) and\ B^* = A_2^* (2\theta_2 + A_2^*)$

Expand (3.36) and retain e's to degree of order two

$$t^* = t_{(a,b)} \left[1 + ae_0^* + \frac{b}{2} e_1^* - A_1^* e_2^* - A_2^* e_3 + \frac{a(a-1)}{2} e_0^{*2} + \frac{b(b-2)}{8} e_1^{*2} + \frac{ab}{2} e_0^* e_1^* - A_1^* \left(ae_0^* e_2^* + \frac{1}{2} e_1^* e_2^*\right) - A_2^* \left(ae_0^* e_3^* + \frac{1}{2} e_1^* e_3^*\right) + A_1^* A_2^* e_2^* e_3^* + \frac{1}{2} B_1^* e_2^{*2} + \frac{1}{2} B_2^* e_3^{*\,2}\right] - w\bar{X} e_2^* + A_1^* w\bar{X} e_2^{*2} + A_2^* w\bar{X} e_2^* e_3^* \quad (3.37)$$

Subtracting $t_{(a,b)}$ from both side of (3.37) and taking the expectation, we obtain the bias of the proposed estimator up to first order approximations as follow.

$$Bias(t^*) = E\left(t^* - t_{(a,b)}\right)$$

$$= \frac{1}{2} t_{(a,b)} \left[a(a-1)V_{20}^* + abV_{30}^* + \frac{b(b-2)}{4} V_{40}^* + B_1^* V_{02}^* + B_2^* V_{04}^* + 2A_1^* \left\{aV_{11}^* + \frac{b}{2} V_{21}^*\right\} - 2A_2^* \left\{aV_{12}^* + \frac{b}{2} V_{22}^*\right\} + 2A_1^* A_2^* V_{03}^* + 2wR(A_1^* V_{02}^* + A_2^* V_{03}^*)\right]$$

$$Bias(t^*) = \frac{1}{2} t_{(a,b)} [M_4^*(a,b) + B_1^* V_{02}^* + B_2^* V_{40}^* + 2A_1^* A_2^* V_{03}^* - 2A_1^* M_2^*(a,b) - 2A_2^* M_3^*(a,b) + 2wR\{A_1^* V_{02}^* + A_2^* V_{03}^*\}], \quad (3.38)$$

Squaring (3.37) we obtain the mean squared error (MSE) of the proposed estimator up to first order approximation as follows.

$$\left(t^* - t_{(a,b)}\right)^2 = t_{(a,b)}^2 \left[ae_0^* + \frac{1}{2} be_1^* - A_1^* e_2^* - A_1^* e_3^* - wRe_2^*\right]^2 \quad (3.39)$$

Expanding (3.39), we get

$$\left(t^* - t_{(a,b)}\right)^2 = t_{(a,b)}^2 \left[a^2 e_0^{*2} + \frac{1}{4} b^2 e_1^{*2} + A_1^{*2} e_2^{*2} + A_2^{*2} e_3^{*2} + abe_0^* e_1^* - 2aA_1^* e_0^* e_2^* - 2aA_2^* e_0^* e_3^* - bA_1^* e_1^* e_2^* - bA_2^* e_1^* e_3^* + 2A_1^* A_2^* e_2^* e_3^* - 2awRe_0^* e_2^* - bwRe_1^* e_2^* + 2aA_1^* wRe_2^{*2} + 2A_2^* wRe_2^* e_3^* + w^2 R^2 e_2^{*2}\right]$$

$$= t_{(a,b)}^2 \left[a^2 e_0^{*2} + \frac{1}{4} b^2 e_1^{*2} + abe_0^* e_1^* + A_1^{*2} e_2^{*2} + A_2^{*2} e_3^{*2} - 2A_1^* \left\{ae_0^* e_2^* + \frac{1}{2} be_1^* e_2^*\right\} - 2A_2^* \left\{ae_0^* e_3^* + \frac{1}{2} be_1^* e_3^*\right\} + 2A_1^* A_2^* e_2^* e_3^* + 2wR\{aA_1^* e_2^{*2} + A_2^* e_2^* e_3^*\} - 2wR \left\{ae_0^* e_2^* + \frac{1}{2} be_1^* e_2^*\right\} + w^2 R^2 e_2^{*2}\right] \quad (3.40)$$

Follow is the expectation of (3.40)

$$MSE(t^*) = t_{(a,b)}^2 \Big[a^2V_{20}^* + abV_{30}^* + \frac{b^2}{4}V_{40}^* + A_1^{*2}V_{02}^* + A_2^{*2}V_{40}^* - 2A_1^* \left\{aV_{11}^* + \frac{b}{2}V_{21}^*\right\} - 2A_2^* \left\{aV_{12}^* + \frac{b}{2}V_{22}^*\right\} + 2A_1^* A_2^* V_{03}^* + 2wR\{aA_1^* V_{02}^* + A_2^* V_{03}^*\} - 2wR\left\{aV_{11}^* + \frac{b}{2}V_{21}^*\right\} + w^2R^2V_{02}^{*2}\Big] \tag{3.41}$$

$$MSE(t^*) = t_{(a,b)}^2[M_1^*(a,b) + A_1^{*2}V_{02}^{*2} + A_2^{*2}V_{04}^* - 2A_1^* M_2^*(a,b) - 2A_2^* M_3^*(a,b) + 2A_1^* A_2^* V_{03}^* + 2wR\{aA_1^* V_{02}^* + A_2^* V_{03}^*\} - 2wRM_2^*(a,b) + w^2R^2V_{02}^*] \tag{3.24}$$

The optimum values of A_1^* and A_2^* which minimizes $MSE(t)$ are obtained by differentiating (3.42) partially with respect to A_1^* and A_2^* and given by

$$A_{1(opt)}^* = \frac{\delta_{04}^*{}^*_{04} M_2^*(a,b) - V_{03}^* M_3^*(a,b)}{V_{02}^* V_{04}^* - V_{03}^{*2}} - wR \qquad A_{2(opt)}^* = \frac{V_{02}^* M_3^*(a,b) - V_{03}^* M_2^*(a,b)}{V_{02}^* V_{04}^* - V_{03}^{*2}} \tag{3.43}$$

Substituting (3.43) into (3.42) the minimum MSE of estimator t up to the first order approximation given by

$$MSE_{min}(t^*) = t_{(a,b)}^2 \left[M_1^*(a,b) - \frac{\{V_{04}^*\{M_2^*(a,b)\}^2 - 2V_{03}^* M_2^*(a,b) M_3^*(a,b) + \{M_2^*(a,b)\}^2\}}{V_{02}^* V_{04}^* - V_{03}^{*2}}\right] \tag{3.44}$$

On putting $v = 0$, when $\bar{X}$ is known, the difference -cum exponential ratio type estimator in (3.33) reduces to the following estimator of parameter $\hat{t}_{(a,b)}^*$ under nonresponse.

$$t_1^* = \left[\hat{t}_{(a,b)}^* + w(\bar{X} - \bar{x}^*)\right] exp\left[\frac{u(\bar{X} - \bar{x}^*)}{\bar{X} + (\alpha - 1)\bar{x}^*}\right] \tag{3.45}$$

where u is a scalar having real values $(-1, 0, 1)$, k and α are arbitrary constants. The bias and MSE of the estimator t_1^* in (3.45) up to first order approximation are given as follows

$$Bias(t_1^*) = \frac{1}{2} t_{(a,b)}[M_4^*(a,b) + B_1^* V_{02}^* - 2A_1^* M_2^*(a,b)] \tag{3.46}$$

$$MSE(t_1^*) = t_{(a,b)}^2[M_1^*(a,b) + A_1^{*2}V_{02}^* - 2A_1^* M_2^*(a,b)] \tag{3.47}$$

Estimating the optimum value of $A_1^*{}_{1(opt)} = M_2^*(a,b)/V_{02}^*$, we have the minimum value of $MSE(t_1^*)$ as

$$MSE_{min}(t_1^*) = t_{(a,b)}^2 \left[M_1^*(a,b) - \frac{\{M_2^*(a,b)\}^2}{V_{02}^*}\right] \tag{3.48}$$

Table 1.1 shows the existing estimators obtained from (3.45) using appropriate values for a, b, w, u and α accordingly.

Considered k and u to be zero for difference-cum exponential ratio-type estimator in (3.33) when the S_x^2 is known, we have ratio-type exponential estimator of the parameter $t^*_{(a,b)}$ as

$$t^*_{(1)} = \hat{t}^*_{(a,b)} exp\left[\frac{v(S_x^2 - s_x^{*2})}{S_x^2 + (\beta-1)s_x^{*2}}\right] \quad (3.49)$$

where v is a scalar having real values $(-1, 0, 1)$, and β is a suitably chosen constant. The bias and MSE of the estimator $t^*_{(1)}$ are given as follows.

$$Bias(t^*_{(1)}) = \frac{1}{2} t_{(a,b)}[M_4^*(a,b) + B_2^* V_{04}^* - 2A_2^* M_3^*(a,b)] \quad (3.50)$$

$$MSE(t^*_{(1)}) = t^2_{(a,b)}[M_1^*(a,b) + A_2^{*2} V_{04}^* - 2A_2^* M_2^*(a,b)] \quad (3.51)$$

Estimating the optimum value, $A^*_{2(opt)} = M_3^*(a,b)/V_{04}^*$, we have the minimum value of $MSE(t^*_{(1)})$ as

$$MSE_{min}(t^*_{(1)}) = t^2_{(a,b)}\left[M_1^*(a,b) - \frac{\{M_3^*(a,b)\}^2}{V_{04}^*}\right] \quad (3.52)$$

Table 1.2 shows special cases of the estimator $t_{(1)}$ obtained from (3.18) considering appropriate values for a, b, w, v and β accordingly.

3.2.2. **Relative Efficiency of t^***

$$MSE(\hat{t}^*_{(a,b)}) - MSE(t^*) = t^2_{(a,b)}\left[\frac{V_{04}^*\{M_2^*(a,b)\}^2 - 2V_{03}^* M_2^*(a,b) M_3^*(a,b) + V_{02}^*\{M_2^*(a,b)\}^2}{(V_{02}V_{04} - V_{03}^2)}\right] \geq 0 \quad (3.53)$$

$$MSE_{min}(t_1^*) - MSE_{min}(t^*) = t^2_{(a,b)}\left[\frac{\{V_{03}^* M_2^*(a,b) - V_{02}^* M_3^*(a,b)\}^2}{V_{02}(V_{02}V_{04} - V_{03}^2)}\right] \geq 0 \quad (3.54)$$

$$MSE_{min}(t^*_{(1)}) - MSE_{min}(t^*) - t^2_{(a,b)} \frac{\{V_{03}^* M_3^*(a,b) - V_{04}^* M_2^*(a,b)\ C_x\}^2}{V_{02}(V_{02}V_{04} - V_{03}^2)} \geq 0 \quad (3.55)$$

If $\delta_{04}^* - \delta_{03}^{*2} - 1$ is always greater than zero, and the relative efficiency of the estimators $\hat{t}^*_{(a,b)}, t_1^*, t^*_{(1)}$ with respect to proposed estimator t^* satisfied the condition stated in (3.53), (3.54), and (3.55), the proposed estimator t^* is more efficiently performed than the estimators $\hat{t}^*_{(a,b)}, t_1^*, t^*_{(1)}$.

3.2.3. **Estimation of Population Mean of study Variable *Y***

when $(a, b, w, u, v, \alpha, \beta) = (1, 0, w, u, v, \alpha, \beta)$, the estimator t^* in (3.33) becomes a class of difference-cum exponential ratio-type estimator t_μ for population mean $\bar{Y}$ given by

$$t_\mu^* = [\bar{y}^* + w(\bar{X} - \bar{x}^*)] exp\left[\frac{u(\bar{X} - \bar{x}^*)}{\bar{X} - (\alpha-1)\bar{x}^*}\right] exp\left[\frac{v(S_x^2 - s_x^{*2})}{S_x^2 - (\beta-1)s_x^{*2}}\right] \quad (3.56)$$

Substituting $(1, 0, w, u, v, \alpha, \beta)$ into (3.38) and (3.42), we obtain the bias and MSE of the estimator t_{μ}^{*}, respectively, as

$$Bias(t_{\mu}^{*}) = \frac{1}{2}\lambda_1 \bar{Y}[B_1^* V_{02}^* + B_2^* V_{04}^* + 2A_1^* A_2^* V_{03}^* - 2A_1^* V_{11}^* - 2A_2^* V_{12}^* + 2wR(A_1^* V_{02}^* + A_2^* V_{03}^*)] \tag{3.57}$$

$$MSE(t_{\mu}^{*}) = \bar{Y}^2[V_{21}^* + A_1^{*2} V_{02}^* + A_2^{*2} V_{04}^* - 2A_1^* V_{11}^* - 2A_2^* V_{12}^* + 2A_1^* A_2^* V_{03}^* + 2wR\{A_1^* V_{02}^* + A_2^* V_{03}^*\} - 2wRV_{11}^* + w^2 R^2 V_{02}^*] \tag{3.58}$$

where, $R = \frac{\bar{X}}{\bar{Y}}$, the MSE of t_{μ}^{*} in (3.58) is minimized for values

$$A_{1(opt)}^* = \frac{\{V_{04}^* V_{11}^* - V_{03}^* V_{12}^*}{V_{02}^* V_{04}^* - V_{03}^{*2}} - wR \qquad A_{2(opt)}^* = \frac{V_{02}^* V_{12}^* - V_{03}^* V_{11}^*}{V_{02}^* V_{04}^* - V_{03}^{*2}} \tag{3.59}$$

Substituting the optimal values in (30), the minimum MSE of estimator t_{μ}^{*} is given by

$$MSE_{min}(t_{\mu}^{*}) = \bar{Y}^2 \left[V_{20}^* - \left\{\frac{V_{04}^* V_{11}^{*2} - 2V_{03}^* V_{11}^* V_{12}^* + V_{02}^* V_{12}^{*2}}{V_{02}^* V_{04}^* - V_{03}^{*2}}\right\}\right] \tag{3.60}$$

The special cases of the estimator t_{μ}^{*} are presented with their corresponding value of minimum MSEs in Table 1.4

3.2.4 **Relative Efficiency**

$$V(t_{1.*}^*) - MSE_{min}(t_{\mu}^{*}) = \bar{Y}^2 \left\{\frac{V_{04}^* V_{11}^{*2} - 2V_{03}^* V_{11}^* V_{12}^* + V_{02}^* V_{12}^{*2}}{V_{02}^* V_{04}^* - V_{03}^{*2}}\right\} \geq 0 \tag{3.61}$$

$$MSE_{min}(t_{1.2}^*) - MSE_{min}(t_{\mu}^{*}) = \bar{Y}^2 \left\{\frac{(V_{03}^* V_{11}^* - V_{02}^* V_{12}^*)^2}{V_{02}^*(V_{02}^* V_{04}^* - V_{03}^{*2})}\right\} \geq 0 \tag{3.62}$$

$$MSE_{min}(t_{1.R}^*) - MSE_{min}(t_{\mu}^{*}) = \bar{Y}^2 \left\{\frac{V_{04} V_{11}^2 - 2V_{03} V_{11} V_{12} + V_{02} V_{12}^2}{(V_{02}^* V_{04}^* - V_{03}^{*2})} - \frac{1}{4} V_{02}^* + V_{11}^*\right\} \geq 0 \tag{3.63}$$

$$MSE_{min}(t_{1.d}^*) - MSE_{min}(t_{\mu}^{*}) = \bar{Y}^2 \left\{\frac{(V_{03}^* V_{11}^* - V_{02}^* V_{12}^*)^2}{V_{02}^*(V_{02}^* V_{04}^* - V_{03}^{*2})}\right\} \geq 0 \tag{3.64}$$

If $V_{02}^* V_{04}^* - V_{03}^{*2}$ is always greater than zero, and the relative efficiency of the estimators $\hat{t}_{(a,b)}^*, t_1^*, t_{(1)}^*$ with respect to proposed estimator t^* satisfied the condition stated in (3.53), (3.54), and (3.55), the proposed estimator t^* is more efficiently performed than the estimators $\hat{t}_{(a,b)}^*, t_1^*, t_{(1)}^*$.

Table 1.1: Special Cases of the proposed estimator t_1 and t_1^*

S/N	a,b,w,u,v,α,β	Estimators t_1	Estimators t_1^*
1	$a,b,0,1,0,\alpha,\beta$	$t_{1.1} = \hat{t}^*_{(a,b)} exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right]$	$t_{1.1} = \hat{t}^*_{(a,b)} exp\left[\frac{(\bar{X}-\bar{x}^*)}{\bar{X}+(\alpha-1)\bar{x}^*}\right]$
2	$1,0,0,1,0,\alpha,\beta$	$t_{1.2} = \bar{y}^* exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right]$	$t_{1.2} = \bar{y}^* exp\left[\frac{(\bar{X}-\bar{x}^*)}{\bar{X}+(\alpha-1)\bar{x}^*}\right]$
3	$1,0,0,1,0,2,\beta$	$t_{1.R} = \bar{y}^* exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+\bar{x}}\right]$ Singh et al. (2009)	$t_{1.R} = \bar{y}^* exp\left[\frac{(\bar{X}-\bar{x}^*)}{\bar{X}+\bar{x}^*}\right]$ Singh et al. (2009)
4	$1,0,w,0,0,\alpha,\beta$	$t_{1.d} = \bar{y}^* + k(\bar{X}-\bar{x})$ Difference Estimator	$t_{1.d} = \bar{y}^* + k(\bar{X}-\bar{x}^*)$ Difference Estimator

Table 1.2: Special Cases of the $t_{(1)}$ and $t^*_{(1)}$

S/N	a,b,w,u,v,α,β	Estimators $t_{(1)}$	Estimators $t^*_{(1)}$
1	$0,2,0,0,1,\alpha,\beta$	$t_{(1)} = \hat{t}^*_{(a,b)} exp\left[\frac{S_x^2-s_x^2}{S_x^2+(\beta-1)s_x^2}\right]$	$t^*_{(1)} = \hat{t}^*_{(a,b)} exp\left[\frac{S_x^2-s_x^{*2}}{S_x^2+(\beta-1)s_x^{*2}}\right]$
2	$0,2,0,0,1,\alpha,\beta$	$t_{(1).1} = s_y^{*2} exp\left[\frac{S_x^2-s_x^2}{S_x^2+(\beta-1)s_x^2}\right]$	$t^*_{(1).1} = s_y^{*2} exp\left[\frac{S_x^2-s_x^{*2}}{S_x^2+(\beta-1)s_x^{*2}}\right]$
3	$0,2,0,1,0,\alpha,2$	$t_{(1).R} = s_y^{*2} exp\left[\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right]$	$t^*_{(1).R} = s_y^{*2} exp\left[\frac{S_x^2-s_x^{*2}}{S_x^2+s_x^2}\right]$

Table 1.3: Special Cases of the estimator t_μ with corresponding MSEs

S/N	a,b,w,u,v,α,β	Estimators t_μ	MSE
1	$1,0,0,0,0,\alpha,\beta$	$t_{1.v} = \bar{y}^*$	$\bar{Y}^2\left[\lambda_1 C_y^2 + \lambda_2 C_{y_2}^2\right] = \bar{Y}^2 V_{20}^*$
2	$1,0,0,1,0,\alpha,\beta$	$t_{1.2} = \bar{y}^* exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right]$	$\bar{Y}^2\left[\lambda_1 C_y^2\{1-\rho_{yx}^2\} + \lambda_2 C_{y_2}^2\right] = \bar{Y}^2\left[V_{20}^* - \frac{V_{11}^2}{V_{02}}\right]$
3	$1,0,0,1,0,2,\beta$	$t_{1.R} = \bar{y}^* exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+\bar{x}}\right]$ Singh et al. (2009)	$\bar{Y}^2\left[\lambda_1\left\{C_y^2 - \frac{1}{4}C_x^2 + \rho_{yx}C_yC_x\right\} + \lambda_2 C_{y_2}^2\right]$ $= \bar{Y}^2\left[V_{20}^* - \frac{1}{4}V_{02} + V_{11}\right]$
4	$1,0,w,0,0,\alpha,\beta$	$t_{1.d} = \bar{y}^* + k(\bar{X}-\bar{x})$ Difference Estimator	$\bar{Y}^2\left[\lambda_1 C_y^2\{1-\rho_{yx}^2\} + \lambda_2 C_{y_2}^2\right] = \bar{Y}^2\left[V_{20}^* - \frac{V_{11}^2}{V_{02}}\right]$

Table 1.4: Special Cases of the estimator t_{μ}^{*} with corresponding MSEs

S/N	$a, b, w, u, v, \alpha, \beta$	Estimators t_{μ}^{*}	MSE
1	$1, 0, 0, 0, 0, \alpha, \beta$	$t_{1.v}^{*} = \bar{y}^{*}$	$\bar{Y}^{2}\left[\lambda_1 C_y^2 + \lambda_2 C_{y_2}^2\right] = \bar{Y}^{2} V_{20}^{*}$
2	$1, 0, 0, 1, 0, \alpha, \beta$	$t_{1.2}^{*} = \bar{y}^{*} exp\left[\frac{(\bar{X}-\bar{x}^{*})}{\bar{X}+(\alpha-1)\bar{x}}\right]$	$\bar{Y}^{2}\left[\lambda_1 C_y^2\{1-\rho_{yx}^2\} + \lambda_2 C_{y_2}^2\{1-\rho_{yx(2)}^2\}\right] = \bar{Y}^{2}\left[V_{20}^{*} - \frac{V_{11}^{*2}}{V_{02}^{*}}\right]$
3	$1, 0, 0, 1, 0, 2, \beta$	$t_{1.R}^{*} = \bar{y}^{*} exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+\bar{x}}\right]$ Singh et al. (2009)	$\bar{Y}^{2}\left[\lambda_1\left\{C_y^2 - \frac{1}{4}C_x^2 + \rho_{yx}C_yC_x\right\} + \lambda_2\left\{C_{y_2}^2 - \frac{1}{4}C_x^2 + \rho_{yx}C_yC_x\right\}\right] = \bar{Y}^{2}\left[V_{20}^{*} - \frac{1}{4}V_{02}^{*} + V_{11}^{*}\right]$
4	$1, 0, w, 0, 0, \alpha, \beta$	$t_{1.d}^{*} = \bar{y}^{*} + k(\bar{X} - \bar{x})$ Difference Estimator	$\bar{Y}^{2}\left[\lambda_1 C_y^2\{1-\rho_{yx}^2\} + \lambda_2 C_{y_2}^2\{1-\rho_{yx(2)}^2\}\right] = \bar{Y}^{2}\left[V_{20}^{*} - \frac{V_{11}^{2}}{V_{02}}\right]$

4.0 Empirical Study

To ascertain the efficiency of the proposed class of estimators t_{μ} and t_{μ}^{*} for the population mean $\bar{Y}$, against the traditional estimators using approximation of first order for the two situations, four real population data sets are considered.

Population-1: Cochran (1977, pg. 34): the data comprising of the income and food cost of 33 families. Let y be the Food cost of family's employment and x be the weekly income of families. The 20% units of the data set is treated as group of non-responding units, that is taken last 6 units of the population. The statistics description for the data set is: $N = 33$, $n = 5$, $\bar{Y} = 27.49$, $\bar{X} = 72.55$, $S_y = 10.13$, $S_x = 10.58$, $C_y = 0.37$, $C_x = 0.15$, $\rho_{yx} = 0.25$, $\delta_{40} = 5.55$, $\delta_{04} = 2.08$, $\delta_{30} = 0.89$, $\delta_{03} = 0.51$, $\delta_{12} = 0.39$, $\delta_{21} = 0.54$, $\delta_{22} = 2.22$, $C_{y(2)} = 0.41$, $C_{x(2)} = 0.16$, $\rho_{yx(2)} = 0.95$, $\delta_{40(2)} = 2.37$, $\delta_{04(2)} = 1.75$, $\delta_{30(2)} = 0.78$, $\delta_{03(2)} = 0.24$, $\delta_{12(2)} = 0.39$, $\delta_{21(2)} = 0.57$, $\delta_{22(2)} = 1.97$, $N_2 = 6$

Population-2: Sarndal et al. (1992, pg. 653): the population consisting of the 284 municipalities with several variables. Here considered, 1985 population (P85) as y and revenue from 1985 (RMT85) municipal taxation in million of kronor as x. The last 47 units of the population which is the 20% units is treated as group of nonresponding population. The statistic description for the population is:

$N = 284$, $n = 35$, $\bar{Y} = 29.3627$, $\bar{X} = 245.088$, $S_y = 51.5567$, $S_x = 596.3325$; $C_y = 1.7559$, $C_x = 2.4331$, $\rho_{yx} = 0.9607$, $\delta_{12} = 8.0238$, $\delta_{21} = 7.879$, $\delta_{22} = 78.7848$ $\delta_{30} = 8.2178$, $\delta_{03} =$

8.7709; δ_{40} = 88.9175, δ_{04} = 88.8759, $C_{y(2)}$ = 1.0088; $C_{x(2)}$ = 1.1024; $\rho_{yx(2)}$ = 0.9954, $\delta_{12(2)}$ = 1.8298, $\delta_{21(2)}$ = 1.7857, $\delta_{22(2)}$ = 5.4822, $\delta_{30(2)}$ = 1.7525, $\delta_{03(2)}$ = 1.885, $\delta_{40(2)}$ = 5.2714, $\delta_{04(2)}$ = 5.8045 ,

Population 3: Murthy (1967): the data comprising the number of workers and output of 80 factories. y represents the output and x represent the number of workers from the factories. in this population, the last 20% units has been considered as nonresponse group. The statistical description of the population is: N = 80, n = 30, $\bar{Y}$ = 5182.637, $\bar{X}$ = 285.125, S_y = 1835.659, S_x = 270.4294, C_y = 0.35419, C_x = 0.94846,
ρ_{yx} = 0.91498, δ_{40} = 2.2383, δ_{04} = 3.5360, δ_{30} = 0.129, δ_{03} = 1.268, δ_{12} = 0.902, δ_{21} = 0.5427, δ_{22} = 2.2943, $C_{y(2)}$ = 0.08378, $C_{x(2)}$ = 0.2328, $\rho_{yx(2)}$ = 0.9805, $\delta_{40(2)}$ = 2.3462, $\delta_{04(2)}$ = 1.9077, $\delta_{30(2)}$ = 0.73298, $\delta_{03(2)}$ = 0.4072, $\delta_{12(2)}$ = 0.5286, $\delta_{21(2)}$ = 0.6344, $\delta_{22(2)}$ = 2.0463, N = 16

Table 2.1: PRE of different mean estimator respect to $\bar{y}^*$ at different value of k

			SITUATION I			SITUATION II		
Data Sets	$1/k$	$\bar{y}^*$	t_μ	$t_{\mu 1}$ or $t_{\mu d}$	$t_{\mu R}$	t^*_μ	$t^*_{\mu 1}$ or $t^*_{\mu d}$	$t^*_{\mu R}$
1	1/5	100	120.6764	107.8836	93.41781	116.2156	101.9049	98.60256
	1/4	100	122.1865	108.3944	93.05171	117.4088	100.8229	100.4888
	1/3	100	123.0274	108.6754	92.85295	118.4861	100.4257	101.5499
	1/2	100	123.5633	108.8533	92.72815	119.3308	100.2431	102.2301
2	1/5	100	3909.334	2441.989	53.06471	1656.977	1337.373	53.89213
	1/4	100	7870.546	3536.181	52.73768	1675.659	1352.147	53.84767
	1/3	100	16286.45	4583.553	52.57244	1685.358	1359.866	53.82502
	1/2	100	46324.7	5587.048	52.47275	1691.297	1364.609	53.8113
3	1/5	100	1191.738	638.8996	60.15364	1254.749	610.7327	60.37042
	1/4	100	1225.338	647.6661	60.09349	1327.584	609.572	60.38303
	1/3	100	1242.926	652.1572	60.06335	1370.162	608.9911	60.38937
	1/2	100	1253.745	654.8874	60.04525	1398.086	608.6424	60.39318

To study the percentage relative efficiency (PRE) of the proposed class of estimators t_μ and t^*_μ against the existing estimators, the PREs of the estimators $t_\mu, t_{\mu 1}$ or $t_{\mu d}, t_{\mu R}$ and $t^*_\mu, t^*_{\mu 1}$ or $t^*_{\mu d}, t^*_{\mu R}$ at different values of k for situation I and II respectively have been estimated and presented in Table 2.1. the PREs of all the considered estimators are computed with respect to unit estimator $\bar{y}^*$. For both situations, it is observed that the performance of the proposed estimator t_μ and t^*_μ are more efficient in comparison to the unit estimator $\bar{y}^*$.and the remaining existing estimators $t_{\mu 1}$ or $t_{\mu d}, t_{\mu R}$ and $t^*_{\mu 1}$ or $t^*_{\mu d}, t^*_{\mu R}$ due to the PRE of the proposed estimator that is higher than the existing estimators.

6. **Simulation Study**

Here, the performance of the estimators which are special cases of the proposed class of estimators at (3.25) and (3.56) respectively for the two situations are considered. Our interest is to draw a sample integrated with the study and auxiliary variables (y, x) from pseudo-population. Following Reddy et al. (2010) algorithm, the study and auxiliary variable are generated as $Y = \rho_{yx}X + \sqrt{(1-\rho_{yx}^2)}Y_1$ where $X \sim N(5,3)$ and $Y_1 \sim N(5,2)$ are independent variates of normal distribution, such that ρ_{yx} represent different selection of correlation coefficients 0.4, 0.6 and 0.8. thus, the process is repeated 500 times. Employing simple random sampling without replacement (SRSWOR) method, 500 samples (x_i and y_i), for $i = 1,2,...,n$ from population size $N = 5000$ units of size n=40, 60, and 80. Suppose that in the first attempt 125 units out of 500 did not respond. Hence, by putting more effort, some of the outstanding units are obtained by interview based on Hansen and Hurwitz (1946) approach. It is obvious that the study variable y is generally follow normal distribution. Since the occurrence of nonresponse in the sample is an intrinsic part of any kind of survey. We computed the percentage relative efficiency of the proposed estimators t_μ and t_μ^* with respect to $\bar{y}^*$ at different values of $k = (5,4,3,\ 2)$, correlation coefficient $\rho_{yx} = (0.4, 0.6, 0.8)$ and sample size $n = (40, 60, 80)$ for both situation I and II and the simulation results are presented in Table 3.1 to 3.3. From Table 3.1, 3.2 and 3.3, it can be observed that the proposed estimators t_μ and t_μ^* are outperformed the unit estimator $\bar{y}^*$ and other remaining estimators $t_{\mu 1}$ or $t_{\mu d}, t_{\mu R}$ and $t_{\mu 1}^*$$or$ $t_{\mu d}^*, t_{\mu R}^*$ under situation I and II, irrespective of the variation in the rate of inverse sampling k at second phase, correlation coefficients and sample size in the simulated data.

Table 3.1: PRE of different mean estimator respect to $\bar{y}^*$at different value of k for $n = 40$

			SITUATION I			SITUATION II		
ρ_{yx}	$1/k$	$\bar{y}^*$	t_μ	$t_{\mu 1}$ or $t_{\mu d}$	$t_{\mu R}$	t_μ^*	$t_{\mu 1}^*$$or$ $t_{\mu d}^*$	$t_{\mu R}^*$
0.4	1/5	100	167.9683	167.7113	76.5943	158.7956	158.2311	77.5187
	1/4	100	166.8795	166.6282	76.7668	158.4261	157.8980	77.6283
	1/3	100	165.1404	164.8982	77.0487	157.8331	157.3609	77.8069
	1/2	100	161.9200	161.6943	77.5924	156.7262	156.3491	78.1500
0.6	1/5	100	318.6934	318.1325	59.53492	301.8336	299.722	60.04177
	1/4	100	312.7096	312.1743	59.74646	297.3911	295.509	60.22055
	1/3	100	303.4326	302.9357	60.09415	290.4836	288.93	60.51396
	1/2	100	287.1113	286.6789	60.77183	278.2702	277.2147	61.08425
0.8	1/5	100	1146.049	1142.895	52.93966	1330.099	1293.896	53.25314
	1/4	100	1069.163	1066.436	53.11148	1209.382	1182.4	53.40513
	1/3	100	963.3947	961.2055	53.39509	1054.488	1037.365	53.65576
	1/2	100	808.7075	807.1985	53.95214	848.4845	840.995	54.14707

Table 3.2: PRE of different mean estimator respect to $\bar{y}^*$at different value of k for $n = 60$

			SITUATION I			SITUATION II		
ρ_{yx}	$1/k$	$\bar{y}^*$	t_μ	$t_{\mu1}$ or $t_{\mu d}$	$t_{\mu R}$	t_μ^*	$t_{\mu1}^*$ or $t_{\mu d}^*$	$t_{\mu R}^*$
0.4	1/5	100	168.7827	168.5213	76.46727	159.0711	158.4785	77.43793
	1/4	100	167.6181	167.3629	76.64948	158.6769	158.1242	77.55378
	1/3	100	165.7622	165.5168	76.94698	158.0455	157.5537	77.74248
	1/2	100	162.3403	162.1124	77.51979	156.8713	156.4825	78.1043
0.6	1/5	100	323.2611	322.6803	59.37967	305.218	302.922	59.91042
	1/4	100	316.7533	316.2007	59.60247	300.3943	298.3587	60.09888
	1/3	100	306.7109	306.2006	59.96841	292.9274	291.2615	60.40792
	1/2	100	289.181	288.7406	60.68077	279.8234	278.7103	61.00775
0.8	1/5	100	1210.097	1206.564	52.8139	1210.097	1206.564	52.8139
	1/4	100	1120.294	1117.287	52.99446	1120.294	1117.287	52.99446
	1/3	100	999.0743	996.7105	53.29236	999.0743	996.7105	53.29236
	1/2	100	826.4556	824.8749	53.87697	826.4556	824.8749	53.87697

Table 3.3: PRE of different mean estimator respect to $\bar{y}^*$at different value of k for $n = 80$

			SITUATION I			SITUATION II		
ρ_{yx}	$1/k$	$\bar{y}^*$	t_μ	$t_{\mu1}$ or $t_{\mu d}$	$t_{\mu R}$	t_μ^*	$t_{\mu1}^*$ or $t_{\mu d}^*$	$t_{\mu R}^*$
0.4	1/5	100	169.6973	169.431	76.32653	159.3798	158.7548	77.3483
	1/4	100	168.4459	168.1863	76.51962	158.9573	158.3763	77.47123
	1/3	100	166.4571	166.208	76.83454	158.2824	157.7682	77.67123
	1/2	100	162.8072	162.5769	77.4397	157.0323	156.6301	78.05384
0.6	1/5	100	328.488	327.8839	59.20826	309.0837	306.5667	59.76528
	1/4	100	321.3622	320.7897	59.44358	303.8117	301.5933	59.9645
	1/3	100	310.4249	309.8991	59.82979	295.6922	293.8941	60.29092
	1/2	100	291.501	291.0517	60.58056	281.5629	280.3833	60.92351
0.8	1/5	100	1289.892	1285.856	52.67539	1576.822	1517.67	53.01913
	1/4	100	1182.866	1179.497	52.86563	1390.538	1349.22	53.18762
	1/3	100	1041.696	1039.115	53.17932	1167.965	1143.837	53.46511
	1/2	100	846.9428	845.2773	53.79435	897.3151	887.9033	54.00802

7. Conclusion

On advantage of auxiliary information, an improved difference-cum exponential ratio-type in estimator SRSWOR for general parameter estimation have been proposed. In section 3 of the study, we have adopted estimators t and t^* in which varieties of population parameters can be estimated. Tables 3.1 and 3.2 present the special class of estimators for estimating the mean and variance of the study variable in the presence of nonresponse under to situations, respectively, using difference choices of arbitrary constants. Moreover, we have shown the performance of the proposed estimators to estimate the mean of the study variable under two situations of nonresponse. The effectiveness of the estimators is tested based on real and simulated data sets for population mean estimators t_{μ} and t_{μ}^{*} the results of the numerical illustration are presented in Table 4.1 which shows that the proposed estimators outperformed the other existing estimators under the situations of nonresponse. We can find from the result of simulation study conducted that the PRE of the proposed estimators is greater than all other estimators for different values of sampling inverse rate, correlation coefficients and sample size. Hence, the proposed estimators are more efficient than other class of estimators. The proposed general class of estimator can be used to estimate different population parameters of study variable in the presence of nonresponse.

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