Original Research Article

A Link between Hydrogenic and Nuclear Properties in Furtherance of Bohr's Theory

ABSTRACT

Background: Atomic physics and nuclear matter physics are often exclusively studied. However, atomic properties are direct function of the nuclear properties. Establishing a link between nuclear and atomic properties could serve the interest of nuclear and atomic engineers. Nuclear and atomic based instrumentation engineering and nuclear medicine (and perhaps atomic medicine) applications could be the benefits.

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Objectives: The research is undertaken in order to 1) link nuclear property, mass radius of the nucleon andionisation energy of hydrogen via the derivation of appropriate equation and 2) determine the mass radii of the nucleons and some leptons.

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Methods: Theoreticaland computational methods.

Results and discussion: As applicable to previousresult in the literature, the larger the mass of the elementary particles, the longer the radii. For the particles investigated, the order of the radius is muon(μ^-)<proton (p^+)< neutron (n)< tauon(τ^-) corresponding to increasing mass, $\mu^- < p^+ < n < \tau^-$.

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Conclusion: Nuclear property such as the radius of any nucleon (Γ_N) can be mathematically linked to atomic property such as ionisation energy of hydrogen via equation which shows that Γ_N is inversely proportional to ionisation energy of hydrogen and directly proportional to the rest mass of the particle.

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Keywords: Hydrogenic ions, nuclear and hydrogenic properties, nucleon and Bohr's radii, hydrogenic ionisation energy.

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1. INTRODUCTION

There is no doubt that a lot of great works had been done in the areas of atomic and nuclear physics characterised with very strong mathematical exposition giving the impression that the facts and principles of physics are not amenable to basic mathematical elucidation. For instance, the proton and neutron root mean square radii can be defined as [1]

$$R_{n(P)}^{r.m.s} = \langle r_{n(P)}^2 \rangle^{½} = \left(\frac{\int r_{n(P)}^2 \rho_{n(P)}(ŕ)\, dŕ}{\int \rho_{n(P)}(ŕ)\, dŕ} \right)^{½} \approx \left(\frac{3}{5} R_{on(P)}^2 + \frac{7\pi^2}{5} a_{n(P)}^2 \right)^{½} \quad (1)$$

Where, $R_{on(P)}$, $a_{n(P)}$, and $\rho_{n(P)}(ŕ)$ are the half-density radius, diffuseness of the neutron (proton) density distributions, and local proton (neutron) density respectively. The issue about Eq. (1) is that the procedural steps from $\langle r_{n(P)}^2 \rangle^{½} = \left(\frac{\int r_{n(P)}^2 \rho_{n(P)}(ŕ)\, dŕ}{\int \rho_{n(P)}(ŕ)\, dŕ} \right)^{½}$ to the final equation are not given; the steps could constitute a major part of the research that produces a separate paper (This could be located in the appendix section) such that student at any level can learn both the “mathematical language” and the principle of physics being discussed or analysed. Besides, Eq. (1) gives the impression that r.m.s. radius and by extension mass radii of proton and neutron are the same. The issue of mathematical complexity in almost all theoretical and experimental theses on elementary particle properties is very much contrary to Bohr’s theory which had met uncomplimentary remarks as discussed in the literature [2]. This has led to what has been known as new physics, wave/quantum mechanics, whose mathematical formalism is rather, to the layman, too complex and sometimeslackingstepwise approach in the derivational process. An aspect of this new physics, the uncertainty principle and Schrödinger theory has been harshly criticised as being too difficult and nonsensical by a scholar [3] who is, no doubt, endowed with highly advanced postdoctoral mathematics. In this research, classical model characterisedwithNewtonian principle is adopted.

Thus, this part of the introductory section can be described as an overview of the literature pieces of information about issues connected with nuclear matter, proton density distribution, and neutron halo or

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neutron skin thickness [1, 4]. Here it has become very imperative to humbly offer a scientific advice which can foster pedagogical principles that enable comprehensibility to the advantage of undergraduates, interested scholars as laymen like me who are at the borderline between core physics/chemical physics, core physical chemistry and biochemistry or any other biological science. Physical concepts or terms as above including surface nucleons which are hardly explained ought to be exhaustively defined. A layman may give the impression that the nucleus is an enclosed 3-D space with both internal (or intra-) nuclear nucleons and external or the surface nucleons.

Atomic physics and nuclear matter physics are often exclusively treated or studied. However, atomic properties are direct function of the nuclear properties. For instance the nuclear charge density is a function of the total number of protons and the nuclear volume. Thus, the modern definitions of periodic law such as [5] (a) the properties of the elements are a periodic function of their atomic numbers; and (b) the properties of the elements depend upon their total electron configuration are the very evidence-based reasons why there is a need to link nuclear matter properties with atomic matter properties.

The stability of the electronic configuration expressed in terms of ionisation energy is a function of the nuclear electronic configuration. Incidentally there seems to be emerging interest in what is referred to as density distribution of finite nuclei as if to imply that infinite nuclei may exist. Indeed to the core giants in physics, there may be infinite nuclear stated in reference to the solution of some problem involving nuclear density distribution which is seen to be minimal at a certain density of nucleus corresponding to *infinite matter*[6]. "Perhaps, it seems the universe is implied". Any investigation ondensity distribution of the so-called finite nuclei requires according to Seif and Mansour [1] accurate information about the root-mean-square (r.m.s.) radii of proton and neutron density distribution, surface diffuseness, and neutron skin thickness. Diffuseness parameter is defined as the fall-off of the nuclear potential in the tail region of the Coulomb barrier [7]. Thus, this research is undertaken in order to 1) link nuclear property, mass radius of the nucleon toionisation energy of hydrogen via the derivation of appropriate equation and 2) determine the mass radii of the nucleons and some leptons.

2.0 Review of theory and new derivations

Previous research [8] has shown that the mass radius of the nucleons and subatomic particles whose mass is greater than the mass of the nucleons can be determined. However, the equation is

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applicable to all particles whose individual mass is greater than the mass of any lepton. The greater the mass of the different particles, the longer the lengths of the mass radii become. This cannot be unreasonable given the fact that while the exact density of the moon may not be known, its radius must be much less than the radius of the earth for reason that is not farfetched. The equation in the literature [8] which is anchored on Newtonian, de Broglie, and Einstein's mass-energy principles is given as follows.

$$\Gamma_P = \frac{e^6\, m_P}{4\,\pi\varepsilon_0^3\, m_e^2 h^2 c^4} \qquad (2)$$

Where, Γ_P, e, m_P, ε_0, m_e, h, and c are the mass radius of any particle whose mass is > the mass of the electron, charge of an electron, mass of the particle, permittivity in free space, mass of an electron, Planck constant, and velocity of light in a vacuum respectively. In line with the policy of a simple step-by-step approach(this simply means that phrase such as after some algebra is avoided) the velocity of light in free space raised to the 4th power is given as

$$c^4 = \frac{e^6 m_P}{4\,\pi\varepsilon_0^3 m_e^2 h^2 \Gamma_P} \qquad (3)$$

The square of the velocity of light is given as

$$c^2 = \frac{e^3}{m_e h \varepsilon_0^{3/2}} \times \sqrt[2]{\frac{m_P}{4\pi\Gamma_P}} \qquad (4)$$

In the literature [9], is the equation as follows.

$$a_x = \frac{1}{m_e}\sqrt[2]{\frac{\xi_H}{\xi_x}\frac{n_x h^2}{\mu_0 e^2 \pi c^2}} \qquad (5)$$

Where as usual, x denotes any element or atom and a_x, ξ_x, n_x, μ_0 and ξ_H are the radius of any atom, ionisationenergy of any atom except hydrogen atom, principal quantum number otherwise referred to as energy level of any atom other than hydrogen atom, magnetic constant, and the ionisation energy of hydrogen atom respectively. If $\xi_x = \xi_H$, the usual Bohr's symbol, a_0 for the radius of hydrogen otherwise known as Bohr's radius for hydrogen applies and n_x must be = 1.

Substitution of Eq. (4) into Eq. (5) gives

$$a_x = \frac{1}{m_e}\sqrt[2]{\frac{\xi_H}{\xi_x}\frac{n_x h^2}{\mu_0 e^2 \pi}}\sqrt[2]{\frac{4\pi\Gamma_P}{m_P}}\, m_e h \frac{\varepsilon_0^{3/2}}{e^3} \qquad (6)$$

Simplification of Eq. (6) gives

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$$a_X = \sqrt[2]{\frac{4\,\xi_H}{\xi_X}\frac{\pi\Gamma_P}{m_P}} \times \frac{n_X\, h^3 \varepsilon_0^{3/2}}{\mu_0\, e^5 \pi} \tag{7}$$

It is known in the literature [2] that for any element, both hydrogenic and non-hydrogenic elements the equation as follows holds.

$$a_X = \frac{n_X h}{\pi(8\, m_e)^{½}} \frac{1}{\xi_X^{½}} \tag{8}$$

It follows based on Eqs (7) and (8) that

$$\frac{n_X h}{\pi(8\, m_e)^{½}} \frac{1}{\xi_X^{½}} = \sqrt[2]{\frac{4\,\xi_H}{\xi_X}\frac{\pi\Gamma_P}{m_P}} \times \frac{n_X\, h^3 \varepsilon_0^{3/2}}{\mu_0 e^5 \pi} \tag{9a}$$

Simplification of Eq. (9a) gives

$$\frac{1}{m_e} = 32\xi_H \frac{\pi\Gamma_P}{m_P} \frac{h^4 \varepsilon_0^3}{\mu_0^2\, e^{10}} \tag{9b}$$

Making $\Gamma_P$subject of the formula in Eq. (9b) gives the equation that reproducesresult equal to what can be obtained from Eq. (1). Thus,

$$\Gamma_P = \frac{m_P\, \mu_0^2\, e^{10}}{32 h^4 \varepsilon_0^3 \pi m_e \xi_H} \tag{10}$$

Apart from hydrogen atom, ξ_H is always the same because for every hydrogenic atom of the form ${}_Z^AX^{(Z-1)+}$ where,*A* and *Z* are the mass number and atomic number respectively, $n^2\xi_{(Z-1)}/Z^2$ (where *n* =1) as well as$n_X^2\xi_X/Z_{eff}^2$ is always = ξ_H where Z_{eff} is the effective nuclear charge.It is very obvious that the constant in Eq. (10) is difficult to choose because ξ_H though experimentally and theoretically determinable, it is nevertheless a constant parameter going by the preceding analysis. Thus, if all the parameters in Eq. (10) are constant, then the dependent parameter must be a constantas long as the rest mass of the particle in question is constant.

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3. MATERIALS AND METHODS

There was no need for materials and equipment because the research entails purely theoretical and computational or calculational methods.

4. RESULTS AND DISCUSSION

As stated earlier, recent results reported for the length of the radii of nucleons in the literature [8] are totally different from results reported in older literature [10, 11]. Such pieces of information include the observation that "the proton charge radii in the literature are 0.84 ± 0.05 fm [9], 0.856 fm [10], 0.84 ± 0.0004 fm [12-15], 0.84087 fm [16] and finally for the purpose of this research, 0.831 ± 0.012 fm [17]. This latter figure for the proton charge radius refutes the claim that 0.84 fm may be the most accurate if lower figures seem to point to a more accurate values "[8]. Yet, again the value of 0.831 ± 0.012 fm [17] may not be the shortest going by the paper by Hare and Papini [18] which showed a value equal to 0.7 fm.

Other reports are those based on unfamiliar chiral bag model which views the radius of the nucleon to be a bag of radius ≈ 0.8 fm [19] and that of Bochkarev*et al*[4] which is also 0.8 fm being according to them, ther.m.s radius of the nucleon. According to the latter authors [4], proton point densities instead of charge densities can be used to determine the finite size of the proton by the prescription, $r_P^2 = r_C^2 - r_N^2$, where r_P, r_C, and r_N(where$r_N$is = 0.8 fm) are the r.m.s radii of proton, charge, and nucleon density distribution respectively.

The Γ_P values for the purpose of illustration are determined for the nucleons and muon in muonic hydrogen only as shown in Table 1. The muon is a lepton whose mass is ≈ 207 times heavier than the electron, a negatively charged lepton; the muon-proton mass ratio, η according to Pohl *et al* [20] is 0.1126095272.

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Table 1. The masses and radii of Baryons and Leptons

Baryons	**Symbols**	**Mass/exp (–27) kg**	**Radii / fm**
Proton	p^+	1.672621777$^{(C)}$	1.101171175
Neutron	n	1.674927351$^{(C)}$	1.102689051
Leptons	**Symbols**	**Mass/exp (–27) kg**	**Radii / fm**
Muon	μ^-	0.188427357	0.123979254
Tauon	τ^-	3.167790098	2.085515799

The superscript (C) means that the values were from CODATA recommended values [21]. Other mass values were calculated using the equation of mass-energy equivalence (GeV/c^2) with data in the literature (www.Sciencedirect.com /topics/chemistry/leptons). Ionisationenergy of hydrogen atom was calculated using a_0 (=5.2917721092 exp (–11) m), e (=1.602176565 exp (–19) C), and ε_0 (=8.854187817 exp (–12) F/m). The usual equation is $\xi_H = e^2/8\,\pi\varepsilon_0 a_0$.

Based on Eq. (10), the radius of any elementary particle is mainly a function of its mass. Expectedly, therefore, the larger particle with higher mass has longer radius similar to report elsewhere [8]. Substitution of ionisation energy of hydrogen into Eq. (10) however, gave results whose difference from previous results for the proton and neutron are ≈ 0.03406 % of the results in the literature [8].

In a recent literature,elastic electron-proton scattering (e-p) and the spectroscopy of hydrogen atoms are the two methods traditionally used to determine the proton charge radius, r_p. Another method, using muonic hydrogen atom, in which measurement of Lamp-shift was taken, found a substantial discrepancy compared with previous results [17]. The shorter length of the proton radius led to what has been termed proton radius puzzle [20]. But a greater puzzle ought to be expected if despite the freely available literature materials the scholars did not notice the values of proton radius shorter than 0.831 fm. The proton radius in question was obtained from the measurement of the Lamb shift in the muonic hydrogen atom.

The term Lamb shift came to reality after the notion of the degeneracy of $2s_{½}$ and $2p_{½}$ states of the hydrogen atompopularised by Dirac's unfamiliar one-particle relativistic theory was replaced with the observation that the $2s_{½}$is actually higher than$2p_{½}$(www.sciencedirect.com/topics/chemistry/lambshift). Nevertheless, one may need to know if kinetic energy is referred to at least from layman's perspective otherwise given what s and p stand for, if not mistaken, the potential energy of p should be higher than s. No extra information in this regard is available in the literature for clarification. Lamb shift is the shift of atomic energy level given suitable conditions, e.g. the interaction of the electron with the virtual photon and vacuum electric current (www.sciencedirect.com/topics/chemistry/lambshift), conditions that may not be unexpected in the experimental process involving muonic hydrogen.

It has been explained in the literature [8] why the r.m.s.radius of the proton will continue to shorten on the bases of the implication of experimental procedure, the unusual electron-proton scattering (e-p) approach involving particles of opposite charge as against positron-proton (p-p) scattering approach. The current approach in this research has dual characteristics in the sense that, as Eq. (10) shows, Γ_P being the mass radius of any particle whose mass is equal to the mass of any of the nucleon or larger particles, is inversely proportional to the ionisation energy of hydrogen atom, a parameter which

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can be determined experimentally and theoretically. Besides, current equation Eq. (10) differs from Eq. (2) due largely to the presence of magnetic constant as nominator while Eq. (2) contains the velocity of light.

5. CONCLUSION

Nuclear property such as the radius of any nucleon (Γ_N) can be mathematically linked to atomic property such as ionisation energy of hydrogen via equation which shows that Γ_N is inversely proportional to ionisation energy of hydrogen and directly proportional to the rest mass of the particle. The chemistry of any atom is a function of its nuclear property. Thus, a link between hydrogenic and nuclear properties in furtherance of Bohr's theory is not out of place and it is not intended to be restricted to hydrogenic atom as long as ionisation energy is known.

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COMPETING INTERESTS DISCLAIMER:

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

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