Some Efficient Exponential Ratio Type Estimators in Adaptive Cluster Sampling

ABSTRACT

In this paper three efficient exponential ratio type estimators of finite population mean in the Adaptive Cluster Sampling design have been proposed using one known auxiliary variable. The expressions of bias and Mean Squared Error of the proposed estimators are derived up to the first order of approximation. A simulation study has been conducted on two different populations to examine the performance of the proposed estimator over similar existing estimators in the Adaptive Cluster Sampling design. The simulation study showed that the proposed estimators perform better than other related estimators discussed in this article.

KEYWORDS

Adaptive Cluster Sampling, Simulation, Exponential estimator, Within-network variance, Ratio estimator

1 Introduction

When the population under study is rare or clumped or hidden clustered, the biggest issue is how to collect the data in that situation. Using conventional sampling design for example Simple Random Sampling (SRS) is not advised as it is highly likely that most units drawn in the random sample, would not provide meaningful information due to the nature of the population being studied. Adaptive Cluster Sampling (ACS) design is an adaptive scheme that provides a better estimate of the population's parameter of interest in such a situation. In ACS, if the observed unit satisfies some researcher-specified condition, then its neighbourhood (which is pre-defined by the researcher) is also selected for estimating the parameter, more simply "if you find what you are looking for at a particular location, you sample in the vicinity of that location with the hope of obtaining even more information"[1].

ACS was proposed by [2] in 1990. Since then, it has received considerable attention and has been widely used as it allows esearchers to collect data and get acceptable estimates which otherwise would not have been possible. It has been used in variety of fields such as Ecological Science [3, 5], Environmental Science [7, 8] and Epidemiology study and Social Sciences [9, 10].

Due to the wide applicability of this design, many researchers have developed different estimators to estimate the unknown population parameter of interest. It is common in sample surveys that information regarding a variable related to the survey variable is known in advance from past surveys and can be availed by incurring much less cost. If it is highly correlated with the survey variable then using such a variable would increase the precision of the estimator. Such a variable is called an auxiliary variable. Researchers in sample surveys have been utilising auxiliary variables for a long time. [4] using a known auxiliary variable, proposed a modified ratio estimator. Using the known auxiliary variable and some known population parameters [6] proposed some ratio estimators to estimate the population mean. Using known coefficient of skewness and kurtosis [11] proposed some improved ratio type estimators and studied its properties. Using a single auxiliary variable [13] proposed their transformed ratio type estimator.

The use of exponential type ratio estimator is well established in sampling theory and there are a number of estimators present in non-adaptive design but there is a lack of such estimators in Adaptive Cluster Sampling design and in this paper, we address this problem by proposing some efficient exponential ratio type estimators using just the population mean of the auxiliary variable.

In this article, we propose three ratio-type exponential estimators of finite population mean in the ACS design. Section 2 provides a brief introduction and methodology of Adaptive Cluster Sampling. Some similar existing estimators in the Adaptive Cluster Sampling are presented in Section 3 of this article. The bias and Mean Squared Error (MSE) of the proposed estimators have been derived up to the first order of approximation and are presented in Section 4. To demonstrate the performance of the proposed estimator over all the estimators presented in this article, a simulation study is conducted on two different populations, the results of which are presented in Section 5. The final conclusion of this article along with future research ideas are presented in Section 6.

2 Adaptive Cluster Sampling methodology

ACS is an adaptive sampling design in which, the units in the final sample depends on all the units which have been observed during the survey. Initially, a sample of size n_1 is drawn from the population of size N using any conventional sampling design (usually SRSWOR) and if these selected units satisfy some researcher-specific condition C, then additional units are drawn from a pre-defined neighbourhood.

So, before conducting the survey, two things should be clearly defined:

- the neighbourhood of a unit (or observation)
- the researcher-specific condition (C)

This researcher-specific condition for selecting the observation on survey variable y is usually $y_i > 0$. In ACS, the used choice of neighbourhood is 4 unit first order in which, if any i^{th} unit selected in the initial sample is greater than 0, the units adjacent to this i^{th} unit in its East, West, North and South directions are also selected. This process of selecting the neighbourhood keeps on going until no further additional unit satisfies the condition C.

The units satisfying condition C form a network, and units not satisfying it are called edge units and are considered to be a network of size 1. The selection of any unit of a network leads to the selection of the entire network. These networks and edge units together form a cluster (Fig. 1).

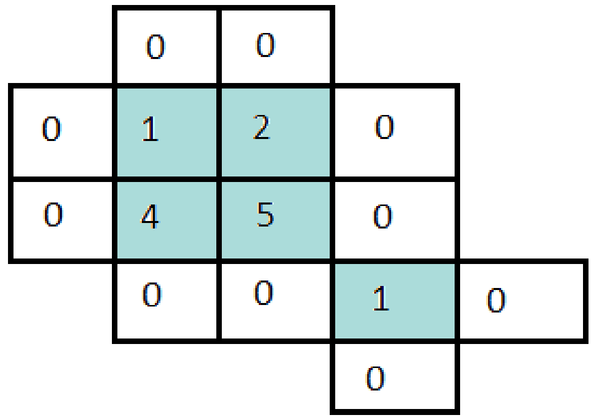

Figure 1: An example of a hypothetical cluster with pre-defined condition (C) $y_i > 0$. The units having y-values 1, 2, 4, 5 and 1 form a network of size five. The edge units are the units with y values 0 and are adjacent to the y values greater than 0. Together they form a cluster.

The clusters are obviously not disjoint due to overlapping edge units but the units of a network are non-overlapping and thus the entire population can be partitioned as a set of networks and edge units.

Once there are no more additional units satisfying condition C, ACS terminates and the sample obtained consists of units selected in the initial sample and adaptively selected units.
Once the population is divided into networks and edge units, we make a transformed population by assigning the average value of a network to all the units of this network but edge units stay the same. [4] stated that once a transformed population is obtained, and we consider averages of networks then ACS can be regarded as either SRSWOR or SRSWR.

3 Some related estimators in Adaptive Cluster Sampling

[2] proposed an unbiased estimator of a population mean in ACS. The estimator proposed by Thompson is as follows

$$t_{Th} = \frac{1}{n}\sum_{i=1}^{n} w_{y_i}, \qquad (1)$$

where w_{y_i} is the network mean of a network ψ_i which contains the i^{th} unit. So,

$$w_{y_i} = \frac{1}{m_i}\sum_{j\epsilon\Psi_i} (y_j), \qquad (2)$$

where m_i is the number of units in network ψ_i.

Variance of Thompson's estimator is given by

$$V(t_{Th}) = (\frac{1}{n} - \frac{1}{N})\bar{W_y}^2 C^2_{w_y} \quad (3)$$

[4] proposed a modified ratio estimator given by

$$t_{DC} = \bar{W}_x \frac{\sum_{i=1}^{n} w_{y_i}}{\sum_{i=1}^{n} w_{x_i}}, \quad (4)$$

where

$$w_{y_i} = \frac{1}{m_i} \sum_{j \epsilon \Psi_i} (y_j),$$

$$w_{x_i} = \frac{1}{m_j} \sum_{k \epsilon \Psi_j} (x_k),$$

and m_i and m_j are number of units in the network ψ_i and ψ_j repectively.
The MSE up to the first order of approximation of the estimator is:

$$MSE(t_{DC}) = \left(\frac{1}{n} - \frac{1}{N}\right) \bar{W}_y^2 (C^2_{w_y} + C^2_{w_x} - 2\rho_{w_x w_y} C_{w_x} C_{w_y}), \quad (5)$$

where $C^2_{w_x} = \frac{S^2_{w_x}}{\bar{W_x}^2}$, $S^2_{w_x} = \frac{1}{N-1} \sum_{i=1}^{N} (w_{xi} - \bar{W}_x)^2$,

$\rho_{w_x w_y} = \frac{S_{w_x w_y}}{S_{w_x} S_{w_y}}$,

and $S_{w_x w_y} = \frac{1}{N-1} \sum_{i=1}^{n} (w_{x_i} - \bar{W}_x)(w_{y_i} - \bar{W}_y)$.

[6] proposed some modified ratio type estimators as follows

$$t_{CH_1} = \bar{w}_y \left(\frac{\bar{W}_x + C_{w_x}}{\bar{w}_x + C_{w_x}}\right), \quad (6)$$

and

$$t_{CH_2} = \bar{w}_y \left(\frac{\bar{W}_x + \beta_2(w_x)}{\bar{w}_x + \beta_2(w_x)}\right). \quad (7)$$

The MSE up to first order of approximation of their estimators are

$$MSE(t_{CH_1}) = \left(\frac{1}{n} - \frac{1}{N}\right) \bar{W}_y^2 (C^2_{w_y} + \theta^2_{w_1} C^2_{w_x} - 2\rho_{w_x w_y} \theta_{w_1} C_{w_x} C_{w_y}), \quad (8)$$

and

$$MSE(t_{CH_2}) = \left(\frac{1}{n} - \frac{1}{N}\right) \bar{W}_y^2 (C^2_{w_y} + \theta^2_{w_2} C^2_{w_x} - 2\rho_{w_x w_y} \theta_{w_2} C_{w_x} C_{w_y}), \quad (9)$$

where $\theta_{w_1} = \frac{\bar{W}_x}{\bar{W}_x + C_{w_x}}$ and $\theta_{w_2} = \frac{\bar{W}_x}{\bar{W}_x + \beta_2(w_x)}$.

[11] proposed some improved ratio estimators given by

$$t_{SY_1} = \bar{w}_y (\frac{\bar{W}_x \beta_1(w_x) + \beta_2(w_x)}{\bar{w}_x \beta_1(w_x) + \beta_2(w_x)}), \quad (10)$$

and

$$t_{SY_2} = \bar{w}_y (\frac{\bar{W_x}^2}{\bar{w_x}^2}). \quad (11)$$

MSE of their estimators up to first order of approximation are given by

$$MSE(t_{SY_1}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{W_y}^2(C_{w_y}^2 + \theta_{w_3}C_{w_x}^2 - 2\theta_{w_3}\rho_{w_xw_y}C_{w_y}C_{w_x}), \tag{12}$$

and

$$MSE(t_{SY_2}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{W_y}^2(C_{w_y}^2 + 4C_{w_x}^2 - 4\rho_{w_xw_y}C_{w_y}C_{w_x}), \tag{13}$$

where $\theta_{w3} = \frac{\bar{W}_x\beta_1(w_x)}{\bar{W}_x\beta_1(w_x)+\beta_2(w_x)}$.

Using known coefficient of variation and population mean square of the auxiliary variable, [13] proposed some transformed ratio estimators given by

$$t_{SM_1} = \bar{w}_y\left(\frac{\bar{W}_x\beta_1(w_x)+C_{w_x}}{\bar{w}_x\beta_1(w_x)+C_{w_x}}\right), \tag{14}$$

and

$$t_{SM_2} = \bar{w}_y\left(\frac{\bar{W}_x\beta_1(w_x)+S_{w_x}}{\bar{w}_x\beta_1(w_x)+S_{w_x}}\right). \tag{15}$$

The bias and MSE of their estimators are given by

$$Bias(t_{SM_1}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{W}_y\left(\frac{C_{w_x}^2}{\theta_{w_4}^2} - \frac{\rho_{w_xw_y}C_{w_x}C_{w_y}}{\theta_{w_4}}\right), \tag{16}$$

$$Bias(t_{SM_2}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{W}_y\left(\frac{C_{w_x}^2}{\theta_{w_5}^2} - \frac{\rho_{w_xw_y}C_{w_x}C_{w_y}}{\theta_{w_5}}\right), \tag{17}$$

$$MSE(t_{SM_1}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{W_y}^2(C_{w_y}^2 + \frac{C_{w_x}^2}{\theta_{w_4}^2} - 2\frac{1}{\theta_{w_4}}\rho_{w_xw_y}C_{w_y}C_{w_x}), \tag{18}$$

and

$$MSE(t_{SM_2}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{W_y}^2(C_{w_y}^2 + \frac{C_{w_x}^2}{\theta_{w_5}^2} - 2\frac{1}{\theta_{w_5}}\rho_{w_xw_y}C_{w_y}C_{w_x}), \tag{19}$$

where $\theta_{w_4} = 1 + \frac{C_{w_x}}{\beta_1(w_x)\bar{W}_x}$ and $\theta_{w_5} = 1 + \frac{S_{w_x}}{\beta_1(w_x)\bar{W}_x}$.

In SRSWOR, [14] proposed a generalized exponential ratio estimator as follows

$$t_{SG_{SRS}} = \bar{y}A^{\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)} \tag{20}$$

where A is any constant.

The bias and MSE of their estimator up to first order of approximation is

$$Bias(t_{SG_{SRS}}) = f\bar{Y}\left(\frac{C_x^2}{4}log_eA + \frac{C_x^2}{8}(log_eA)^2 + \frac{\rho C_xC_y}{2}log_eA\right) \tag{21}$$

and

$$MSE(t_{SG_{SRS}}) = f\bar{Y}\left(C_y^2 + \frac{C_x^2}{4}(log_eA)^2 - \rho C_xC_ylog_eA\right). \tag{22}$$

4 Proposed efficient exponential ratio type estimators

Aim of this article is to propose highly efficient efficient exponential ratio type estimators of finite population mean in the Adaptive Cluster Sampling design. Thus in this section we propose three efficient exponential ratio type estimators as follows

i. First proposed estimator

Motivated by [4] and [6] we propose

$$t_p^{(1)} = r\bar{w}_y exp\left(\frac{\bar{W}_x - \bar{w}_x}{\bar{W}_x + \bar{w}_x}\right) \tag{23}$$

where r is optimum scalar which minimises the MSE. In order to obtain the expressions of bias and MSE, we re-write the above expression as

$$t_p^{(1)} = r\bar{W}_y(e_{w_y}+1)exp\left(\frac{\bar{W}_x - \bar{W}_x(e_{w_x}+1)}{\bar{W}_x + \bar{W}_x(e_{w_x}+1)}\right) \tag{24}$$

where

$e_{w_y} = \frac{\bar{w}_y}{\bar{W}_y} - 1,\; e_{w_x} = \frac{\bar{w}_x}{\bar{W}_x} - 1,$

$E(e_{w_x}^2) = \frac{1-f}{n}C_{w_x}^2,\; E(e_{w_y}^2) = \frac{1-f}{n}C_{w_y}^2,$

and $E(e_{w_x}e_{w_y}) = \frac{1-f}{n}\rho_{w_x w_y}C_{w_x}C_{w_y}.$

Simplifying (24) we get

$$t_p^{(1)} = r\bar{W}_y(e_{w_y}+1)exp\left(\frac{-e_{w_x}}{2}\left(1+\frac{e_{w_x}}{2}\right)^{-1}\right) \tag{25}$$

Expanding and simplifying (25) we get

$$t_p^{(1)} = r\bar{W}_y(e_{w_y}+1)exp\left(1-\frac{e_{w_x}}{2}+\frac{3}{8}e_{w_x}^2\right). \tag{26}$$

Further expanding and subtracting $\bar{W}_y$ from both the sides we get

$$t_p^{(1)} - \bar{W}_y = r\bar{W}_y(1+e_{w_y}-\frac{e_{w_x}}{2}+\frac{3}{8}e_{w_x}^2-\frac{1}{2}e_{w_x}w_{w_y}) - \bar{W}_y \tag{27}$$

Taking expectation on both sides in (27) we get

$$Bias(t_p^{(1)}) = \bar{W}_y(r(1+\frac{3}{8}fC_{w_x}^2-\frac{1}{2}f\rho_{w_x w_y}C_{w_y}C_{w_x})-1). \tag{28}$$

Squaring and taking expectation on both sides of (27) we get

$$MSE(t_p^{(1)}) = \bar{W}_y{}^2(1+r^2A-2rB) \tag{29}$$

where $A = 1+fC_{w_y}^2+fC_{w_x}^2-2f\rho_{w_x w_y}C_{w_y}C_{w_x},$

$B = 1+\frac{3}{8}fC_{w_x}^2-\frac{1}{2}f\rho_{w_x w_y}C_{w_y}C_{w_x},$

and $f = \frac{1}{n}-\frac{1}{N}$. Differentiating the expression of $MSE(t_p^{(1)})$ to obtain the optimum value of r that minimizes the MSE, we get

$$r_{opt} = B/A. \tag{30}$$

Putting r_{opt} in (29) we get

$$MSE(t_{p_{min}}^{(1)}) = \bar{W}_y^{\ 2}\left(1 - \frac{B^2}{A}\right). \tag{31}$$

ii. Second proposed estimator

Motivated by [14] we propose

$$t_p^{(2)} = (r_1\bar{w}_y + r_2(\bar{W}_x - \bar{w}_x))exp\left(\frac{\bar{W}_x - \bar{w}_x}{\bar{W}_x + \bar{w}_x}\right) \tag{32}$$

where r_1 and r_2 are optimum scalars which minimizes the MSE. Following the same line of expansion and simplification the expression of bias and MSE can be obtained. So we directly write

$$Bias(t_p^{(2)}) = r_1\bar{W}_y(1 + \tfrac{3}{8}fC_{w_x}^2 - \tfrac{1}{2}f\rho_{w_xw_y}C_{w_y}C_{w_x}) + \tfrac{1}{2}r_2\bar{W}_xfC_{w_x}^2 - \bar{W}_y, \tag{33}$$

$$MSE(t_p^{(2)}) = \bar{W}_y^{\ 2} + r_1^2A_1 + r_2^2B_1 - 2r_1r_2C_1 - 2r_1D_1 - 2r_2E_1, \tag{34}$$

where

$A_1 = \bar{W}_y^{\ 2}(1 + fC_{w_y}^2 + fC_{w_x}^2 - 2f\rho_{w_xw_y}C_{w_y}C_{w_x}),$

$B_1 = \bar{W}_x^{\ 2}fC_{w_x}^2,$

$C_1 = \bar{W}_y\bar{W}_x(f\rho_{w_xw_y}C_{w_y}C_{w_x} - fC_{w_x}^2),$

$D_1 = \bar{W}_y^{\ 2}(1 + \frac{3}{8}fC_{w_x}^2 - \frac{1}{2}f\rho_{w_xw_y}C_{w_y}C_{w_x})$ and

$E_1 = \frac{1}{2}\bar{W}_y\bar{W}_xfC_{w_x}^2$. The optimum values of r_1 and r_2 upon partially differentiating (34) are

$$r_{1opt} = \frac{E_1C_1 + B_1D_1}{A_1B_1 - C_1^2} \tag{35}$$

and

$$r_{2opt} = \frac{C_1D_1 + A_1E_1}{A_1B_1 - C_1^2}. \tag{36}$$

Upon putting these values in (34), we get

$$MSE(t_{p_{min}}^{(2)}) = \bar{W}_y^{\ 2} + r_{1opt}^2A_1 + r_{2opt}^2B_1 - 2r_{1opt}r_{2opt}C_1 - 2r_{1opt}D_1 - 2r_{2opt}E_1. \tag{37}$$

iii. Third proposed estimator

Using the condition $r_1 + r_2 = 1$ we propose

$$t_p^{(3)} = (r^*\bar{w}_y + (1 - r^*)(\bar{W}_x - \bar{w}_x))exp\left(\frac{\bar{W}_x - \bar{w}_x}{\bar{W}_x + \bar{w}_x}\right) \tag{38}$$

where r^* is optimum scalar which minimizes the MSE. To obtain the expressions of bias and

MSE_{min} replace r_1 with r^* and r_2 with $1-r^*$ in (33) and (34) respectively. The expresions obtained are

$$Bias(t_p^{(3)}) = r^*\bar{W}_y(1+\frac{3}{8}fC_{w_x}^2 - \frac{1}{2}f\rho_{w_xw_y}C_{w_y}C_{w_x}) + \frac{1}{2}(1-r^*)\bar{W}_xfC_{w_x}^2 - \bar{W}_y, \quad (39)$$

and

$$MSE(t_p^{(3)}) = \bar{W}_y^{\ 2} + r^{*2}A_1 + (1-r^*)^2B_1 - 2r^*(1-r^*)C_1 - 2r^*D_1 - 2(1-r^*)E_1. (40)$$

Differentiating (40) we get

$$r_{opt}^* = \frac{B_1+C_1+D_1-E_1}{A_1+B_1+2C_1}, \quad (41)$$

where A_1, B_1, C_1, D_1 and E_1 have been defined above. The expression of MSE_{min} is

$$MSE(t_{p_{min}}^{(3)}) = \bar{W}_y^{\ 2} + (A_1+B_1+2C_1)r_{opt}^* - 2(B_1+C_1+D_1-E_{E1})r^* + B_1 - 2E_1. \quad (42)$$

5 Simulation study

In this section, we have conducted two simulation studies to demonstrate the performance of our proposed estimators. The performance of the proposed estimators has been compared with similar existing estimators discussed in this article. The comparison is made on the basis of the Relative efficiency (RE) of all the estimators.

The first population is generated using the model

$$y_i = 2x_i + e_i, \quad (43)$$

where $e \sim N(0, x_i)$ and the auxliliary variable X has been taken from [?]. The population generated is presented in Figure-1. The second population is generated from the model

$$y_i = 4x_i + e_i, \quad (44)$$

where $e \sim N(0, x_i)$ and the auxliliary variable X has been taken from [?].

In simulation, the MSE is

$$MSE(t_i) = \frac{1}{r}\sum_{i=1}^{r} (t_i - \bar{W}_y)^2, \quad (45)$$

and the Relative root Mean Square Error is given by

$$RRMSE(t_i) = \frac{1}{\bar{W}_y}\sqrt{\frac{1}{r}\sum_{i=1}^{r} (t_i - \bar{W}_y)^2}, \quad (46)$$

and the Relative efficiency is given by

$$RE = \frac{Var(t_{Th})}{t_i}, \quad (47)$$

where r is the number of iteration which is 10,000 and t_i are the appropriate estimators(t_{Th}, CH_{1-2}, SY_{1-2} and SM_{1-2} respectively).

The following algorithm is used to conduct the simulation study:

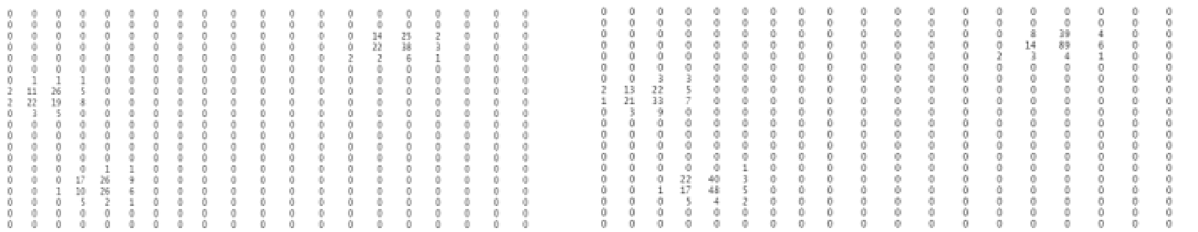

Figure 2: Population-1(X,Y)

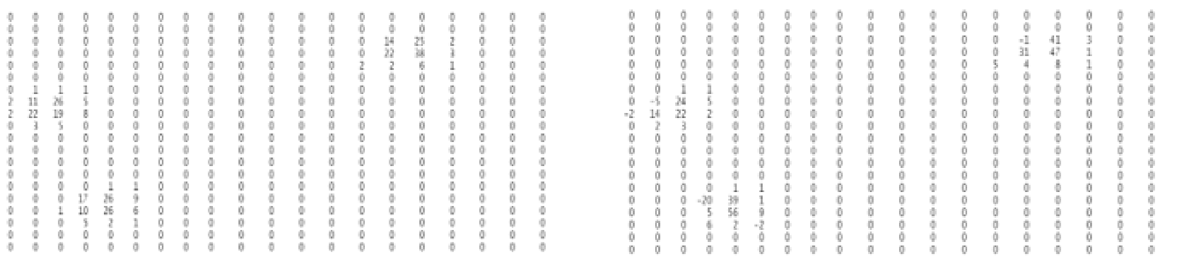

Figure 3: Population-1(X,Y)

1. Using the model in equation (43), the population of survey variable Y for population-1 is generated.
2. Using the model in equation (44), the population of survey variable Y for population-2 is generated.
3. Using sample sizes 150, 160, 170, 180 and 190 the sampling procedure of ACS is repeated ten thousand times to calculate several values of all the estimators.
4. For each sample size, RRMSEs and REs are obtained for each estimator and are presented (Tables 1-4).

Table 1: Relative root Mean Square Errors of all estimators in case of population-1

n	150	160	170	180	190
t_{Th}	0.214331122	0.203252508	0.193011579	0.183527467	0.174458709
t_{CH_1}	0.173992864	0.164645648	0.154472326	0.149125634	0.139566967
t_{CH_2}	0.194059993	0.18396917	0.172822758	0.166605784	0.156040613
t_{SY_1}	0.163904553	0.154996852	0.145544995	0.140436552	0.131482158
t_{SY_2}	0.163904553	0.154996852	0.145544995	0.140436552	0.131482158
t_{SM_1}	0.128674386	0.121538176	0.113991557	0.10996897	0.102718507
t_{SM_2}	0.118152045	0.111435287	0.104481203	0.100723783	0.094056815
$t_p^{(1)}$	0.115019327	0.108108108	0.101125875	0.097447332	0.090986531
$t_p^{(2)}$	0.042255998	0.038221988	0.034352569	0.032118096	0.028890699
$t_p^{(3)}$	0.042638413	0.038433746	0.03438799	0.032168597	0.028913165

Table 2: Relative root Mean Square Errors of all estimators in case of population-2

n	150	160	170	180	190
t_{Th}	0.228246698	0.216329842	0.205872869	0.198002185	0.185571296
t_{CH_1}	0.196210273	0.184617196	0.175797429	0.168618971	0.158357147
t_{CH_2}	0.214279276	0.201975869	0.192117373	0.184617196	0.173267838
t_{SY_1}	0.189805407	0.178785702	0.170182704	0.163298415	0.153256646
t_{SY_2}	0.189805407	0.178785702	0.170182704	0.163298415	0.153256646
t_{SM_1}	0.15667543	0.146782206	0.140008688	0.13421269	0.12607087
t_{SM_2}	0.141264396	0.132224244	0.126838963	0.120337344	0.113541578
$t_p^{(1)}$	0.143250636	0.133553164	0.127602433	0.119601329	0.113541578
$t_p^{(2)}$	0.100330026	0.092069146	0.088149496	0.08256332	0.078921663
$t_p^{(3)}$	0.100417996	0.092069146	0.088349609	0.082776938	0.07893285

Table 3: Relative efficiencies of all the estimators in case of population-1

n	150	160	170	180	190
t_{Th}	1	1	1	1	1
t_{CH_1}	1.517426273	1.523952096	1.56122449	1.51459854	1.5625
t_{CH_2}	1.219827586	1.220623501	1.247282609	1.213450292	1.25
t_{SY_1}	1.709969789	1.719594595	1.75862069	1.70781893	1.76056338
t_{SY_2}	1.709969789	1.719594595	1.75862069	1.70781893	1.76056338
t_{SM_1}	2.774509804	2.796703297	2.866958151	2.785234899	2.884615385
t_{SM_2}	3.290697674	3.326797386	3.412639405	3.32	3.440366972
$t_p^{(1)}$	3.472392638	3.534722222	3.642857143	3.547008547	3.676470588
$t_p^{(2)}$	25.72727273	28.27777778	31.56808803	32.65145555	36.46441074
$t_p^{(3)}$	25.26785714	27.96703297	31.50308854	32.54901961	36.40776699

Table 4: Relative efficiencies of all the estimators in case of population-2

n	150	160	170	180	190
t_{Th}	1	1	1	1	1
t_{CH_1}	1.353211009	1.373056995	1.371428571	1.378881988	1.373239437
t_{CH_2}	1.134615385	1.147186147	1.148325359	1.150259067	1.147058824
t_{SY_1}	1.446078431	1.464088398	1.463414634	1.470198675	1.466165414
t_{SY_2}	1.446078431	1.464088398	1.463414634	1.470198675	1.466165414
t_{SM_1}	2.122302158	2.172131148	2.162162162	2.176470588	2.166666667
t_{SM_2}	2.610619469	2.676767677	2.634467618	2.707317073	2.671232877
$t_p^{(1)}$	2.538726334	2.623762376	2.603036876	2.740740741	2.671232877
$t_p^{(2)}$	5.175438596	5.520833333	5.454545455	5.751295337	5.528777998
$t_p^{(3)}$	5.166374781	5.520833333	5.429864253	5.721649485	5.527210884

6 Conclusion

Proposed estimators $t_p^{(1)} - t_p^{(3)}$ have been developed using only the known value population mean of the auxiliary variable which in practice may be easily available. The proposed estimators have been compared with similar existing estimators in the ACS design. It is evident (from Tables 1-4) that the proposed estimators $t_p^{(1)} - t_p^{(3)}$ resulted in lower RRMSE as compared to all the estimators discussed in this article. It is observed that for each sample size, proposed estimator $t_p^{(2)}$ resulted in much higher relative efficiency as compare to proposed estimators $t_p^{(1)}$ and $t_p^{(3)}$. It is due to the fact that the proposed estimator $t_p^{(2)}$ is developed using the condition $r_1 + r_2 \neq 1$.

In this article, our aim was to develop some efficient exponential ratio type estimators of finite population mean in the ACS design using a single auxiliary variable. We proposed three such estimators and derived their expressions of bias and MSE up to the first order of approximation. Further, their efficiency has been demonstrated by two simulation studies where the proposed estimators have been compared with similar existing estimators in the ACS designs. From the results tabulated in (Tables 1-4), it is clear that all the three proposed estimators $t_p^{(1)} - t_p^{(3)}$ can be used when the ACS design has to be applied to estimate the finite population mean but for higher efficiency, it is recommended that proposed estimator $t_p^{(2)}$ should be used.

Some lucrative future areas of research include extending these estimators in stratified adaptive cluster sampling and using multi-auxiliary information.

7 References

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