ON CONTINUITY OF GRILL TOPOLOGICAL
SPACES VIA REGULAR GENERALIZED
G! CLOSED SETS

Abstract
In this paper, we study the continuity property in grill topological spaces via generalized G! closed sets and regular generalized G! closed sets. These notions are generalized G! continuous functions which is weak form of g! continuous functions, generalized G! irresolute functions, regular generalized G! continuous functions which is weak form of rg! continuous functions, regular generalized G! irresolute functions and investigates some of its properties in grill topological spaces.
Keywords: Grill topological spaces; continuous functions.
AMS classi_cation: Primary 54C08, 54C05

1 Introduction

The continuity property is one of the fundamental concepts in point set topology. In 1982, Hdeib [7] introduced the notion of ! continuous functions. In 1983, [8] Mashhour, et al. introduced the notions of _ continuity functions. In 1993, Palaniappan and Rao [11] introduced the notion of regular generalized continuous functions. In 2005, Al Zoubi [3] introduced the notion of generalized ! continuous functions. In 2007, Al Omari and Noorani [1] introduced the notion of regular generalized ! continuous functions. In 2009, [10] Noiri, et al. introduced the class of _ ! continuous functions.

For the studied of grill topological spaces. In 2011, Al-Omari and Noiri [2] introduced the notion of G _ continuous function in grill topological spaces. In 2020, [13] Saif, et al. introduced the notions of G! open sets and generalized G! closed sets in grill topological spaces. In 2021, [14] Saif, et al. introduced the notion of g! continuous property in grill topological spaces. In 2021, [16] Saif, et al. introduced the notion of G_ ! continuous property in grill topological spaces. In 2021, [17] Saif, et al. introduced the notion of G! rg closed sets in grill topological spaces.

In this paper, we introduce and investigate new notions of generalized G! closed sets and regular generalized G! closed sets in grill topological spaces. These notions are called generalized G! continuous functions and regular generalized G! continuous functions in grill topological spaces. In Section 3, we introduce and investigate the notions of generalized G! closed sets called generalized G! continuous functions, generalized G! closed functions, generalized G! irresolute functions and introduce investigate the notion of G! irresolute functions. In Section 4, we introduce and investigate new the notions of generalized G! continuous

functions, generalized G! closed functions and generalized G! irresolute functions called regular generalized G! continuous functions, regular generalized G! closed functions and regular generalized G! irresolute functions, respectively.

2 Preliminaries

De_nition 2.1. A function f : (X; _) ! (Y; _) of a topological space (X; _) into a topological space (Y; _) is called:

1. g continuous function [4] if f 1(F) is g closed set in (X; _) for every closed set F of (Y; _);
2. ! continuous function [7] if f 1(V) is ! open set in (X; _) for every open set V of (Y; _);
3. g! continuous function [3] if f 1(F) is g! closed set in (X; _) for every closed set F of (Y; _);
4. g! closed function [3] if the image of every closed set of (X; _) is g! closed set in

(Y; _);
5. rg!□continuous function [1] if f□1(F) is rg!□closed set in (X; _) for every !□closed set F of (Y; _).
By Cl(A) and Int(A), we mean the closure set and the interior set of A in topological space (X; _), respectively.
A collection G of subsets of a topological space (X; _) is said to be a grill [5] on X if G satis es the following conditions:
1. ; =2 G;
2. A 2 G and A _ B implies that B 2 G;
3. A;B _ X and A [B 2 G implies that A 2 G or B 2 G.
For a grill G on a topological space X, an operator from the power set P(X) of X to P(X) was de_ned in [12] the following manner : For any A 2 P(X),
_(A) = fx 2 X : U \ A 2 G; for each open neighborhoodU of xg:
Then the operator : P(X) ! P(X), given by (A) = A [_(A), for A 2 P(X), was also shown in [12] to be a Kuratowski closure operator, de_ning a unique topology _G on X such that _ _ _G. This topology de_ned by:
_G = fU _ X : (X □ U) = X □ Ug;
where _ _ _G and for any A _ X, (A) = GCl(A) such that GCl(A) denotes the set of all closure points of A in a topological space (X; _G). The set of all interior points of A in a topological space (X; _G) denoted by GInt(A).
2
De_nition 2.2. A function f : (X; _; G) ! (Y; _) of a grill topological space (X; _; G) into a topological space (Y; _) is called:
1. G!□continuous function [14] if f□1(U) is G!-open set in (X; _; G) for every open set U in (Y; _);
2. G!-closed function [14] if f(F) is a closed set in (Y; _) for every G!-closed set F in (X; _; G);
3. G_
!□continuous function [16] if f□1(U) is G_
!□open set in (X; _; G) for every open set
U in (Y; _);
4. G_
!□closed function [16] if f(F) is a closed set in Y for every G_
!□closed set F in
(X; _; G).
De_nition 2.3. A subset A of a topological space (X; _) is called:
1. _□open set [9] if A _ Int(Cl(Int(A))). The complement of _□open set is called _□closed set;
2. !□open set [6] if for each x 2 A, there is an open set Ux 2 _ containing x such that Ux □ A is a countable set. The complement of !□open set is called !□closed set;
3. _□!□open set [10] if A _ Int!(Cl(Int!(A))). The complement of _□!□open set is called _ □ !□closed set.
De_nition 2.4. A subset A of a grill topological space (X; _; G) is called:
1. G□_□open set [2] if A _ Int((Int(A)). The complement of G□_□open set is called G □ _□closed set;
2. G!□open set [13] if A _ Cl(Int!((A))). The complement of G!□open set is called G!□closed set;
3. G_
!□open set [15] if A _ Int((Int!(A))). The complement of G_
!□open set is called
G_

!□closed set. The set of all G_
!□open sets in X is denoted by G_
!O(X; _) and the set
of all G_
!□closed sets in X is denoted by G_
!C(X; _);
4. G!
g □closed set [13] if G!Cl(A) _ U whenever A _ U and U is open set of (X; _; G).
The complement of G!
g □closed set is called G!
g □open set;
5. G!
rg□closed set [17] if G!Cl(A) _ U whenever A _ U and U is a regular□open set of
(X; _; G). The complement of G!
rg□closed set is called G!
rg□open set.
3
From the de_nition stated above we obtain the following _gure of implications:
continuous function
_
!□continuous function /
_
g!□continuous function / rg!□continuous function
G!□continuous function
Figure 1:

3 G!
g □Continuous functions

De_nition 3.1. A function f : (X; _; G) ! (Y; _) of a grill topological space (X; _; G) into
a space (Y; _) is called G!
g □continuous function if f□1(U) is G!
g □closed set in (X; _; G) for
every closed set U in (Y; _).
Example 3.2. Any function f : (X; _; G) ! (Y; _) of any countable grill topological space
(X; _; G) into any space (Y; _) is G!
g □continuous function.
De_nition 3.3. A function f : (X; _; G) ! (Y; _;H) of a grill topological space (X; _; G)
into a grill topological space (Y; _;H) is called:
1. G!
g □irresolute function if f□1(U) is G!
g □closed set in (X; _; G) for every G!
g □closed set
in (Y; _;H).
2. G!□irresolute function if f□1(U) is G!□closed set in (X; _; G) for every G!□closed set
in (Y; _;H).
Theorem 3.4. Let f : (X; _; G) ! (Y; _) be a function of a grill topological space (X; _; G)
into a space (Y; _). Then f is G!
g □continuous function if f□1(U) is G!
g □open set in (X; _; G)
for every open set U in (Y; _).
Proof. Let f be G!

g continuous function. Let U be open set in (Y; _) then Y F is closed set
in (Y; _). Since f is G!
g continuous function. Then f1(Y U) = X f1(U) is G!
g closed
set in a grill topological space (X; _; G). Therefore f1(U) is G!
g open set in (X; _; G). Hence
f is G!
g continuous function.
It is clear that every G!continuous function is G!
g continuous function but the converse of
this fact need not be true.
Example 3.5. Let f : (R; _; G) ! (Y; _) be a function de_ned by:
f(x) =
_
1; x 2 (0; 1
2]
2; x 2 R (0; 1
2]
4
where Y = f1; 2g,
_ = f;;R;R (0; 1)g; G = P(R) f;g and _ = f;; Y; f1gg:
The function f is G!
g continuous, since f1(f2g) = R(0; 1
2] and f1(Y) = R are G!
g closed
sets in (R; _; G). The function f is not G!continuous function, since f1(f2g) = R (0; 1
2]
is not G!closed set.
It is clear that every g!continuous function is G!
g continuous function but the converse of
this fact no need to be true.
Example 3.6. Let f : (X; _; G) ! (Y; _) be a function de_ned by:
f(x) =
_
1; x 2 X B
2; x 2 B
where X, B and A are uncountable sets, B _ A and Y = f1; 2g,
_ = f;;X;Ag; G = P(X) f;g and _ = f;; Y; f1gg:
Then the function f is G!
g continuous but f is not g!continuous. Since f1(f2g) = B and
f1(Y) = X are G!
g closed sets in (X; _; G) but f1(f2g) = B is not g!closed set.
It is clear that every G_
!continuous function is G!
g continuous function but the converse of
this fact need not be true.
Example 3.7. In Example (3.5), we note that f is G!
g continuous function but the function
f is not G_
!continuous function. Since f1(f2g) = R(0; 1
2] is not G_

!□closed set in (X; _; G).
It is clear that every G!□irresolute function is G!□continuous function but the converse of this fact need not be true.
Example 3.8. Let f : (R; _; G) ! (R; _;H) be the identity function where
_ = f;;R;R □ (0; 1)g; G = P(R) □ f;g; H = P(R) □ f;g and _ = f;;Rg:
The function f is G!□continuous function since f□1(;) = ; and f□1(R) = R are G!□closed sets in (R; _; G) but f it is G!□irresolute function. Since R □ (0; 1) is G!□closed set in (R; _;H) but it is not G!□closed set in (R; _; G).
It is clear that every G!
g □irresolute function is G!
g □continuous function but the converse of
this fact need not be true.
Example 3.9. In Example (3.8), we note that f is G!
g □continuous function, since f□1(;) = ;
and f□1(R) = R are G!□closed sets in (R; _; G) but the function f is not G!
g □irresolute
function, since R□(0; 1) is G!
g □closed set in (R; _;H) but it is not G!
g □closed set in (R; _; G).

It is clear that every G!□irresolute function is G!
g □irresolute function but the converse of
this fact need not be true.
Example 3.10. In Example (3.5), if f : (R; _; G) ! (R; _;H), we note that f is G!
g □irresolute
function, but the function f is not G!□irresolute function, since R □ (0; 1
2] is G!□closed set
in (Y; _;H) but f□1(f2g) = R □ (0; 1
2] is not G!□closed set in (X; _; G).
The following _gure is an enlargement of _gure(1) and an theorems above we have the following _gure:

continuous function
_
!□continuous function /
_
g!□continuous function /
_
rg!□continuous function
G!□continuous function / G!
g □continuous function
G!□irresolute function /
O
G!
g □irresolute function
O

Figure 2:

Theorem 3.11. If f : (X; _; G) ! (Y; _) is G!
g □continuous, then for each x 2 X and each
open set V in (Y; _) with f(x) 2 V , there exists G!
g □open set U in (X; _; G) such that x 2 U
and f(U) _ V .

Proof. Lit f is G!
g □continuous function. Let x 2 X and V be any open set in (Y; _) containing
f(x). Put U = f□1(V). Since f is G!
g □continuous then U is G!
g □open set in (X; _; G) such
that x 2 U and f(U) _ V .
The converse of the above Theorem is not true in general as the following Example shows.
Example 3.12. In Example (3.6), if
_ = f;;X;Bg; G = P(X) □ f;g and _ = f;; Y; f1gg:
We note that f is satis_es the property stated in Theorem(3.11). The function f is not
G!
g □continuous function. Since f2g is a closed set in Y but f□1(f2g) = B is not G!
g □closed
set in (X; _; G).
Theorem 3.13. Let f : (X; _; G) ! (Y; _) be a G!
g □continuous function and let A be a
closed subset of (X; _; G). Then, the restriction fjA : (A; _A) ! (Y; _) is G!
g □continuous.

Proof. Let F be a closed subset of (Y; _). Then (fjA)□1(F) = f□1(F) \ A. Since f is
G!
g □Continuous, f□1(F) 2 G!
g C(X; _; G) and since It is clear that if A 2 G!
g C(X; _; G) and
B is closed set in (X; _; G) then A \ B 2 G!
g C(X; _; G). Therefore, since It is clear that if
A 2 G!
g C(X; _; G), then A 2 G!
g C(Y; _Y); (fjA)□1(F) 2 G!
g C(Y; _Y).
Theorem 3.14. For a function f : (X; _; G) ! (Y; _), the following holed:
1. f is G!
g □continuous function.
2. For each x 2 X and each open set V in (Y; _) with f(x) 2 V , there exists G!
g □open set
U in (X; _; G) such that x 2 U and f(U) _ V .
De_nition 3.15. A function f : (X; _; G) ! (Y; _) of a grill topological space (X; _; G) into
a space (Y; _) is called:
1. G!
g □closed function if f(G) is a closed set in (Y; _) for every G!
g □closed set G in
(X; _; G).
2. G!
g □open function if f(G) is a open set in (Y; _) for every G!
g □open set G in (X; _; G).
Example 3.16. f : (R; _; G) ! (R; _D) be a any function, with
_ = f;;R; f1; 3gg and G = P(R) □ f;g:
Since f1; 3g is a countable set then by Remark(??) any subset of X is a both G!
g □open set
and G!
g □closed set. Therefore f is a both G!

g □closed function and G!
g □open function.
Remark 3.17. Let (X; _I ; G) be a grill topological space with any set X then any subset of
X is a both G!
g □open set and G!
g □closed set.
Theorem 3.18. Let f : (X; _I ; G) ! (Y; _) be any function then f is G!
g □continuous
function.
Proof. Let A _ Y then f□1(A) is a both G!
g □open set and G!
g □closed set. Therefore f is
G!
g □continuous function.
Theorem 3.19. Every G!
g □closed function is G_
!□closed function.
The proof is obvious.
The converse above Theorem need not be true.
Example 3.20. Let f : (X; _; G) ! (Y; _) be a function de_ned by:
f(x) =
_
b; x 2 f2; 3g
a; x 2 f1g
where X = f1; 2; 3g; Y = fa; bg,
_ = f;;X; f1gg; G = P(X) □ f;g and _ = f;; Y; fagg:
We note that f is G_
!□closed function but it is not G!
g □closed function. Since f1g is G!
g □closed
set in (X; _; G) but f(f1g) = fag it is not closed set in (Y; _).

Theorem 3.21. Every G!
g □closed function is G!□closed function.
The proof is obvious.
The converse above Theorem need not be true.
Example 3.22. Let f : (R; _; G) ! (Y; _) be a function de_ned by:
f(x) =
8<
:
1; x 2 B;B _ (0; 1)
3; x 2 R □ B; x 2 (0; 1)
2; x 2 R □ C
where (0; 1) _ C; Y = f1; 2; 3g,
_ = f;;R;R □ (0; 1)g; G = P(R) □ f;g and _ = f;; Y; f1; 3g; f2; 3g; f3gg:
We note that f is G!□closed function but it is not G!
g □closed function. Since R □ B is
G!
g □closed set in (X; _; G) but f(R □ B) = f3g it is not closed set in (Y; _).
Theorem 3.23. Let f : (X; _; G) ! (Y; _) and g : (Y; _;H) ! (Z;) two functions. Then
the following hold:

1. g _ f is G!
g continuous function if g is continuous function and f is G!
g continuous
function.
2. g _ f is G!
g continuous function if g is g continuous function and (Y; _) is T1j2 space
and f is G!
g continuous function.
Theorem 3.24. Let f : (X; _; G) ! (Y; _; I) and g : (Y; _;H) ! (Z; ;K) two functions.
Then the following hold:
1. g _ f is G!
g continuous function if g is G!
g continuous function and f is G!
g irresolute
function.
2. g _ f is G!
g irresolute function if f and g are G!
g irresolute function.
Theorem 3.25. Let f : (X; _; G) ! (Y; _) and g : (Y; _) ! (Z;) be two functions. Then
g _ f is G!
g closed function if g is a closed function and f is G!
g closed function.
Proof. Let U be G!
g closed set in (X; _; G). Since f is G!
g closed function then f(U) is a
closed set in Y . Since g is closed function then g[f(U)] = (g _ f)(U) is closed set in (Z;).
That is, g _ f is G!
g closed function.
Corollary 3.26. Let f : (X; _; G) ! (Y; _) and g : (Y; _) ! (Z;) be two functions. Then
g _ f is G!
g open function if g is a open function and f is G!
g open function.
The following Example shows that the composition of two G!
g continuous functions need
not be G!
g continuous.

Example 3.27. Let f : (R; _; G) ! (Y; _) and g : (Y; _;H) ! (Y;) be two functions de_ned
by:
f(x) =
_
0; x 2 R (0; 1)
1; x 2 (0; 1)
And g is identity function. Where Y = f0; 1g,
_ = f;;R;R (0; 1)g; G = P(R) f;g; H = P(Y) f;g and _ = f;; Y; f0gg:
And = f;; f1g; Y g. Then f and g are G!
g continuous functions but g_f is not G!
g continuous
function. Since f0g is a closed set in (Y;) but f1(f0g) = R(0; 1) is not G!
g closed set in
(R; _; G).

The following Example shows that the composition of two f is G!
g □continuous function
and g is g!□continuou function need not be G!
g □continuous.
Example 3.28. In Example(3.27), we note that f is G!
g □continuous function and g is
g!□continuou function, but g _ f is not G!
g □continuous function Since f0g is a closed set in
(Y;) but f□1(f0g) = R □ (0; 1) is not G!
g □closed set in (R; _; G).
Theorem 3.29. Let f : (X; _; G) ! (Y; _) be a G!
g □closed function and A is g□closed subset
in (X; _; G). Then f(A) is rg!□closed subset in (Y; _).
Proof. Let f be G!
g □closed function and A _ X suth that A is g□closed set. Since f
is G!
g □closed function and A is g□closed set. Since it is clear that every g□closed set is
G!
g □closed set then f(A) is a closed set in (Y; _). Since any closed set is rg!□closed set.
Therefore f(A) is rg!□closed set in (Y; _).
Theorem 3.30. Let f : (X; _; G) ! (Y; _) be a G!
g □continuous function and (Y; _) is
T1j2□space and A is g□closed subset in (Y; _). Then f□1(A) is G!
g □_________closed subset in (X; _; G).
Proof. Let f be G!
g □continuous function and (Y; _) is T1j2□space. Let A be g□closed subset
in (Y; _). Since (Y; _) is T1j2 □space, then for every g□closed subset in (Y; _) is closed subset
in (Y; _). Since f is G!
g □continuous function then for every A closed subset in (Y; _) implies
f□1(A) is G!
g □closed subset in (X; _; G).

4 G!
rg□Continuous functions

De_nition 4.1. A function f : (X; _; G) ! (Y; _;H) of a grill topological space (X; _; G) into
a grill topological space (Y; _;H) is called G!
rg□continuous function if f□1(U) is G!
rg□closed
set in (X; _; G) for every G!□closed set U in (Y; _;H).
De_nition 4.2. A function f : (X; _; G) ! (Y; _;H) of a grill topological space (X; _; G)
into a grill topological space (Y; _;H) is called G!
rg□irresolute if f□1(U) is G!
rg□closed set in
(X; _; G) for every G!
rg□closed set in (Y; _;H).

Remark 4.3. Let (X; _; G) be any grill topological space with _ = f;;X;Ag, G = P(X) □
f;g, where A is an uncountable set. Then any subset of (X; _; G) is a both G!
rg□open set and
G!
rg□closed set.

Example 4.4. Let f : (R; _; G) ! (R; _D;H) be a any function with
_ = f;;R; (0; 1)g; H = P(R) □ f;g and G = P(R) □ f;g:
Since every f□1(U) is G!
rg□closed set in (R; _; G) for every G!□closed set U in (R; _D;H).
Then f is G!
rg□continuous function.
Remark 4.5. Let (X; _; G) be any grill topological space with
_ = f;;X;Ag [fA [Bi : A _ Bi for every i 2 Ng and G = P(X) □ f;g:
Where A be any subset of X. Then any subset of (X; _; G) is a both G!
rg□open set and
G!
rg□closed set.
Example 4.6. Let f : (R; _; G) ! (Y; _;H) be a any function where (Y; _;H) is any grill topological space and
_ = f;;R; (0; 1]; (0; 2); (0; 7)g; H = P(Y) □ f;g and G = P(R) □ f;g:
Then f is G!
rg□continuous function by Remark (4.5).
It is clear that every rg!□continuous function is G!
rg□continuous function but the converse
of this fact need not be true.
Example 4.7. Let f : (R; _; G) ! (R; _;H) be the identity function with
_ = f;;R; (0; 1];R□(1; 3);R□(0; 3)g; H = P(R)□f;g; G = P(R)□f;g and _ = f;;R;R□(0;
1
2
]g
We note that f is G!
rg□continuous function. Since (0; 1
2] and R are closed sets in (R; _;H).
Then f□1((0; 1
2]) = (0; 1
2] and f□1(R) = R are G!
rg□closed sets but it is not rg!□continuous
function. Since (0; 1
2] is closed set in (R; _;H) but f□1((0; 1
2]) = (0; 1
2] is not rg!□closed set
in (R; _; G).
It is clear that every G!
g □continuous function is G!
rg□continuous function but the converse of
this fact need not be true.
Example 4.8. Let f : (R; _; G) ! (R; _;H) be the identity function with
_ = f;;R;R □ (0; 1)g; H = P(R) □ f;g; G = P(R) □ f;g and _ = f;;R; (0; 1)g:
The function f is G!
rg□continuous, since f□1(fR □ (0; 1)g) = R □ (0; 1) and f□1(R) = R are
G!
rg□closed sets in (R; _; G). The function f is not G!
g □continuous function, since f□1(fR □
(0; 1)g) = R □ (0; 1) is not G!
g □closed set in (R; _; G).
It is clear that every G!

rg□irresolute function is G!
rg□continuous function but the converse of
this fact need not be true.
10
Example 4.9. Let f : (R; _; G) ! (R; _I ;H) be the identity function with
_ = f;;R; (0; 1); (0; 1];R□(0; 3);R□(1; 3);R□[1; 3)g; G = P(R)□f;g and H = P(R)□f;g:
We note that f is G!
rg□continuous function but it is not G!
rg□irresolute function. Since (0; 1)
is G!
rg□closed set in (R; _I ;H) but f□1((0; 1)) = (0; 1) is not G!
rg□closed set in (R; _; G).
It is clear that every G!
g □irresolute function is G!
rg□irresolute function but the converse of
this fact need not be true.
Example 4.10. Let f : (R; _; G) ! (R; _I ;H) be the identity function with
_ = f;;R;R □ (1; 3)g; H = P(R) □ f;g and G = P(R) □ f;g:
We note that f is G!
rg□irresolute function but it is not G!
g □irresolute function. Since the set
R□(1; 3) is G!
g □closed set in (R; _I ;H) but f□1(R□(1; 3)) = R□(1; 3) is not G!
g □closed set
in (R; _; G).
The following _gure is an enlargement of _gure (2) and an theorems above we have the
following _gure:
continuous function
_
!□continuous function /
_
g!□continuous function /
_
rg!□continuous function
_
G!□continuous function / G!
g □continuous function / G!
rg□continuous function
G!□irresolute function /
O
G!
g □irresolute function
O
/ G!
rg□irresolute function
O
Figure 3:
De_nition 4.11. A function f : (X; _; G) ! (Y; _;H) of a grill topological space (X; _; G)
into a grill topological space (Y; _;H) is called:
1. G!
rg□closed function if f(G) is a closed set in (Y; _) for every G!

rg□closed set G in
(X; _; G).
2. G!
rg□open function if f(G) is a open set in (Y; _) for every G!
rg□open set G in (X; _; G).
3. pre□G!□closed function if f(V) is G!□closed in (Y; _;H) for every G!□closed subset
V of (X; _; G).
Example 4.12. In Example (4.4), we note that f is a both G!
rg□closed function and
G!
rg□open function.

De_nition 4.13. A grill topological space (X; _; G) is a regular generalized G! □T1j2□space
(simply, G!
rg □ T1j2□space) if every G!
rg□closed set in (X; _; G) is G!□closed set.
Theorem 4.14. Let f : (X; _; G) ! (Y; _;H) be a function of a grill topological space
(X; _; G) into a grill topological pace (Y; _;H). Then f is G!
rg□continuous function if f□1(U)
is G!
rg□open set in (X; _; G) for every G!□open set in (Y; _;H).
Proof. Let f be G!
g □continuous function. Let U be G!□open set in (Y; _;H) then Y □ F
is G!□closed set in (Y; _;H). Since f is G!
rg□continuous function. Then f□1(Y □ U) =
X □ f□1(U) is G!
g □closed set in a grill topological space (X; _; G). Therefore f□1(U) is
G!
rg□open set in (X; _; G). Hence f is G!
rg□continuous function.
Theorem 4.15. Let f : (X; _; G) ! (Y; _; I) and g : (Y; _;H) ! (Z; ;K) two functions.
Then the following hold:
1. g _ f is G!
rg□continuous function if g is G!□irresolute function and f is G!
rg□continuous
function.
2. g _ f is G!
rg□continuous function if g is G!
rg□continuous function and f is G!
rg□irresolute
function.
3. g _ f is G!
rg□irresolute function if f and g are G!
rg□irresolute functions.
4. Let (Y; _) be G!
rg□T1=2□space. Then, g_f is G!
rg□continuous function if g is G!
rg□continuous
function and f is G!
rg□continuous function.
The following Example shows that the composition of two G!

rg□continuous functions need
not be G!
rg□continuous.
Example 4.16. Let f : (R; _; G) ! (Y; _;H) and g : (Y; _;H) ! (Y; ;K) be two functions
de_ned by:
f(x) =
_
0; x 2 (0; 1)
1; x 2 R □ (0; 1)
:
And g is identity function. Where Y = f0; 1g,
_ = f;;R; (0; 1); (0; 1];R□(0; 3);R□(1; 3);R□[1; 3)g; G = P(R)□f;g; K = P(Y)□f;g; H = P(Y)□f;g;
_ = f;; Y; f0gg and = f;; f1g; Y gg:
Then f and g are G!
rg□continuous function but g _ f is not G!
rg□continuous function. Since
f0g is G!□closed set in (Y; ;H) but f□1(f0g) = (0; 1) is not G!
rg□closed set in (R; _; G).
Theorem 4.17. Let (X; _) be any door topological space and f : (X; _; G) ! (Y; _D;H)
be any function. Then f is both G!
rg□open function and G!
rg□closed function and it is
G!
rg□continuous function.

Proof. Let (X; _) be any door topological space. Let A _ X, since X is a door topological
space, then A is an either open set or a closed set. If A is an open set of X, it is clear that
every open set is G!
rg□open set implies A is G!
rg□open set. Then f(A) is an open set in Y .
Therefore f is G!
rg□open function. If A is a closed set of X, it clear that every closed set is
G!
rg□closed set implies A is G!
rg□closed set. Then f(A) is a closed set in Y . Therefore f is
G!
rg□closed function.
Let A _ Y , since X is a door topological space then f□1(A) is an either open set or a closed
set in X. If f□1(A) is G!
rg□open set, since Y is (Y; _D) then A is an open set. If f□1(A)
is G!
rg□closed set since Y is (Y; _D) then A is a closed set. Therefore f is G!
rg□continuous
function.
In Theorem (4.17), if X is a countable set and (Y; _) be any topological space then f is
G!
rg□continuous function.
Theorem 4.18. Let f : (X; _I ; G) ! (Y; _D;H) be any function. Then f is G!
rg□continuous
function.
Proof. Let A _ Y and since Y is (Y; _D;H), then any subset of Y is a both open set and closed

set. Now f□1(A) is a both G!
rg□open set and G!
rg□closed set. Therefor f is G!
rg□continuous
function by Remark(3.17).
Theorem 4.19. Let f : (X; _I ; G) ! (Y; _;H) be any function. Then f is G!
rg□continuous
function.
Proof. Let A be an open subset of Y . Then f□1(A) is G!
rg□open set by Remark(3.17). Let
A be a closed subset of Y . Then f□1(A) is G!
rg□closed set by Remark(3.17). Therefore f is
G!
rg□continuous function.
Theorem 4.20. Every G!
rg□closed function is G!
g □closed function but the converse of this
fact need not be true.
Theorem 4.21. A grill topological space (X; _; G) is called a regular generalized G! □
T1j2□space (simply G!
rg□T1j2□space) if and only if every G!
rg□open set in (X; _; G) is G!□open
set.
Theorem 4.22. A grill topological space (X; _; G) is called a regular generalized G! □
T1j2□space (simply G!
rg □ T1j2□space) if and only if every singleton set in (X; _; G) is either
regular□closed set or G!□open set.
Proof. Suppose that (X; _; G) is G!
rg □ T1j2□space and fxg is not regular□closed subset of
X for some x 2 X. Then X □ fxg is not regular□open set in X. Hence X is the only
regular□open set containing X □ fxg. That is, X □ fxg is G!
rg□closed set in X. Since
(X; _; G) is G!
rg □T1j2□space then X □fxg is G!□closed set in X. That is, fxg is a G!□open
set in X.
Conversely, suppose that every singleton set in X is either regular□closed or G!□open Let
A be any G!
rg□closed set in X and x 2 G!Cl(A). We show that x 2 A. By the hypothesis
fxg is either regular□closed set or G!□open set in X. The set fxg is regular□closed set and
x =2 A then
x 2 G!Cl(A) □ A _ X □ A:

Then fxg _ X □ A and so A _ X □ fxg. Since A is G!
rg□closed set in X contained in
regular□open set X □ fxg then G!Cl(A) _ X □ fxg and so fxg_ X □ G!Cl(A). Therefore
fxg _ G!Cl(A) \ [X □ G!Cl(A)] = 0;
and this is a contradiction. Hence fxg 2 A, that is, G!Cl(A) = A and so A is G!□closed set.
If fxg is
G!□open set and fxg 2 G!Cl(A) then we have fxg \ A 6= ;. Hence fxg 2 A, that is,
G!Cl(A) = A and so A is G!□closed set.
Theorem 4.23. If f : (X; _; G) ! (Y; _;H) is G!

rg□continuous function and X is G!
rg □
T1j2□space, then f is G!□continuous function.
Proof. Let f be any G!
rg□continuous function and X is G!
rg□T1j2□space, then every G!
rg□closed
subset of X is G!□closed set. Let A be any closed set in Y . Since any closed set is G!□closed set, f is G!
rg□continuous function and G!
rg□T1j2□space, then f□1(A) is G!□closed set. There-
fore f is G!□continuous function.
Theorem 4.24. Let (X; _; G) be any grill topological space with a countable set X. Then every G!
rg□closed subset of X is G!
g □closed set.
Theorem 4.25. Let f : (X; _; G) ! (Y; _;H) be any function and X is countable set. Then f is G!
rg□continuous function and it is G!
g □continuous function.
Theorem 4.26. If f : (X; _; G) ! (Y; _;H) is G!
rg□continuous, then for each x 2 X and
each G!□pen set V in (Y; _;H) with f(x) 2 V , there exists G!
rg□open set U in (X; _; G) such
that x 2 U and f(U) _ V .
Proof. Let f is G!
rg-continuous function. Let x 2 X and V be any G!□open set in (Y; _;H)
containing f(x). Take U = f□1(V). Since f is G!
rg-continuous then U is G!
rg-open set in
(X; _; G) such that x 2 U and f(U) _ V .
The converse of the above Theorem is not true in general as the following Example shows.
Example 4.27. In Example (4.16), we note that f is satis_es the property stated in Theorem (4.26). The function f is not G!
rg□continuous function. Since f0g is G!□closed set in Y but
f□1(f0g) = (0; 1) is not G!
rg□closed set in (R; _; G).
Theorem 4.28. Let f : (X; _; G) ! (Y; _;H) be a surjective, G!
rg□irresolute, and pre□G!□closed
function. if X is G!
rg □ T1=2□space, then Y is also an G!
rg □ T1=2□space.
Proof. Let A be G!
rg□closed subset of Y . Since f is an G!
rg□irresolute function, then f□1(A)
is an G!
rg□closed subset of X. Since X is G!
rg □ T1=2□space, then f□1(A) is an G!□closed
subset of X. Since f is a pre□G!□closed function, then f(f□1(A)) = A is an G!□closed
subset of Y . Therefore Y is also G!
rg □ T1=2□space.

Theorem 4.29. Let f : (X; _; G) ! (Y; _;H) be ro□preserving and G!□irresolute function, if B is G!
rg□closed set in Y , then f□1(B) is G!
rg□closed set in X.
14
Proof. Let U be a regular□open subset of X such that f□1(B) _ U. Then B _ f(U) and f(U) is regular□open. Since B is G!
rg□closed set, then G!Cl(A) _ f(U) and f□1(G!Cl(B)) _
U. Since f is G!□irresolute then f□1(G!Cl(B)) is G!□closed and G!Cl(f□1(G!Cl(B))) = f□1(G!Cl(B)), therefore G!Cl(f□1(B)) _ G!Cl(f□1(G!Cl(B))) _ U thus f□1(B) is G!
rg□closed
set in X.
De_nition 4.30. A function f : (X; _; G) ! (Y; _) is said to be G!□contra□R□function if for every regula□open subset V of Y; f□1(V) is G!□closed set.
Example 4.31. Let f : (R; _; G) ! (Y; _) be function de_ned by:
f(x) =
_
c; x 2 R □ (0; 1]
a; x 2 (0; 1]
;
where Y = fa; b; cg,
_ = f;;R;R □ (0; 1]g; G = P(R) □ f;g; and _ = f;; fag; fbg; fa; bgg:
Then f is G!□contra□R□function. But the function f(x) de_ned by:
f(x) =
_
b; x 2 R □ (0; 1]
a; x 2 (0; 1]
is not G!□contra□R□function. Since fbg is regula□open in Y and f□1(fbg) = R □ (0; 1] is not G!□closed.
Theorem 4.32. Let f : (X; _; G) ! (Y; _) be G!
rg□closed function. Then every U is
G!
rg□closed set in X exists V is closed set in Y such that f□1(V) 2 G!
rgC(X; _; G).
Proof. Let U be any G!
rg□closed set in (X; _; G). Since f is G!
rg□closed function. Then every
G!
rg□closed set in (X; _; G) such that f(U) = V is a closed set in (Y; _). Therefore f□1(V) = U is G!
rg□closed set.

References

[1] A. Al-Omari and M. Noorani, Regular Generalized !□Closed Sets, Int. J. Math. and Math. Sciences, 16(1), (2007), 1-11.
[2] A. Al-Omari and T. Noiri, Decompositions of continuity via grills, Jordan J. Math. Stat., 4(1), (2011), 33-46.
[3] K. Al-Zoubi, On generalized !□closed sets, International Journal of Mathematics and Mathematical Sciences, 13, (2005), 2011-2021.
[4] K. Balachandran, P. Sundaram, and H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. KochiUniv. Ser. A Math. 12, (1991), 5{13.

[5] G. Choquet, Sur les notions de _ltre et grille, Comptes Rendus Acad. Sci. Paris, 224, (1947), 171-173.
15
[6] H. Z. Hdeib, !closed mappings, Revista Colombiana de Matematicas, 16, (1982), 65-78.
[7] H. Z. Hdeib, !continous functions, Dirasat 16, (1989), 136{142.
[8] A. Mashhour, I. Hasanien and S. ElDeep, _continuity and _open mappings, Acta Math. Hungar, 41, (1983), 213-218.
[9] O. Njastad, On some classes of nearly open sets, Paci_c J. Math, 15, (1965), 961-970.
[10] T. Noiri, A. Al-omari and M. Noorani, Weak forms of !open sets and decompositions of continuity, European Journal of Pure and Applied Mathematics, 1, (2009), 73-84.
[11] N. Palaniappan and K. C. Rao, Regular generalized closed sets, Kyungpook Mathematical Journal, 33, no. 2, (1993), 211{219.
[12] B. Roy and M. N. Mukherjee; On a typical topology induced by a grill, Soochow J. Math., 33, (2007), 771-786.
[13] A. Saif, M. Al-Hawmi and B. Al-refaei, On G!open sets in grill topological spaces, Journal of Advance in Mathematics and Computer Science, 35(6), (2020), 132-143.
[14] A. Saif, M. Al-Hawmi and B. Al-refaei, The continuity property via, g!open sets in grill topological spaces, Intemational Journal Of Scienti_c, 12(4), (2021), 2229-5518.
[15] A. Saif, A. Mahdi and Khaled M. Hamadi, G_
! -open sets in grill topological spaces,
International Journal of Computer Applications, 183(13), (2021), 1-5.
[16] A. Saif, A. Mahdi and Khaled M. Hamadi, On Continuity Property of Grill Topological Spaces into Topological Spaces via G_
!Open Sets, International Journal of Applied
Mathematical Sciences, 14(1), (2021), 1-16.
[17] A. Saif, A. M. Al-Audhahi and K. M. Al-Hamadi, regular generalized G!closed sets in grill topological spaces, International Journal of Computer Applications, 183(26), (2021), 1-4.