Original Research Article

On The Comparison of Adam-Bashforth and Adam Moulton Methods for Non-Stiff Differential Equations

Abstract

This paper presents the comparison of the two Adams methods using extrapolation for the best method suitable for the approximation of the solutions. The two methods (Adams Moulton and Adams Bashforth) of step k = 3 to k = 4 are considered and their equations derived. The extrapolation points, order, error constant, stability regions were also derived for the steps. More importantly, the consistency and zero stability are also investigated and finally, the derived equations is used to solve some non-stiff differential equations for best in efficiency and accuracy.

Keyword: Adam-Bashforth; Adam Moulton; Accuracy; Stability region; Consistency; Zero stability

1. Introduction

Most scientific and engineering problems such as the study of vibration, chemical reactions and elasticity are modeled mathematically using ordinary differential equations. It may be interesting to know that most of these problems do not have exact solutions hence, numerical methods is employed.

There are many methods for finding direct approximate solution to differential solution. These methods are referred to by a variety of different names including numerical methods, numerical integration or approximate solutions. The numerical method for solving ordinary differential equation are method of integrating a system of first order differential equation since higher ordinary differential equation can be reduced to a set of first order differential equation.

Linear multi-step method is one of the ways of getting approximate solution to a certain differential equation when the exact or analytical can be derived or not. Linear multi-step method also increases efficiency Haire and Wannal [8] by using the information from several previous solution approximations. Examples of linear multi-step method are Adams-Bashforth method, Adams-Moulton method, Nytrom method and Milne-Simpson method. This can be used as Predictor Corrector method.

In this work, we compare the Adams Moulton method of step k=3 and 4 with the Adams Bashforth method of the same step for best in efficiency and implementation. We use Maple software to solve the problem and the results are presented in the tables.

2. Problem definition and Methodology

The continuous and discrete form of the Adams Moulton and Adams Bashforth extrapolation points, are derived as follows:

$$y(x) = \sum_{j=0}^{t-1} \alpha_j(x) y(x_j) + h \sum_{j=0}^{m-1} \beta_j(x) f(x_j, \bar{y}(x_j)) \quad (1)$$

Where $\alpha_j(x)$ and $\beta_j(x)$ are the continuous coefficient define as follows:

$$\alpha_j(x) = \sum_{i=0}^{t+m-1} \alpha_{j,i+1} x^i \quad (2)$$

and

$$h\beta_j(x) = \sum_{i=0}^{t+m-1} \alpha_{j,i+1} x^i \quad (3)$$

Also, we shall use DC = I

where I is the identity matrix of dimension (t+m) x (t+m) as used by Sirisena [3] to determine the coefficients $\alpha_j(x)$ and $\beta_j(x)$.

D and C are derived as:

$$D = \begin{vmatrix} 1 & X_n & X_n^2 & \cdots & X_n^t & \cdots & X_n^{t+m-1} \\ 1 & X_{n+1} & X_{n+1}^2 & \cdots & X_{n+1}^t & \cdots & X_{n+1}^{t+m-1} \\ \vdots & \vdots & \vdots & & & & \\ 1 & X_{n+t-1} & X_{n+t-1}^2 & \cdots & X_{n+t-1}^t & \cdots & X_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\bar{X}_0 & \cdots & t\bar{X}_0^{t-1} & \cdots & (t+m-1)t\bar{X}_0^{t+m-2} \\ \vdots & \vdots & \vdots & & & & \vdots \\ 0 & 1 & 2\bar{X}_{m-1} & \cdots & t\bar{X}_{m-1}^{t-1} & \cdots & (t+m-1)t\bar{X}_{m-1}^{t+m-2} \end{vmatrix}$$

Where m is the number of collocation points and t is the number of interpolation points and k is the step number. C is also of dimension (t + m) x (t + m) where c is given as:

$$C = \begin{vmatrix} & \alpha_{1,1} & \alpha_{t-1,1} & \cdots & h\beta_{0,2} & \cdots & h\beta_{m-1,2} \\ \alpha_{0,1} & \alpha_{1,1} & \alpha_{t-1,1} & \cdots & h\beta_{0,2} & \cdots & h\beta_{m-1,2} \\ \vdots & \vdots & \vdots & & \alpha_{0,1} \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ \alpha_{0,t+m} & \alpha_{1,t+m} & \alpha_{t-1,1,t+m} & \cdots & h\beta_{0,1,t+m} & \cdots & h\beta_{m-1,1,t+m} \end{vmatrix}$$

If C = (c_{ij}), i= j = 1, D = (d_{ij}), I = j = 1,. . . n, and I = (e_{ij}), I = j =1, …, n

3. Derivation of Adams Moulton and Adams Bashforth Methods

3.1 Derivation of Adams Moulton Explicit Methods for Case k=3

We consider first, the derivation of step k = 3, which has the general form

$$y(x)=\alpha_1 y_{n+3}+h(\beta_0(x)f_n\beta_1(x)f_{n+1}+\beta_2(x)f_{n+2}+\beta_3(x)f_{n+3}) \qquad (4)$$

using D above, we get

$$D:=\begin{bmatrix} 1 & x_n+2h & (x_n+2h)^2 & (x_n+2h)^3 & (x_n+2h)^4 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 \\ 0 & 1 & 2x_n+2h & 3(x_n+h)^2 & 4(x_n+h)^3 \\ 0 & 1 & 2x_n+4h & 3(x_n+2h)^2 & 4(x_n+2h)^3 \\ 0 & 1 & 2x_n+6h & 3(x_n+3h)^2 & 4(x_n+3h)^3 \end{bmatrix}$$

We use maple software to generate the matrix D above for case k=3. The inverse of the matrix D is obtain using maple software as

$$C:=\Bigg[1, -\frac{1}{24}\frac{(x_n+2h)(4h^3+10x_nh^2+x_n^3+6x_n^2h)}{h^3},$$

$$-\frac{1}{24}\frac{(x_n+2h)(-8x_nh^2-14x_n^2h-3x_n^3+16h^3)}{h^3},$$

$$-\frac{1}{24}\frac{(x_n+2h)(10x_n^2h+3x_n^3-2x_nh^2+4h^3)}{h^3}, \frac{1}{24}\frac{(x_n+2h)^2x_n^2}{h^3}\Bigg]$$

$$\Bigg[0, \frac{1}{6}\frac{6h^3+11x_nh^2+x_n^3+6x_n^2h}{h^3}, -\frac{1}{2}\frac{x_n(6h^2+5x_nh+x_n^2)}{h^3},$$

$$\frac{1}{2}\frac{x_n(4x_nh+x_n^2+3h^2)}{h^3}, -\frac{1}{6}\frac{x_n(2h^2+3x_nh+x_n^2)}{h^3}\Bigg]$$

$$\left[0, -\frac{1}{12}\frac{11h^2+3x_n^2+12x_n h}{h^3}, \frac{1}{4}\frac{6h^2+10x_n h+3x_n^2}{h^3}, -\frac{1}{4}\frac{8x_n h+3x_n^2+3h^2}{h^3}, \frac{1}{12}\frac{3x_n^2+6x_n h+2h^2}{h^3}\right]$$

$$\left[0, \frac{1}{6}\frac{x_n+2h}{h^3}, -\frac{1}{6}\frac{5h+3x_n}{h^3}, \frac{1}{6}\frac{4h+3x_n}{h^3}, -\frac{1}{6}\frac{x_n+h}{h^3}\right]$$

$$\left[0, -\frac{1}{24}\frac{1}{h^3}, \frac{1}{8}\frac{1}{h^3}, -\frac{1}{8}\frac{1}{h^3}, \frac{1}{24}\frac{1}{h^3}\right]$$

To obtain the continuous scheme of the method, we use C which is evaluated at the collocation point $x = x_{n+3}$ to obtain:

$$y_{n+2}+\frac{1}{24}h f_n-\frac{5}{24}h f_{n+1}+\frac{19}{24}h f_{n+2}+\frac{3}{8}h f_{n+3}$$

$$y_{n+3} := y_{n+2}+\frac{1}{24}h f_n-\frac{5}{24}h f_{n+1}+\frac{19}{24}h f_{n+2}+\frac{3}{8}h f_{n+3}$$

Extrapolating at $x = x_{n+4}$ yields

$$y_{n+4} := y_{n+2}-\frac{1}{3}h f_n+\frac{4}{3}h f_{n+1}-\frac{5}{3}h f_{n+2}+\frac{8}{3}h f_{n+3}$$

3.2 Derivation of Adam Moulton Method for case k=4

The derivation of the Adams Moulton method of case k=4 uses the general form

$$y(x)=\alpha_1 y_{n+4}+h(\beta_0(x)f_n\beta_1(x)f_{n+1}+\beta_2(x)f_{n+2}+\beta_3(x)f_{n+3}+\beta_4(x)f_{n+4})$$

which results into the D matrix below:

$$D := \begin{bmatrix} 1 & x_n+3h & (x_n+3h)^2 & (x_n+3h)^3 & (x_n+3h)^4 & (x_n+3h)^5 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\ 0 & 1 & 2x_n+2h & 3(x_n+h)^2 & 4(x_n+h)^3 & 5(x_n+h)^4 \\ 0 & 1 & 2x_n+4h & 3(x_n+2h)^2 & 4(x_n+2h)^3 & 5(x_n+2h)^4 \\ 0 & 1 & 2x_n+6h & 3(x_n+3h)^2 & 4(x_n+3h)^3 & 5(x_n+3h)^4 \\ 0 & 1 & 2x_n+8h & 3(x_n+4h)^2 & 4(x_n+4h)^3 & 5(x_n+4h)^4 \end{bmatrix}$$

The inverse of the above matrix for case k=4 is obtain as follows;

$$C := \left[1,\ -\frac{1}{720}\,\frac{(x_n+3\,h)\,(57\,x_n^3\,h+213\,x_n\,h^3+179\,x_n^2\,h^2+6\,x_n^4+81\,h^4)}{h^4},\ \frac{1}{360}\,\frac{(x_n+3\,h)\,(51\,x_n\,h^3+223\,x_n^2\,h^2+99\,x_n^3\,h+12\,x_n^4-153\,h^4)}{h^4},\ -\frac{1}{60}\,\frac{(x_n+3\,h)\,(32\,x_n^2\,h^2+21\,x_n^3\,h+3\,x_n^4-6\,x_n\,h^3+18\,h^4)}{h^4},\ \frac{1}{360}\,\frac{(x_n+3\,h)\,(69\,x_n^3\,h+12\,x_n^4+73\,x_n^2\,h^2+21\,x_n\,h^3-63\,h^4)}{h^4},\ -\frac{1}{720}\,\frac{(x_n+3\,h)\,(3\,x_n\,h^3+29\,x_n^2\,h^2+6\,x_n^4+27\,x_n^3\,h-9\,h^4)}{h^4} \right]$$

$$\left[0,\ \frac{1}{24}\,\frac{24\,h^4+50\,x_n\,h^3+35\,x_n^2\,h^2+10\,x_n^3\,h+x_n^4}{h^4},\ -\frac{1}{6}\,\frac{x_n\,(24\,h^3+26\,x_n\,h^2+9\,x_n^2\,h+x_n^3)}{h^4},\ \frac{1}{4}\,\frac{x_n\,(19\,x_n\,h^2+8\,x_n^2\,h+x_n^3+12\,h^3)}{h^4},\ -\frac{1}{6}\,\frac{x_n\,(7\,x_n^2\,h+x_n^3+14\,x_n\,h^2+8\,h^3)}{h^4},\ \frac{1}{24}\,\frac{x_n\,(6\,h^3+11\,x_n\,h^2+x_n^3+6\,x_n^2\,h)}{h^4} \right]$$

$$\left[0,\ -\frac{1}{24}\,\frac{25\,h^3+35\,x_n\,h^2+15\,x_n^2\,h+2\,x_n^3}{h^4},\ \frac{1}{12}\,\frac{24\,h^3+52\,x_n\,h^2+27\,x_n^2\,h+4\,x_n^3}{h^4},\ -\frac{1}{4}\,\frac{19\,x_n\,h^2+12\,x_n^2\,h+2\,x_n^3+6\,h^3}{h^4},\ \frac{1}{12}\,\frac{21\,x_n^2\,h+4\,x_n^3+28\,x_n\,h^2+8\,h^3}{h^4},\ -\frac{1}{24}\,\frac{2\,x_n^3+9\,x_n^2\,h+11\,x_n\,h^2+3\,h^3}{h^4} \right]$$

$$\left[0, \frac{1}{72}\frac{35h^2+30x_n h+6x_n^2}{h^4}, -\frac{1}{18}\frac{26h^2+27x_n h+6x_n^2}{h^4}, \frac{1}{12}\frac{19h^2+24x_n h+6x_n^2}{h^4}, -\frac{1}{18}\frac{21x_n h+6x_n^2+14h^2}{h^4}, \frac{1}{72}\frac{6x_n^2+18x_n h+11h^2}{h^4}\right]$$

$$\left[0, -\frac{1}{48}\frac{5h+2x_n}{h^4}, \frac{1}{24}\frac{9h+4x_n}{h^4}, -\frac{1}{4}\frac{x_n+2h}{h^4}, \frac{1}{24}\frac{7h+4x_n}{h^4}, -\frac{1}{48}\frac{2x_n+3h}{h^4}\right]$$

$$\left[0, \frac{1}{120}\frac{1}{h^4}, -\frac{1}{30}\frac{1}{h^4}, \frac{1}{20}\frac{1}{h^4}, -\frac{1}{30}\frac{1}{h^4}, \frac{1}{120}\frac{1}{h^4}\right]$$

To obtain the continuous scheme of the method, we use C which is evaluated at the collocation point $x = x_{n+4}$ to obtain:

$$y_{n+3} - \frac{19}{720}h f_n + \frac{53}{360}h f_{n+1} - \frac{11}{30}h f_{n+2} + \frac{323}{360}h f_{n+3} + \frac{251}{720}h f_{n+4}$$

$$y_{n+4} := y_{n+3} - \frac{19}{720}h f_n + \frac{53}{360}h f_{n+1} - \frac{11}{30}h f_{n+2} + \frac{323}{360}h f_{n+3} + \frac{251}{720}h f_{n+4}$$

3.3 Derivation of Adams Bashforth method for k=3

The general form for Adams Bashforth method for order k = 3is given by

y(x)=$\alpha_1 y_{n+3}$+h(β_0(x)$f_n\beta_1$(x)f_{n+1}r k=3 is given as+β_2(x)f_{n+2}+β_3(x)f_{n+3})

such that the D matrix is given as:

$$D=\begin{bmatrix} 1 & x_n+2h & (x_n+2h)^2 & (x_n+2h)^3 \\ 0 & 1 & 2x_n & 3x_n^2 \\ 0 & 1 & 2x_n+2h & 3(x_n+h)^2 \\ 0 & 1 & 2x_n+4h & 3(x_n+2h)^2 \end{bmatrix}$$

The inverse of the matrix D for k=3, Adams Bashforth method is obtain as

$$C := \left[1, -\frac{1}{12}\frac{(x_n+2h)(2h^2+5x_n h+2x_n^2)}{h^2}, -\frac{1}{3}\frac{(x_n+2h)^2(-x_n+h)}{h^2}, -\frac{1}{12}\frac{(x_n+2h)(2x_n^2-x_n h+2h^2)}{h^2}\right]$$

$$\left[0, \frac{1}{2}\frac{x_n^2+3x_n h+2h^2}{h^2}, -\frac{x_n(x_n+2h)}{h^2}, \frac{1}{2}\frac{x_n(x_n+h)}{h^2}\right]$$

$$\left[0, -\frac{1}{4}\frac{3h+2x_n}{h^2}, \frac{x_n+h}{h^2}, -\frac{1}{4}\frac{2x_n+h}{h^2}\right]$$

$$\left[0, \frac{1}{6}\frac{1}{h^2}, -\frac{1}{3}\frac{1}{h^2}, \frac{1}{6}\frac{1}{h^2}\right]$$

Using C we obtain the continuous scheme of the method which we evaluate at the collocation point $x = x_{n+3}$ to obtain:

$$y_{n+3} := \frac{23}{12} f_{n+2}\, h + y_{n+2} - \frac{4}{3} f_{n+1}\, h + \frac{5}{12} f_n\, h$$

Extrapolating at $x = x_{x+4}$ we get

$$y_{n+4} := \frac{7}{3} f_n\, h + y_{n+2} + \frac{19}{3} f_{n+2}\, h - \frac{20}{3} f_{n+1}\, h$$

3.4 Derivation of Adams Bashforth Method for case k=4

The Adams Bashforth method foor case k=4 have the general form

$$y(x)=\alpha_1 y_{n+4}+h(\beta_0(x) f_n \beta_1(x) f_{n+1}+\beta_2(x) f_{n+2}+\beta_3(x) f_{n+3}+\beta_4(x) f_{n+4})$$

which gives the following matrix D

$$D := \begin{bmatrix} 1 & x_n+3h & (x_n+3h)^2 & (x_n+3h)^3 & (x_n+3h)^4 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 \\ 0 & 1 & 2x_n+2h & 3(x_n+h)^2 & 4(x_n+h)^3 \\ 0 & 1 & 2x_n+4h & 3(x_n+2h)^2 & 4(x_n+2h)^3 \\ 0 & 1 & 2x_n+6h & 3(x_n+3h)^2 & 4(x_n+3h)^3 \end{bmatrix}$$

The inverse of the matrix D for Adams Bashforth of case k=4 is obtain as follows:

$$C := \left[1, -\frac{1}{24}\frac{(x_n+3h)(3h^3+7x_nh^2+x_n^3+5x_n^2h)}{h^3}, -\frac{1}{24}\frac{(x_n+3h)(-3x_nh^2-11x_n^2h-3x_n^3+9h^3)}{h^3}, -\frac{1}{24}\frac{(x_n+3h)(7x_n^2h+3x_n^3-3x_nh^2+9h^3)}{h^3}, -\frac{1}{24}\frac{(x_n+3h)(-x_n^3-x_n^2h-x_nh^2+3h^3)}{h^3}\right]$$

$$\left[0, \frac{1}{6}\frac{6h^3+11x_nh^2+x_n^3+6x_n^2h}{h^3}, -\frac{1}{2}\frac{x_n(6h^2+5x_nh+x_n^2)}{h^3}, \frac{1}{2}\frac{x_n(4x_nh+x_n^2+3h^2)}{h^3}, -\frac{1}{6}\frac{x_n(x_n^2+3x_nh+2h^2)}{h^3}\right]$$

$$\left[0, -\frac{1}{12}\frac{11h^2+3x_n^2+12x_nh}{h^3}, \frac{1}{4}\frac{6h^2+10x_nh+3x_n^2}{h^3}, -\frac{1}{4}\frac{8x_nh+3x_n^2+3h^2}{h^3}, \frac{1}{12}\frac{3x_n^2+6x_nh+2h^2}{h^3}\right]$$

$$\left[0, \frac{1}{6}\frac{x_n+2h}{h^3}, -\frac{1}{6}\frac{5h+3x_n}{h^3}, \frac{1}{6}\frac{4h+3x_n}{h^3}, -\frac{1}{6}\frac{x_n+h}{h^3}\right]$$

$$\left[0, -\frac{1}{24}\frac{1}{h^3}, \frac{1}{8}\frac{1}{h^3}, -\frac{1}{8}\frac{1}{h^3}, \frac{1}{24}\frac{1}{h^3}\right]$$

the continuous scheme of the method evaluated at the collocation point $x = x_{n+4}$ is obtain as:

$$y_{n+4} := y_{n+3} - \frac{3}{8} f_n h + \frac{37}{24} f_{n+1} h - \frac{59}{24} f_{n+2} h + \frac{55}{24} f_{n+3} h$$

3.5 Analysis of the Methods

3.5.1 Order and Error constants

To find the order and error constants of the derived equations, the following equation is used:

$$c_q := \frac{\alpha_1 + 2^q \alpha_2 + 3^q \alpha_3 + 4^q \alpha_4 + 5^q \alpha_5 + 6^q \alpha_6 + 7^q \alpha_7 + 8^q \alpha_8}{q!} - \frac{\beta_1 + 2^{(q-1)} \beta_2 + 3^{(q-1)} \beta_3 + 4^{(q-1)} \beta_4 + 5^{(q-1)} \beta_5 + 6^{(q-1)} \beta_6}{(q-1)!}$$

Then we obtain below the error constant and order of the extrapolated equations from our derivations, using the error constant equation stated above as shown in the table:

Table 1 Order and Error Constant

	Adams Bashforth New Explicit methods		Adams Moulton Explicit Methods	
	Order	Error Constant	Order	Error Constant
-	-	-	-	-
y_{n+3}	3	⅓	4	1/3
y_{n+4}	4	3	5	29/90

3.5.2 Stability Region of the New Methods

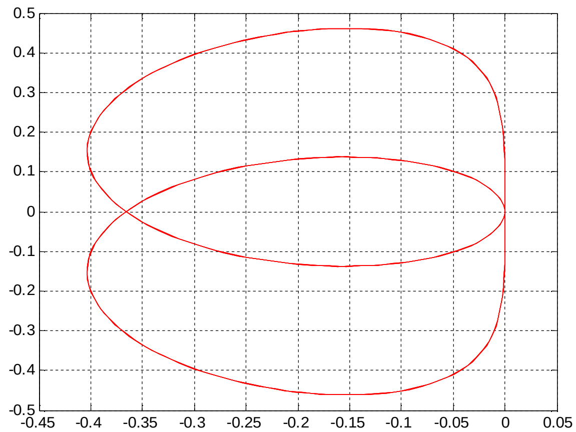

Figure 1 Stability region of Adams Bashforth explicit method K=3

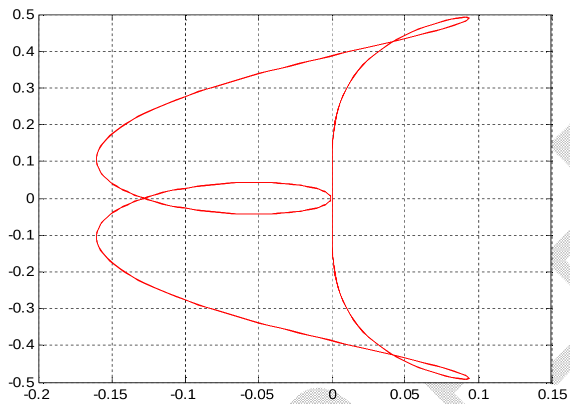

Figure 2 Stability region for Adams Bashforth New explicit method k=4

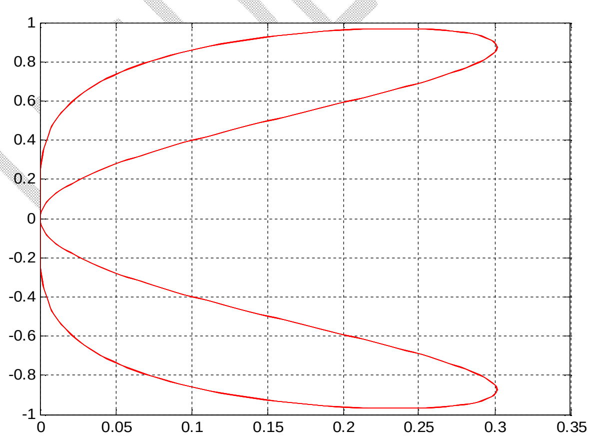

Figure 3 Stability Region of Adams Moulton explicit method for k=3

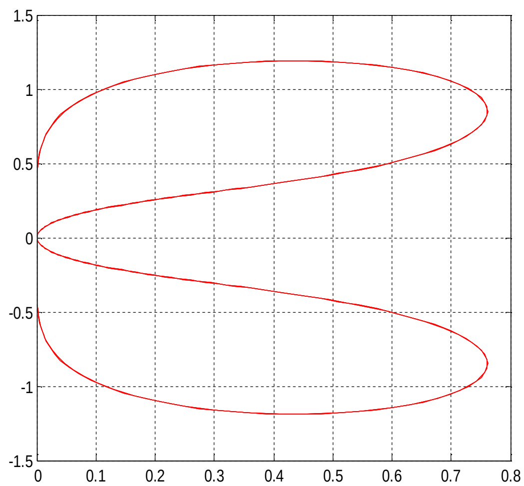

Figure 4. Stability Region for Adams Moulton explicit method for k=4

3.5.3 Zero stability and Consistency (Adams Moulton Explicit method)

i. $y_{n+3} = y_{n+1} - 2hf_n + 4hf_{n+1}$

$\rho(\xi) = \xi^3 - \xi = 0 \Rightarrow \xi(\xi^1 - 1) = 0 \Rightarrow \xi = 0,\ \xi^2 - 1 = 0,\ \xi = \pm 1 \Rightarrow |\xi| \leq 1$

ii. $y_{n+4} = y_{n+2} - \frac{7h}{3} f_n + \frac{20h}{3} f_{n+1} + \frac{19h}{3} f_{n+2}$

$\rho(\xi) = \xi^4 - \xi^2 = \xi^2(\xi^2 - 1) = 0 \Rightarrow \xi =$ twice, $\xi^2 - 1 = 0 \Rightarrow \xi = \pm 1 \Rightarrow |\xi| = 1$

$\xi = 0$ four times on $\xi = \pm 1 \Rightarrow |\xi| = 1 \Rightarrow |\xi| \leq 1$

3.5.3 Zero stability and Consistency (Adams Bashforth New Explicit Methods:

iii. $y_{n+3} = y_{n+1} - 2hf_n + 4hf_{n+1}$

$\rho(\xi) = \xi^3 - \xi = 0 \Rightarrow \xi(\xi^1 - 1) = 0 \Rightarrow \xi = 0,\ \xi^2 - 1 = 0,\ \xi = \pm 1 \Rightarrow |\xi| \leq 1$

iv. $y_{n+4} = y_{n+2} - \frac{7h}{3} f_n + \frac{20h}{3} f_{n+1} + \frac{19h}{3} f_{n+2}$

v. $\rho(\xi) = \xi^4 - \xi^2 = \xi^2(\xi^2-1) = 0 \Rightarrow \xi =$ twice, $\xi^2 - 1 = 0 \Rightarrow \xi = \pm 1 \Rightarrow |\xi| = 1$

vi. $\xi = 0$ four times on $\xi = \pm 1 \Rightarrow |\xi| = 1 \Rightarrow |\xi| \leq 1$

since the derived equations satisfies the requirement for consistency and zero stability, then by Dalquist definition the equations are convergent.

4.0 Numerical Examples

This section presents some numerical examples of the new explicit methods which are applied on non-stiff initial value problems, to examine their performances

Example 1.

Solve the following problem

$$y' = x-y, \quad y(0) = 0, \quad h = 0.1, \quad 0 \leq x \leq 1$$

$$y(x) = x + 1 + e^{-x}$$

Using Adams Moulton and Adams Bashforth explicit Method

Solution

The results are summarize in table 2 below

Table 2. Results of example 1 using the Explicit Methods

X	Exact Solution	Explicit Adams Moulton	New Explicit Adams Bashforth
0.0	0.00000000	0.00000000	0.00000000
0.1	0.00487418	0.004837418	0.004837418
0.2	0.018730753	0.018730753	0.18730753
0.3	0.040818220	0.053477777	0.04081220
0.5	0.106530659	0.103932536	0.023986143
0.6	0.148811636	0.087168084	0.215214229
0.4	0.196585303	0.192976084	-0.052780229
0.8	0.249328964	0.249330734	0.403027339

0.9	0.306569659	0.306571267	0.446717573

Example 2

Solve the following

$$y^1 = -y$$

$$y(x) = e^{-x},\ y(0) = 1,\ 0 \leq x \leq 1,\ h = 0.1$$

using Adam Moulton and Adams Bashforth Explicit method

Solution

The result are summarize in table 3 below

Results of example 2. using Adams Moulton explicit and Adams Bashforth new explicit Methods

Table 3

Results of example 2 using the Derived Explicit Methods

X	Exact Solution	Explicit Adams Moulton	New Explicit Adams Bashforth
0.0	1.000000000	1.000000000	1.000000000
0.1	0.904837418	0.904837418	0.904837418
0.2	0.818730753	0.818730753	0.818730753
0.3	0.74081822	0.74081822	0.74081822
0.4	0.670320046	0.471765048	0.670092888
0.5	0.606530659	0.53604304	0.606325112
0.6	0.548811636	0.559106238	0.548625653
0.7	0.496585303	0.438868246	0.496417017
0.8	0.449328964	0.397104407	0.449386137
0.9	0.406569659	0.359314909	0.406431879
1.0	0.367879441	0.325121591	0.36775479

Example 3

Solve the following

$$y^1 = -8(y-x) + 1,\ y(0) = 2,\ h = 0.2,\ 0 \leq x \leq 2$$

Giving the exact solution is $y(x) = x + 2e^{-8x}$

Solution:

Using the new Adams Moulton explicit and the Adam Bashforth new explicit methods. The results are presented in table 4 below

Table 4

Results of example 3 using the derived Explicit Methods

X	Exact Solution	Explicit Adams Moulton	New Explicit Adams Bashforth
0.0	2.000000000	2.000000000	2.000000000
0.2	0.603793036	9.583654937	0.603793036
0.4	0.4181524408	0.474914869	0.817523081
0.6	0.616459494	0.61437073	0.141991824
0.8	0.803323114	0.080280611	1.460029255
1.0	1.000670925	1.000538289	1.474491255
1.2	1.200135457	1.20010511	1.19920843
1.4	1.400027348	1.330638466	1.400012409
1.6	1.600005522	1.693625009	1.739118981
1.8	1.800001115	1.800023111	1.800000916
2.0	2.000000225	2.00003548	2.000003998

Error of example 1

X	Error of Adams Moulton extrapolation	Error of Adams Bashforth extrapolation
0.1	0.00	0.00
0.2	0.00	0.00
0.3	1.33×10^{-2}	0.00
0.4	4.03×10^{-2}	6.40×10^{-1}
0.5	2.60×10^{-3}	8.25×10^{-2}
0.6	6.16×10^{-2}	6.64×10^{-2}
0.7	3.61×10^{-3}	2.49×10^{-1}

0.8	1.77 x 10^{-6}	1.54 x x 10^{-1}
0.9	1.61 x 10^{-6}	1.40 x 10^{-1}
1.0	2.00 x 10^{-3}	2.10 x 10^{-1}

The result in table 4 reveals that Adams Moulton explicit method performed better than Adams Bashforth new explicit method.

Errors of example 2

X	Error Of Adams Moulton Explicit Methods	Error of Adams Bashforth New Explicit Method
0.1	0.00	0.00
0.2	0.00	0.00
0.3	1.33 x 10^{-2}	0.00
0.4	1.98 x 10^{-1}	2.27 x 10^{-4}
0.5	7.05 x 10^{-2}	2.06 x 10^{-4}
0.6	1.03 x 10^{-2}	1.86 x 10^{-4}
0.7	5.77 x 10^{-2}	1.68 x 10^{-4}
0.8	5.22 x 10^{-2}	5.32 x 10^{-4}
0.9	4.73 x 10^{-2}	1.38 x 10^{-4}
1.0	4.28 x 10^{-2}	1.25 x 10^{-4}

The result reveals that the Adams Bashforth extrapolation performed better than the new Adams Moulton Extrapolation

The absolute error using the two method is displayed on table 5 for illustration 4.3

Table 5

Errors of example 3

	Error of Adams Moulton Explicit Method	Error of Adams Bashforth New Explicit Method
0.2	2.01 x 10^{-2}	0.00
0.4	6.61 x 10^{-3}	3.36 x 10^{-1}
0.6	2.09 x 10^{-3}	4.74 x 10^{-1}
0.8	5.17 x 10^{-4}	6.57 x 10^{-1}
1.0	1.33 x 10^{-4}	4.74 x 10^{-1}
1.2	3.03 x 10^{-4}	2.15 x 10^{-4}
1.4	6.94 x 10^{-2}	1.49 x 10^{-5}
1.6	9.36 x 10^{-2}	1.39 x 10^{-1}
1.8	2.31 x 10^{-5}	1.99 x 10^{-7}

2.0	3.54 x 10^{-5}	1.75 x 10^{-8}

The result reveals that the Adams Moulton explicit method performed better than the new Adams Bashforth explicit method.

Discussion:

From the numerical examples considered to justify the methods, it is observed that both methods are suitable for solving non-stiff problems and mostly from the tables 3 and 5 that the Adams Moulton explicit method performs better than the new Adam Bashforth method.

Conclusion:

In this work, the Adams Moulton and Adams Bashforth extrapolation methods were formulated for steps k = 3; and 4: The methods yielded a class of explicit methods and both methods were apply to solve non-stiff initial value problems. The results obtained show that both methods are efficient and suitable for such problems.

References:

[1] Burden. F. (1993). Numerical Analysis, *P.W.S publishing compass Boston*.

[2] Fantula, S. O. (1988). Computer science and science computational numerical Method for initial value problems in ordinary different equation. *Academic press Inc.*

[3] Sirisena U.W. Computer science and science computational numerical Method for initial value problems in ordinary different equation Academic press Inc.

[4] Joshua, Pam Chollom (2004). A study of block. Hybrid Adams methods with tink to two step Runge Kutta method for first order ordinary differential equation.

[5] Lambert, J.D (1972) Computational method ordinary differential equations. *John Willey*

[6] Onumanyi P. A, Jator N.S. and Sirisena U. W. (1994). A reformation of the continuous general linear multi-step method by matrix inversion in the falks order initial value problem *University of Ilorin, (unpublished),*

[7] Onumanyi P. A, Jator N.S. and Sirisena U. W. (1994). Non-linear multi-step method with continuous coefficient for first order N P journal of the mathematical society of Nigeria vol 1, 67-81

[8] Hairer Ernst and Wannal Gerhard “Linear multistep method” *scholarpedia,* 5(4), p 4591, 2010.