

# Chi-Square Test in Time Series Data

**Abstract:** This study examines the application of Chi-Square test in time series data which considers the mixed model structure and linear trending curve. The test is applied to the seasonal variances of the Buys-Ballot table. The emphasis is to assess the validity of the Chi-Square test using empirical examples. One hundred simulated numerical examples are used to illustrate the applicability of the Chi-Square test. Using empirical example, the Chi-Square test successfully recorded 100% of times for the mixed model

**Keywords:** Buys-Ballot method, descriptive time series, mixed model, linear trend, seasonal variance, choice of model

## 1 Introduction

The purpose of descriptive time series analysis is to isolate the four time series components available in the series. That is to de-compose an observed series ( $X_t, t = 1, 2, \dots, n$ ) into components consisting the trend ( $T_t$ ), the seasonal ( $S_t$ ), the cyclical ( $C_t$ ) and irregular ( $e_t$ ) (Kendal and Ord [1], Chatfield [2]).

The decomposition models are the additive, multiplicative and mixed models. For short series, the cyclical component is superimposed into trend (Chatfield [2]) and observed series ( $X_t, t = 1, 2, \dots, n$ ) can be decomposed into the trend-cycle component ( $M_t$ ), seasonal component ( $S_t$ ) and irregular component ( $e_t$ ). Hence, the decomposition models are

$$\text{Additive Model } X_t = M_t + S_t + e_t \quad (1)$$

$$\text{Multiplicative Model } X_t = M_t \times S_t \times e_t \quad (2)$$

$$\text{Mixed Model } X_t = M_t \times S_t + e_t \quad (3)$$

It is assumed that the seasonal indices, when exist, has period  $s$ , that is it repeats after  $s$  time periods.  $S_{t+s} = S_t$ , for all  $t$  (4)

For equation (1), the assumption is that, the sum of seasonal indices over a complete period is zero, ie.

$$\sum_{j=1}^s S_{t+j} = 0 \quad (5)$$

Again, for equations (2) and (3), the assumption is that, the sum of seasonal indices over a complete period is s.

$$\sum_{j=1}^s S_{t+1} = s \quad (6)$$

The assumption is that, the irregular component  $e_t$  is the Gaussian  $N(0, \sigma_1^2)$  white noise for equations (1) and (3), while for equation (2),  $e_t$  is the Gaussian  $N(1, \sigma_2^2)$  white noise and that  $Cov(e_t, e_{t+k}) = 0, \forall k \neq 0$

One greatest problems identified in the use descriptive time series analysis is choice of suitable model for decomposition of any study series. That is when to use any of the three time series is uncertain. It is important to note that, use of wrong model will definitely lead to erroneous estimate of the components.

To select among additive, multiplicative and mixed models, many scholars have suggested different approaches. Puerto and Rivera [3] proposed the use of coefficient of variation of seasonal differences CV(d) and seasonal quotient CV(c) for choice of model. According to them, additive model is appropriate, if CV(c) is greater than CV(d) and it is multiplicative if CV(c) is less than or equal to CV(d). However, they did not provide any statistical test to justify the use. Chatfield [2] suggested the use of time plot to choose between additive and multiplicative model. However, no theoretical basis was proposed for the decision rule. Iwueze, et al. [4] proposed the use of the relationship between the seasonal average ( $\bar{X}_j, j=1,2,\dots,s$ ) and the seasonal standard deviations ( $\hat{\sigma}_j, j=1,2,\dots,s$ ) to choose the appropriate model for decomposition.

In proposing the Chi-Square test Nwogu, et al. [5] and Dozie, et al [6] assumed that (i) the underlying distribution of the variable,  $X_{ij}$ ,  $i = 1, 2, \dots, m$ ,  $j = 1, 2, \dots, s$ , under study is normal. (ii) the observations in each column,  $X_{ij}$ ,  $i = 1, 2, \dots, m$  are independent and (iii) that the s-column are independent. Therefore, they proposed use of Chi-Square test as a basis for choosing between mixed and multiplicative models. The seasonal variance for the mixed model is  $\sigma_{.j}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$ . Therefore, the test for choice between mixed and multiplicative model may be reduced to identify the mixed model whose seasonal variance is simply constant multiply of the square of seasonal effect.

## 2. Methodology

Chi-Square test is applied in the seasonal variance of the Buys-Ballot table and the model structure is mixed. Hence, the summary of the seasonal variance of the Buys-Ballot table for the mixed model derived by Nwogu, et al. [5] and Dozie, et al. [6] is shown in Table 1 and the linear trending curve

Table 1: Linear trending curve Seasonal Variance ( $\sigma_{.j}^2$ )

|          |  |
|----------|--|
| $a + bt$ | $\frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$ |
|----------|--|

In this arrangement,  $n$  is the total number of observations,  $s$  is the seasonal lag (number of columns),  $b$  is the slope,  $s_j$  is the seasonal indices,  $\sigma_1^2$  is the error variance, assumed equal to 1. Both the  $b$  and  $s_j$  are derivable from the seasonal average.

$$\bar{X}_{.j} = \left[ a + b \left( \frac{n-s}{2} \right) + b_j \right] * S_j \quad (7)$$

$$\equiv [\alpha + \beta_j] \times S_j \quad (8)$$

$$\text{where, } \alpha = a + b \left( \frac{n-s}{2} \right), \beta = b \quad (9)$$

Estimates of  $\alpha$  and  $\beta$  are derivable from the regression of  $\bar{X}_{.j}$  on  $j$  and estimates of  $S_j$

$$\text{is } \hat{S}_j = \frac{\bar{X}_{.j}}{\hat{\alpha} + \hat{\beta}_j} \quad (10)$$

For details of Buys-Ballot procedure, see Iwueze *et.al* [4], Nwogu *et.al* [5], Dozie *et.al* [6], Akpanta and Iwueze [7], Dozie and Ijeomah [8], Dozie and Nwanya [9], Dozie [10], Dozie and Uwaezuoke[11], Dozie and Ibebuogu [12], Dozie and Ihekuna [13]

## 2.1 Chi-Square Test

The seasonal variance for the mixed model is  $\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$ . Therefore, Chi-Square test proposed by Nwogu, et al. [5] and Dozie, et al. [6] when trend-cycle component is linear reduces to that of test null hypothesis.

$$H_0 : \sigma_j^2 = \sigma_{zj}^2$$

and the suitable model is mixed

$$H_1 : \sigma_j^2 \neq \sigma_{zj}^2$$

and the suitable model is not mixed

$\sigma_j^2 = (j = 1, 2, \dots, s)$  is the true variance of the  $j$ th season.

$$\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \quad (11)$$

and  $\sigma_1^2$  is the error variance assumed to be equal to 1

$$\text{Therefore, the statistic is } \chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{zj}^2} \quad (12)$$

follows the chi-square distribution with  $m-1$  degree of freedom,  $m$  is the number of observations in each column and  $s$  is the seasonal lag.

The interval  $\left[ \chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)} \right]$  contains the statistic (12) with  $100(1-\alpha)\%$  degree of confidence.

## 2.2 Levene's Test for Constant Variance

The Levene's test statistic for the null hypothesis

$$H_0 : \sigma_i^2 = \sigma_j^2$$

$H_1 : \sigma_i^2 \neq \sigma_j^2$  for at least one  $i \neq j$  is defined as

$$W = \frac{(N - K) \sum_{i=1}^k N_i \left( \bar{z}_i - \bar{z}_{..} \right)^2}{(k - 1) \sum_{i=1}^k \sum_{j=1}^{N_i} \left( z_{ij} - \bar{z}_i \right)^2} \quad (13)$$

where  $k$  is the number of different groups,  $N_i$  is the number of cases in the  $i$ th group,  $Y_{ij}$  is the value of the  $j$ th observation in the  $i$ th group.

$z_{ij}$  may be defined as deviation of  $y_{ij}$  from the mean ( $\bar{y}_i$ ) or from the median ( $\bar{y}_i$ ). That is

$$z_{ij} = y_{ij} - \bar{y}_i \text{ or } y_{ij} - \bar{y}_i \quad (14)$$

$$\bar{z}_i = \frac{1}{N_i} \sum_{j=1}^{N_i} z_{ij} \text{ is the mean of the } z_{ij} \text{ for group } i \quad (15)$$

$$\bar{z}_{..} = \frac{1}{N} \sum_{i=1}^k \sum_{j=1}^{N_i} z_{ij} \text{ is mean of all } z_{ij}. \quad (16)$$

The test statistic  $W$  approximately follows the F-distribution with  $k - 1$  and  $N - K$  degree of freedom. To suit the Buys-Ballot procedure, the Levene's test statistic is modified with

$$N = ms, \quad k = s, \quad N_i = m \text{ as}$$

$$W = \frac{(ms - s)}{s - 1} \left[ \frac{\sum_{j=1}^s m(\bar{z}_i - \bar{z}_{..})^2}{\sum_{j=1}^s \sum_{i=1}^m (z_{ij} - \bar{z}_{.j})^2} \right] \quad (17)$$

$$= \frac{s(m-1)}{s-1} \left[ \frac{m \sum_{i=1}^s (\bar{z}_i - \bar{z}_{..})^2}{\sum_{i=1}^s \sum_{j=1}^m (z_{ij} - \bar{z}_{.j})^2} \right] \quad (18)$$

### 3.0 Empirical Examples

This section discusses some of the empirical examples to illustrate the validity of the Chi-Square test. The empirical examples consist of both simulated and real life data. Sections 3.1 and 3.2 contain simulations results for the mixed model and real life example respectively.

#### 3.1 Simulations Results using the Mixed Model

The simulated series used consists 100 data set of 120 observations each simulated from

$$X_t = (a + bt) \times S_t + e_t, \text{ with } a = 1, b = 0.02, e_t \sim N(0,1). \text{ with } S_1 = 0.98, S_2 = 0.80, S_3 = 0.88,$$

$$S_4 = 1.04, S_5 = 0.96, S_6 = 1.22, S_7 = 1.27, S_8 = 1.32, S_9 = 0.96, S_{10} = 0.80, S_{11} = 0.88, S_{12} = 0.89$$

Using the MINITAB 17.0 version. The results of calculated value of the statistic from the simulated series are given in Table 2. The critical values at 5% level of significance and which for  $m-1=9$  degree of freedom, equal to 2.7 and 19.0. Under the null hypothesis that the appropriate model is mixed, the calculated value of the statistic in (12) is expected to lie within the interval, otherwise, it will be concluded that the data does not admit mixed model.

**Table 2:** Calculated Chi-Square for Mixed Model: The critical values for  $m - 1 = 9$  degree of freedom are 2.7 and 19

| Col | Series  |        |        |        |        |        |        |        |        |        |
|-----|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|     | 1       | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     |
| 1   | 10.4821 | 5.0921 | 7.5963 | 7.7977 | 7.1774 | 9.3257 | 9.4257 | 3.1520 | 3.7203 | 9.0872 |

|          |         |         |         |         |         |         |         |         |         |         |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 2        | 9.9157  | 10.8037 | 14.0504 | 3.8469  | 6.9436  | 5.4959  | 5.4413  | 12.9076 | 15.0742 | 12.7306 |
| 3        | 13.0416 | 5.1566  | 11.2798 | 8.6702  | 13.7272 | 8.7078  | 13.9401 | 6.8329  | 8.4794  | 6.9740  |
| 4        | 6.9085  | 12.2505 | 12.4492 | 14.8293 | 4.9423  | 7.2650  | 7.9545  | 7.3044  | 12.3286 | 12.3963 |
| 5        | 16.1748 | 10.4191 | 8.4801  | 10.5545 | 4.2798  | 17.1571 | 3.9333  | 5.2740  | 4.9656  | 12.9938 |
| 6        | 8.2899  | 16.4968 | 6.5897  | 3.3076  | 7.5989  | 9.7034  | 7.8557  | 15.1125 | 7.5076  | 6.6820  |
| 7        | 8.0266  | 10.3324 | 9.5820  | 16.5750 | 16.5760 | 4.9320  | 6.0845  | 9.1149  | 6.6597  | 12.0505 |
| 8        | 16.9952 | 2.5788  | 15.9770 | 9.7204  | 7.0275  | 13.3809 | 14.6644 | 9.7687  | 17.1803 | 9.2279  |
| 9        | 7.5575  | 3.3121  | 4.8592  | 7.8997  | 3.3922  | 4.2077  | 8.2510  | 4.6430  | 8.8565  | 8.9871  |
| 10       | 7.0924  | 14.5750 | 5.2015  | 8.1676  | 15.4410 | 11.3758 | 8.8120  | 13.1021 | 6.3318  | 3.9990  |
| 11       | 15.7355 | 12.9050 | 12.1345 | 6.8173  | 12.7856 | 4.7580  | 6.8402  | 6.0106  | 5.0302  | 7.8786  |
| 12       | 4.8516  | 8.6963  | 10.7143 | 2.7000  | 18.1478 | 5.0776  | 3.2408  | 12.7302 | 6.2707  | 8.8515  |
| Decision | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  |

**Table 2:** Calculated Chi-Square for Mixed Model (continue.)

| Col | Series  |         |         |         |         |         |         |         |         |         |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|     | 11      | 12      | 13      | 14      | 15      | 16      | 17      | 18      | 19      | 20      |
| 1   | 9.6094  | 14.2685 | 8.9455  | 4.3441  | 7.5303  | 13.2527 | 10.9166 | 5.9109  | 11.4996 | 4.7858  |
| 2   | 11.2284 | 10.0976 | 5.4781  | 12.1527 | 6.7144  | 8.8954  | 7.5931  | 7.3266  | 10.3937 | 6.3749  |
| 3   | 7.0312  | 5.5312  | 6.2797  | 15.7758 | 9.4296  | 5.5272  | 4.8595  | 10.4684 | 5.0037  | 12.0421 |
| 4   | 5.9895  | 9.5869  | 10.2811 | 9.0821  | 12.0272 | 8.1651  | 5.9885  | 11.6237 | 8.5224  | 7.9510  |
| 5   | 6.5379  | 4.7852  | 6.8220  | 7.7652  | 3.9335  | 6.2330  | 8.0709  | 4.5609  | 7.6014  | 13.8613 |
| 6   | 13.8824 | 5.7359  | 6.9559  | 6.2243  | 13.9755 | 9.0818  | 7.1403  | 13.1169 | 4.8691  | 5.9653  |
| 7   | 6.1482  | 7.9960  | 14.7312 | 13.7218 | 8.2492  | 11.2175 | 10.9352 | 16.8010 | 6.7297  | 8.7209  |
| 8   | 15.7446 | 5.4835  | 7.6453  | 6.5535  | 12.3519 | 5.6445  | 8.2929  | 5.1867  | 4.7716  | 17.7810 |
| 9   | 6.3454  | 6.6425  | 6.7641  | 13.7449 | 5.9355  | 8.0525  | 8.9194  | 4.1364  | 10.5184 | 7.6881  |
| 10  | 9.0031  | 7.5444  | 12.1063 | 5.8258  | 8.6373  | 11.5941 | 9.8175  | 15.5784 | 5.3195  | 4.6014  |
| 11  | 4.7313  | 16.6570 | 7.5044  | 3.1541  | 15.2454 | 6.3591  | 4.1399  | 4.2341  | 7.3711  | 10.6682 |
| 12  | 9.2021  | 7.9693  | 15.0332 | 9.3116  | 5.2108  | 6.6369  |         | 10.6694 | 18.1081 | 7.5191  |

|          |        |        |        |        |        |        |         |        |        |        |
|----------|--------|--------|--------|--------|--------|--------|---------|--------|--------|--------|
|          |        |        |        |        |        |        | 10.7285 |        |        |        |
| Decision | Accept | Accept | Accept | Accept | Accept | Accept | Accept  | Accept | Accept | Accept |

**Table 2:** Calculated Chi-Square for Mixed Model (continue.)

| Col      | Series  |         |         |         |         |         |         |         |         |         |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|          | 21      | 22      | 23      | 24      | 25      | 26      | 27      | 28      | 29      | 30      |
| 1        | 4.9925  | 6.6872  | 9.3019  | 8.4440  | 16.1231 | 4.6607  | 5.6206  | 12.8225 | 7.5324  | 7.2493  |
| 2        | 9.9520  | 7.6571  | 5.1852  | 3.8616  | 5.1936  | 16.0880 | 6.3393  | 9.3057  | 7.9772  | 11.2456 |
| 3        | 15.7739 | 9.2675  | 7.5613  | 7.3520  | 9.5426  | 3.5049  | 7.1022  | 7.4936  | 6.2753  | 13.7839 |
| 4        | 7.9491  | 8.8898  | 6.4351  | 3.4607  | 4.3826  | 8.9271  | 5.1558  | 6.6446  | 5.9960  | 4.3721  |
| 5        | 15.4095 | 7.8317  | 12.7285 | 2.6768  | 5.3428  | 8.9074  | 8.3408  | 14.2036 | 6.3303  | 5.2609  |
| 6        | 4.4069  | 11.7922 | 7.8317  | 16.1478 | 8.3204  | 10.8279 | 17.3949 | 7.7486  | 8.6958  | 6.9722  |
| 7        | 10.5531 | 10.0569 | 9.3096  | 14.4791 | 7.5278  | 4.7670  | 7.9232  | 10.5640 | 13.9507 | 8.6635  |
| 8        | 9.1184  | 5.5631  | 12.5454 | 7.8327  | 11.9395 | 9.5254  | 8.5931  | 9.3800  | 10.2167 | 8.2705  |
| 9        | 8.0141  | 8.2588  | 7.5039  | 6.1646  | 9.3666  | 5.1337  | 5.9292  | 8.7776  | 6.7719  | 7.2804  |
| 10       | 7.0412  | 3.1853  | 8.2615  | 11.9400 | 7.4869  | 5.5111  | 7.5531  | 13.3698 | 15.8669 | 10.7257 |
| 11       | 2.3845  | 8.1368  | 4.6635  | 3.9207  | 6.7324  | 14.2902 | 6.2990  | 8.0857  | 17.5143 | 10.1542 |
| 12       | 8.7773  | 15.5007 | 9.8291  | 3.6141  | 10.8442 | 10.7897 | 12.2908 | 10.2846 | 13.0388 | 10.5793 |
| Decision | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  |

**Table 2:** Calculated Chi-Square for Mixed Model (continue.)

| Col | Series  |         |         |         |         |         |         |         |         |         |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|     | 31      | 32      | 33      | 34      | 35      | 36      | 37      | 38      | 39      | 40      |
| 1   | 4.3301  | 5.2052  | 11.6108 | 8.4196  | 13.2948 | 8.7634  | 9.7346  | 7.9785  | 7.0880  | 5.2840  |
| 2   | 12.5402 | 8.0585  | 4.8702  | 8.7460  | 3.7832  | 3.6429  | 6.8156  | 7.5950  | 12.7919 | 18.6303 |
| 3   | 5.3371  | 6.7496  | 7.6270  | 5.8716  | 6.0977  | 4.2432  | 13.7521 | 13.4111 | 8.3039  | 3.9887  |
| 4   | 9.4842  | 14.6409 | 2.7816  | 6.4014  | 6.2389  | 12.0595 | 15.8052 | 5.1196  | 6.9820  | 13.5517 |
| 5   | 3.4534  | 8.2087  | 11.4106 | 4.0613  | 5.8798  | 6.6919  | 8.6991  | 12.1725 | 12.3841 | 10.2771 |
| 6   | 19.0150 | 8.7136  | 5.6496  | 14.7791 | 12.0007 | 17.3150 | 10.3676 | 17.1883 | 8.5542  | 5.1544  |

|          |         |         |         |         |         |         |         |         |         |         |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 7        | 13.7678 | 17.1864 | 9.5216  | 11.3132 | 5.1484  | 7.9064  | 13.3484 | 6.0798  | 10.6810 | 14.6176 |
| 8        | 11.9285 | 9.2793  | 9.5300  | 10.9811 | 6.4705  | 8.3198  | 10.1218 | 4.7634  | 8.9523  | 7.3645  |
| 9        | 12.2726 | 7.0765  | 12.9150 | 11.2232 | 9.3524  | 6.5209  | 5.8827  | 10.9141 | 14.8175 | 14.1636 |
| 10       | 4.3265  | 9.1949  | 6.7857  | 10.5474 | 10.8313 | 17.9135 | 2.8196  | 8.8829  | 7.6546  | 6.6831  |
| 11       | 10.4583 | 3.4714  | 9.7626  | 5.9101  | 10.5449 | 6.0276  | 2.9715  | 12.7119 | 7.4249  | 4.9223  |
| 12       | 8.4716  | 4.9938  | 9.8488  | 5.7529  | 8.1471  | 4.4695  | 5.8214  | 3.4978  | 3.6641  | 7.1728  |
| Decision | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  |

**Table 2:** Calculated Chi-Square for Mixed Model (continue.)

| Col      | Series  |         |         |         |         |         |         |         |         |         |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|          | 41      | 42      | 43      | 44      | 45      | 46      | 47      | 48      | 49      | 50      |
| 1        | 10.3691 | 8.5130  | 7.0330  | 4.3074  | 7.3176  | 6.8423  | 6.7538  | 9.7079  | 6.4727  | 7.0871  |
| 2        | 11.6339 | 7.8723  | 6.2565  | 12.1941 | 11.3870 | 3.4437  | 16.2822 | 10.9314 | 5.9619  | 7.6211  |
| 3        | 4.4322  | 9.6867  | 14.3807 | 3.1714  | 9.9120  | 13.7780 | 10.9761 | 7.3828  | 10.6245 | 8.2764  |
| 4        | 8.5585  | 4.6870  | 4.5882  | 9.7055  | 2.9632  | 8.8073  | 14.5320 | 7.9623  | 5.9960  | 7.0251  |
| 5        | 5.6940  | 4.8856  | 10.7079 | 6.3863  | 10.2996 | 4.3244  | 5.4493  | 10.7636 | 12.7787 | 5.6149  |
| 6        | 7.5931  | 11.1882 | 5.7294  | 4.9469  | 9.5238  | 6.4484  | 9.4625  | 6.7796  | 5.4770  | 7.0564  |
| 7        | 15.1459 | 3.8121  | 10.5215 | 14.1749 | 11.2227 | 12.5596 | 8.8130  | 6.9940  | 18.6699 | 17.7016 |
| 8        | 6.3324  | 8.1706  | 2.8706  | 14.4109 | 4.4109  | 14.4099 | 7.0864  | 14.2041 | 11.1815 | 8.7890  |
| 9        | 7.8135  | 3.4254  | 9.6682  | 9.4238  | 8.8953  | 7.4470  | 6.5467  | 5.0319  | 5.9216  | 11.6033 |
| 10       | 3.0301  | 10.6712 | 6.3942  | 15.3995 | 7.6932  | 10.7740 | 4.9069  | 14.8724 | 3.6535  | 18.5999 |
| 11       | 11.0178 | 18.1801 | 8.6928  | 11.8529 | 16.1330 | 12.1989 | 12.4033 | 4.3272  | 5.8165  | 5.5348  |
| 12       | 12.5979 | 10.8027 | 7.3551  | 12.0797 | 6.6074  | 7.7126  | 8.1004  | 5.4474  | 8.1823  | 10.3745 |
| Decision | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  | Accept  |

### 3.2 Real Life Example.

The real life example is based on the monthly time series data on church marriages in Owerri, Imo State, Nigeria, for the period 2008 to 2019 given in Appendix A while the time plots are in Figures 3.1 and 3.2. The column (monthly) variances are shown in Table 3. The first step is to check whether the data admits the additive model. The modified Levene's test statistic shown in (18) is used. The null hypothesis that the data admits additive model is reject if  $W$  is greater than the tabulated value, for which  $\alpha = 0.05$  level of significance and  $m-1=11$  degree of freedom equal to 1.82 or do not reject  $H_0$  otherwise. When compared with the critical value (1.82).  $W$  is greater, suggesting that the data does not admit the additive model.

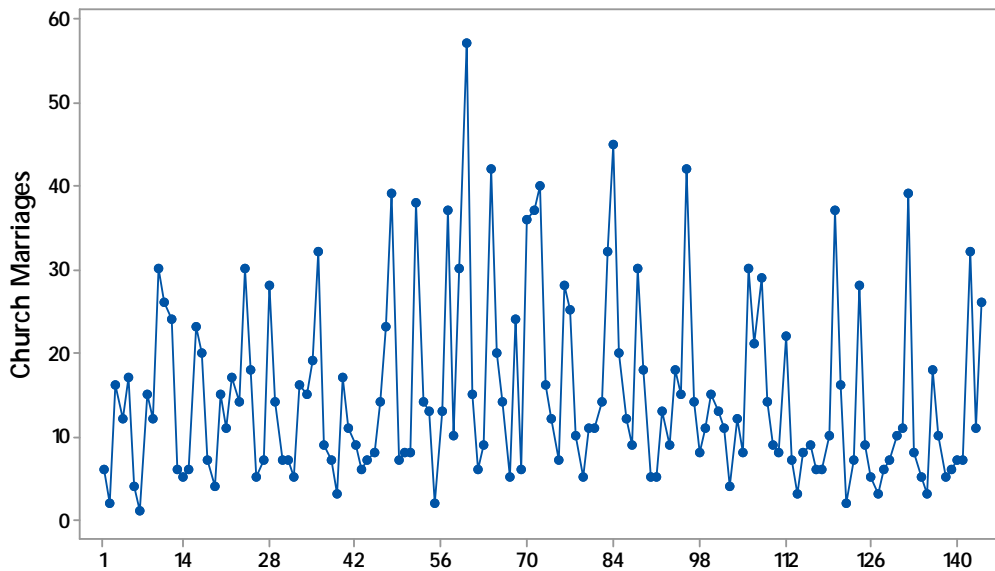


Fig 3.1: Time plot of original series of church marriages

From appendix A and Table 5

$$W = \frac{12 * (12 - 1) (685.66)}{(12 - 1) (1987.05)} = \frac{90507.12}{21857.55} = 4.14$$

Table 3: Deviations of the Observed Values from Means ( $Z_{ij} = \left| y_{ij} - \bar{y}_{.j} \right|$ )

|             | Jan          | Feb          | Mar          | Apr          | May          | Jun          | Jul          | Aug          | Sep          | Oct           | Nov          | Dec          | total         | $\bar{z}_i$ | $\sigma_i$  |
|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|---------------|--------------|--------------|---------------|-------------|-------------|
| 2008        | 6.42         | 4.75         | 8.17         | 13.08        | 2.17         | 3.75         | 3.67         | 3.58         | 0.50         | 10.67         | 5.25         | 12.67        | 74.67         | 6.22        | 4.09        |
| 2009        | 6.42         | 1.75         | 1.83         | 2.08         | 5.17         | 0.75         | 0.67         | 3.58         | 0.50         | 2.33          | 6.75         | 6.67         | 38.50         | 3.21        | 2.42        |
| 2010        | 5.58         | 1.75         | 0.83         | 2.92         | 0.83         | 0.75         | 2.33         | 6.42         | 4.50         | 4.33          | 1.75         | 4.67         | 36.67         | 3.06        | 1.98        |
| 2011        | 3.42         | 0.25         | 4.83         | 8.08         | 3.83         | 1.25         | 1.33         | 4.42         | 3.50         | 5.33          | 2.25         | 2.33         | 40.83         | 3.40        | 2.14        |
| 2012        | 5.42         | 1.25         | 0.17         | 12.92        | 0.83         | 5.25         | 2.67         | 1.58         | 25.50        | 9.33          | 9.25         | 20.33        | 94.50         | 7.88        | 8.13        |
| 2013        | 2.58         | 0.75         | 1.17         | 16.92        | 5.17         | 6.25         | 0.33         | 12.58        | 5.50         | 16.67         | 16.25        | 3.33         | 87.50         | 7.29        | 6.50        |
| 2014        | 3.58         | 5.25         | 0.83         | 2.92         | 10.17        | 2.25         | 0.33         | 0.42         | 0.50         | 5.33          | 11.25        | 8.33         | 51.17         | 4.26        | 3.87        |
| 2015        | 7.58         | 5.25         | 1.17         | 4.92         | 3.17         | 2.75         | 0.33         | 1.58         | 2.50         | 1.33          | 5.75         | 5.33         | 41.67         | 3.47        | 2.25        |
| 2016        | 1.58         | 1.25         | 3.17         | 10.08        | 1.83         | 3.25         | 0.67         | 0.58         | 3.50         | 10.67         | 0.25         | 7.67         | 44.50         | 3.71        | 3.70        |
| 2017        | 1.58         | 2.25         | 0.17         | 3.08         | 7.83         | 4.75         | 3.33         | 2.42         | 5.50         | 13.33         | 10.75        | 0.33         | 55.33         | 4.61        | 4.12        |
| 2018        | 3.58         | 4.75         | 0.83         | 2.92         | 5.83         | 2.75         | 1.67         | 5.42         | 4.50         | 9.33          | 9.75         | 2.33         | 53.67         | 4.47        | 2.80        |
| 2019        | 4.42         | 1.75         | 4.83         | 7.08         | 4.83         | 2.75         | 1.33         | 4.42         | 4.50         | 12.67         | 9.75         | 10.67        | 69.00         | 5.75        | 3.58        |
| Total       | <b>52.17</b> | <b>31.00</b> | <b>28.00</b> | <b>87.00</b> | <b>51.67</b> | <b>36.50</b> | <b>18.67</b> | <b>47.00</b> | <b>61.00</b> | <b>101.33</b> | <b>89.00</b> | <b>84.67</b> | <b>688.00</b> |             |             |
| $\bar{z}_j$ | <b>4.35</b>  | <b>2.58</b>  | <b>2.33</b>  | <b>7.25</b>  | <b>4.31</b>  | <b>3.04</b>  | <b>1.56</b>  | <b>3.92</b>  | <b>5.08</b>  | <b>8.44</b>   | <b>7.42</b>  | <b>7.06</b>  |               | <b>4.78</b> |             |
| $\sigma_j$  | <b>1.96</b>  | <b>1.86</b>  | <b>2.46</b>  | <b>4.99</b>  | <b>2.82</b>  | <b>1.74</b>  | <b>1.19</b>  | <b>3.31</b>  | <b>6.69</b>  | <b>4.72</b>   | <b>4.63</b>  | <b>5.52</b>  |               |             | <b>4.32</b> |

Table 4: Square of Deviations of the Observed Values from Seasonal Means

$$\left( z_{ij} - \bar{z}_j \right)^2$$

|           | Jan.  | Feb   | Mar   | Apr.   | Ma<br>y | Jun.  | Jul.  | Aug.   | Sept.  | Oct.   | Nov.   | Dec.   | total          |
|-----------|-------|-------|-------|--------|---------|-------|-------|--------|--------|--------|--------|--------|----------------|
| 2008      | 4.28  | 4.69  | 34.03 | 34.03  | 4.57    | 0.50  | 4.46  | 0.11   | 21.01  | 4.94   | 4.69   | 31.48  | 148.80         |
| 2009      | 4.28  | 0.69  | 0.25  | 26.69  | 0.74    | 5.25  | 0.79  | 0.11   | 21.01  | 37.35  | 0.44   | 0.15   | 97.76          |
| 2010      | 1.53  | 0.69  | 2.25  | 18.78  | 12.06   | 5.25  | 0.60  | 6.25   | 0.34   | 16.90  | 32.11  | 5.71   | 102.47         |
| 2011      | 0.87  | 5.44  | 6.25  | 0.69   | 0.22    | 3.21  | 0.05  | 0.25   | 2.51   | 9.68   | 26.69  | 22.30  | 78.17          |
| 2012      | 1.14  | 1.78  | 4.69  | 32.11  | 12.06   | 4.88  | 1.23  | 5.44   | 416.84 | 0.79   | 3.36   | 176.30 | 660.63         |
| 2013      | 3.11  | 3.36  | 1.36  | 93.44  | 0.74    | 10.29 | 1.49  | 75.11  | 0.17   | 67.60  | 78.03  | 13.85  | 348.58         |
| 2014      | 0.58  | 7.11  | 2.25  | 18.78  | 34.35   | 0.63  | 1.49  | 12.25  | 21.01  | 9.68   | 14.69  | 1.63   | 124.46         |
| 2015      | 10.47 | 7.11  | 1.36  | 5.44   | 1.30    | 0.09  | 1.49  | 5.44   | 6.67   | 50.57  | 2.78   | 2.97   | 95.69          |
| 2016      | 7.64  | 1.78  | 0.69  | 8.03   | 6.11    | 0.04  | 0.79  | 11.11  | 2.51   | 4.94   | 51.36  | 0.37   | 95.38          |
| 2017      | 7.64  | 0.11  | 4.69  | 17.36  | 12.45   | 2.92  | 3.16  | 2.25   | 0.17   | 23.90  | 11.11  | 45.19  | 130.95         |
| 2018      | 0.58  | 4.69  | 2.25  | 18.78  | 2.33    | 0.09  | 0.01  | 2.25   | 0.34   | 0.79   | 5.44   | 22.30  | 59.86          |
| 2019      | 0.05  | 0.69  | 6.25  | 0.03   | 0.28    | 0.09  | 0.05  | 0.25   | 0.34   | 17.83  | 5.44   | 13.04  | 44.29          |
| Tot<br>al | 42.14 | 38.17 | 66.33 | 274.17 | 87.21   | 33.23 | 15.63 | 120.83 | 492.92 | 244.96 | 236.17 | 335.30 | <b>1987.05</b> |

Table 5: Calculation of  $m \left( \bar{z}_{.j} - \bar{z}_{..} \right)^2$   $m = 12$

| $\bar{z}_{.j}$ | $\bar{z}_{..}$ | $\bar{z}_{.j} - \bar{z}_{..}$ | $\left( \bar{z}_{.j} - \bar{z}_{..} \right)^2$ | $12 \times \left( \bar{z}_{.j} - \bar{z}_{..} \right)^2$ |
|----------------|----------------|-------------------------------|--|--|
| <b>4.35</b>    | 4.78           | -0.43                         | 0.18   | 2.22   |
| <b>2.58</b>    | 4.78           | -2.20                         | 4.84   | 58.08  |
| <b>2.33</b>    | 4.78           | -2.45                         | 6.00   | 72.03  |
| <b>7.25</b>    | 4.78           | 2.47                          | 6.10   | 73.21  |

|             |      |       |       |        |
|-------------|------|-------|-------|--------|
| <b>4.31</b> | 4.78 | -0.47 | 0.22  | 2.65   |
| <b>3.04</b> | 4.78 | -1.74 | 3.03  | 36.33  |
| <b>1.56</b> | 4.78 | -3.22 | 10.37 | 124.42 |
| <b>3.92</b> | 4.78 | -0.86 | 0.74  | 8.88   |
| <b>5.08</b> | 4.78 | 0.30  | 0.09  | 1.08   |
| <b>8.44</b> | 4.78 | 3.66  | 13.40 | 160.75 |
| <b>7.42</b> | 4.78 | 2.64  | 6.97  | 83.64  |
| <b>7.06</b> | 4.78 | 2.28  | 5.20  | 62.38  |
|             |      |       |       | 685.66 |

Having admitted that the data does not additive model, the choice now choose lies between mixed and multiplicative models. According to the chi-square test by Nwogu, *et al*, [5] and Dozie *et al*, [6] the null hypothesis that the data admits the mixed model is rejected, if the statistic defined in (12) lies outside the interval  $\left[ \chi_{\frac{\alpha}{2},(m-1)}^2, \chi_{1-\frac{\alpha}{2},(m-1)}^2 \right]$  which for  $\alpha = 0.05$  level of significance and  $m - 1 = 11$  degrees of freedom, equals (3.8, 21.9) or do not reject  $H_0$  otherwise, and from (12) the calculated values,  $\chi_{cal}^2$  given in Table 6 were obtained. When compared with the critical values (3.8 and 21.9), all the calculated values lie outside the interval, indicating that the data does not admit mixed model.

However, there is indication the choice of model may be affected by violation of the underlying assumptions, therefore, there is need to evaluate data for transformation to meet the constant variance and normality assumptions in the distribution. When the column variances of the transformed series given in Table 7 are subjected to test for constant variance, the calculated Levene's test statistic (0.78) is less than the tabulated (1.82) at  $\alpha = 0.05$  level of significance and  $m - 1 = 11$  degrees of freedom. This indicates that the variance is constant and the transformed series admits additive model.

Table 6: Seasonal effects ( $S_j$ ), estimate of the column variance ( $\hat{\sigma}_j^2$ ) and Calculated Chi-square ( $\chi_{cal}^2$ )

|       |      |      |      |      |      |      |      |      |      |      |      |      |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|
| j     | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   |
| $S_j$ | 1.68 | 0.77 | 0.77 | 2.18 | 1.15 | 0.54 | 0.30 | 0.67 | 0.63 | 0.98 | 0.98 | 1.63 |

|                    |      |      |      |      |      |      |      |      |      |      |      |      |
|--------------------|------|------|------|------|------|------|------|------|------|------|------|------|
| $\hat{\sigma}_j^2$ | 24.5 | 10.8 | 11.9 | 82.3 | 28.2 | 13.1 | 4.1  | 27.7 | 73.0 | 100  | 81.5 | 84.8 |
| $\chi_{cal}^2$     | 0.03 | 0.06 | 0.06 | 0.05 | 0.07 | 0.14 | 0.14 | 0.19 | 0.58 | 0.32 | 0.26 | 0.10 |

From appendix B and Table 6,  $\sigma_1^2 = 1$ ,  $b = 0.1143$ ,  $n = 144$ ,  $m = 12$

Therefore, from (6),  $\sigma_{sj}^2 = (0.1143)^2 \times 144 \left( \frac{144+12}{12} \right) S_j^2 + 1$

Table 7: Transformed series of Deviations of the Observed Values from Means ( $Z_{ij} = \left| y_{ij} - \bar{y}_{.j} \right|$ )

|                | Jan.        | Feb.        | Mar.        | Apr.        | May         | Jun.        | Jul.        | Aug.        | Sept.       | Oct.        | Nov.        | Dec.        | total | $\bar{z}_i$ | $\sigma_i$ |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------|-------------|------------|
| 2008           | 0.64        | 1.08        | 0.81        | 0.67        | 0.20        | 0.56        | 1.42        | 0.37        | 0.19        | 0.57        | 0.32        | 0.40        | 7.22  | 0.60        | 0.36       |
| 2009           | 0.64        | 0.16        | 0.17        | 0.02        | 0.36        | 0.00        | 0.03        | 0.37        | 0.11        | 0.00        | 0.30        | 0.17        | 2.35  | 0.20        | 0.19       |
| 2010           | 0.46        | 0.16        | 0.02        | 0.17        | 0.00        | 0.00        | 0.53        | 0.73        | 0.48        | 0.12        | 0.00        | 0.11        | 2.79  | 0.23        | 0.25       |
| 2011           | 0.24        | 0.17        | 0.86        | 0.33        | 0.24        | 0.25        | 0.38        | 0.40        | 0.21        | 0.19        | 0.19        | 0.09        | 3.55  | 0.30        | 0.20       |
| 2012           | 0.49        | 0.31        | 0.12        | 0.48        | 0.00        | 0.62        | 0.72        | 0.22        | 1.32        | 0.53        | 0.46        | 0.47        | 5.74  | 0.48        | 0.34       |
| 2013           | 0.27        | 0.02        | 0.23        | 0.58        | 0.36        | 0.70        | 0.19        | 0.84        | 0.50        | 0.75        | 0.67        | 0.12        | 5.23  | 0.44        | 0.27       |
| 2014           | 0.34        | 0.71        | 0.02        | 0.17        | 0.58        | 0.36        | 0.19        | 0.06        | 0.11        | 0.19        | 0.52        | 0.23        | 3.49  | 0.29        | 0.22       |
| 2015           | 0.56        | 0.71        | 0.23        | 0.24        | 0.26        | 0.33        | 0.19        | 0.22        | 0.09        | 0.06        | 0.23        | 0.16        | 3.31  | 0.28        | 0.19       |
| 2016           | 0.20        | 0.31        | 0.44        | 0.45        | 0.07        | 0.46        | 0.03        | 0.14        | 0.21        | 0.57        | 0.10        | 0.21        | 3.19  | 0.27        | 0.18       |
| 2017           | 0.20        | 0.42        | 0.12        | 0.07        | 0.69        | 0.84        | 0.66        | 0.14        | 0.50        | 1.04        | 0.64        | 0.04        | 5.37  | 0.45        | 0.33       |
| 2018           | 0.34        | 1.08        | 0.02        | 0.17        | 0.44        | 0.33        | 0.32        | 0.55        | 0.35        | 0.53        | 0.54        | 0.09        | 4.75  | 0.40        | 0.28       |
| 2019           | 0.36        | 0.16        | 0.86        | 0.27        | 0.33        | 0.33        | 0.38        | 0.40        | 0.35        | 0.64        | 0.54        | 0.32        | 4.93  | 0.41        | 0.19       |
| Total          | <b>4.74</b> | <b>5.30</b> | <b>3.90</b> | <b>3.63</b> | <b>3.53</b> | <b>4.80</b> | <b>5.04</b> | <b>4.43</b> | <b>4.42</b> | <b>5.19</b> | <b>4.53</b> | <b>2.40</b> | 51.91 |             |            |
| $\bar{z}_{.j}$ | <b>0.40</b> | <b>0.44</b> | <b>0.32</b> | <b>0.30</b> | <b>0.29</b> | <b>0.40</b> | <b>0.42</b> | <b>0.37</b> | <b>0.37</b> | <b>0.43</b> | <b>0.38</b> | <b>0.20</b> |       | 0.36        |            |
| $\sigma_{.j}$  | <b>0.16</b> | <b>0.37</b> | <b>0.34</b> | <b>0.20</b> | <b>0.21</b> | <b>0.25</b> | <b>0.38</b> | <b>0.24</b> | <b>0.34</b> | <b>0.32</b> | <b>0.22</b> | <b>0.13</b> |       |             | 0.27       |

Table 8: Transformed Series of Square of Deviations of the Observed Values from Seasonal Means

$$\left( z_{ij} - \bar{z}_{.j} \right)^2$$

|              | Jan.        | Feb.        | Mar.        | Apr.        | May         | Jun.        | Jul.        | Aug.        | Sept.       | Oct.        | Nov.        | Dec.        | total       |
|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 2008         | 0.06        | 0.41        | 0.24        | 0.14        | 0.01        | 0.02        | 0.99        | 0.00        | 0.03        | 0.02        | 0.00        | 0.04        | 1.96        |
| 2009         | 0.06        | 0.08        | 0.02        | 0.08        | 0.00        | 0.16        | 0.15        | 0.00        | 0.07        | 0.18        | 0.01        | 0.00        | 0.81        |
| 2010         | 0.00        | 0.08        | 0.09        | 0.02        | 0.08        | 0.16        | 0.01        | 0.13        | 0.01        | 0.10        | 0.14        | 0.01        | 0.84        |
| 2011         | 0.02        | 0.07        | 0.29        | 0.00        | 0.00        | 0.02        | 0.00        | 0.00        | 0.02        | 0.06        | 0.03        | 0.01        | 0.54        |
| 2012         | 0.01        | 0.02        | 0.04        | 0.03        | 0.08        | 0.05        | 0.09        | 0.02        | 0.91        | 0.01        | 0.01        | 0.07        | 1.34        |
| 2013         | 0.01        | 0.18        | 0.01        | 0.08        | 0.00        | 0.09        | 0.05        | 0.22        | 0.02        | 0.10        | 0.08        | 0.01        | 0.85        |
| 2014         | 0.00        | 0.07        | 0.09        | 0.02        | 0.08        | 0.00        | 0.05        | 0.10        | 0.07        | 0.06        | 0.02        | 0.00        | 0.57        |
| 2015         | 0.03        | 0.07        | 0.01        | 0.00        | 0.00        | 0.00        | 0.05        | 0.02        | 0.07        | 0.14        | 0.02        | 0.00        | 0.43        |
| 2016         | 0.04        | 0.02        | 0.01        | 0.02        | 0.05        | 0.00        | 0.15        | 0.05        | 0.02        | 0.02        | 0.08        | 0.00        | 0.47        |
| 2017         | 0.04        | 0.00        | 0.04        | 0.05        | 0.16        | 0.20        | 0.06        | 0.05        | 0.02        | 0.37        | 0.07        | 0.03        | 1.08        |
| 2018         | 0.00        | 0.41        | 0.09        | 0.02        | 0.02        | 0.00        | 0.01        | 0.03        | 0.00        | 0.01        | 0.03        | 0.01        | 0.64        |
| 2019         | 0.002       | 0.08        | 0.29        | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.04        | 0.03        | 0.01        | 0.46        |
| <b>Total</b> | <b>0.28</b> | <b>1.48</b> | <b>1.24</b> | <b>0.46</b> | <b>0.50</b> | <b>0.71</b> | <b>1.63</b> | <b>0.63</b> | <b>1.24</b> | <b>1.10</b> | <b>0.52</b> | <b>0.19</b> | <b>9.99</b> |

Table 9: Calculation of  $m \left( \bar{z}_{.j} - \bar{z}_{..} \right)^2$  m =12

| $\bar{z}_{.j}$ | $\bar{z}_{..}$ | $\bar{z}_{.j} - \bar{z}_{..}$ | $\left( \bar{z}_{.j} - \bar{z}_{..} \right)^2$ | $12 \times \left( \bar{z}_{.j} - \bar{z}_{..} \right)^2$ |
|----------------|----------------|-------------------------------|--|--|
| 0.4            | 0.36           | 0.040                         | 0.002  | 0.019  |
| 0.44           | 0.36           | 0.080                         | 0.006  | 0.077  |
| 0.32           | 0.36           | -0.040                        | 0.002  | 0.019  |
| 0.3            | 0.36           | -0.060                        | 0.004  | 0.043  |
| 0.29           | 0.36           | -0.070                        | 0.005  | 0.059  |
| 0.4            | 0.36           | 0.040                         | 0.002  | 0.019  |
| 0.42           | 0.36           | 0.060                         | 0.004  | 0.043  |
| 0.37           | 0.36           | 0.010                         | 0.000  | 0.001  |
| 0.37           | 0.36           | 0.010                         | 0.000  | 0.001  |
| 0.43           | 0.36           | 0.070                         | 0.005  | 0.059  |
| 0.38           | 0.36           | 0.020                         | 0.000  | 0.005  |
| 0.2            | 0.36           | -0.160                        | 0.026  | 0.307  |
|                |                |                               |  | <b>0.653</b>   |

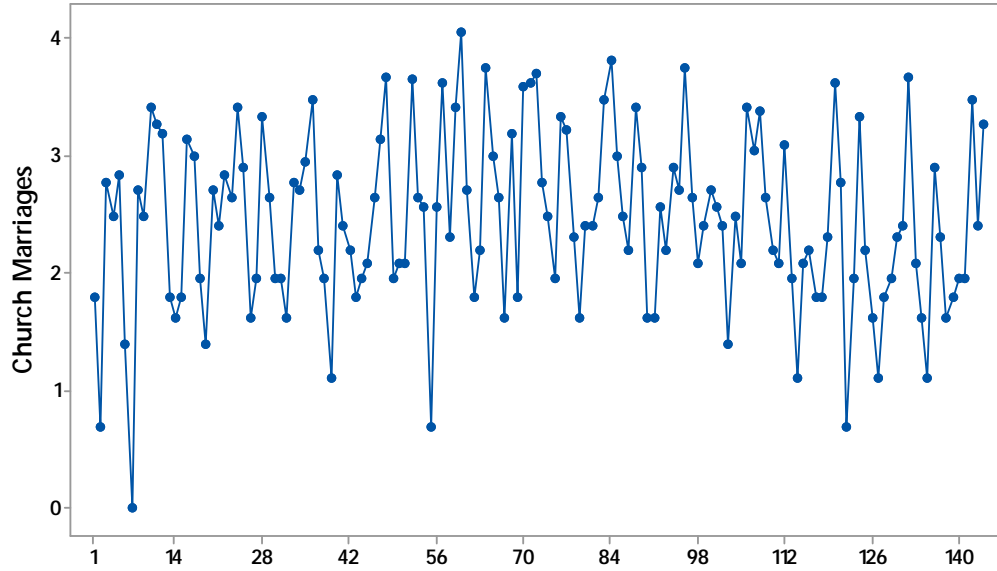


Fig 3.2: Time plot of transformed series of church marriages

From Appendix B and Table 9

$$W = \frac{12*(12-1)(0.653)}{(12-1)(9.99)} = \frac{86.196}{109.89} = 0.78$$

#### 4 Concluding Remarks

This article has discussed the Chi-Square test in time series data. The Chi-Square test is applied in the seasonal variances of the Buys-Ballot table. Results indicate that, 1) the test statistic identified the mixed model successfully in one hundred (100) out of the one hundred (100) simulations, 2) applying the real life data, the transformed series admits additive model. This further confirms that the appropriate model of original data is multiplicative. In view of this, it is recommended that a study data should be evaluated for assumptions of time series model before applying test for choice of model.

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Appendix A: Actual Series of Church Marriages in Imo State, Nigeria (2008 – 2019)

|      | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | total | $\bar{x}_i$ | $\sigma_i$ |
|------|------|------|------|------|-----|------|------|------|-------|------|------|------|-------|-------------|------------|
| 2008 | 6    | 2    | 16   | 12   | 17  | 4    | 1    | 15   | 12    | 30   | 26   | 24   | 165   | 13.75       | 9.53       |
| 2009 | 6    | 5    | 6    | 23   | 20  | 7    | 4    | 15   | 11    | 17   | 14   | 30   | 152   | 13.82       | 8.28       |

|             |              |             |             |              |              |             |             |              |              |              |              |              |     |       |       |
|-------------|--------------|-------------|-------------|--------------|--------------|-------------|-------------|--------------|--------------|--------------|--------------|--------------|-----|-------|-------|
| 2010        | 18           | 5           | 7           | 28           | 14           | 7           | 7           | 5            | 16           | 15           | 19           | 32           | 166 | 15.09 | 9.02  |
| 2011        | 9            | 7           | 3           | 17           | 11           | 9           | 6           | 7            | 8            | 14           | 23           | 39           | 153 | 12.75 | 9.88  |
| 2012        | 7            | 8           | 8           | 38           | 14           | 13          | 2           | 13           | 37           | 10           | 30           | 57           | 229 | 20.82 | 17.16 |
| 2013        | 15           | 6           | 9           | 42           | 20           | 14          | 5           | 24           | 6            | 36           | 37           | 40           | 245 | 22.27 | 14.37 |
| 2014        | 16           | 12          | 7           | 28           | 25           | 10          | 5           | 11           | 11           | 14           | 32           | 45           | 216 | 18.00 | 11.99 |
| 2015        | 20           | 12          | 9           | 30           | 18           | 5           | 5           | 13           | 9            | 18           | 15           | 42           | 187 | 17.00 | 10.96 |
| 2016        | 14           | 8           | 11          | 15           | 13           | 11          | 4           | 12           | 8            | 30           | 21           | 29           | 176 | 14.67 | 8.09  |
| 2017        | 14           | 9           | 8           | 22           | 7            | 3           | 8           | 9            | 6            | 6            | 10           | 37           | 131 | 11.91 | 9.72  |
| 2018        | 16           | 2           | 7           | 28           | 9            | 5           | 3           | 6            | 7            | 10           | 11           | 39           | 136 | 12.36 | 11.42 |
| 2019        | 8            | 5           | 3           | 18           | 10           | 5           | 6           | 7            | 7            | 32           | 11           | 26           | 138 | 11.50 | 9.12  |
| Total       | <b>149</b>   | <b>81</b>   | <b>40</b>   | <b>301</b>   | <b>178</b>   | <b>93</b>   | <b>56</b>   | <b>137</b>   | <b>138</b>   | <b>232</b>   | <b>249</b>   | <b>440</b>   |     |       |       |
| $\bar{y}_j$ | <b>12.42</b> | <b>6.75</b> | <b>8.00</b> | <b>25.08</b> | <b>14.83</b> | <b>7.75</b> | <b>4.67</b> | <b>11.42</b> | <b>11.50</b> | <b>19.33</b> | <b>20.75</b> | <b>36.67</b> |     |       |       |
| $\sigma_j$  | <b>4.94</b>  | <b>3.28</b> | <b>5.57</b> | <b>9.07</b>  | <b>5.31</b>  | <b>3.62</b> | <b>2.02</b> | <b>5.25</b>  | <b>8.54</b>  | <b>10.00</b> | <b>9.03</b>  | <b>9.21</b>  |     |       |       |

Appendix B: Transformed Series of Church Marriages in Imo State, Nigeria (2008 – 2019)

|             | JAN           | FEB           | MAR           | APR           | MAY           | JUN           | JUL           | AUG           | SEP           | OCT           | NOV           | DEC           | $T_i$          | $\bar{X}_i$ | $\hat{\sigma}_i$ |
|-------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|----------------|-------------|------------------|
| <b>2008</b> | 1.7918        | 0.6931        | 2.7726        | 2.4849        | 2.8332        | 1.3863        | 0.0000        | 2.7081        | 2.4849        | 3.4012        | 3.2581        | 3.1781        | 26.9922        | 2.2494      | 1.0693           |
| <b>2009</b> | 1.7918        | 1.6094        | 1.7918        | 3.1355        | 2.9957        | 1.9459        | 1.3863        | 2.7081        | 2.3979        | 2.8332        | 2.6391        | 3.4012        | 28.6358        | 2.3863      | 0.6629           |
| <b>2010</b> | 2.8904        | 1.6094        | 1.9459        | 3.3322        | 2.6391        | 1.9459        | 1.9459        | 1.6094        | 2.7726        | 2.7081        | 2.9444        | 3.4657        | 29.8091        | 2.4841      | 0.6478           |
| <b>2011</b> | 2.1972        | 1.9459        | 1.0986        | 2.8332        | 2.3979        | 2.1972        | 1.7918        | 1.9459        | 2.0794        | 2.6391        | 3.1355        | 3.6636        | 27.9253        | 2.3271      | 0.6728           |
| <b>2012</b> | 1.9459        | 2.0794        | 2.0794        | 3.6376        | 2.6391        | 2.5649        | 0.6931        | 2.5649        | 3.6109        | 2.3026        | 3.4012        | 4.0431        | 31.5622        | 2.6302      | 0.9298           |
| <b>2013</b> | 2.7081        | 1.7918        | 2.1972        | 3.7377        | 2.9957        | 2.6391        | 1.6094        | 3.1781        | 1.7918        | 3.5835        | 3.6109        | 3.6889        | 33.5321        | 2.7943      | 0.796            |
| <b>2014</b> | 2.7726        | 2.4849        | 1.9459        | 3.3322        | 3.2189        | 2.3026        | 1.6094        | 2.3979        | 2.3979        | 2.6391        | 3.4657        | 3.8067        | 32.3738        | 2.6978      | 0.649            |
| <b>2015</b> | 2.9957        | 2.4849        | 2.1972        | 3.4012        | 2.8904        | 1.6094        | 1.6094        | 2.5649        | 2.1972        | 2.8904        | 2.7081        | 3.7377        | 31.2866        | 2.6072      | 0.6448           |
| <b>2016</b> | 2.6391        | 2.0794        | 2.3979        | 2.7081        | 2.5649        | 2.3979        | 1.3863        | 2.4849        | 2.0794        | 3.4012        | 3.0445        | 3.3673        | 30.5509        | 2.5459      | 0.5655           |
| <b>2017</b> | 2.6391        | 2.1972        | 2.0794        | 3.091         | 1.9459        | 1.0986        | 2.0794        | 2.1972        | 1.7918        | 1.7918        | 2.3026        | 3.6109        | 26.825         | 2.2354      | 0.6476           |
| <b>2018</b> | 2.7726        | 0.6931        | 1.9459        | 3.3322        | 2.1972        | 1.6094        | 1.0986        | 1.7918        | 1.9459        | 2.3026        | 2.3979        | 3.6636        | 25.7508        | 2.1459      | 0.8472           |
| <b>2019</b> | 2.0794        | 1.6094        | 1.0986        | 2.8904        | 2.3026        | 1.6094        | 1.7918        | 1.9459        | 1.9459        | 3.4657        | 2.3979        | 3.2581        | 26.3952        | 2.1996      | 0.7051           |
| $T_j$       | <b>29.224</b> | <b>21.278</b> | <b>23.551</b> | <b>37.916</b> | <b>31.621</b> | <b>23.307</b> | <b>17.002</b> | <b>28.097</b> | <b>27.496</b> | <b>33.958</b> | <b>35.306</b> | <b>42.885</b> | <b>351.639</b> |             |                  |

|                     |        |        |        |        |        |        |        |        |        |        |        |        |  |        |        |
|---------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--|--------|--------|
| $\bar{X}_{.j}$      | 2.4353 | 1.7732 | 1.9625 | 3.1597 | 2.6351 | 1.9422 | 1.4168 | 2.3414 | 2.2913 | 2.8299 | 2.9422 | 3.5737 |  | 2.4419 |        |
| $\hat{\sigma}_{.j}$ | 0.4428 | 0.5896 | 0.4773 | 0.3758 | 0.3746 | 0.4889 | 0.5835 | 0.4537 | 0.5108 | 0.5519 | 0.45   | 0.2472 |  |        | 0.7493 |

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