Numerical solution of Landau-Lifshitz Equation in the theory of magnetic elastic

Abstract
In this paper we study Landau-Lifshitz equation which describe a nonlinear magnetic elastic in easy-axis ferromagnets.By means of numerical simulation a numerical solitonic solution to above mentioned equation with Standard Finite Difference(SFD)and NonStandard Finite Difference (NSFD) rules and obtained as a result of solution of Couchy problem using exact analytical solution to Landau-Lifshitz equation as initial data.Then we obtain the error of each method
Keywords: NonStandard Finite Difference scheme; Standard Finite Difference scheme;Landau-Lifshitz equation.
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1 Introduction

Investigation of nonlinear properties of magnetoelastic crystals in condensed matter is very important. Most of this work has been done for linear models. It seems,that nonlinear magnetoelastic waves in nonlinear approximation of phonon mode demonstrate a wide range difference their propagation and

interactions in comparision to a linear one.This paper arise to study nonlinear magnetoelastic waves in nonlinear approximation by phonon mode.The paper is organized as follows.In the section 2 starting from quantum-mechanical Hamiltonian taking into account modulation of exchange integrals as a result of oscillation of each site of spin chain,using generalized coherent states of $SU(2)$ group as a basis of trial functions following paper[5].we receive Landau-Lifshitz equation which describes magnetoelastic waves in nonlinear approximation of phonon modes analytically we were able to our best find the solution to the above mentioned system only in linear approximation of phonon mode ,which requires application of numerical methods.In the following we discuss a numerical approaches,which is not so simple due to singularities arising in the poles of Bloch sphere,as for all types of sigmamodels. To avoid this we use a stereographical projection to a complex plane ,that corresponds to the projection of $SO(3)$ space to a coset $SU(2)/U(1)$ space,due to their isomorphism.In the section 3 introdution NSFD Rules and implementation of NSFD for Lanadu-Lifshitz equation.In the section 4 we solve the problem using the analytical solution with a linear approximation in phonon mode as a initial condition,which could be considered as a perturbed solitonic solution.As a result evolution of the initial perturbation we receive a stable solitonic solution moving with some speed. Let us remind ,that the Landau-Lifshitz equation could be written,in the isotropic case,in the following form

$i\hbar S_t =$
1
2

$[S, S_{xx}]$ (1.1)

Where we introduce $S =$

$\begin{pmatrix} S_z & S_- \\ S_+ & -S_z \end{pmatrix} = S.\sigma$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

are Pauli operators.

This equation $(1, 1)$ give us a macroscopic description of the magnetization dynamic in ferromagnets and represents the equation of motion of the magnetization vector in non-dissipative media. On the other hand, it is well know, that in microscopic level the most popular models for description of magnetic properties of a crystal are spin models of the Heisenberg-Frenkel magnet

$$\hat{H} = -\sum_{i,k} J_{ik}\hat{S}_i\hat{S}_k \quad (1.2)$$

where J_{ik} are exchange integrals , $\hat{S}_j$ are the spin operators of the atom in

2

the jth site.

2 Mathematical Modeling of problem

Let us consider the model of the Heisenberg ferromagnet with single-axis anisotropy in the presence of oscillations of sites of the crystal lattice [5]

$$\hat{H} = \hat{H}_s + \hat{H}_p$$

where

$$\hat{H}_s = -\sum_{j=1}^{N}\left[\frac{J_0}{2}\left(\hat{s}_j^{+}\hat{s}_{j+1}^{-} + \hat{s}_j^{-}\hat{s}_{j+1}^{+}\right) + J_z \hat{s}_j^{z}\hat{s}_{j+1}^{z}\right] = -J_0\sum_{j=1}^{N}\{\hat{s}_j\hat{s}_{j+1} + \triangle \hat{s}_j^{z}\hat{s}_{j+1}^{z}\}$$

is the spin part of the Hamiltonian,

$$H_p = \sum_{j=1}^{N}\left[\frac{p_j^2}{2m} + \frac{k}{2}(y_{j+1} - y_j)^2\right]$$

is the phonon part of the Hamiltonian.Here $\Delta = \frac{(J_z - J_0)}{J_0}$ is the costant of exchange anisotropy,m and p are the mass and momentum of the atom,correspondingly $|y_{j+1} - y_j|$ is the displacement of the jth atom from the equilibrium position,k is the elastic constant,j is the summation index,here $s_{\pm} = s_x \pm i s_y$. Let us pass over to the classical description.In order to make this we average ,by use of the $SU(2)$ generalized coherent states (GCS) [6],[7] .Let us remind the reader that $SU(2)GCS$ in complex parameterization has the form

$$|z\rangle = \prod_j |z_j\rangle = \prod_j \left(1 + |z_j|^2\right)^{-k} \exp\{z_j \hat{S}_j^+\}|k, -k\rangle, \quad (2.1)$$

here k is the number of representation, is the parameter of quasiclassical description.
Spin operators averaged by use of the $SU(2)$ GCS get the following form

$$S_j^+ = \bar{S}_j^- = \langle b S_j^+ \rangle = \frac{2z_j}{1 + |z_j|^2}, \quad S_j^z = \langle b S_j^z \rangle = \frac{1 - |z_j|^2}{1 + |z_j|^2} \quad (2.2)$$

3

Note that the parameterization via more habitual angle variables is possible.
In the case the values of the averaged spin operators have the form

$$S = s\,(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta) \quad (2.3)$$

$$z_j = \tan\left(\frac{\theta}{2}\right)\exp\{i\phi_j\}, \quad (2.4)$$

where the values of the angle parameters are restricted as $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. By use (2.3) we obtained Landau-Lifshitz equation as the following equations [5]

$$a_{20}\,\theta_{xx} -$$

$$\frac{a^2_0}{2}\phi^2_x + 2\Delta\Big) \sin\theta\cos\theta + \hbar J_0 s^2 \sin\theta\,\phi_t = 0,$$

$$a^2_0\left(\sin^2\theta\,\phi_x\right)_x - \hbar J_0 s^2 \sin\theta\,\theta_t = 0 \quad (2.5)$$

By use of the vector (2.1) we carry out the averaging procedure of the spin Hamiltonian $\hat{H}_s$. We have

$$H = \langle z|\hat{H}|z\rangle = H_s + H_p. \quad (2.6)$$

$$z_{j+1} = z_j + a_0 z_{jx} + \frac{a^2_0}{2} z_{jxx} + \ldots$$

$$y_{j+1} = y_j + a_0 y_{jx} + \ldots \quad (2.7)$$

Then rewriting Hamiltonian(2.4) in spherical variables we obtain

$$H = -s^2 \sum_{j=1}^{N} \Big[\frac{J_0}{2}\left(S^+_j S^-_{j+1} + S^-_j S^+_{j+1}\right) + J_z S^z_j S^z_{j+1}\Big] + H_p. \quad (2.8)$$

Introducing Poison brackets in the following form

$$\{A, B\} = \int \Big($$

$-\varepsilon_{ijk}$

δA

δS_i

δB

δS_j

$-\ \delta A$

δP

δB

δy

$+$

δA

δy

δB

δP

$\overline{dx}$

We derive the equations of dynamics of the magnetization vector coupled with the lattice oscillations

$\hbar S_j$

$_t =$

$\overline{H},\ S_j$

$= -\varepsilon_{ijk}$

δH

$\delta S_i S_k$

4

then

$\hbar$

$-\rightarrow S_t = s_2 J_0\, a_{20}$

2

$(2S_y$

$_{xx}S_z - 2S_z$

$_{xx}S_y)$

$-\rightarrow i\ \ - s_2 J_0\, a_{20}$

2

$(2S_x$

$_{xx}S_z - 2S_z$

$_{xx}S_x)$

$-\rightarrow j$

$+s_2 J_0\, a_{20}$

2
$(2S_x$
$_{xx}S_y - 2S_y$
$_{xx}S_x)$
$\vec{k} - s_2 J_0 2\Delta$

$\overline{S_z S_y}$
$\vec{i} - S_z S_x$
$\vec{j}$

$\overline{(2.9)}$
we have $S = (S_x, S_y, S_z)$, $e_z = (0, 0, 1)$ then from $(2, 9)$ obtain
$\hbar$
$\vec{S}_t + s_2 J_0 a_{20}$
$h\vec{S} \times$
$\vec{S}_{xx}$
i
$+ 2s_2 J_0 \Delta S_z$
$h\vec{S} \times$
$\vec{e}_z$
i
$= 0$ (2.10)
or rewriting this equationsin matrixs form we obtain
$i\hbar S_t + s_2 J_0 a_{20}$
2
$[S, S_{xx}] + s_2 J_0 \Delta$
2
$[S, \hat{\sigma}_z] \{S, \sigma_z\} = 0$ (2.11)
$(2, 5)$, $(2, 9)$, $(2, 10)$, $(2, 11)$ are different form of Landau-Lifshitz equation.

2.1) Introduction the stereographical projection

To conduct numerical simulation of magnetoelastic interaction processes it is not convenient the parametrization by spherical variables of by Euler angles, due to the singularities in the poles. In order to prevent that for numerical simulation we use stereographical projection.

a)The relationship between the complex parameters z and S_x, S_y, S_z by stereographical projection for top plane is given with the following relations
$S_x =$
$z + z$
$1 + zz$
$, S_y =$

$z \; - z$

$1 + zz$

$i, \; S_z =$

$1 - zz$

$1 + zz$

with use chain rule we have

S_{zx}

$=$

$-2z$

$(1 + zz)_2 \, z_x +$

$-2z$

$(1 + zz)_2 \, z_x$

S_x

$_x =$

$1 - z_2$

$(1 + zz)_2 \, z_x +$

$1 - z_2$

$(1 + zz)_2 \, z_x$

5

S_y

$_x =$

$-1 - z_2$

$(1 + zz)_2 \, iz_x +$

$1 + z_2$

$(1 + zz)_2 \, iz_x$

S_x

$_{xx} =$

$-2z \; (1 - z_2)$

$(1 + zz)_3 \, (z_x)_2 +$

$-2z \; (1 - z_2)$

$(1 + zz)_3 \, (z_x)_2 +$

$-4z \; - 4z$

$(1 + zz)_3 \, z_x z_x +$

$1 - z_2$

$(1 + zz)_2 \, z_{xx} +$

$1 - z_2$

$(1 + zz)_2 \, z_{xx}$

S_y

$_{xx} =$
$2z\ (1 + z_2)$
$(1 + zz)_3\, i\ (z_x)_2 +$
$-2z\ (1 + z_2)$
$(1 + zz)_3\, i\ (z_x)_2 +$
$4z\ - 4z$
$(1 + zz)_3\, i z_x z_x +$
$1 + z_2$
$(1 + zz)_2\, i z_{xx} +$
$-1 - z_2$
$(1 + zz)_2\, i z_{xx}$
S_z
$_{xx} =$
$4z_2$
$(1 + zz)_3\, (z_x)_2 +$
$4z_2$
$(1 + zz)_3\, (z_x)_2 +$
$4zz\ - 4$
$(1 + zz)_3\, z_x z_x - 2z$
$(1 + zz)_2\, z_{xx} - 2z$
$(1 + zz)_2\, z_{xx}$
S_y
$_{xx}S_z - S_z$
$_{xx}S_y =$
$-2zz_4 - 2z_3 + 2zz_2 + 2z$
$(1 + zz)_4\, i\ (z_x)_2 +$
$2zz_4 + 2z_3 - 2zz_2 - 2z$
$(1 + zz)_4\, i\ (z_x)_2 +$
$zz_3 + z_2 - zz\ - 1$
$(1 + zz)_3\, i\ (z_{xx}) +$
$-zz_3 - z_2 + zz\ + 1$
$(1 + zz)_3\, i\ (z_{xx})$
S_x
$_{xx}S_y - S_y$
$_{xx}S_x =$
$-4z_3 z\ - 4z_2$
$(1 + zz)_4\, i\ (z_x)_2 +$

$4zz_3 + 4z_2$
$(1 + zz)_4 i \ (z_x)_2+$
$2z + 2zz_2$
$(1 + zz)_3 iz_{xx}+$
$-2z - 2zz_2$
$(1 + zz)_3 iz_{xx}$
S_x
$_{xx}S_z - S_z$
$_{xx}S_x =$
$-2z_4z - 2z_3 - 2z_2z - 2z$
$(1 + zz)_4 (z_x)_2+$
$-2z_4z - 2z_3 - 2z_2z - 2z$
$(1 + zz)_4 (z_x)_2 +$
$zz_3 + z_2 + zz + 1$
$(1 + zz)_3 z_{xx} +$
$zz_3 + z_2 + zz + 1$
$(1 + zz)_3 z_{xx}$
from (2, 9) we have $\hbar S_x$
$_t = s_2J_0a_2$
$_0 (S_y$
$_{xx}S_z - S_z$
$_{xx}S_y) - s_2J_02\Delta S_zS_y$ then
$\hbar$

—
$1 - z_2$
$(1 + zz)_2 z_t +$
$1 - z_2$
$(1 + zz)_2 z_t$

—
$= s_2J_0a_{20}$
(
$-2zz_4 - 2z_3 + 2zz_2 + 2z$
$(1 + zz)_4 i \ (z_x)_2 +$
$2zz_4 + 2z_3 - 2zz_2 - 2z$
$(1 + zz)_4 i \ (z_x)_2 +$
$zz_3 + z_2 - zz - 1$
$(1 + zz)_3 i \ (z_{xx})+$

$-z\bar{z}^3 - \bar{z}^2 + z\bar{z} + 1$

$(1 + z\bar{z})^3 \, i \, (\bar{z}_{xx})) - s^2 J_0 2\Delta$

$z - \bar{z}$

$1 + z\bar{z}$

$i \times 1 - z\bar{z}$

$1 + z\bar{z}$

(2.1)

and also $\hbar S^y$

$_t = -s^2 J_0 a^2_0$

$(S^x$

$_{xx} S^z - S^z$

$_{xx} S^x) + s^2 J_0 2\Delta S^z S^x$ then

$\hbar$

_ $-1 - z^2$

$(1 + z\bar{z})^2 \, i z_t +$

$1 + \bar{z}^2$

$(1 + z\bar{z})^2 \, i \bar{z}_t$

_

$= -s^2 J_0 a^2_0$

(

$-2z^4\bar{z} - 2z^3 - 2z^2\bar{z} - 2z$

$(1 + z\bar{z})^4 \, (\bar{z}_x)^2 +$

6

$-2\bar{z}^4 z - 2\bar{z}^3 - 2\bar{z}^2 z - 2\bar{z}$

$(1 + z\bar{z})^4 \, (z_x)^2 +$

$z\bar{z}^3 + \bar{z}^2 + z\bar{z} + 1$

$(1 + z\bar{z})^3 \, z_{xx} +$

$\bar{z}z^3 + z^2 + z\bar{z} + 1$

$(1 + z\bar{z})^3 \, \bar{z}_{xx})$

$+ s^2 J_0 2\Delta$

$z + \bar{z}$

$1 + z\bar{z}$

$\times 1 - z\bar{z}$

$1 + z\bar{z}$

then

$\hbar$

_ $-1 - z^2$

$(1 + z\bar{z})^2 \, z_t +$

$1 + z^2$

$(1 + zz)^2\, z_t$

$=$ $s^2 J_0 a^2_0$

$($

$-2z^4 z \; - 2z^3 - 2z^2 z \; - 2z$

$(1 + zz)^4\, i \; (z_x)^2 +$

$-2z^4 z \; - 2z^3 - 2z^2 z \; - 2z$

$(1 + zz)^4\, i \; (z_x)^2 +$

$zz^3 + z^2 + zz \; + 1$

$(1 + zz)^3\, i z_{xx} +$

$zz^3 + z^2 + zz \; + 1$

$(1 + zz)^3\, i z_{xx})$

$-s^2 J_0 2\Delta$

$z \; + z$

$1 + zz$

$i \; \times 1 - zz$

$1 + zz$

(2.2)

and also $\hbar S^z_t = s^2 J_0 a^2_0$

$(S^x$

$_{xx} S^y - S^y$

$_{xx} S^x)$ then

$\hbar($

$-2z$

$(1 + zz)^2\, z_t +$

$-2z$

$(1 + zz)^2\, z_t = s^2 J_0 a^2_0$

$($

$-4z^3 z \; - 4z^2$

$(1 + zz)^4\, i \; (z_x)^2 +$

$4z^3 z \; + 4z^2$

$(1 + zz)^4\, i \; (z_x)^2 +$

$2z \; + 2zz^2$

$(1 + zz)^3\, i z_{xx} +$

$-2z \; - 2z^2 z$

$(1 + zz)^3\, i z_{xx})$ (2.3)

we deduce relation (2.12) from relation (2.13) and we obtain

$\hbar$

$\overline{}$
2
$(1 + zz)_2\, z_t +$
$-2z_2$
$(1 + zz)_2\, z_t$

$\overline{}$
$= s_2 J_0 a_{20}($
$4zz_2 + 4z$
$(1 + zz)_4\, i\ (z_x)_2 +$
$4zz_4 + 4z_3$
$(1 + zz)_4\, i\ (z_x)_2 +$
-2
$(1 + zz)_2\, i\ (z_{xx}) +$
$-2z_2$
$(1 + zz)_2\, i\ (z_{xx})) - s_2 J_0 2\Delta$
$2z_2 z\ - 2z$
$(1 + zz)_2\, i$ (2.4)

we multiply relation (2.14) in $-z$ and add with relation (2.15) then we will have

$\hbar$

$\overline{}$
2
$(1 + zz)$
z_t

$\overline{}$
$= s_2 J_0 a_{20}$
$($
$4z$
$(1 + zz)_2\, i\ (z_x)_2 +$
-2
$(1 + zz)$
$i z_{xx}) - s_2 J_0 2\Delta$
$2z_2 z\ - 2z$
$(1 + zz)_2\, i$
$i\hbar z_t = s_2 J_0$
$\_ -2a_{20}$
z
$1 + zz$

z_{2x}
$+ a_{20}$
$z_{xx} + \Delta$
$2z(zz\ - 1)$
$1 + zz$

$\overline{}$
7
then the another form of Landau-Lifshitz equation by use stereographical projection for top plane is
$i\hbar z_t + s_2 J_0$

$-a_{20}$
$z_{xx} +$
$2a_{20}$
z
$1 + |z|_2\ z_2$
$_x + \Delta$
$2z(1 - |z|_2)$
$1 + |z|_2$

$= 0$
then this equation after scale transformation can be written as
$iz_t - z_{xx} +$
$2z$
$1 + |z|_2\ z_2$
$_x +$

$s_2 J_0$
$\omega_0 \hbar$
^{2}
Δ
$2z(1 - |z|_2)$
$1 + |z|_2 = 0$ (2.5)
b)The relationship between the complex parameters z and S_x, S_y, S_z by stereographical projection for bottom plane is given with the following relations
$S_x =$
$z\ + z$
$1 + zz$
, $S_y =$

$z - z$

$1 + zz$

$i, \; S_z =$

$zz - 1$

$1 + zz$

with use chain rule we have

S_{zx}

$=$

$2z$

$(1 + zz)^2 z_x +$

$2z$

$(1 + zz)^2 z_x$

S_y

${}_{xx}S_z - S_z$

${}_{xx}S_y =$

$2zz^4 + 2z^3 - 2zz^2 - 2z$

$(1 + zz)^4 i \; (z_x)^2 +$

$-2zz^4 - 2z^3 + 2zz^2 + 2z$

$(1 + zz)^4 i \; (z_x)^2 +$

$-zz^3 - z^2 + zz + 1$

$(1 + zz)^3 i \; (z_{xx}) +$

$zz^3 + z^2 - zz - 1$

$(1 + zz)^3 i \; (z_{xx})$

S_x

${}_{xx}S_z - S_z$

${}_{xx}S_x =$

$2z^4 z + 2z^3 + 2z^2 z + 2z$

$(1 + zz)^4 (z_x)^2 +$

$2z^4 z + 2z^3 + 2z^2 z + 2z$

$(1 + zz)^4 (z_x)^2 +$

$-zz^3 - z^2 - zz - 1$

$(1 + zz)^3 z_{xx} +$

$-zz^3 - z^2 - zz - 1$

$(1 + zz)^3 z_{xx}$

then the another form of Landau-Lifshitz equation by stereographical projection for bottom plane is

$i\hbar z_t + s^2 J_0$

$\overline{a}_{20}$

$z_{xx} - 2a_{20}$

z

$1 + |z|^2 z^2$

$_x + \Delta$

$2z(|z|^2 - 1)$

$1 + |z|^2$

$\overline{=} 0$

then this equation after scale transformation can be written as

$iz_t + z_{xx} - 2z$

$1 + |z|^2 z^2$

$_x +$

$\overline{s_2 J_0}$

$\omega_0 \hbar$

$\overline{}^2$

Δ

$2z(|z|^2 - 1)$

$1 + |z|^2 = 0$ (2.6)

3 Nonstandard Finite Difference Method

3.1) Introduction to NSFD or NFSD Rules

The genesis of nonstandard finite difference (NSFD) modelling procedures began with the 1989 publication of Mickens [2]. Extensions and a summary of the known results up to 1994 are given in Mickens [3]. This class of schemes and their formulation center on two issues. First, how should discrete representations for derivatives be determined, and second, what are the proper forms to be used for nonlinear terms. A brief and first introduction to the discrete derivative issue is discussed in the paper by Mickens et al. [4].we suppose decay equation[1]

du

dt

$= -\lambda u, \quad u(0) = u_0$

Note that the usual forward Euler representation for the first-derivative is

du

dt

$\rightarrow u_{k+1} - u_k$

h

However ,the discrete first-derivative for decay equation is given by the expression

$$\frac{du}{dt} \to \frac{u_{k+1} - u_k}{\phi}$$

where the denominator function ϕ is

$$\phi = \frac{1 - e^{-\lambda h}}{\lambda} \qquad 9$$

and the $denominatorfunction$ satisfies the condition

$$\phi(\lambda, h) = h + O(\lambda h^2)$$

This way of constructing discrete derivatives can be easily extended to partial derivatives. The linear harmonic oscillator is modeled by following secondorder ODE . The linear harmonic oscillator is modeled by the following second-order ODE[1]

$$\frac{d^2u}{dt^2} + \omega^2 = 0$$

where ω is a real constant.the discrete second-derivative for this equation is given by the expression

$$\frac{d^2u}{dt^2} \to \frac{u_{k+1} - 2u_k + u_{k-1}}{\frac{4}{\omega^2}\sin^2\left(\frac{h\omega}{2}\right)}$$

where $h = \Delta t,\ t_k = hk$.the "exact" NSFD scheme is

$$\frac{u_{k+1} - 2u_k + u_{k-1}}{\frac{4}{\omega^2}\sin^2\left(\frac{h\omega}{2}\right)} + \omega^2 u_k = 0.$$

This way of constructing discrete derivatives can be easily extended to partial derivatives In general, the NSFD rules do not lead to a unique discrete model for either ODEs or PDEs. However, this a priori nonuniqueness can often be partially resolved by appeal to various constraints applied to the discrete equations modelling the differential equations. For example, if the ODE or PDE has special solutions, such as fixed points or traveling waves, the requirement that the discrete equations also have these solutions along with the corresponding (linear) stability properties will often force the discrete models to only assume a small set of possible structures.

3.2) Implementation of NSFD for Landau-Lifshitz Equation

we use forward and backward and central difference method(explicit method or SFD) for equation (2.16) then

$$
i\,\frac{z_{i,j+1}-z_{i,j-1}}{2\tau}-\frac{z_{i+1,j}-2z_{i,j}+z_{i-1,j}}{h^2}+\frac{2z_{i,j}}{1+|z_{i,j}|^2}\left(\frac{z_{i+1,j}-z_{i-1,j}}{2h}\right)^2+10\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2\Delta\,\frac{2z_{i,j}(1-|z_{i,j}|^2)}{1+|z_{i,j}|^2}=0 \quad (3.1)
$$

and the NSFD scheme for this equation is

$$
i\,\frac{z_{i,j+1}-z_{i,j-1}}{2\tau}-\frac{z_{i+1,j}-2z_{i,j}+z_{i-1,j}}{\dfrac{4}{\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2\Delta\frac{2(-1+|z_{i,j}|^2)}{1+|z_{i,j}|^2}}\sin^2\left(\dfrac{h\sqrt{\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2\Delta\frac{2(-1+|z_{i,j}|^2)}{1+|z_{i,j}|^2}}}{2}\right)}+\frac{2z_{i,j}}{1+|z_{i,j}|^2}\left(\frac{z_{i+1,j}-z_{i-1,j}}{2h}\right)^2+
$$

$$\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2\Delta\,\frac{2z_{i,j}(1-|z_{i,j}|^2)}{1+|z_{i,j}|^2}=0 \qquad (3.2)$$

we use forward and backward and central difference method(explicit method) for equation (2.17) then

$$i\frac{z_{i,j+1}-z_{i,j-1}}{2\tau}+\frac{z_{i+1,j}-2z_{i,j}+z_{i-1,j}}{h^2}-\frac{2z_{i,j}}{1+|z_{i,j}|^2}\left(\frac{z_{i+1,j}-z_{i-1,j}}{2h}\right)^2+\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2\Delta\,\frac{2z_{i,j}(|z_{i,j}|^2-1)}{1+|z_{i,j}|^2}=0 \qquad (3.3)$$

and the NSFD scheme for this equation is

$$i\frac{z_{i,j+1}-z_{i,j-1}}{2\tau}+\frac{z_{i+1,j}-2z_{i,j}+z_{i-1,j}}{\frac{4}{\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2\Delta\frac{2(-1+|z_{i,j}|^2)}{1+|z_{i,j}|^2}}\sin^2\left(\frac{h}{2}\sqrt{\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2\Delta \ldots}\right)}$$

2(−1+|zi,j
|2)
1+|zi,j
|2
2)
$-2z_{i,j}$
$1+|z_{i,j}|^2$ (
$z_{i+1,j}-z_{i-1,j}$
$2h$
$)^2+$
(
$s_2 J_0$
$\omega_0 \hbar$
$)^2 \Delta$
$2z_{i,j}(-1+|z_{i,j}|^2)$
$1+|z_{i,j}|^2 = 0$ (3.4)

4 Numerical Result

To derive a numerical solution of the Landau-Lifshitz equation we use the analytical solitonic solution [5] of the Landau-Lifshitz equation as initial condition . We know for magnetic solitons moving with velocity υ exact solution is [5]

$tan^2\ \theta$
2
=
μ^2
$\Omega cosh^2 \mu\ \xi\ -[\Omega-\Omega_1]$
2
,
11
$\xi = z - \nu\ \tau = _2$
r
$a_1\Delta - \omega\ -$
_ ν
2
_2 (4.1)

Now for obtain approximation solution at $h = 0.1,\ \ \tau = 0.002, \Delta = 0.1,\ \ \nu = 0.1,\ 0.3,\ 0.6,\ 0.8$ at $T = 10$ for SFD and NSFD scheme that we got and we used $(4,\ 1)$ as initial valu As a result we will receive a magnetoelastic solitonic solution,which could be considered as magnetic polaron. In Fig 1,Fig 2 we show the error of SFD and NFSD scheme at $\nu = 0.1$ and in Fig 3,Fig 4 we show the error of SFD and NFSD scheme at $\nu = 0.3$.

Figure 1:Errore of SFD method for $\nu = 0.1$

Figure 2:Errore of NSFD method for $\nu = 0.1$

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Figure 3:Errore of SFD method for $\nu = 0.3$

Figure 4:Errore of NSFD method for $\nu = 0.3$

In Table 1, 2 we calculated the Maximum absolute errors for $\nu = 0.1,\ 0.3$.

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T	SFD method	NSFD method
1	$5.0000(-5)$	$5.0000(-5)$
2	$1.0000(-4)$	$1.0000(-4)$
3	$1.2000(-4)$	$1.1000(-4)$
4	$1.4000(-4)$	$1.3000(-4)$
5	$1.6000(-4)$	$1.4000(-4)$
6	$1.9000(-4)$	$1.6000(-4)$
7	$2.2000(-4)$	$1.8000(-4)$
8	$2.5000(-4)$	$2.0000(-4)$
9	$2.9000(-4)$	$2.1000(-4)$
10	$3.3000(-4)$	$2.3000(-4)$

Table 1: Maximum absolute errors for $\nu = 0.3$

T	SFD method	NSFD method
1	$5.30000(-5)$	$3.0000(-5)$
2	$5.0000(-5)$	$5.0000(-5)$
3	$6.0000(-5)$	$7.0000(-5)$
4	$8.0000(-5)$	$8.0000(-5)$
5	$1.0000(-4)$	$1.0000(-4)$
6	$1.2000(-4)$	$1.1000(-4)$
7	$1.3000(-4)$	$1.1000(-4)$
8	$1.4000(-4)$	$1.2000(-4)$
9	$1.6000(-4)$	$1.3000(-4)$
10	$1.8000(-4)$	$1.4000(-4)$

Table 2: Maximum absolute errors for $\nu = 0.1$

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We also did this for $\nu = 0.6,\ 0.8$

Figure 5:Errore of SFD method for $\nu = 0.6$

Figure 6:Errore of NSFD method for $\nu = 0.6$

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Figure 7:Errore of SFD method for $\nu = 0.8$

Figure 8:Errore of NSFD method for $\nu = 0.8$

T	SFD method	NSFD method
1	2.0000(−5)	3.000(−5)
2	4.0000(−5)	6.0000(−5)
3	6.0000(−5)	9.0000(−5)
4	7.0000(−5)	1.2000(−4)
5	9.0000(−5)	1.5000(−4)
6	1.1000(−4)	1.8000(−4)
7	1.3000(−4)	2.1000(−4)
8	1.4000(−4)	2.4000(−4)
9	1.5000(−4)	2.7000(−4)
10	1.7000(−4)	3.0000(−4)

Table 3: Maximum absolute errors for $\nu = 0.6$

T	SFD method	NSFD method
1	1.0000(−5)	2.0000(−5)
2	2.0000(−5)	4.0000(−5)
3	3.0000(−5)	6.0000(−5)
4	4.0000(−5)	8.0000(−5)
5	5.0000(−5)	1.0000(−4)
6	7.0000(−5)	1.1000(−4)
7	8.0000(−5)	1.3000(−4)
8	8.0000(−5)	1.5000(−4)
9	9.0000(−5)	1.7000(−4)
10	1.0000(−4)	1.8000(−4)

Table 4: Maximum absolute errors for $\nu = 0.8$

5 Conclusion and Future Works

According to numerical examples,it can be concluded that if the velocity is near the 1 the error of SFD method is less than that of the NSFD method,and $\nu \ll 1$ the error of NSFD method is less than that of the SFD method,but in general ,both of these methods have a close error but to increase the accuracy of calculation,we can select kind of method according to the value of velocity.In this paper we wanted to use two methods to find the approximate solution and compare them. In the future we intend to use this method to solve dynamic systems that is a combination of Landau-Lifshitz equation and wave equation.

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