

Abstract

In this study, we derive a new representation for the Euler-Mascheroni constant and present various expressions for the classical Euler-Mascheroni constant related to the Riemann zeta function. Also, we proved that γ is not algebraic if the Schanuel conjecture is true.

Keywords: Euler-Mascheroni constant; Schanuel Conjecture; Representations
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1 Introduction

The Euler-Mascheroni constant or simply the Euler constant, γ , was first investigated by Euler in 1734 and later by Mascheroni. As a mathematical constant, the Euler-Mascheroni constant appears in the study of special functions such as the gamma and Riemann zeta functions and keeps recurring in number theory and analysis. The Euler-Mascheroni constant is in the class of mathematical constants such as Ludolph's number (π) and Euler's number (e) and has applications in general theory of relativity and quantum theory (Sukenic and Sima, 2018).

It is known that π and e are irrational and transcendental, but little is known about the algebraic properties of the Euler-Mascheroni constant. An algebraic number is a complex number which is a root of a polynomial with rational coefficients and a transcendental number is a complex number which is not algebraic. It remains an open problem whether or not the Euler-Mascheroni constant

UNDER PEER REVIEW

Original Research

Paper

On Some Representations of the Euler-Mascheroni constants

is irrational, transcendental or algebraic.

Euler-Mascheroni constant, γ , is defined as

$$\gamma = \lim_{n \rightarrow \infty} \left(\sum_{j=1}^n \frac{1}{j} - \ln(n) \right) \quad (1.1)$$

Now, other representations of the Euler-Mascheroni constant are given by

$$\gamma = \lim_{n \rightarrow \infty} \left(H_n - \ln(n) \right) \quad (1.2)$$
$$= \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \zeta(n) \quad (1.3)$$

where H_n is the n th harmonic number and $\zeta(n)$ is the Riemann zeta function.

Euler actually introduced (1.1), and later Lorenzo Mascheroni also studied this constant, and gave a numerical approximation for it up to 32 decimal places and this made him to have some resulting in prominence as far as the Euler constant is concerned. Lagarias (2013) examined the Euler-Mascheroni constant, and described various mathematical developments about this constant and its connection with arithmetic functions, the Riemann hypothesis, random permutations, and random matrix products. In an essay on the Euler-Mascheroni constant, Lima (2014) delved into the recurrence of this constant in multiple branches of mathematics and intimated the possibility that γ could be transcendental. He also concluded that based on the computation of γ and its exponential, it strongly indicates that they are irrational.

Wu and Bercu (2012) presented a new sequence that converges to the Euler-Mascheroni constant using an approximation of Pade type. They established lower and upper bound estimates between their sequence and the Euler-Mascheroni constant. A brief survey on the history of the Euler-Mascheroni constant, its applications, and appearances in various mathematical settings can be found in Dence and Dence (2009).

2 Materials and Methods

The Riemann zeta is defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}; \operatorname{Re}(s) > 1 \quad (2.1)$$

$$= \prod_p \frac{1}{1 - p^{-s}}; \operatorname{Re}(s) > 1 \quad (2.2)$$

where p is a prime number.

The derivatives of the Riemann zeta function is given by

$$\zeta_k(s) = (-1)^k \sum_{n=1}^{\infty} \frac{\ln^k n}{n^s} \quad (2.3)$$

The digamma function is given by the logarithmic derivative of the gamma function:

$$\psi(z) = \frac{d}{dz} \ln \Gamma(z)$$

$$\Gamma'(z) = \Gamma(z) \psi(z) \quad (2.4)$$

101

UNDER PEER REVIEW

The digamma function is defined as

$$\psi(z+1) = -\gamma + \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+z} \right); z \neq -2, -3, \dots \quad (2.5)$$

where γ is the Euler-Mascheroni constant.

For $|z| < 1$,

$$\ln \Gamma(z+1) = -\gamma z +$$

$$\sum_{k=2}^{\infty} \frac{(-1)^{k+1} \zeta(k)}{k} z^k$$

$$+ \sum_{k=2}^{\infty} \frac{(-1)^k (\zeta(k) - 1)}{k} z^k \quad (2.6)$$

$$= -\ln(1+z) - (\gamma-1)z +$$

$$\sum_{k=2}^{\infty} \frac{(-1)^k (\zeta(k) - 1)}{k} z^k$$

$$+ \sum_{k=2}^{\infty} \frac{(-1)^k (\zeta(k) - 1)}{k} z^k \quad (2.7)$$

To expand Stirling numbers facilitate expansion of $\ln \Gamma(z+1)$ in a power series about $z=0$, Stirling numbers are utilized.

The Stirling numbers of the first kind, $s(m; j)$, is defined by the generating function as

$$\ln_j(1+nz) = \sum_{n=j}^{\infty} \frac{n!}{j!} s(m; j) n z^m; \quad (2.8)$$

where $|z| < 1$.

Alternatively, Stirling numbers of the first kind are also defined as

$$\ln_j(1+t) = \sum_{n=j}^{\infty} \frac{s(n; j)}{n!} t^n; \quad (2.9)$$

or

$$\ln_j(1-t) = \sum_{n=j}^{\infty} \frac{(-1)^n s(n; j)}{n!} t^n; \quad (2.10)$$

From the above, we see that $s(n; 1) = (-1)^{n-1} (n-1)!$, $s(n; n) = 1$,

$s(n; 2) = (-1)^n n(n-1)! \prod_{j=1}^{n-1} \frac{1}{j}$ and
 $s(n; 3) = \frac{1}{2} (-1)^{n+1} H_{n-2} - H_{n(2)-1}$,
 where H_{n-1} is the $(n-1)$ th harmonic number and $H_{n(2)-1}$ is a harmonic number of order 2.
 Abe-I-kpeng [et al.](#) (2021) established a generalized gamma function as

| | |
|---|--|
| $\Gamma_k(z) = \lim_{n \rightarrow \infty} \frac{n!}{n^z} \prod_{j=1}^n \exp\left(\frac{z}{k+1} \ln\left(\frac{j+1}{j}\right)\right)$ | $\prod_{j=1}^n \exp\left(\frac{z}{k+1} \ln\left(\frac{j+1}{j}\right)\right)$ |
|---|--|

$$\exp\left(\frac{z}{k+1} \ln\left(\frac{j+1}{j}\right)\right) \prod_{j=1}^n \exp\left(\frac{z}{k+1} \ln\left(\frac{j+1}{j}\right)\right) + z^j \quad (2.11)$$

and a functional equation as

$$\Gamma_k(z+1) = \exp\left(\frac{z}{k+1}\right) \Gamma_k(z); \quad (2.12)$$

where $z \in \mathbb{C}$ and $k \in \mathbb{N}_0$.

Euler's formula for calculating $\zeta(2k)$ is expressed as

$$\zeta(2k) = \frac{(-1)^{k+1} (2\pi)^{2k} B_{2k}}{2(2k)!}; \quad (2.13)$$

$$102$$

UNDER PEER REVIEW

where B_{2k} are Bernoulli numbers.

An identity for $\zeta(2k+1)$ was established in Abe-I-kpeng [et al.](#) (2019) as

$$\zeta(2k+1) = \frac{(-1)^{k+1}}{2} \int_0^1 \frac{t^{2k} \cot(\pi t)}{1-t^{2k+1}} dt; \quad (2.14)$$

| | |
|-------------------------------------|-----|
| $\frac{1-k(2\pi)^{2k+1}}{2(2k+1)!}$ | Z |
|-------------------------------------|-----|

$$\int_0^1 \frac{t^{2k} \cot(\pi t)}{1-t^{2k+1}} dt; \quad (2.14)$$

where $k \in \mathbb{N}$.

In Folland (2009), [it is observed/observes](#) that if f_n is a sequence in a [measurable/measurable](#) function space L^+ , then by the monotone convergence theorem

$$\int \liminf_{n \rightarrow \infty} f_n \leq \liminf_{n \rightarrow \infty} \int f_n; \quad (2.15)$$

$$f_n = X$$

$$\int f_n \leq \int f_n; \quad (2.15)$$

Lemma 2.1. (Lindemann-Weierstrass)

Given a positive integer n and distinct algebraic numbers $\alpha_0, \dots, \alpha_n$. The numbers $e^{\alpha_0}, \dots, e^{\alpha_n}$ are linearly independent over the set of algebraic numbers, A for $\beta_0, \dots, \beta_n \in A$ not all zero.

Lemma 2.1 implies that

$$\sum_{k=0}^n \beta_k e^{\alpha_k} = 0; \quad (2.16)$$

$$\beta_k e^{\alpha_k} = 0; \quad (2.16)$$

Lemma 2.2. (Schanuel Conjecture)

Let z_1, \dots, z_n be complex numbers that are linearly independent over the rational numbers \mathbb{Q} . Then

the extension field $\mathbb{Q}(z_1, \dots, z_n, e^{z_1}, \dots, e^{z_n})$ has [transcendence/transcendence](#) degree of at least n where e^z is the complex exponential of z .

The Lemma below was established as a Corollary in Lima (2014).

Lemma 2.3. For any positive integer n , the numbers $\ln \pi$ and $n \ln \pi$ are [linearly/linearly](#) independent.

Remark 2.1. By Lemma 2.2 and taking $n=1$ implies that $\ln \pi$ and π are transcendental.

For $j \in \mathbb{N}$, Abe-I-kpeng [et al.](#) (2021) established that

$$\ln_{k+1}(j+1) - \ln_{k+1} j = \frac{1}{j}$$

$$(k+1) \ln_k j$$

$$+$$

$$\int X_m$$

$$= \frac{1}{j} \prod_{m=1}^m \frac{1}{m!} \prod_{m=1}^m (k+1-k) \ln_{k+1-n} j; \quad (2.17)$$

In Abramowitz and Stegun (1972), the sine function is given as

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}; \quad (2.18)$$

$$x$$

$$=$$

$$\int Y_n$$

$$= \frac{1}{(2n+1)!} x^{2n+1}; \quad (2.18)$$

3 Results and Discussion

We begin by establishing a new representation for the Euler-Mascheroni constant.

Theorem 3.1. The Euler-Mascheroni constant is given by

$$\gamma = \ln \pi - 2$$

$$\prod_{m=2}^{\infty} \zeta(m) = 2m; \quad (3.1)$$

103 UNDER PEER REVIEW

Proof. From (2.11), we have

$$\Gamma_k(z) =$$

| | |
|---|-------------------------|
| $\exp \prod_{k+1}^{\infty} z^{\frac{1}{k+1}}$ | $k+1 \frac{1}{k+1} + j$ |
| $\exp \prod_{k+1}^{\infty} z^{\frac{1}{k+1}}$ | $k+1 \frac{1}{k+1} + j$ |

$$\prod_{j=1}^{\infty} \ln^{k+1} z \prod_{j=1}^{\infty} \ln^{k+1} z; \quad (3.2)$$

By (2.17), we obtain

$$\Gamma_k(z) =$$

$$\exp \prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} (k+1-s(m,n-n)!) \zeta_{(k+1-n)}(m)$$

| | |
|--------------------------------|--|
| $\exp \prod_{m=2}^{\infty} m!$ | $k+1 \frac{1}{k+1} \exp \prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(k+1-n)}(m)$ |
| $\prod_{m=2}^{\infty} m!$ | $k! s(m;n)$ |

$$\prod_{n=1}^{\infty} \ln^{m-n-1}; \quad (3.3)$$

By letting $z = -1/2$ and $k = 0$ yields

| | |
|---------------------------|--|
| $\Gamma_{-1} =$ | $\exp \prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(1-n)}(m)$ |
| $\prod_{m=2}^{\infty} m!$ | $\prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(1-n)}(m)$ |
| $\prod_{m=2}^{\infty} m!$ | $\prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(1-n)}(m)$ |

$$\prod_{m=2}^{\infty} m!$$

| | |
|---------------------------|---|
| $\prod_{m=2}^{\infty} m!$ | $\prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(1-n)}(m)$ |
|---------------------------|---|

$$\prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(1-n)}(m);$$

$$=$$

$$\exp \prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(1-n)}(m)$$

$$\exp \prod_{m=2}^{\infty} m! \prod_{n=1}^{\infty} \zeta_{(1-n)}(m);$$

$$-2\pi = -2 \exp 1$$

$$2$$

$$\prod_{m=2}^{\infty} m!$$

$$=$$

$$(-1)_m \zeta(m)$$

$$m$$

$$+$$

$$\prod_{m=2}^{\infty} m!$$

$$=$$

$$(-1)_{2m} \zeta(m)$$

$$m \prod_{m=2}^{\infty} m! :$$

Further simplifying gives

$$\rho\pi = \exp \gamma$$

$$2$$

$$+$$

$$\prod_{m=2}^{\infty} m!$$

$$=$$

$$\zeta(m)$$

$$m \prod_{m=2}^{\infty} m! : \quad (3.4)$$

By applying logarithm on both sides of (3.4) and making γ the subject completes the proof.

Remark 3.1. If $k = 0$ and $z = 1$

in (3.3), we obtain

$$\Gamma_{-1} = 2 \exp \prod_{m=2}^{\infty} m!$$

$$m$$

$$+$$

$$\prod_{m=2}^{\infty} m!$$

$$=$$

$$(-1)_m \zeta(m)$$

$$m \prod_{m=2}^{\infty} m! : \quad (3.5)$$

Simplifying further gives

$$\rho\pi = 2 \exp -\gamma$$

$$2$$

$$+$$

$$\sum_{m=2}^{\infty} \frac{(-1)^m \zeta(m)}{m 2^m} \quad (3.6)$$

Taking logarithm on both sides of (3.6) and making γ the subject gives

$$\gamma = \ln 4$$

$$\sum_{m=2}^{\infty} \frac{(-1)^m \zeta(m)}{m 2^m} \quad (3.7)$$

which is a known representation of the Euler-Mascheroni constant also found in Bonnar (2017).

We observe that the series in (3.1) converges faster than that of the Euler-Mascheroni constant given by (1.3). Since the Euler-Mascheroni constant, as a mathematical constant, keeps reoccurring in the study of special functions and in analysis, we foresee the suitability of the constant $\lambda = \sum_{m=2}^{\infty} \frac{(-1)^m \zeta(m)}{m 2^m}$ in numerical calculations where γ converges slowly.

By the use of [infinite series calculator](http://www.wolframalpha.com/widgets) (www.wolframalpha.com/widgets), the constant λ converged to 0.283757.

Next, we present a new representation of the constant, e_γ , in the corollary below in corollary 3.2.

104

UNDER PEER REVIEW

Corollary 3.2.

$$e_\gamma = \sum_{m=2}^{\infty} \frac{(-1)^m \zeta(m)}{m 2^m} \quad (3.8)$$

where γ is the Euler-Mascheroni constant and $\lambda = \sum_{m=2}^{\infty} \frac{(-1)^m \zeta(m)}{m 2^m}$

Proof. From (3.1), we obtain

$$\pi = e_\gamma \quad (3.9)$$

By making e_γ the subject in (3.9) completes the proof.

The constant e_γ is vital in number theory and relates to the Marten's third theorem in the following form:

| | | |
|--|---|-----------------------|
| $e_\gamma = \lim_{n \rightarrow \infty} \frac{\ln p_n}{p_n}$ | 1 | $\frac{p_i}{p_i - 1}$ |
|--|---|-----------------------|

$$\sum_{i=1}^{\infty} \frac{1}{p_i} \quad (3.10)$$

where p_n is the n th prime number.

Following Theorem 3.1, we present various representations of the Euler-Mascheroni constant.

Theorem 3.3.

$$\gamma = \ln \ln(i) - 2 \sum_{m=2}^{\infty} \frac{(-1)^m \zeta(m)}{m 2^m} \quad (3.11)$$

where $i_2 = -1$.

Proof. From Given the Euler's identity, we obtain

$$\pi = \ln(i) - 2i \quad (3.12)$$

Substituting (3.12) into (3.1) completes the proof.

Theorem 3.4.

$$\gamma = \ln 1 - 2 \sum_{m=2}^{\infty} \frac{(-1)^m \zeta(m)}{m 2^m}$$

$$\zeta(m) \\ m^{2m} \\ : (3.13)$$

Proof. Choudhury (1995) established that

$$\zeta(0) = -\frac{1}{2}$$

$$\ln 2\pi: (3.14)$$

By making $\ln \pi$ the subject, we get

$$\ln \pi = -\ln 2 - \zeta_0(0): (3.15)$$

Substituting (3.15) into (3.1) completes the proof.

Theorem 3.5.

$$\gamma = \ln 1 \\ 2$$

$$+ 2$$

$$1 X m$$

$$-2 \ln m - \zeta_m(2m): (3.16)$$

$$105$$

UNDER PEER REVIEW

Proof. For $k = 1$ and $z = 0$, (2.3) becomes

$$\zeta_0(0) = -$$

$$1 X m$$

$$-1$$

$$\ln m: (3.17)$$

which yields

$$-$$

$$1 2$$

$$\ln 2\pi =$$

$$1 X m$$

$$-1$$

$$\ln m: (3.18)$$

Making $\ln \pi$ the subject, we obtain

$$\ln \pi = \ln 1$$

$$2$$

$$+$$

$$1 X m$$

$$-1$$

$$\ln m_2: (3.19)$$

Substituting (3.19) into (3.1) completes the proof.

Theorem 3.6.

$$\gamma = \ln p^6 + 1$$

$$2$$

$$\ln Y$$

$$1 - p^{2-1} - 2$$

$$1 X m$$

$$-2$$

$$\zeta(m)$$

$$m^{2m}$$

$$: (3.20)$$

Proof. Substituting $s = 2$ into (2.1) gives

$$\zeta(2) = Y$$

$$1 - p^{2-1}: (3.21)$$

and further yields

$$\pi^2$$

$$6$$

$$= Y$$

$$1 - p^{2-1}: (3.22)$$

Applying logarithm on both sides of (3.22), we get

$$\ln \pi = 1$$

$$\ln 2 + \frac{1}{2} \ln 2 + \frac{1}{4} \ln 2 + \dots$$

$$1 - p_{2^{-1}}! : (3.23)$$

Substituting (3.23) into (3.1) ends the proof.

Remark 3.2. Comparing (3.13) and (3.23), we get

$$\zeta(2) = \frac{1}{6}$$

$$2 \zeta(2) = \frac{1}{3} \ln 2 + \frac{1}{4} \ln 2 + \dots : (3.24)$$

Theorem 3.7.

$$\zeta(2) = \frac{1}{6} = \frac{1}{2} \ln 2 + \frac{1}{4} \ln 2 + \frac{1}{8} \ln 2 + \dots : (3.25)$$

Proof. By letting $x = \pi^2$ and substituting into (2.18), we obtain

$$\ln \pi = \ln 2 + \frac{1}{2} \ln 2 + \frac{1}{4} \ln 2 + \dots : (3.26)$$

is a [Wallis product](#) formula for π^2 .

106

UNDER PEER REVIEW

Taking logarithm on both sides of (3.26) gives

$$\ln \pi = \ln 2 + \frac{1}{2} \ln 2 + \frac{1}{4} \ln 2 + \dots : (3.27)$$

Substituting (3.27) into (3.1) ends the proof.

Theorem 3.8. The constant λ is given by

$$\lambda = \frac{1}{2} \ln 2 - \ln 2 + \frac{1}{4} \ln 2 - \frac{1}{8} \ln 2 + \dots : (3.28)$$

Proof. Integrating (2.5) [and](#) using (2.15), we get

$$\ln \Gamma(z+1) + \gamma z = \frac{1}{2} \ln z - \ln(m+z) + \ln(m) : (3.29)$$

By letting $z = -1/2$, (3.33) yields

$$\ln \pi - \gamma = \frac{1}{2} \ln 2 - \ln m - \frac{1}{2} \ln m + \ln m : (3.30)$$

By substituting (3.30) into (3.1) completes the proof.

[We further give](#) new series representations [emerge](#) for the Euler-Mascheroni constant involving Bernoulli numbers, Bernoulli polynomials, and generalized [Clausen](#) functions.

Theorem 3.9.

| | | |
|--|--|--------|
| $\gamma = \ln \pi - \sum_{k=1}^{\infty} \frac{(-1)^k (\pi)^{2k}}{2k(2k)!}$ | $+\sum_{k=1}^{\infty} \frac{(-1)^{1-k} (\pi)^{2k}}{(2k+1)(2k+1)!} Z$ | |
| B_{2k} | B_{2k+1} | $2k+1$ |

$$\int_0^1 B_{2k+1}(t) \cot(\pi t) dt; (3.31)$$

where B_{2k} and $B_{2k+1}(t)$ are Bernoulli numbers and polynomials, respectively.

Proof. For even values of m , (3.1) becomes

$$\gamma = \ln \pi - 2$$

$$\sum_{m=2}^{\infty} \frac{X_m}{2^m m}$$

$$= \zeta(m)$$

$$2^m m$$

-

$$\sum_{m=2}^{\infty} \frac{X_m}{2^{m+1} (m+1)!} \quad (3.32)$$

$$= \zeta(m+1)$$

$$2^{m+1} (m+1)! \quad (3.32)$$

Let $m = 2k$ for $k \in \mathbb{N}$, then we obtain

$$\gamma = \ln \pi - 2$$

$$\sum_{k=1}^{\infty} \frac{X_k}{2^{2k} 2k}$$

$$= \zeta(2k)$$

$$2^{2k} 2k$$

$$-$$

$$\sum_{k=1}^{\infty} \frac{X_k}{2^{2k+1} (2k+1)!} \quad (3.33)$$

$$= \zeta(2k+1)$$

$$2^{2k+1} (2k+1)! \quad (3.33)$$

$$2^{2k+1} (2k+1)! \quad (3.33)$$

Substituting (2.13) and (2.14) into (3.33), the proof is complete.

In Theorem 3.1, $\ln \pi$ is a term in the new representation for the Euler-Mascheroni constant as well as the constant $\lambda = \sum_{m=2}^{\infty} \frac{X_m}{2^m m \zeta(2m)}$. Understanding the nature of these constants can reveal the nature of the Euler-Mascheroni constant. Already, it is observed that the constant λ is algebraic in nature. The transcendental nature of $\ln \pi$ is shown in the lemma below 3.10.

Lemma 3.10. *Let the Schanuel conjecture be true. Then, $\ln \pi$ is transcendental.*

107

UNDER PEER REVIEW

Proof. Assuming Lemma 2.2 and Lemma 2.3 are true. This implies that π and $\ln \pi$ are algebraically independent over the set of rational numbers.

Thus, if Lemma 2.2 and Lemma 2.3 are true, then $\ln \pi$ is transcendental.

Claim

γ is transcendental.

Proof. By (3.1) and Lemma 3.10, the proof is complete.

Remark 3.3. The Euler-Mascheroni constant is not algebraic.

4 Conclusions

A new series representation of the Euler-Mascheroni constant has also been derived and various representations given. We also present a new representation for e_γ , which is useful in number theory and proved that if $\ln \pi$ is transcendental, then the Euler-Mascheroni constant is also transcendental.

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108

UNDER PEER REVIEW

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109

UNDER PEER REVIEW