Original Research Article

Improved Estimators of Population Coefficient of Variation under Simple Random Sampling

ABSTRACT

In this article, we suggest some novel estimators of population Coefficient of Variation (CV) of the study variable using the known information on an auxiliary variable like population mean and population variance. . Up to the first order of approximation, formulas for the bias and Mean squared Errors (MSE) of the proposed estimators are obtained. The efficiencies of proposed and competing estimators are evaluated by comparing their MSEs. A real and two simulated data sets are used to verify the efficiency conditions. The results showed that the proposed estimators were more efficient than the other existing estimators considered in the study.

Keywords: *Study variable, auxiliary variable, bias, mean square error, and coefficient of variation.*

1. INTRODUCTION

In theory of Sampling Survey, use of auxiliary information improves the efficiency of the estimator. Use of auxiliary information can be done at various stages, it helps in improving the precision of the estimator. Cochran (1940) was the first to introduce a ratio estimator of Population Mean using auxiliary information. Shabbir and Gupta (2007), Singh et al. (2007) , Koyuncu and Kadilar (2009) and Chaudhary et al.(2009) have considered the problem of estimating population mean taking into consideration information on auxiliary variable.

The Coefficient of Variation (CV) is a well-known measure of dispersion, which is defined as the ratio of the standard deviation to the mean of the characteristic under study. Whenever the population is very large, the complete enumeration is very time consuming and costly then the population CV is estimated through sample CV by using auxiliary information as it improves precision. McKay (1931) was the first to estimate population CV. Archana & Rao (2011) gave some new estimator of CV for the enhancement of the estimation of CV. Shabbir and Gupta (2017) used two auxiliary variables to improve the estimation of population CV in simple and stratified random sampling under a two-phase sampling technique. Singh et al. (2018) proposed various improved and more enhanced estimators based on the arithmetic mean, geometric mean, and harmonic mean of these estimators. Singh and Mishra (2019) proposed estimating the population CV using a single auxiliary variable. Audu et al. (2021) proposed difference cum ratio type estimators for estimating population CV under SRS and demonstrated that their estimators are more efficient than the existing estimators.

To estimate any parameter under study we need more and more efficient estimation. In search of more and more efficient estimator, we proposed new estimators of population CV under SRS using known auxiliary parameters. These new estimators are expected to give a precise and efficient estimate of the population CV than the existing estimators considered in this paper.

Let us consider a finite population U = (U_1, U_2……… U_N) of size 'N' consisting of distinct and identifiable units. Let Y and X denotes the study and auxiliary variables and let Y_i and X_i be their values corresponding to i^{th} unit in the population (i = 1, 2………. N). For the population observations, we define:

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \ and\ \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

as the population means for the study and Auxiliary Variables.

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2 \ and \ S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$

as the population mean squares for the study and auxiliary variables.

$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})\,(X_i - \bar{X})$ as the population covariance.

Let us consider that a sample of size 'n' has been drawn from this population of size 'N' units using SRSWOR. For this sample let y_i and x_i denote the value of the i^{th} sample unit corresponding to study variable Y and auxiliary variable X. For the sample observations we have:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \ and \ \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

as the sample means for the study and Auxiliary Variables.

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \ and \ s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

as the sample mean squares for the study and auxiliary variables.

$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})\,(x_i - \bar{x})$ as the sample covariance.

Now let us define

$$\in_0 = \frac{\bar{y}}{\bar{Y}} - 1, \in_1 = \frac{\bar{x}}{\bar{X}} - 1, \ \in_2 = \frac{s_y^2}{S_y^2} - 1 \ and \quad \in_3 = \frac{s_x^2}{S_x^2} - 1$$

Such that

$$E(\in_0) = E(\in_1) = E(\in_2) = E(\in_3) = 0$$

$$E(\in_0^2) = \gamma C_y^2 \, , E(\in_1^2) = \gamma C_x^2 \, , E(\in_2^2) = \gamma(\lambda_{40} - 1), E(\in_3^2) = \gamma(\lambda_{04} - 1)$$

$$E(\in_0 \in_1) = \gamma C_{yx} = \gamma \rho_{yx} C_y C_x \, , E(\in_0 \in_2) = \gamma C_y \lambda_{30} \, , E(\in_0 \in_3) = \gamma C_y \lambda_{12} \, , E(\in_1 \in_2) = \gamma C_x \lambda_{21}$$

$$E(\in_1 \in_3) = \gamma C_x \lambda_{03} \, , E(\in_2 \in_3) = \gamma(\lambda_{22} - 1)$$

Where

$\gamma = \left(\frac{1}{n} - \frac{1}{N}\right), C_y = \frac{S_y}{\bar{Y}} \ and \ C_x = \frac{S_x}{\bar{X}}$ are the population coef icient of variation for the study variable Y and auxiliary variable X . Also ρ_{yx} denotes the correlation coefficient between X and Y.

$$\lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\,\mu_{02}^{s/2}} \, , \mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^r\,(X_i - \bar{X})^s$$

2. EXISTING ESTIMATORS

•The usual unbiased estimator to estimate the population coefficient of variation is given by:

$$t_0 = \hat{C}_y = \frac{s_y}{\bar{y}} \tag{2.1}$$

The mean square error (MSE) expression of the estimator t_0 is given by:

$$MSE(t_0) = \gamma C_y^2\left(C_y^2 + 0.25(\lambda_{04} - 1) - C_y\lambda_{30}\right) \tag{2.2}$$

• Archana & Rao (2011) proposed ratio type estimator for the population coefficient of variation is given by:

$$t_{AR} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right) \tag{2.3}$$

The mean square error (MSE) expression of the estimator t_{AR} is given by:

$$MSE(t_{AR}) = \gamma C_y^2\left(C_y^2 + 0.25(\lambda_{04} - 1) - C_y\lambda_{30} - C_x\lambda_{21} + 0.25(\lambda_{40} - 1)\right) \tag{2.4}$$

•Singh R et al. (2018) proposed ratio-type, exponential ratio-type and difference-type estimators for coefficient of variation of the study variable Y using mean of auxiliary variable and are given below with their MSEs as

$$t_1 = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} \tag{2.5}$$

$$t_2 = \hat{C}_y exp\left\{\beta\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right\} \tag{2.6}$$

$$t_3 = \hat{C}_y + d_1(\bar{X} - \bar{x}) \tag{2.7}$$

$$MSE(t_1) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \alpha^2 C_x^2 - C_y\lambda_{30} + 2\rho_{yx}C_yC_x - \alpha C_x\lambda_{21}\right] \tag{2.8}$$

$$MSE(t_2) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\beta^2 C_x^2}{4} - C_y\lambda_{30} + \beta\rho_{yx}C_yC_x - \frac{\beta}{2}C_x\lambda_{21}\right] \tag{2.9}$$

$$MSE(t_3) = \gamma\left[C_y^2\left(C_y^2 - C_y\lambda_{30} + \frac{\lambda_{40} - 1}{4}\right) + d_1^2\bar{X}^2C_x^2 + 2d_1\bar{X}\rho_{yx}C_yC_x - d_1\bar{X}C_xC_y\lambda_{21}\right] \tag{2.10}$$

Where

$$\alpha = \frac{\lambda_{21} - 2\rho_{yx}C_y}{2C_x}, \beta = \frac{\lambda_{21} - 2\rho_{yx}C_y}{C_x}, d_1 = \frac{C_y\lambda_{21} - 2\rho_{yx}C_y^2}{2\bar{X}C_x}$$

•Singh R et al. (2018) proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_1 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_4^{AM} = \frac{\hat{C}_y}{2}\left[1 + \left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\right] \tag{2.11}$$

$$t_4^{GM} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha/2} \tag{2.12}$$

$$t_4^{GM} = 2\hat{C}_y\left[1 + \left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\right]^{-1} \tag{2.13}$$

$$MSE(t_4^{j}) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\alpha^2 C_x^2}{4} - C_y\lambda_{30} + \alpha\rho_{yx}C_yC_x - \frac{\alpha}{2}C_x\lambda_{21}\right] \tag{2.14}$$

Where $\alpha = \frac{\lambda_{21} - 2\rho_{yx}C_y}{2C_x}$, $j = AM, GM, HM$

• Singh R et al. (2018) proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_2 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_5^{AM} = \frac{\hat{C}_y}{2}\left[1 + exp\left\{\beta\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right\}\right] \tag{2.15}$$

$$t_5^{GM} = \hat{C}_y\left\{\frac{\beta}{2}\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right\} \tag{2.16}$$

$$t_5^{HM} = 2\hat{C}_y\left[1 + exp\left\{\beta\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right\}\right]^{-1} \tag{2.17}$$

$$MSE\left(t_5^j\right) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\beta^2 C_x^2}{16} - C_y\lambda_{30} + \frac{\beta}{2}\rho_{yx}C_yC_x - \frac{\beta}{4}C_x\lambda_{21}\right] \tag{2.18}$$

Where , $\beta = \frac{2(\lambda_{21} - 2\rho_{yx}C_y)}{C_x}$

• Singh R et al. (2018) proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_1 and t_2 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_6^{AM} = \frac{\hat{C}_y}{2}\left[\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} + exp\left\{\beta\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right\}\right] \tag{2.19}$$

$$t_6^{GM} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)^{\frac{\alpha}{2}} exp\left\{\beta\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right\} \tag{2.20}$$

$$t_6^{HM} = 2\hat{C}_y\left[\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} + exp\left\{-\beta\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\right\}\right]^{-1} \tag{2.21}$$

$$MSE\left(t_5^j\right) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{1}{4}\left(\alpha + \frac{\beta}{2}\right)^2 C_x^2 - C_y\lambda_{30} + \left(\alpha + \frac{\beta}{2}\right)\rho_{yx}C_yC_x - \frac{1}{4}\left(\alpha + \frac{\beta}{2}\right)C_x\lambda_{21}\right] \tag{2.22}$$

Where , $\beta = 2\left(\frac{\lambda_{21} - 2\rho_{yx}C_y}{C_x} - \alpha\right)$

• Audu et al. (2021) suggested the following two difference-cum-ratio type estimators of C_y utilizing the known $\bar{X}$ as,

$$t_{a1} = \left[\frac{\hat{C}_y}{2}\left(\frac{\bar{X}}{\bar{x}} + \frac{\bar{x}}{\bar{X}}\right) + w_1(\bar{X} - \bar{x}) + w_2\hat{C}_y\right]\left(\frac{\bar{X}}{\bar{x}}\right) \tag{2.23}$$

$$MSE\ (t_{a1}) = C_y^2(a + w_1^2 b + w_2^2 c + 2w_1 d - 2w_2 e - 2w_1w_2 f) \tag{2.24}$$

Where

$$a = \gamma\left(C_x^2 + C_y^2 + 2\rho_{yx}C_yC_x - C_x\lambda_{21} - C_y\lambda_{30} + \frac{(\lambda_{40} - 1)}{4}\right), \qquad b = \gamma\delta^2 C_x^2, \ \ \delta = \frac{\bar{X}}{C_y}$$

$$c = 1 + \gamma\left(3C_x^2 + 3C_y^2 + 4\rho_{yx}C_yC_x - 2C_x\lambda_{21} - 2C_y\lambda_{30}\right), d = \gamma\,\delta\left(C_x^2 + \rho_{yx}C_yC_x - \frac{C_x\lambda_{21}}{2}\right)$$

$$e = \gamma\left(\frac{3C_x\lambda_{21}}{2} - 3\rho_{yx}C_yC_x - \frac{5C_x^2}{2} - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} + \frac{(\lambda_{40} - 1)}{8}\right)$$

$$f = \gamma\,\delta\left(\frac{C_x\lambda_{21}}{2} - \rho_{yx}C_yC_x - 2C_x^2\right)$$

• Adichwal et al. (2016) suggested the following estimator for estimating C_y using the known S_x^2 as,

$$t_7 = \delta_1 \left[\frac{(1-\eta)s_x^2 + \eta S_x^2}{\eta s_x^2 + (1-\eta)S_x^2}\right]\hat{C}_y + (1-\delta_1)\left[\frac{\eta s_x^2 + (1-\eta)S_x^2}{(1-\eta)s_x^2 + \eta S_x^2}\right] \quad (2.25)$$

Where δ_1 and η are the characterizing constants to be determined such that the MSEs of the estimators t_7 is least.

The minimum MSEs of the estimator t_7 for the optimum values of these constants is,

$$MSE\ (t_7) = MSE(t_0) - \frac{1}{4}\gamma \frac{\left[(\lambda_{22}-1) - 2C_y\lambda_{12}\right]^2}{(\lambda_{04}-1)} C_y^4 \quad (2.26)$$

• Singh R et al. (2018) proposed ratio type, exponential ratio-type and difference-type estimators for estimating coefficient of variation of the study variable Y using variances of the auxiliary variables and are given below:

$$t_8 = \hat{C}_y \left(\frac{S_x^2}{s_x^2}\right)^{\alpha} \quad (2.27)$$

$$t_9 = \hat{C}_y exp\left\{\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\} \quad (2.28)$$

$$t_{10} = \hat{C}_y + d_2(S_x^2 - s_x^2) \quad (2.29)$$

$$MSE(t_8) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40}-1}{4} + \alpha^2(\lambda_{04}-1) - C_y\lambda_{30} - \alpha(\lambda_{22}-1) + 2\alpha C_y\lambda_{12}\right] \quad (2.30)$$

$$MSE(t_9) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{\beta^2(\lambda_{04}-1)}{4} - C_y\lambda_{30} + \beta C_y\lambda_{12} - \frac{\beta}{2}(\lambda_{22}-1)\right] \quad (2.31)$$

$$MSE(t_{10}) = \gamma\left[C_y^2\left(C_y^2 - C_y\lambda_{30} + \frac{\lambda_{40}-1}{4}\right) + d_2^2S_x^4(\lambda_{04}-1) + 2C_y^2d_2S_x^2\lambda_{12} - d_2C_yS_x^2(\lambda_{22}-1)\right] \quad (2.32)$$

Where

$$\alpha = \frac{\lambda_{22} - 1 - 2C_y\lambda_{21}}{2(\lambda_{04}-1)}, \beta = \frac{\lambda_{22} - 1 - 2C_y\lambda_{21}}{(\lambda_{04}-1)}, d_2 = \frac{C_y(\lambda_{22}-1) - 2C_y^2\lambda_{12}}{2\,S_x^2(\lambda_{04}-1)}$$

• Singh R et al. (2018) proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_8 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{11}^{AM} = \frac{\hat{C}_y}{2}\left[1 + \left(\frac{S_x^2}{s_x^2}\right)^{\alpha}\right] \quad (2.33)$$

$$t_{11}^{GM} = \hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)^{\alpha/2} \quad (2.34)$$

$$t_{11}^{GM} = 2\hat{C}_y\left[1 + \left(\frac{S_x^2}{s_x^2}\right)^{\alpha}\right]^{-1} \quad (2.35)$$

$$MSE(t_{11}^{j}) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{\alpha^2(\lambda_{04}-1)}{4} - C_y\lambda_{30} + \alpha C_y\lambda_{12} - \frac{\alpha}{2}(\lambda_{22}-1)\right] \quad (2.36)$$

Where $\alpha = \frac{\lambda_{22}-1-2C_y\lambda_{21}}{(\lambda_{04}-1)}$, $j = AM, GM, HM$

• Rajyaguru A et al. (2002) proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_9 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{12}^{AM} = \frac{\hat{C}_y}{2}\left[1 + exp\left\{\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\}\right] \quad (2.37)$$

$$t_{12}^{GM} = \hat{C}_y \; exp\left\{\frac{\beta}{2}\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\} \quad (2.38)$$

$$t_{12}^{HM} = 2\hat{C}_y\left[1 + exp\left\{-\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\}\right]^{-1} \quad (2.39)$$

$$MSE(t_{12}^{j}) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\beta^2(\lambda_{04} - 1)}{16} - C_y\lambda_{30} + \frac{\beta}{2}C_y\lambda_{12} - \frac{\beta}{4}(\lambda_{22} - 1)\right] \quad (2.40)$$

Where $\beta = \frac{2(\lambda_{22}-1)-4C_y\lambda_{21}}{(\lambda_{04}-1)}$, $\quad j = AM, GM, HM$

• Singh R et al. (2018) proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_8 and t_9 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{13}^{AM} = \frac{\hat{C}_y}{2}\left[\left(\frac{S_x^2}{s_x^2}\right)^{\alpha} + exp\left\{\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\}\right] \quad (2.41)$$

$$t_{13}^{GM} = \hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)^{\alpha/2} exp\left\{\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\} \quad (2.42)$$

$$t_{13}^{HM} = 2\hat{C}_y\left[\left(\frac{s_x^2}{S_x^2}\right)^{\alpha} + exp\left\{-\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\}\right]^{-1} \quad (2.43)$$

$$MSE(t_{13}^{j}) = \gamma C_y^2\left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{1}{4}\left(\alpha + \frac{\beta}{2}\right)^2(\lambda_{40} - 1) - C_y\lambda_{30} + \frac{1}{2}\left(\alpha + \frac{\beta}{2}\right)(\lambda_{22} - 1) + \left(\alpha + \frac{\beta}{2}\right)C_y\lambda_{12}\right] \quad (2.44)$$

Where , $\beta = 2\left(\frac{\lambda_{22}-1-2C_y\lambda_{12}}{(\lambda_{40}-1)} - \alpha\right)$

• Audu et al. (2021) suggested the following two difference-cum-ratio type estimators of C_y utilizing the known S_x^2 as,

$$t_{a2} = \left[\frac{\hat{C}_y}{2}\left(\frac{S_x^2}{s_x^2} + \frac{s_x^2}{S_x^2}\right) + w_3(S_x^2 - s_x^2) + w_4\hat{C}_y\right]\left(\frac{S_x^2}{s_x^2}\right) \quad (2.45)$$

$$MSE\,(t_{a2}) = C_y^2(a_1 + w_3^2 b_1 + w_4^2 c_1 + 2w_3 d_1 - 2w_4 e_1 - 2w_3 w_4 f_1) \quad (2.46)$$

Where

$$a_1 = \gamma\left((\lambda_{04} - 1) + C_y^2 + 2C_y\lambda_{12} - C_y\lambda_{30} - (\lambda_{22} - 1) + \frac{(\lambda_{40} - 1)}{4}\right), b_1 = \gamma\delta_1^2 C_x^2, \qquad \delta_1 = \frac{S_x^2}{C_y}$$

$$c_1 = 1 + \gamma\left(3(\lambda_{04} - 1) + 3C_y^2 + 4C_y\lambda_{12} - 2(\lambda_{22} - 1) - 2C_y\lambda_{30}\right)$$

$$d_1 = \gamma\,\delta_1\left((\lambda_{04} - 1) + C_y\lambda_{12} - \frac{(\lambda_{22} - 1)}{2}\right)$$

$$e_1 = \gamma \left(\frac{3(\lambda_{22}-1)}{2} - 3C_y\lambda_{12} - \frac{5(\lambda_{04}-1)}{2} - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} - \frac{(\lambda_{40}-1)}{8} \right)$$

$$f_1 = \gamma\delta_1 \left(\frac{(\lambda_{22}-1)}{2} - C_y\lambda_{12} - (\lambda_{04}-1) \right)$$

• Yunusa et al. (2021) suggested the following log type ratio estimator of C_y using the known S_x^2 as,

$$t_{14} = \hat{C}_y \left(\frac{Ln(S_x^2)}{Ln(s_x^2)} \right) \tag{2.47}$$

The MSE of the estimator t_{14}, up to the first order of approximation is,

$$MSE(t_{14}) = \gamma C_y^2 \left[C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{\lambda_{40}-1}{\{Ln(S_x^2)\}^2} - \frac{\{(\lambda_{22}-1) - 2C_y\lambda_{12}\}}{Ln(S_x^2)} - C_y\lambda_{30} \right] \tag{2.48}$$

3. PROPOSED ESTIMATORS

Having studied the estimators in section 2, we proposed four estimators for coefficient of variation based on information on a single auxiliary variable.

$$t_{p1} = \left[\frac{\hat{C}_y}{2}\left(\frac{\bar{X}}{\bar{x}} + \frac{\bar{x}}{\bar{X}} \right) + k_1(\bar{X} - \bar{x}) + k_2\hat{C}_y \right] \left\{ 2 - \left(\frac{\bar{x}}{\bar{X}} \right) \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right\} \tag{3.1}$$

$$t_{p2} = \left[\frac{\hat{C}_y}{2}\left(\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right) + k_3(\bar{X} - \bar{x}) + k_4\hat{C}_y \right] \left\{ 2 - \left(\frac{\bar{x}}{\bar{X}} \right) \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right\} \tag{3.2}$$

$$t_{p3} = \left[\frac{\hat{C}_y}{2}\left(\frac{S_x^2}{s_x^2} + \frac{s_x^2}{S_x^2} \right) + k_5(S_x^2 - s_x^2) + k_6\hat{C}_y \right] \left(2 - \frac{s_x^2}{S_x^2} \right) \tag{3.3}$$

$$t_{p4} = \left[\frac{\hat{C}_y}{2}\left(\exp\left(\frac{S_x^2 - s_x^2}{s_x^2 + S_x^2} \right) + \exp\left(\frac{s_x^2 - S_x^2}{s_x^2 + S_x^2} \right) \right) + k_7(S_x^2 - s_x^2) + k_8\hat{C}_y \right] \left(2 - \frac{s_x^2}{S_x^2} \right) \tag{3.4}$$

Expressing the estimators t_j, j= p1, p2, p3, p4 in terms of e_i, i = 0, 1, 2, 3 and simplifying respectively, we have

$$t_{p1} = \left[\begin{array}{l} \dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{2\bar{Y}(1+\epsilon_0)} \left[\dfrac{\bar{X}}{\bar{X}(1+\epsilon_1)} + \dfrac{\bar{X}(1+\epsilon_1)}{\bar{X}} \right] \\ +k_1\left(\bar{X} - \bar{X}(1+\epsilon_1)\right) + k_2 \dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{\bar{Y}(1+\epsilon_0)} \end{array} \right] \left[2 - \frac{\bar{X}(1+\epsilon_1)}{\bar{X}} exp\left(\frac{\bar{X}(1+\epsilon_1) - \bar{X}}{\bar{X}(1+\epsilon_1) + \bar{X}} \right) \right] \tag{3.5}$$

$$t_{p2} = \left[\begin{array}{l} \dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{2\bar{Y}(1+\epsilon_0)} \left[\begin{array}{r} exp\left(\dfrac{\bar{X}(1+\epsilon_1) - \bar{X}}{\bar{X}(1+\epsilon_1) + \bar{X}} \right) + \\ exp\left(\dfrac{\bar{X} - \bar{X}(1+\epsilon_1)}{\bar{X} + \bar{X}(1+\epsilon_1)} \right) \end{array} \right] \\ +k_3\left(\bar{X} - \bar{X}(1+\epsilon_1)\right) + k_4 \dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{\bar{Y}(1+\epsilon_0)} \end{array} \right] \left[2 - \frac{\bar{X}(1+\epsilon_1)}{\bar{X}} exp\left(\frac{\bar{X}(1+\epsilon_1) - \bar{X}}{\bar{X}(1+\epsilon_1) + \bar{X}} \right) \right] \tag{3.6}$$

$$t_{p3} = \left[\begin{array}{l} \dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{2\bar{Y}(1+\epsilon_0)} \left[\dfrac{S_x^2}{S_x^2(1+\epsilon_3)} + \dfrac{S_x^2(1+\epsilon_3)}{S_x^2} \right] \\ +k_5\left(S_x^2 - S_x^2(1+\epsilon_3)\right) + k_6 \dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{\bar{Y}(1+\epsilon_0)} \end{array} \right] \left[2 - \frac{S_x^2(1+\epsilon_3)}{S_x^2} \right] \tag{3.7}$$

$$t_{p4} = \begin{bmatrix} \dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{2\bar{Y}(1+\epsilon_0)}\left[exp\left(\dfrac{S_x^2(1+\epsilon_3)-S_x^2}{S_x^2\,(1+\epsilon_3)+S_x^2}\right)+exp\left(\dfrac{S_x^2-\;S_x^2\,(1+\epsilon_3)}{S_x^2+\;S_x^2\,(1+\epsilon_3)}\right)\right] \\ +k_7\left(S_x^2-\;S_x^2\,(1+\epsilon_3)\right)+k_8\,\dfrac{S_y(1+\epsilon_2)^{\frac{1}{2}}}{\bar{Y}(1+\epsilon_0)} \end{bmatrix}\left[2-\frac{S_x^2(1+\epsilon_3)}{S_x^2}\right] \tag{3.8}$$

$$t_{p1} = C_y\begin{bmatrix} \left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}+\dfrac{\epsilon_1^2}{8}-\dfrac{3\epsilon_1}{2}+\dfrac{3\epsilon_0\epsilon_1}{2}-\dfrac{3\epsilon_1\epsilon_2}{4}\right)-k_1\dfrac{\bar{X}}{C_y}\left(\epsilon_1-\dfrac{3\epsilon_1^2}{2}\right) \\ +k_2\left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}-\dfrac{3\epsilon_1}{2}+\dfrac{3\epsilon_0\epsilon_1}{2}-\dfrac{3\epsilon_1\epsilon_2}{4}-\dfrac{3\epsilon_1^2}{8}\right) \end{bmatrix} \tag{3.9}$$

$$t_{p2} = C_y\begin{bmatrix} \left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}-\dfrac{\epsilon_1^2}{4}-\dfrac{3\epsilon_1}{2}+\dfrac{3\epsilon_0\epsilon_1}{2}-\dfrac{3\epsilon_1\epsilon_2}{4}\right)-k_3\dfrac{\bar{X}}{C_y}\left(\epsilon_1-\dfrac{3\epsilon_1^2}{2}\right) \\ +k_4\left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}+\dfrac{\epsilon_1^2}{2}-\dfrac{3\epsilon_1}{2}+\dfrac{3\epsilon_0\epsilon_1}{2}-\dfrac{3\epsilon_1\epsilon_2}{4}-\dfrac{3\epsilon_1^2}{8}\right) \end{bmatrix} \tag{3.10}$$

$$t_{p3} = C_y\begin{bmatrix} \left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}+\dfrac{\epsilon_3^2}{2}-\epsilon_3+\epsilon_0\epsilon_3-\dfrac{\epsilon_2\epsilon_3}{2}\right)-k_5\dfrac{S_x^2}{C_y}\left(\epsilon_3-\epsilon_3^2\right) \\ +k_6\left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}-\epsilon_3+\epsilon_0\epsilon_3-\dfrac{\epsilon_2\epsilon_3}{2}\right) \end{bmatrix} \tag{3.11}$$

$$t_{p4} = C_y\begin{bmatrix} \left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}+\dfrac{\epsilon_3^2}{8}-\epsilon_3+\epsilon_0\epsilon_3-\dfrac{\epsilon_2\epsilon_3}{2}\right)-k_7\dfrac{S_x^2}{C_y}\left(\epsilon_3-\epsilon_3^2\right) \\ +k_8\left(1-\epsilon_0+\epsilon_0^2+\dfrac{\epsilon_2}{2}-\dfrac{\epsilon_0\epsilon_2}{2}-\dfrac{\epsilon_2^2}{8}-\epsilon_3+\epsilon_0\epsilon_3-\dfrac{\epsilon_2\epsilon_3}{2}\right) \end{bmatrix} \tag{3.12}$$

Subtracting C_y from all above four equations, we have, and taking expectations on both sides and putting values of different expectations, we get the biases of t_{p1}, t_{p2}, t_{p3} and t_{p4} up to the approximation of order one respectively as,

$$Bias(t_{p1}) = \begin{bmatrix} \gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}+\dfrac{C_x^2}{8}+\dfrac{3C_{xy}}{2}-\dfrac{3C_x\lambda_{21}}{4}\right)-k_1\dfrac{\bar{X}}{C_y}\gamma\left(-\dfrac{3C_x^2}{2}\right) \\ +k_2\left(1+\gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}-\dfrac{3C_x^2}{8}+\dfrac{3C_{xy}}{2}-\dfrac{3C_x\lambda_{21}}{4}\right)\right) \end{bmatrix} \tag{3.13}$$

$$Bias(t_{p2}) = \begin{bmatrix} \gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}-\dfrac{C_x^2}{4}+\dfrac{3C_{xy}}{2}-\dfrac{3C_x\lambda_{21}}{4}\right)-k_3\dfrac{\bar{X}}{C_y}\gamma\left(-\dfrac{3C_x^2}{2}\right) \\ +k_4\left(1+\gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}-\dfrac{3C_x^2}{8}+\dfrac{3C_{xy}}{2}-\dfrac{3C_x\lambda_{21}}{4}\right)\right) \end{bmatrix} \tag{3.14}$$

$$Bias(t_{p3}) = \begin{bmatrix} \gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}+\dfrac{(\lambda_{04}-1)}{2}+C_y\lambda_{12}-\dfrac{(\lambda_{22}-1)}{2}\right)-k_5\dfrac{S_x^2}{C_y}\gamma\left(-(\lambda_{40}-1)\right) \\ +k_6\left(1+\gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}+C_y\lambda_{12}-\dfrac{(\lambda_{22}-1)}{2}\right)\right) \end{bmatrix} \tag{3.15}$$

$$Bias(t_{p4}) = \begin{bmatrix} \gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}+\dfrac{(\lambda_{04}-1)}{8}+C_y\lambda_{12}-\dfrac{(\lambda_{22}-1)}{2}\right)-k_7\dfrac{S_x^2}{C_y}\gamma\left(-(\lambda_{40}-1)\right) \\ +k_8\left(1+\gamma\left(C_y^2-\dfrac{C_y\lambda_{30}}{2}-\dfrac{(\lambda_{40}-1)}{8}+C_y\lambda_{12}-\dfrac{(\lambda_{22}-1)}{2}\right)\right) \end{bmatrix} \tag{3.16}$$

Subtracting C_y from (3.9), (3.10), (3.11), & (3.12), squaring and taking expectation, we get the MSEs of the suggested estimators as

$$MSE\left(t_{p1}\right) = C_y^2\left(A_1 + k_1^2 B_1 + k_2^2 C_1 + 2k_1 D_1 - 2k_2 E_1 - 2k_1 k_2 F_1\right) \tag{3.17}$$

$$MSE\left(t_{p2}\right) = C_y^2\left(A_2 + k_3^2 B_2 + k_4^2 C_2 + 2k_3 D_2 - 2k_4 E_2 - 2k_3 k_4 F_2\right) \tag{3.18}$$

$$MSE\left(t_{p3}\right) = C_y^2\left(A_3 + k_5^2 B_3 + k_6^2 C_3 + 2k_5 D_3 - 2k_6 E_3 - 2k_5 k_6 F_3\right) \tag{3.19}$$

$$MSE\left(t_{p4}\right) = C_y^2\left(A_4 + k_7^2 B_4 + k_8^2 C_4 + 2k_7 D_4 - 2k_8 E_4 - 2k_7 k_8 F_4\right) \tag{3.20}$$

Where

$$A_1 = \gamma\left(C_y^2 + \frac{9}{4}C_x^2 + \frac{(\lambda_{40}-1)}{4} - C_y\lambda_{30} + 3C_{xy} - \frac{3C_x\lambda_{21}}{2}\right), B_1 = \gamma g^2 C_x^2, \quad g = \frac{\bar{X}}{C_y}$$

$$C_1 = 1 + \gamma\left(3C_y^2 + \frac{3}{2}C_x^2 + 6C_{xy} - 2C_y\lambda_{30} - 3C_x\lambda_{21}\right), \quad D_1 = \gamma g\left(\frac{3}{2}C_x^2 + C_{xy} - \frac{C_x\lambda_{21}}{2}\right)$$

$$E_1 = \gamma\left(\frac{9C_x\lambda_{21}}{4} - \frac{9C_{xy}}{2} - \frac{19}{8}C_x^2 - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} - \frac{(\lambda_{40}-1)}{8}\right), F_1 = \gamma g\left(\frac{C_x\lambda_{21}}{2} - C_{xy} - 3C_x^2\right)$$

$$A_2 = \gamma\left(C_y^2 + \frac{9}{4}C_x^2 + \frac{(\lambda_{40}-1)}{4} - C_y\lambda_{30} + 3C_{xy} - \frac{3C_x\lambda_{21}}{2}\right), B_2 = \gamma g^2 C_x^2, \quad g = \frac{\bar{X}}{C_y}$$

$$C_2 = 1 + \gamma\left(3C_y^2 + \frac{3}{2}C_x^2 + 6C_{xy} - 2C_y\lambda_{30} - 3C_x\lambda_{21}\right), \quad D_2 = \gamma g\left(\frac{3}{2}C_x^2 + C_{xy} - \frac{C_x\lambda_{21}}{2}\right)$$

$$E_2 = \gamma\left(\frac{9C_x\lambda_{21}}{4} - \frac{9C_{xy}}{2} - 2C_x^2 - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} - \frac{(\lambda_{40}-1)}{8}\right), F_2 = \gamma g\left(\frac{C_x\lambda_{21}}{2} - C_{xy} - 3C_x^2\right)$$

$$A_3 = \gamma\left(C_y^2 + (\lambda_{04}-1) + \frac{(\lambda_{40}-1)}{4} - C_y\lambda_{30} + 2C_y\lambda_{12} - (\lambda_{22}-1)\right), B_3 = \gamma g_1^2(\lambda_{04}-1),$$

$$C_3 = 1 + \gamma\left(3C_y^2 + (\lambda_{04}-1) + 4C_y\lambda_{12} - 2C_y\lambda_{30} - \frac{3(\lambda_{22}-1)}{2}\right)$$

$$D_3 = \gamma g_1\left((\lambda_{04}-1) + C_y\lambda_{12} - \frac{(\lambda_{22}-1)}{2}\right), \quad g_1 = \frac{S_x^2}{C_y}$$

$$E_3 = \gamma\left(\frac{3(\lambda_{22}-1)}{2} - \frac{3(\lambda_{04}-1)}{2} - 3C_y\lambda_{12} - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} - \frac{(\lambda_{40}-1)}{8}\right),$$

$$F_3 = \gamma g_1\left(\frac{(\lambda_{22}-1)}{2} - C_y\lambda_{12} - 2(\lambda_{04}-1)\right)$$

$$A_4 = \gamma\left(C_y^2 + (\lambda_{04}-1) + \frac{(\lambda_{40}-1)}{4} - C_y\lambda_{30} + 2C_y\lambda_{12} - (\lambda_{22}-1)\right), B_4 = \gamma g_1^2(\lambda_{04}-1),$$

$$C_4 = 1 + \gamma\left(3C_y^2 + (\lambda_{04}-1) + 4C_y\lambda_{12} - 2C_y\lambda_{30} - \frac{3(\lambda_{22}-1)}{2}\right),$$

$$D_4 = \gamma g_1\left((\lambda_{04}-1) + C_y\lambda_{12} - \frac{(\lambda_{22}-1)}{2}\right), \quad g_1 = \frac{S_x^2}{C_y}$$

$$E_4 = \gamma\left(\frac{3(\lambda_{22}-1)}{2} - \frac{9(\lambda_{04}-1)}{8} - 3C_y\lambda_{12} - 2C_y^2 + \frac{3C_y\lambda_{30}}{2} - \frac{(\lambda_{40}-1)}{8}\right),$$

$$F_4 = \gamma g_1\left(\frac{(\lambda_{22}-1)}{2} - C_y\lambda_{12} - 2(\lambda_{04}-1)\right)$$

Differentiating (3.17) partially with respect k_1 and k_2, equate to zero and solve for k_1 and k_2 simultaneously, we obtained $k_1 = \frac{C_1D_1-E_1F_1}{F_1^2-B_1C_1}$ $and\ k_2 = \frac{D_1F_1-B_1E_1}{F_1^2-B_1C_1}$

Substituting the results in (3.17), we obtained the minimum mean square error of t_{p1} denoted by $MSE(t_{p1})_{min}$

$$MSE(t_{p1})_{min} = C_y^2\left[A_1 + \frac{(C_1D_1^2 + B_1E_1^2 - 2D_1E_1F_1)}{F_1^2 - B_1C_1}\right]$$

Differentiating (3.18) partially with respect k_3 and k_4, equate to zero and solve for k_3 and k_4 simultaneously, we obtained $k_3 = \frac{C_2D_2-E_2F_2}{F_2^2-B_2C_2}$ $and\ k_4 = \frac{D_2F_2-B_2E_2}{F_2^2-B_2C_2}$

Substituting the results in (3.18), we obtained the minimum mean square error of t_{p2} denoted by $MSE(t_{p2})_{min}$

$$MSE(t_{p2})_{min} = C_y^2\left[A_2 + \frac{(C_2D_2^2 + B_2E_2^2 - 2D_2E_2F_2)}{F_2^2 - B_2C_2}\right]$$

Differentiating (3.19) partially with respect k_5 and k_6, equate to zero and solve for k_5 and k_6 simultaneously, we obtained $k_5 = \frac{C_3D_3-E_3F_3}{F_3^2-B_3C_3}$ $and\ k_6 = \frac{D_3F_3-B_3E_3}{F_3^2-B_3C_3}$

Substituting the results in (3.19), we obtained the minimum mean square error of t_{p3} denoted by $MSE(t_{p3})_{min}$

$$MSE(t_{p3})_{min} = C_y^2\left[A_3 + \frac{(C_3D_3^2 + B_3E_3^2 - 2D_3E_3F_3)}{F_3^2 - B_3C_3}\right]$$

Differentiating (3.20) partially with respect k_1 and k_2, equate to zero and solve for k_7 and k_8 simultaneously, we obtained $k_7 = \frac{C_4D_4-E_4F_4}{F_4^2-B_4C_4}$ $and\ k_8 = \frac{D_4F_4-B_4E_4}{F_4^2-B_4C_4}$

Substituting the results in (3.20), we obtained the minimum mean square error of t_{p4} denoted by $MSE(t_{p4})_{min}$

$$MSE(t_{p4})_{min} = C_y^2\left[A_4 + \frac{(C_4D_4^2 + B_4E_4^2 - 2D_4E_4F_4)}{F_4^2 - B_4C_4}\right]$$

4. EMPIRICAL STUDY

In this section, we carry out an empirical study to elucidate the performance of our proposed estimators with respect to some existing related estimators using data set below.

Population: [Murthy (1967) p.399]

X: Area under wheat in 1963, Y: Area under wheat in 1964

N=34, n=15, $\bar{X}$ = 208.88, $\bar{Y}$ = 199.44, C_x=0.72, C_y=0.75, ρ=0.98, λ_{21} = 1.0045, λ_{12} = 0.9406,

λ_{40} = 3.6161, λ_{04} = 2.8266, λ_{30} = 0.9206, λ_{03} = 2.52, λ_{22} = 3.0133

Table 1. MSEs and PREs of proposed and other estimators in the study

Estimators	MSE	PRE
Auxiliary Information : $\bar{X}$, $\bar{x}$		
t_0	0.008003575	100.00
t_{ar}	0.02715658	29.47
t_1	0.006868341	116.53
t_2	0.006868341	116.53
t_3	0.006868341	116.53
t_4^j	0.006868341	116.53
t_5^j	0.006868341	116.53
t_6^j	0.006868341	116.53
t_{a1}	0.006737495	118.79
t_{p1}	0.006033	132.66
t_{p2}	0.005659	141.43
Auxiliary Information : S_x^2, s_x^2		
t_7	0.00696301	114.94
t_8	0.006962763	114.95
t_9	0.006962763	114.95
t_{10}	0.006962763	114.95
t_{11}^j	0.006962763	114.95
t_{12}^j	0.006962763	114.95
t_{13}^j	0.006962763	114.95
t_{14}	0.00712551	112.32
t_{a2}	0.006013652	133.09
t_{p3}	0.006417	124.72
t_{p4}	0.004996	160.19

The formula for Percent Relative Efficiency (PRE) is

PRE (estimators) $=\frac{MSE(t_{RSS})}{MSE(estimator)} \times 100$

Table 1 shows the MSEs and PREs of the proposed and other estimators considered in the study. Results obtained revealed that proposed estimators has minimum MSEs and higher PREs compared to other competing existing estimators.

5. SIMULATION STUDY

We perform some simulation experiments to check the proposed estimator's relative efficiency (RE) with the other existing estimators.

The following steps have been used for the simulation:

1. We generated bivariate random observations of size N=1000 units from a bivariate normal distribution with parameters μ =100, σ =11, andμ_1 = 120, σ_1=14 and ρ= 0.9.

2. Similarly, generate the population-II with the parameters μ =3, σ =2, μ_1=5 and σ_1=3

3. A sample of size n = 20 has been selected from this simulated population.

4. Sample statistics that is the sample mean, sample variance, and the values of the suggested and competing estimators t_i of population CV are calculated for this sample.

5. Steps (3) and (4) are repeated m=10, 000 times.

6. The MSE of every estimator t_i is calculated through the formula, $MSE(t_i = \frac{1}{m}\sum_{j=1}^{m}(t_{ij} - \bar{T}_i)^2)$

7. The Percentage Relative Efficiency (PRE) of the estimator t_i over the mean per unit estimator t_0 given by,

PRE (t_i) $=\frac{V(t_0)}{MSE(t_i)} \times 100$ i=ar, 1, 2… p4

Table 2: MSE of competing and suggested estimators and PRE with respect to $\hat{C}_y$ for symmetric simulated population

Estimators	PRE for Population 1	PRE for Population 2
Auxiliary Information : $\bar{X}$, $\bar{x}$		
t_0	100	100.00
t_{ar}	101.02	99.44
t_1	115.63	118.98
t_2	115.56	118.76
t_3	115.54	118.77
t_4^j	115.63	118.36
t_5^j	115.34	118.76
t_6^j	115.83	118.35
t_{a1}	133.43	136.67
t_{p1}	141.67	161.29
t_{p2}	172.43	174.76
Auxiliary Information : S_x^2, s_x^2		
t_7	114.16	116.62
t_8	114.56	116.95
t_9	114.64	116.95
t_{10}	114.54	116.95
t_{11}^j	114.45	116.95
t_{12}^j	114.67	116.95
t_{13}^j	114.33	116.95
t_{14}	113.22	115.32
t_{a2}	135.29	133.09
t_{p3}	133.09	145.72
t_{p4}	175.99	180.19

Table 2 also shows that our proposed estimators perform better than the existing estimators as our proposed estimators have greater PRE.

6. CONCLUSION

• In this article we have proposed estimators for the coefficient of Variation (CV) using the information of auxiliary variables. The expressions for Bias and MSE of the suggested estimators have been derived up to the first order of approximation.

•Empirical approach and simulation study for comparing the efficiency of the proposed estimators with other estimators have been used.

•The results have been shown in the Tables 1 & 2. The Tables show that the proposed estimators turn out to be more efficient as compared to the other estimators for both populations.

•The proposed estimators are found to be rather improved in terms of lesser MSE and greater PRE as compared to the existing estimators in both real and simulated data sets.

•Based on our empirical study and simulation study, we can conclude that our proposed estimators can be preferred over the other estimators taken in this paper in several real situations.

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