Original Research Article

A Coherent Approach Towards Quantum Gravity

Abstract

This paper focuses on a phase transition from the asymptotic safety approach of renormalizing the quantum gravity (QG) to a more granular approach of the loop quantum gravity (LQG) and then merging it with the Regge calculus for deriving the spin-(2) graviton. From loop-(2) onwards, the higher derivative curvatures make the momentum go to infinity which assaults a problem in renormalizing the QG. If the Einstein-Hilbert (E-H) action, is computed, and a localized path integral (or partition functions) is defined over a curved space, then that action is shown to be associated with the higher order dimension in a more compactified way, resulting in an infinite winding numbers being accompanied through the exponentiality coefficients of the partition integrals in the loop expansions of the second order term onwards. Based on that localization principle, the entire path integral got collapsed to discrete points that if corresponds the aforesaid actions, results in negating the divergences' with an implied bijections' and reverse bijections' of a diffeormorphism of a continuous differentiable functional domains. If those domains are being attributed to the spatial constraints, Hamiltonian constraints and Master constraints then, through Ashtekar's variables, it can be modestly shown that the behavior of quantum origin of asymptotic safety is similar to the LQG granules of spinfoam spacetime. Then, we will proceed with the triangulation of the entangled-points that results in the inclusion of Regge poles via the quantum number $(+2, -2, 0)$ as the produced variables of the spin-(2) graviton and spin-(0) dilaton.

Keywords – Localization; Partition Functions; Asymptotic Safety; Loop Quantum Gravity; Einstein-Hilbert Action; Renormalized Fixed Flow; Master Constraints: Regge Poles; Yukawa Potential

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Introduction

Description of the universe at the smallest approach is rationalized by QG. LQG [1] being consistent with general relativity [2] describes a modest approach of gravity where quantum effects cannot be ignored. One concept that arises out of time, with regards to QG is the asymptotic safety (or nonperturbative renormalizability) in quantum field theory (QFT) which asserts a consistent and predictive theory of gravity in the Plank regime. The asymptotic safety [3] with relation to the QG describes the coupling constants to have UV-finite fixed points. This is allowed in the renormalizable approach [4], where it is logarithmic to the gravity [5, 6, 7, 8]. The non-trivial part in a RG-flow [9] is that, after the UV cut-off [10] the theory is safe from divergences. To generalize the notion of the perturbative theory of QG in the background of a Minkowski space is totally gauge invariant and unitary [11] in its scheme. The application of the RG-fixed points can be a brevity in its approach of generating the asymptotic safety schemes. Now, it is not at all permissible compulsorily in the theory that, the momentum becomes zero or they tends to take an arbitrarily minute values in the high energy limit, instead what is needed is that, it takes up a finite value in the area of a non-trivial UV-fixed point [12]. The coupling tendency of the gravity in the high energy limits could be subject to non-trivial compactified approach of the dimensions as extended through the partition functions through the highly-warped space where each identical but discrete collapsed points could provide a bijective diffeormorphism, rather than a continuous continuum of pathways that allows an inert mapping between two neighboring points to preserve and protect the information's of the infinite domains that otherwise gets washed away beyond the cut-off point, thus rendering and preserving a finitely valued LQG theory. In the running couplings of the RG-flow, there preserved a scale dependency, or scale invariance (as regards to the continuous anomaly between quantum theory and general relativity(GR)), paving the UV limit to take dimensionless combinations, that are finite. The scattering amplitudes being free from any unphysical divergences, although from loop-(2) these divergences couldn't be controlled because of the higher derivatives of the curvature tensors. This requirement restricts the bare action principles from the UV-cut off schemes.

In case of a Newton's constant, the negative mass dimensions of the expansion parameters always failed the standardized procedure of perturbative renormalizations, which renders the GR to be perturbatively renormalizable. Thus arises the need to control the parameters of the expansions and the non-perturbative approaches have arrived where the asymptotic gravity is arriving at a fixed point, upon localizations of a curved spacetime, that curbs the continuous flows when the exponential terms get towards zero, because of the exponential coefficients of the winding numbers, making distinct but discrete entanglement channels where the spacetime granules itself is an emergent points of gravity [13, 14, 15, 16]. Through the UV cut-off the discrete geometry is comparable to the uniqueness of the LQG, before which the higher order curvature derivatives has been suppressed from the E-H loop-(2) actions with a correlation function related to UV cut-off at Λ_{UV}. The E-H truncations that acts on the principle of least actions through a curved spacetime, that reduces (over a background geometry for the localization purpose) to ensemble of points rather than a continuous pathways making space for the spinfoams and discreteness (granular emerging structures of spacetime). Considering the distinct points as a unique singular manifold, we will extend the concepts of the dynamic triangulations over that "zoomed in" points, to make the hinge limit a goes to zero, while the n-dimensional triangle limit T goes to infinity, thereby with the continuous pressure of the limit, the extended notion creates a cone with a high deficit angle, and when the critical deficit angle ϑ_c is reached, the tension T_N builds up on the surface area of the n-dimensional cone $\partial C^{(n)}$ while, the gradient increased to the maximum and the circumference of the right circular conic base being minimum, that critical angle ϑ_c is marked on the excess of the tension force being accumulated on the boundary, from the (warped) potential, which can't proceed beyond that angle, the tension will flow down from the gradient with a closed band of vibrating (kinetic) energy as a tension string with a excessive decrement of energy due to the increase in circumference of the closed band as it moves down from the vertex (tip) to the base, thereby escapes as the spin-(2) graviton and spin-(0) dilation, resulting in the emergence of gravity. To compute this we will extensively apply Scattering matrix and Regge poles given the parametric values from the negative energy of the Schrödinger equation and its radical limits to produce a non-trivial geometry taking the $+2, -2, 0$ spin chirality as the closed string graviton and dilaton.

A seemingly simple question to ask is that: How does the elementary particle in the microscopic domain interact via the gravitational field? Well, the simple fact that those particles that falls beyond the regime of the standard model like the 'dark matter' stuff, also interacts via the gravitational field, this is analogues to the stars, planets and black holes and their mutual interaction with the gravity in the macroscopic scales. Although the microscopic scales are difficult to comprehend in today's technology due to the 'large coupling' of gravity, the macroscopic domain is simplified in just one field equations and its numerous solutions, that is, the field equations of the GR, which states as,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N * T_{\mu\nu} \tag{1}$$

(The units that have been worked in this paper are $\hbar = 1 = c$ and G_N being the Newton's gravitational constant), however, for the quantum effects the treatment is difficult because of the scale mismatch between GR and QM like, if we try to derive the quantum supremacy of a particle eigenstate, we have to search for a new theory, therefore, a proper space-time understanding of quantum geometry is vital to correlate gravity of GR with quantum gravity of QM. For, any theories to construct, there lies an underline motivation and this motivation is the existence of singularities in the GR and QM, although the nature of the singularities are different in their aspects but the affine connections between them call for a theory where, the proposed model of both GR and QM breaks down and the need for 'new physics' arises. In case of black holes, the 'old physics; GR' breaks down at the 'singularities' which is a point in time rather than a space of immense density stocked in a zeroed regions, quantum effects become prominent in this points, and if we take those effects then the gravitational warping demands for a quantum theory of gravity. So, in a way, the macro GR and micro QM are always connected and the theory should come from the Planckian scales $M_{Plank} = \sqrt{\hbar c G_N^{-1}} \approx 10^{19}\ GeV$. For the SM, there exists a screening model, called as the quantum fluctuations, which when considered in the 'energy couplings', they hits the high energy domain and gets to Landau Poles which signals for the new physics to come and simplify the landscape. After Higgs particle has been discovered at a mass of $125\ GeV$, it has been prominent to physicists that Landau Poles lies well above the Planckian regime, implying that SM is well, up (and beyond) the scale, where one can expect the quantum domain of gravity, giving a strong hint that UV-completion could arise as the minimalistic feature for the inclusion of QG into the standard model (SM).

Asymptotic safety

One of the most confirmed theories of particle physics in the QED [17], which has been tested experimentally with fruitful results, therefore, quantization of gravity is also possible but it cannot be performed through perturbative renormalization. Through power counting in the perturbative scheme, the mass dimension of Newton's constant is already -2. Now, the renormalization schemes that have been implemented by the counter terms-inclusion, should cancel the divergences arising out of 'loop integrals'. But, when, this method is applied to gravity-bound interactions, the counter terms ensemble to infinity. Therefore, it is no longer considered as a low energy effective theory due to infinite filed theory parameters and the problem goes on increasing as the loop expands more and more. Then, it returns back with a divergence at the quantization of GR at loop-(1) level which are being non-absorbed in counter terms expansions in the presence of matter fields. At loop-(2) level, the divergence becomes more prominent in the 'gamma integrals' even in the case of pure gravity. Therefore, nonperturbative schemes are required for the negation of this conceptual difficulty, providing the idea of 'asymptotic safety' that abandons the traditional concept of perturbative renormalization [18].

Now, concerning this new study, the traditional way of approaching to the effect of renormalization were performed in $d = 2 + \epsilon$ dimensions, however, its not completely clear that, how a renormalization could be done from $d = 2 + \epsilon$ to $d = 4$ dimensions, yielding the value of $\epsilon = 2$ as calculations to be done such that the parameters of ϵ should be very small. So, the nonperturbative treatment of the computational methods were not in hand at this time, so, it has been put aside till 90's when it has been chalked out in various books but still, the ϵ has remained very small. With the advent of 'functional renormalization group' methods, the situation improved by the effective average action, where the cutoff scale of the 'flow equation' can be taken care in presence of gauge symmetries also. Presently, the truncations of the $f(R)$ [19] and Weyl tensor [20] shows that 'asymptotic safety' has been possible in actual scenario where with each truncation there exists a 'nontrivial fixed point'. This shows us way that 'asymptotic gravity' approach could show a consistent and renormalizable, effective field theory of gravity in the quantum field theory. In the 'asymptotic safety' program, the quantum field theory has been looked upon from Wilsonian viewpoint where firstly, the fields have a degree of freedom and secondly, the symmetries underlying behind the fields. Here, in the theory space, singular points can be treated as actions where the space is spanning by monomials where the correspondent coefficients are coupling constants $\{g_a\}$ which are dimensionless. The microscopic quantity that arises in the integral are smoothen out to a lower resolution by the RG flow. A mapping is done from each RG transformations to the nearby ones, thus spanning a vector field flow. The scale dependence running coupling constants can be parameterized by $\{g_a\} = \{g_a(k)\}$ where k, the free momenta gives rise to RG trajectory having taken the values of $k \to 0$ in the IR limit and $k \to \infty$ in the UV limit. Asymptotic safety deals with the limit $k \to \infty$ with the existence of a RG fixed point. In the fixed point of the theory space, all fixed points $\{g_a{}^*\}$ taking the value of the beta functions, $\beta_\gamma\{g_a{}^*\} = 0$ for all the possible values of γ. The UV attractive directions of the fixed point ensures that all the directions of the RG flow are permitted through trajectories in the increasing scale. As the points go to larger scale referred as "UV critical surface" in the limit of $k \to \infty$, the asymptotic theory could be extended for a consistent quantum field theory. In the RG flow-Fixed point, the UV attractive regions are relevant than the UV repulsive regions where the relevant couplings dimensionality is equal to the dimensionality of the UV critical surface. Therefore, more predictive theory arises when the dimensionality of the UV critical surface is smaller. The studies mainly focus on the existence of the non-Gaussian fixed points, in the search of quantum gravity where $\{g_a{}^*\} \neq 0$ for the least value of g_a. Using the effective field theory framework, the correlation functions of the metric space $g_{\mu\nu}$ which is the modulo dieffornisms with a UV cutoff as Λ_{UV} is possible with the ultraviolet framework, which is ultraviolet complete and predictive as described by the equation [21, 22, 23, 24, 24, 26, 27],

(2) $$\sigma = \int_{\Lambda_{\text{UV}}} \mathcal{F} g_{\mu\nu}\, e^{iS[g_{\mu\nu}]}$$

GR having 2-free parameters (the Newton's constant and the cosmological constant), to elucidate why predictivity is important, let us describe the E-H action for the perturbative quantization of gravity given as [28],

(3) $$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R$$

Where one could expand,

(4) $$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G_N} * h_{\mu\nu}$$

Where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ is the spin-2 fields of graviton. Now, the divergences can be generated through the expansions of $\sqrt{-g}R$ where below, the loop-(1) and loop-(2) are given in details.

(5) $$\Gamma^{(1)}_{k\to\infty} = \frac{1}{(4\pi)^2 \varepsilon} \int d^4x \sqrt{-g} \left(-\frac{53}{180} \varepsilon^{\alpha\beta\gamma\delta} \varepsilon_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\rho\sigma}{}_{\gamma\delta} + \frac{7}{20} R^{\mu\nu} R_{\mu\nu} + \frac{1}{120} R^2 \right)$$

(6) $$\Gamma^{(2)}_{k\to\infty} = \frac{1}{(4\pi)^4} \int d^4x \sqrt{-g} \left(\frac{209}{2880} \frac{1}{\varepsilon} R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} \right)$$
$$- \left(\frac{5}{18} \frac{1}{\varepsilon^2} + \frac{5771}{4800} \frac{1}{\varepsilon} \right) R^{\alpha\beta} D^2 R_{\alpha\beta}$$
$$+ \left(\frac{1255}{54} \frac{1}{\varepsilon^2} - \frac{703049}{64800} \frac{1}{\varepsilon} \right) R_{\alpha\beta} R_{\gamma\delta} R^{\alpha\beta\gamma\delta}$$
$$- \left(\frac{551}{27} \frac{1}{\varepsilon^2} - \frac{833}{16200} \frac{1}{\varepsilon} \right) R^{\alpha}{}_{\beta} R^{\beta}{}_{\gamma} R^{\gamma}{}_{\alpha}$$
$$+ \left(\frac{1033}{108} \frac{1}{\varepsilon^2} - \frac{47417}{8100} \frac{1}{\varepsilon} \right) R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\sigma} R^{\delta}{}_{\sigma}$$
$$+ (\textit{terms involving the scalar})$$

At the core origins of the quantum gravity through the singularity (points), which is in the Planckian regime, it has been assessed that the corresponding dimensions are higher and warped up due to infinite spatial and temporal curvature terms which if applied in a logical way then the winding or warp number of the corresponding dimensions extending to infinity, thereby dragging all the frames, here apart from $s + t = 3 + 1$ dimensions, the frame dragging is preserved on a quantum segmentation of the winding parameters given the dimensional configurations as expressed by [29],

(7) $$\zeta = 2\pi R, \omega$$

Where,

(8) $$\omega = \infty$$

As the E-H actions gives the principle of the least action of motion, the least paths between events could be described in presence of curved space-times (with gravity) including other possible paths through a partition functions provided that the path integrals should correspond on a background metric preserving the Localizations [30] where each $(fermions)^2 = bosons$ and the partition integral carrying an exponent ω should involve the integral as,

(9) $$\sigma = \int \mathcal{F} \varphi_{(0)} * e^{-S\omega\varphi_{(0)}} \frac{1}{S\, Det'(QV)^{(2)}_{\varphi_{(0)}}}$$

And when,

$$\omega = \infty$$

The partition function collapsed to zero as,

$$\sigma = 0$$

This shows that on a background localization, the bosonic gravity collapsed to certain isolated points instead of a straight line which can only be applied to the second order terms of the loop-(2) actions, if and only if, there exists a specified integrand O through some affine valued parameters Ψ given by the integral as,

(10) $$\int O \, d\Psi$$

And, the components of this integral would be,

(11) $$\int O \, d\Psi = \{Second\ order\ term\ onwards\ from\ Loop - (2)\}$$

Now, to combine equation (6 with the integrand, we need to modify the equation of loop-(2) as,

(12) $$\Gamma^{(2)}_{k\to\infty} = \frac{1}{(4\pi)^4}\int d^4x\sqrt{-g}\left(\frac{209}{2880}\frac{1}{\varepsilon}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}\right) + \sigma \times \left(\int O \, d\Psi\right)$$

This giving us discrete points where one point is connected to others by means of an entanglement quantity that preserves the bijective property of functions mapping through domain from codomain (with respect to point pairs) that dictates an affine connectivity through the equation,

(13) $$\boldsymbol{f} : X \to Y$$

Then,

(14) $$\forall y \in Y, \exists!\, x \in X\ such\ that\ y = f(x)$$

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This can be extended to many-valued points that are isolated by means of the collective mapping through,

(15) $$\boldsymbol{f} : X \to Y \to Z \to A \to B \to C \ldots$$

Where,

(16) $$\boldsymbol{f} : (\boldsymbol{Points}) \ \propto \ (\sigma(\textstyle\int O \, d\Psi)\ terms)$$

This clearly shows the localization of bosonic gravity on a curved space-time upto the first order terms and after the UV cutoff, the higher order terms are segmented into discrete points which are an ensemble of localizations that preserves the bijection relation with each other but are nonetheless divergent.

To investigate the RG-Flow of gravity, upto the energy scale k to the nonperturbed level, the effective average action for gravity can be given by Γ_k where the momenta which is covariant below the level k are suppressed in the flow, the rest being integrated out. Therefore, if the dynamical field is given by the parameter Π and the background field given by $\widetilde{\Pi}$, the FRGE equation can be given by,

(17) $$k\partial_k\Gamma_k\left[\Pi,\widetilde{\Pi}\right] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)}\left[\Pi,\widetilde{\Pi}\right] + \mathcal{R}_k\left[\widetilde{\Pi}\right]\right)^{-1} k\partial_k\Gamma_k\left[\widetilde{\Pi}\right]\right]$$

Here, the second functional derivative of the Γ_k being $\Gamma_k^{(2)}$ with the mode quantum fields Π at fixed background $\widetilde{\Pi}$. The operator for the mode suppression being $\mathcal{R}_k\left[\widetilde{\Pi}\right]$ depending on the background fixed space metric, the momentum fluctuations become covariant at $p^2 \ll k^2$ and vanishes in $p^2 \gg k^2$. Both the IR and UV SuperTrace (STr) peaking momentum resides at $p^2 \approx k^2$. The effective solutions between the bare at $k \to \infty$ and effective $k \to 0$ satisfies the trajectory of the theory space as,

(18) $$\Gamma[\Pi] = \Gamma_{\mathrm{k}=0}\left[\Pi,\widetilde{\Pi} = \Pi\right]$$

Therefore, for the asymptotically safe theory, $k \to \infty$ (the bare action) can be determined by a fixed point,

(19) $$\Gamma_* = \Gamma_{k\to\infty}$$

While assuming the truncations of the theory space, the basis could be taken as $\{P_a[\cdot]\}$ with the approximation of the finite dimensional subspace as the full theory space with $\alpha = 1,2,3,\ldots..N$ suffices to the ansatz,

(20) $$\Gamma_k\left[\Pi,\widetilde{\Pi}\right] = \sum_{\mathrm{a}=1}^{\mathrm{N}} g_a(k)P_a\left[\Pi,\widetilde{\Pi}\right]$$

Leading to the relation of the beta functions containing all the powers of the coupling as $\beta_a(g_1, g_2, \ldots\ldots, g_N)$ showing the nonpertrurbative character of the FRGE equations.

Therefore, the E-H Truncation can be given by terms of the metric potential $g_{\mu\nu}$ and the background metric $\widetilde{g_{\mu\nu}}$ with the Newtons constant $G_{N(k)}$ and Cosmological constant Λ_k in arbitrary space-time dimension d could be written as,

(21) $$\Gamma_k[g, \tilde{g}, \Phi, \widetilde{\Phi}] = \frac{1}{16\pi G_{N(k)}} \int d^d \sqrt{g}(-R(g) + 2\Lambda_k) + \Gamma_k^{\text{gauge fixing}}[g, \tilde{g}] + \Gamma_k^{\text{ghost action}}[g, \tilde{g}, \Phi, \widetilde{\Phi}]$$

Where $\Phi, \widetilde{\Phi}$ are the ghost fields of the scalar curvature $R(g)$.

$\beta - functions$ takes the values of,

- Newton Constant $k^{d-2} G_{N(k)}$
- Dimensionless cosmological constant $k^{-2}\Lambda_k$

If $d = 4$ dimensions, then this gives rise to RG-Flow equations which ultimately shows the non-Gaussian fixed point relating to asymptotic safety which is UV-attractive in both the Newton's constant and cosmological constant (directional trajectories). As presented before $d = 2 + \epsilon$ in the Beta functions, when $\epsilon = 2$, d becomes 4 which is the resultant conclusion of the nonperturbative flow equation without any further extrapolation of ϵ.

Loop quantum gravity

LQG being a strong candidature for a tough competition of superstring theory has been established for the purpose of incorporating GR into QM by incorporating the SM into the framework for the QG case. As a strong approach for QG, this LQG dictates the theory which shows the geometric formulation of the Einstein's GR rather than treating gravity, only as a force. LQG quantizes space into nodes, links and spinfoam networks that acts as a discrete and emerging properties of the analogue to classical gravity. The theory takes place on the smallest possible lengths of the matter at 10^{-35} meters where the space itself has been treated as an atomic quantity.

Now, coming back to the calculations, this can be easily realized that followed by the bijection, there arises a inverse bijection as oppose to equation (15) and there is no harm in considerating that. Which suffices the equation as,

(22) $$\boldsymbol{f^{-1}} : \ldots C \to B \to A \to Z \to Y \to X$$

Which satisfies the diffeormorphism [see Appendix B for the additional property called holonomy] *connecting the asymptotic safety formalism to the LQG formalism* which is a continuously differentiable function and can be called as a r times differentiable via C^r diffeomorphism. We will take the spatial diffeomorphism constraints $C^r{}_a(x)$ along with a shift function $\vec{N}$ to give a smeared spatial diffeomorphism constraints as

(23) $$(C^r{}_a(x))(\vec{N}) = \int C^r{}_a(x) N^a(x)\, d^3x$$

The Hamiltonian constraints [31] H with the smeared lapse in shift functions given by,

(24) $$H(\vec{N}) = \int H(x)\, N^a(x)\, d^3x$$

Using the Ashtekar's variable [32] [see Appendix A for detailed derivation] in the form of an electric field triad (which is actually the conjugate momentum as opposed to normal momentum q) takes the form $\tilde{E}_i^a(x)$ that in turn represents the Hamilton H with F_{ab}^i is the field strength tensor of the gauge connection A_a^i and $\beta = \pm i$ as,

(25) $$H = \frac{\epsilon_{ijk}\, F_{ab}^k \tilde{E}_i^a \tilde{E}_j^b}{\sqrt{\det(q)}} + 2\frac{\beta^2+1}{\beta^2} \frac{\tilde{E}_i^a \tilde{E}_j^b - \tilde{E}_j^b \tilde{E}_i^a}{\sqrt{\det(q)}} (A_a^i - \Gamma_a^i)(A_b^j - \Gamma_b^j) = H_E + H'$$

Here from onwards we will introduce a curvature terms of the background geometry as Euclidean as $\sigma_V{}^O$ where the background geometry σ^O acts over a volume space V to give the Euclidean Hamiltonian constraint functional by means of lapse function N as,

(26) $$H_E[N]|_{\sigma_V{}^O} = 2\int_\Sigma N(x) \epsilon_{abc} F_{ab}^k \{A_c^k, V|_{\sigma^O}\}$$

Where the $A_c^k, V|_{\sigma^O}, F_{ab}^k$ has been designated as a well defined operator in the loop representation, where upon quantization, the Poisson bracket is replaced by a commutator taking care of the first term with the introduction of the new quantity $\mathcal{K}$ as the trace of the extrinsic curvature to give four parallel equations with the final equations depicting $\mathcal{K}$ the "negative time derivative of the volume" as,

(27) $$\mathcal{K} = \int d^3x\, \mathcal{K}_a^i \tilde{E}_i^a, \qquad \mathcal{K}_a^i = \{A_a^i, \mathcal{K}\}, \qquad A_a^i - \Gamma_a^i = \beta\mathcal{K}_a^i = \beta\{A_a^i, \mathcal{K}\}$$

$$\mathcal{K} = -\left\{V, \int d^3x\, H_E\right\}$$

The Hamiltonian constraint takes the diffeomorphism invariant states and maps into non-diffeomorphism states in order to negate the diffeomorphism through the Hilbert space $\mathcal{H}_{\text{Diff}}$ over the manifold $\mathcal{M}$ through the Lagrangian of the shift operator $\mathcal{L}_{\vec{N}}$ taking the spin network ϱ_S given by,

(28) $$\varrho_S \in \mathcal{H}_{\text{Diff}}, \qquad (\vec{C}^r{}_a(x))(\vec{N})\varrho_S = 0$$

To find the equivalence relation with a constant ϑ_k is,

$$(\vec{C}^r{}_a(x))(\vec{N})[\hat{H}(\mathcal{M})\varrho_S] = \vartheta_k \hat{H}(\mathcal{L}_{\vec{N}}\mathcal{M})\varrho_S \neq 0 \tag{29}$$

So, $\hat{H}(\mathcal{M})\varrho_S$ is not in $\mathcal{H}_{\text{Diff}}$ thus indicating that, to solve the problem of spatial diffeomorphism constraint and the Hamiltonian constraint one needs to accommodate a special type of constraint called the Master constraint to circumvent the problem by constructing the inner product of $\mathcal{H}_{\text{Diff}}$.

Now, its time to introduce the spin-foam network of the quantum state which in actual is the 3D surface representation of the gravitational field where equivalent spin networks are countable to construct a basis of LQG Hilbert space. This spin represents a topological structure that summed upon to get the Feynman's path integral (or partition functions) over a loop of quantum gravitational states indicating a new description of quantum gravity. The Master constraint program of LQG takes an infinite number of Hamiltonian constraint and correlates them to a unified Master equation $\mathcal{W}$ with the assumption $H(x) = 0$ as,

$$\mathcal{W} = \int d^3x \left([H(x)]^2 \Big/ \sqrt{\det(q(x))} \right) \tag{30}$$

Where the square of the constraints takes place, with the Master constraint [33] being only one, provides the knowledge that, if $\mathcal{W}$ vanishes then so do $H(x)'s$ where as if $H(x)'s$ vanish then $\mathcal{W}$ providing that, they are equivalent. Whereas $\mathcal{W}$ involves every 'smeared' spatial diffeomorphism constraint $\vec{C}^r{}_a(x)$ through Poisson brackets given,

$$\{\mathcal{W}, \vec{C}^r{}_a(x)\} = 0 \tag{31}$$

This makes it a single constrains as $\{\mathcal{W}, \mathcal{W}\} = 0$ which is $su(2)$ invariant as well, this is a dramatic simplification of the Poisson bracket analogy of LQG over the semi-classical limit. From the initial analogy, it seems that the Master constraint does not include any observables, however, this has been imposed that on a phase space conditions this includes the Dirac's observable $\mathcal{D}$ with a realization given as,

$$\{\{\mathcal{W}, \mathcal{D}\}\mathcal{D}\}_{\mathcal{W}=0} = 0 \tag{32}$$

The Master constraint on a quantum level is defined by the set of 2 equations for all $\widehat{\mathcal{W}}\Psi = 0$ as,

$$\mathcal{W}_{Quantum\ \ limit} = \int \left(\hat{H} \Big/ \sqrt[4]{\det(q(x))} \right)(x) \left(\hat{H} \Big/ \sqrt[4]{\det(q(x))} \right)^{\dagger} (x)\ d^3x \tag{33}$$

$$\left(\hat{H} \Big/ \sqrt[4]{\det(q(x))} \right)(x(\Psi) = 0$$

As the Master constraint operator $\widehat{\mathcal{W}}$ is defined densely on the Hilbert space $\mathcal{H}_{\text{Diff}}$ with $\widehat{\mathcal{W}}$ being a positively defined symmetric operator existing on $\mathcal{H}_{\text{Diff}}$. then, the physical inner product of the spin-foam network is defined by the following,

$$\langle \phi, \varrho_S \rangle_{Phys} = \lim_{\mathcal{g} \to \infty} \langle \phi, \int_{-\mathcal{g}}^{\mathcal{g}} dt e^{it \int_\Sigma H(x)^2 - q^{ab} V_a(x) V_b(x) \big/ \sqrt{\det(q)}\, d^3x}, \varrho_S \rangle \tag{34}$$

Where by splitting the $\mathcal{g}$ - parameter discrete steps can be given through χ approaching infinity by the relation,

$$e^{it \int_\Sigma H(x)^2 - q^{ab} V_a(x) V_b(x) \big/ \sqrt{\det(q)}\, d^3x} = \lim_{\chi \to \infty} \left[e^{it \int_\Sigma H(x)^2 - q^{ab} V_a(x) V_b(x) \big/ \sqrt{\det(q)}\, d^3x} \Big/ \chi \right]^{\chi} = \lim_{\chi \to \infty} \left[1 + it \int_\Sigma H(x)^2 - q^{ab} V_a(x) V_b(x) \Big/ \sqrt{\det(q)}\, d^3x \Big/ \chi \right]^{\chi} \tag{35}$$

where, the interconnectivity of the linear combination of spins could be given by the extended Master constraints $\widehat{\mathcal{W}}_{extended} = \int_\Sigma H(x)^2 - q^{ab} V_a(x) V_b(x) \Big/ \sqrt{\det(q)}\, d^3x$, summing over all spatial and Hamiltonian constraints of semi-classical coherent states.

To develop a concrete notion of entanglement over the spin loops, it is needed to carefully analyze the 2-neighbouring nodes of the spin network, where the nodes could be expressed as X and Y where we will use the convention used in [34]. Assumed that, X and Y are linked with a fixed spin $\mathcal{J}$ with the edges of the vertex of $\mathcal{J}$ are connected with spins $\mathcal{J}_1, \dots, \mathcal{J}_P$ while another edges Y got connected with spins $k_1, \dots, k_Q$, then the(bounded) Hilbert states for a given spin network could take the form,

$$\mathcal{H}_{XY}^0 = \mathcal{H}_X^0 \otimes \mathcal{H}_Y^0 \tag{36}$$

Where each respective intertwiners $\mathcal{H}_X^0$ and $\mathcal{H}_Y^0$ with the diagonalizing notations V^i are given by,

$$\begin{cases} \mathcal{H}_X^0 = \mathrm{Inv}_{\mathrm{SU}(2)}[V^{\jmath_1} \otimes .. \otimes V^{\jmath_p} \otimes V^{\jmath}] \\ \mathcal{H}_Y^0 = \mathrm{Inv}_{\mathrm{SU}(2)}[V^{k_1} \otimes .. \otimes V^{k_q} \otimes V^{\jmath}] \end{cases} \tag{37}$$

The subscripts 0 indicates the $SU(2)$ invariant states whereupon considering a (pure) state $|\psi\rangle \in \mathcal{H}_{XY}^0$ the entanglement between X and Y bounded by the dimension of the states, with the Von Neumann entropy density $\rho^{VN} = |\psi\rangle\langle\psi|$ with E as the entanglement-entropy are,

$$E(X|Y) = -\mathrm{Tr}\,\rho^{VN}{}_A \mathrm{In}\,\rho^{VN}{}_A = -\mathrm{Tr}\,\rho^{VN}{}_B \mathrm{In}\,\rho^{VN}{}_B \leq \min(\mathrm{In}\dim \mathcal{H}_X^0, \mathrm{In}\dim \mathcal{H}_Y^0) \tag{38}$$

With the Hilbert spin sates boundary $\mathcal{H}_{XY}^{\partial}$ given by,

$$\mathcal{H}_{XY}^{\partial} = [V^{\jmath_1} \otimes .. \otimes V^{\jmath_p}][V^{k_1} \otimes .. \otimes V^{k_q}] \tag{39}$$

Considering two basis as $\Delta_{\mathrm{X}} \otimes \Delta_{\mathrm{Y}}$ for $\mathcal{H}_{XY}^{\partial}$ where the holonomy maps – between the intermediate states with a $SU(2)$ invariant projections $\mathcal{P}$ as,

$$\mathcal{P}:\ \Delta_{\mathrm{X}} \otimes \Delta_{\mathrm{Y}}\ \in \mathcal{H}_{XY}^{\partial} \mapsto \frac{1}{dim\ (2j+1)} \sum_{a,b} (-1)^{j-a} Q_{ab}^{j}(g) \langle j, -a | \Delta_{\mathrm{X}} \otimes j \Delta_{\mathrm{Y}} \in \mathcal{H}_{XY}^{\partial} \tag{40}$$

Where Q_{ab}^{J} is the Wigner matrix for $\boldsymbol{spin-j}$ which can straightforwardly extended over some density matrix $\mathcal{H}_{XY}^{\partial}$ with a (pure) state on $\mathcal{P}$ as $|\psi\rangle$ on $\mathcal{H}_{XY}^0$ having the spin state entanglements $\tilde{E}_g(X|Y)$ for any $g \in SU(2)$ with the bounded dimensions of the Hilbert spaces given as,

$$\tilde{E}_g(X|Y) \leq \min(\mathrm{In}\dim \mathcal{H}_X^{\setminus(XY)}, \mathrm{In}\dim \mathcal{H}_Y^{\setminus(XY)}) \tag{41}$$

$$\dim\ \mathcal{H}_X^{\setminus(XY)} = \prod_{i=1}^{P} (2j_i + 1)$$

$$\dim\ \mathcal{H}_Y^{\setminus(XY)} = \prod_{i=1}^{Q} (2k_i + 1)$$

Thus, the notion of entanglement between the nodes satisfies.

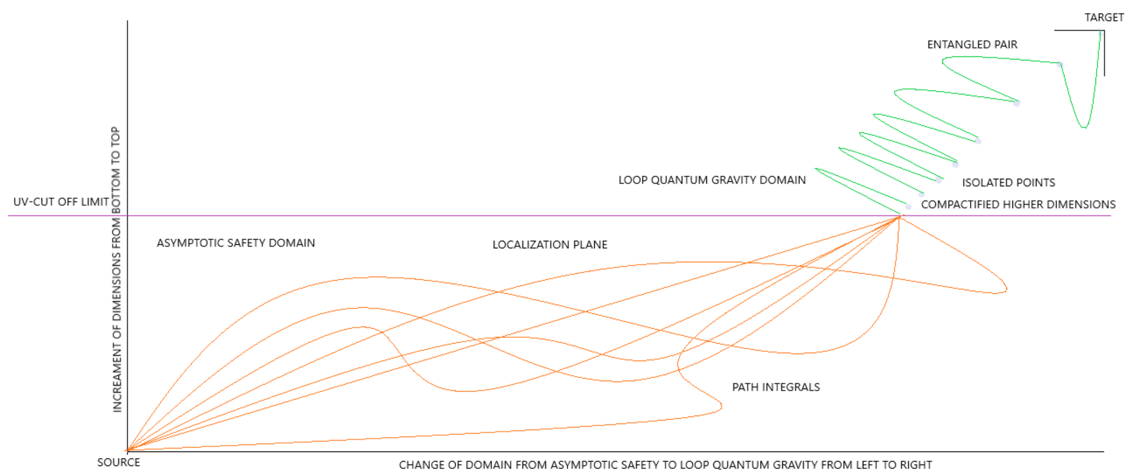

Figure 1: This picture illustrates the fact that beyond the UV-cut off plane, there is a collapse of path integrals (which is the FRGE here) and that insists the discrete geometry of the LQG where the entanglement between spin states are located on the networks links. LQG forms the purely spin network states which although doesn't have any physical bias, but the true entanglement arises from the spin or intertwiner superpositions [34]. One thing that is taken as a shift is "*to reconcile asymptotic safety in Quantum origins of gravity through renormalizable fixed point flows and linking it with (LQG)*". Initially the difficulty arises as gravity always involves quadratic divergences, higher order derivatives, higher order curvatures and higher order scalars. So, renormalizing is a good procedure but [informations are washed away with the UV cutoff]. So, to prevent this and to preserve informations, we have induced the compactification schemes from Superstring theory into the partition functions (or Feynman's path integrals over curved space). Then, we used the localization schemes which when acquired the infinite curvature terms from singularity, then the E-H action reduced to localized granular points with entanglements. There the the bijection and inverse bijection rules act as to preserve diffeomorphisms that *connects* the LQG of the Hamiltonian with the master constraints which allows the [phase transitions from asymptotic safety to distinct spin-foam granular spacetime]. Here it will not be completely correct to assert that, beyond the UV-cut off region (or plane) where there lies the crtical surface is the only domain for the playground of LQG. No, that's not the logic induced in this paper, rather it has to be said that, LQG has been considered in this case where there is extreme curvature zones of gravity like for e.g., singularity. If in that domain the $k \to \infty$ then combined with the extreme curvatures, the compactification plays the main role in the exponential coefficient of the partition function that leads the RG-flow points collapsed into isolated but diffeomorphisms domains where LQG beyond the critical limit gets a connectivity with the asymptotic safety below the critical limit, however, the RG-flow is continuous below the crtical limit, but it has been shown that, the RG-flow collapsed to ensemble of points beyond the critical limit. This again does not consider the fact that, beyond the critical limit RG-flow is non-continuously differentiable through bijective functions, no, they can be continuous but, that's the scope of others or previous research works based on a different viewpoint.

Regge Calculus and the birth of graviton

Regge calculus is another approach of QG which incorporates Einstein's GR and makes simplicial approximations of the spacetimes that are analogous to the Einstein field equation. This theory admits the triangulation of spaces into simplices, where the triangles meet at vertices with either a positive, negative or null curvature (null where there are no deficit angles). As this Regge calculus takes into account, the time orientable four dimensional Lorentzian manifolds into simplices, the deficit angles that can be computed directly from the edges (or hinges) is equivalent to the Riemann curvature tensors of GR.

To proceed further with the theory, we will consider that, the granules of space in LQG to be of the order $\sim 2\pi r, \omega$. Having very close or approximately equal to the higher dimensional circle, this will help us in realizing a further important problem for consideration, the dynamic triangulations of the space and the deficit angle constants (which is a powerful tool applied here for the formation of graviton). Therefore, lets consider the dynamical constraints of the triangulations that is the deficit angle ϵ is given by [40],

(42) $$\epsilon = 2\pi - \sum_{conic\ angle\ i \supset hinge\ h} \alpha_i$$

This can be safely assumed that, we have "zoomed in" those granules to inspect the triangulations with the deficit angular constraint. This equation (42) gives us 3-types of curvature coefficients presumably [40],

(43) $$\sum_{i \supset h} \alpha_i = 2\pi \quad \hookrightarrow Flat(Minkowski)$$
$$\sum_{i \supset h} \alpha_i < 2\pi \quad \hookrightarrow Positive(Gaussian)$$
$$\sum_{i \supset h} \alpha_i > 2\pi \quad \hookrightarrow Negative(Saddle)$$

We will take the positive (Gaussian) curvature and proceeds accordingly. In case of the higher dimensions, as in the case of the warped singular granules, let us define the Euler characteristics' as,

(44) $$E^0 = \sum_{\sigma^0} \epsilon_0^{(\omega)} - \frac{1}{2}\sum_{\sigma^2} \epsilon_2^{(4)}$$

Where $\epsilon_0^{(\omega)}$ is the solid deficit angle at the vertex σ^0 and $\epsilon_2^{(4)}$ is the deficit angle as appeared in the 4-dimensional Regge calculus, here (ω) is taken as a analogue of higher dimensions. Now, our aim is to denote the nonperturbative path integrals in terms of dynamic triangulations which is given by the parameter $\mathcal{Z}^{(R)}$ as,

(45) $$\mathcal{Z}^{(R)} = \lim_{\substack{a \to 0 \\ n_T \to \infty}} \sum_{\substack{inequivalent \\ T \in J_{(n)}}} \frac{1}{C^T} e^{i * S^{Regge}\ \{T\}}$$

Where $J_{(n)}$ is the causal triangulations with $\leq N$ building blocks and C^T is the automorphism groupof T. So, what we are doing is that, we have a higher dimensional manifold of the order $\sim 2\pi r, \omega$ and higher dimensional analogue of triangles has been chalked out with the hinges a limits to zero, while the n-dimensional triangles n_T tends to infinity. Therefore, considering two limits as the state when $a \ll 0$ and $n \ll \infty$ denoted by the initial state $S|\Phi_i\rangle$ and the state where $a < 0$ and $n < \infty$ by the final state $\langle\Phi_f|$S the equation suffices to form,

(46) $$S_{i \to f} = \langle\Phi_f|\mathrm{S}|\Phi_i\rangle = \langle\Phi_f \left| \sum_{n_T=0}^{\infty} \left[{(-i)^{n_T}}/{n_T!} \right] \int_{-\infty}^{\infty} dx_1^4 \dots \dots \int_{-\infty}^{\infty} dx_n^4 T[\mathcal{H}(t_1) \dots \dots \mathcal{H}(t_n)] \right| \Phi_i\rangle$$

Here, through time evolution from t_1 to t_n we have showed the scattering matrix $S_{i \to f}$ taking the value as showed in $\mathcal{Z}^{(R)}$. However, one important thing is to be mentioned that a will never reaches to zero, and n will never reaches to infinity. This is because, there will be a critical deficit angle measured by the parameter ϑ_C beyond which it cannot contract (whatever extreme limits we take!). therefore, where will the contraction energy goes to? It will create a tension force T_N over the surface area of the n-dimensional cone $\partial C^{(n)}$ and as no more deficit occurs beyond the critical angle ϑ_C, the gradient becomes too steep, the buildup strong-tension T_N will flow through the body downwards and leaves it as a circular band of energy. That's the string tension and that tension while flowing downwards looses most of its energy to become a closed vibrating string having some extra tensions left in it. And we can generate a suitable Hamiltonian for the case of the closed energy band having intervals $[0,2\pi]$ where if the energy band can be thing of a closed vibrating string, then the net tension force $\vec{F}^{T_N}$ would be approximately equal to $\pm k[2\pi r, \omega]$ with the $\pm k$ being the stretching and restoration of the string (as a spring of Hook's parameter k) taking positive real values over the space of $2\pi r, \omega$. Therefore keeping the string parameters (σ, τ) and with k, the Hamiltonian is given over the interval $[0,2\pi]$ as,

(47) $$\mathcal{H}^{(strings\ )} = \int_0^{2\pi} d\sigma \left[{\dot{X}^2 \sigma}/{2} + k \left({\partial(X)}/{\partial\sigma} \right)^2 \right]$$

Giving the Coulomb potential as $V_C(r) = -e^2/(4\pi\epsilon_0 r)$, the binding energy of the string can be given by the expression,

(48) $$E \to E_N = -{2m'\pi^2 e^4}/{h^2 N^2 (4\pi\epsilon_0)^2}$$

Where h is the Plank's constant and ϵ_0 be the permittivity of the vacuum, by solving the radial Schrödinger equation, the principle quantum number N takes the value as $N = n + \ell + 1$ where ℓ is the Regge pole or the quantum number of the orbital angular momentum with n the radial quantum number, the complex function of E given via ℓ plane path called the Regge treajectory, which appears in the modified S-matrix given by,

(49)
$$S^{(mod)} = \left[\Gamma\left(\ell - \left(-1 + i\frac{\pi e^2}{4\pi\epsilon_0 h}\right)\sqrt{\frac{2m'}{E}}\right) \Big/ \Gamma\left(\ell + \left(-1 + i\frac{\pi e^2}{4\pi\epsilon_0 h}\right)\sqrt{\frac{2m'}{E}}\right) \right] e^{-i\pi\ell}$$

Where in the gamma function $\Gamma(x)$, the the simple poles occur at $x = -\ell$ with $\ell = 0,1,2 \dots$ however in certain circumstances of the smoothen out of singularity in the Born approximation Regge poles can also appear in $-1, -2$ for the values of ℓ [see Appendix C for detailed calculations].

Considering the quantum number for the orbital angular momentum or spin, we can take 3 values as, $0, -2, +2$ and ignore the rest. This 0 would be the dilaton, -2 would be the left handed polarized graviton and $+2$ would be the right handed polarized graviton. This suffices to the equation of string theory for a closed string (ignoring the dilaton) [39],

(50)
$$\frac{1}{2}\left(\frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial \sigma}\right) = -\frac{1}{2}\left(\frac{\partial x}{\partial \tau} - \frac{\partial x}{\partial \sigma}\right)$$

With σ, τ being the string parameter satisfying the above wave equation which concludes that we would get 4 states with the ground state creation operators $|O\rangle$, labeling by α and β yields [39],

(51)
$$\begin{cases} (a_1^+ + i\beta_1^+)(a_{-1}^+ + i\beta_{-1}^+)|O\rangle \rightsquigarrow +2\, units\ spin\ angular\ momentum \mapsto Right\ handed\ polarized\ graviton \\ (a_1^+ - i\beta_1^+)(a_{-1}^+ - i\beta_{-1}^+)|O\rangle \rightsquigarrow -2\, units\ spin\ angular\ momentum \mapsto Left\ handed\ polarized\ graviton \\ (a_1^+ + i\beta_1^+)(a_{-1}^+ - i\beta_{-1}^+)|O\rangle \rightsquigarrow 0\, units\ spin\ angular\ momentum \mapsto dilaton \\ (a_1^+ - i\beta_1^+)(a_{-1}^+ + i\beta_{-1}^+)|O\rangle \rightsquigarrow 0\, units\ spin\ angular\ momentum \mapsto dilaton \end{cases}$$

Discussions

The fixed points of the RG-Flow may provide a UV complete description of gravity, but it is in no way comparable to the discrete geometry of the LQG. Because, space-time itself originates from the LQG which appears as a spinfoam dynamics. The UV complete description is necessary to tackle the higher order curvature derivatives, that gives rise from the Einstein-Hilbert loop-(2) actions with the correlation function showing the UV-Cutoff at Λ_{UV}. The main ingredient of the paper is the approach of a curved space-time that acts through the least action of the E-H truncation which ultimately reduces the equation to discrete points after the solutions' of the higher derivatives could be collapsed as an ensemble of isolated points rather than a localization line through the background plane and this tasks have been successfully completed through the notional mathematics of the winding number. This winding number having a parametric value ω also appears to the exponential factors of the path integrals as $e^{-S\omega\varphi_{(0)}}$ that showcases the collapsed lines to a point like ensembles of the divergent lines when ω approaches infinity. It is very safe to assume that, the infinite notions of the winding number gives it a topological twist of many warps that acts as a higher degree curvatures from the compactification of extra dimensions that appears throughout the singularity identity inside the black holes, that acts as a core companion of the commencement of the need of 'new physics' through quantum gravity. One thing needs to be mentioned here that, the more the dimensions, the less the coupling strength, or it can be expressed as $\{d^{-1} \propto g(s)\}$ resulting in curbing down the energies of the gravity at $\sim 1\, TeV$ scale thereby attaining the grand unification or the unification of the 4-Fundamental forces of nature, namely the strong and weak nuclear force, electromagnetism and gravity through the conjunction of $U(1) \times SU(2) \times SU(3) \subset SU(5) \subset SO(10) \subset E(8)$ and the gravity. This will open a totally new window of physics where the hints for the suggestive 'new physics' peaks into existence by quantifying GR and merging with QM. And at the very end, the LQG generates through the bijective functions (both injective and surjective) where one discrete point is entangled to the other discrete points providing a network of entanglement suitable for the spinfoam-connected domain of space-time at the quantum realm. After that, dynamic triangulations have been computed over the granules, then Regge limits have been taken and along with the $S - matrix$ the Γ function tends to provide the Regge poles for which the spin angular momentum gives the corresponding values of the graviton and dilaton.

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Appendix A

From Ref. [35, 36] the Ashtekar formalism can be derived from the self-dual action by taking into account the Patalini action which reads,

$$S(\varepsilon,\varpi) = \int d^4x\, \varepsilon\varepsilon_I^a \varepsilon_J^b\, \gamma_{ab}{}^{IJ}[\varpi]$$

Where the Ricci tensor $\gamma_{ab}{}^{IJ}$ is purely constructed from the connection ϖ_a^{IJ} which suffices the compatibility relation through the equation,

$$\mathcal{D}_b \varepsilon_a^I = 0$$

From this, the tetrad connection is determined through the covariant derivative $\mathcal{D}$ satisfying the usual Ricci tensor $\gamma_{ab}{}^{IJ}$, where the self-dual form for the GR is given with respect to the curvature $\mathcal{F}$ of the variable A as,

$$S(\varepsilon,A) = \int d^4x\, \varepsilon\varepsilon_I^a \varepsilon_J^b\, \mathcal{F}_{ab}{}^{IJ}[A]$$

Thus representing the self-dual part of ϖ with respect to A_a^{IJ} as,

$$A_a^{IJ} = \frac{1}{2}\left(\varpi_a^{IJ} - \frac{i}{2} e^{IJ}MN\varpi_a^{MN}\right)$$

Subject to the condition that, $\mathcal{F}[A]$ is the self-dual part of $\gamma[\varpi]$. Let, $\mathcal{T}_b^a = \delta_b^a \varepsilon_I^b$ projects into a triad with the corresponding vector fields are,

$$E_I^a = \mathcal{T}_b^a \varepsilon_I^b$$

Where the orthogonality is maintained as a projection to n^a such that,

$$E_I^a = (\delta_b^a + n^a n_b)\varepsilon_I^b$$

Then it can be defined as,

$$\begin{aligned}
&\int d^4x\big(\varepsilon E_I^a E_J^b \mathcal{F}_{ab}{}^{IJ} - 2\varepsilon E_I^a \varepsilon_J^d n^b n_d \mathcal{F}_{ab}{}^{IJ}\big)\\
&= \int d^4x\big(\varepsilon(\delta_c^a + n_a n^c)\varepsilon_I^c\big(\delta_d^b + n_d n^b\big)\varepsilon_J^d \mathcal{F}_{ab}{}^{IJ} - 2\varepsilon(\delta_c^a + n_c n^a)\varepsilon_I^c \varepsilon_J^d n_d n^b \mathcal{F}_{ab}{}^{IJ}\big)\\
&= \int d^4x\left(\varepsilon\varepsilon_I^a \varepsilon_J^b \mathcal{F}_{ab}{}^{IJ} + \varepsilon n^a n_c \varepsilon_I^c \varepsilon_J^b \mathcal{F}_{ab}{}^{IJ} + \varepsilon\varepsilon_I^a n_d n^b \varepsilon_J^d \mathcal{F}_{ab}{}^{IJ} + \varepsilon n_c n^a n_d n^b E_I^c E_J^d \mathcal{F}_{ab}{}^{IJ} - 2\varepsilon\varepsilon_I^a \varepsilon_J^d n_d n^b \mathcal{F}_{ab}{}^{IJ} - 2n^a n_c \varepsilon_I^c \varepsilon_J^d n_d n^b \mathcal{F}_{ab}{}^{IJ}\right)\\
&= \int d^4x\, \varepsilon_I^a \varepsilon_J^b \mathcal{F}_{ab}{}^{IJ}\\
&= S(E, A)
\end{aligned}$$

Where it has been provided $\mathcal{F}_{ab}{}^{IJ} = \mathcal{F}_{ba}{}^{JI}$ and $n^a n^b = \mathcal{F}_{ab}{}^{I} = 0$ with the computed action given by,

$$S(E, A) = \int d^4x\big(\varepsilon E_I^a E_J^b \mathcal{F}_{ab}{}^{IJ} - 2\varepsilon E_I^a \varepsilon_J^d n_d n^b \mathcal{F}_{ab}{}^{IJ}\big)$$

We have $\varepsilon = N\sqrt{\mathcal{T}}$, we can now define $\tilde{E}_I^a = \sqrt{\mathcal{T}} E_I^a$ with an internal tensor $*\,\mathcal{S}^{IJ} := \frac{1}{2}\varepsilon^{IJ}MN\mathcal{S}^{MN} = i\mathcal{S}^{IJ}$ with the given curvature $\mathcal{F}_{ab}{}^{IJ}$ being self dual if and only if,

$$\mathcal{F}_{ab}{}^{IJ} = -i\frac{1}{2}\varepsilon^{IJ}MN\mathcal{F}_{ab}{}^{MN}$$

Substituting this into the lastly computed action, we get,

$$S(E, A) = \int d^4x\left(-i\frac{1}{2}\left(\frac{N}{\sqrt{\mathcal{T}}}\right)\tilde{E}_I^a \tilde{E}_J^b e^{IJ}MN\mathcal{F}_{ab}{}^{MN} - 2Nn^b \tilde{E}_I^a n_J \mathcal{F}_{ab}{}^{IJ}\right)$$

Where, we denoted $n_J = \varepsilon_J^d n_d$ and the gauge has been chosen $\tilde{E}_0^a = 0$ and $n^I = \delta_0^I$ which indicates $n_I = \mathcal{V}_{IJ} n^J = \mathcal{V}_{00}\delta_0^I = -\delta_0^I$ which can be written as $e_{IJKL}n^D = e_{IJK}$ which represents the gauge $e_{IJK0} = e_{IJK}$, then the same action simplifies to,

$$\begin{aligned}
S(E, A) \qquad &= \int d^4x\left(-i\frac{1}{2}\left(\frac{N}{\sqrt{\mathcal{T}}}\right)\tilde{E}_I^a \tilde{E}_J^b\big(e^{IJ}{}_{M0}\mathcal{F}_{ab}{}^{M0} + e^{IJ}{}_{0M}\mathcal{F}_{ab}{}^{0M}\big) - 2Nn^b \tilde{E}_I^a n_J \mathcal{F}_{ab}{}^{IJ}\right)\\
&= \int d^4x\left(-i\frac{1}{2}\left(\frac{N}{\sqrt{\mathcal{T}}}\right)\tilde{E}_I^a \tilde{E}_J^b e^{IJ}{}_M \mathcal{F}_{ab}{}^{M0} + 2Nn^b \tilde{E}_I^a n_J \mathcal{F}_{ab}{}^{I0}\right)
\end{aligned}$$

$1, 2, 3$ are the range values of I, J, M which will be denoted by lowercase letters in a moment. Through self-duality over A_a^{IJ}, the resultant equation suffices to,

$$A_a^{i0} = -i\frac{1}{2}e_{jk}^{i0}A_a^{jk} = i\frac{1}{2}e_{jk}^i A_a^{jk} = iA_a^i$$

Where,

$$e_{jk}^{i0} = -e_{0jk}^i = -e_{jk0}^i = -e_{jk}^i$$

This implies the conditional relation,

$$\begin{aligned}
\mathcal{F}_{ab}{}^{i0} &= \partial_a A_b^{i0} - \partial_b A_a^{i0} + A_a^{ik}A_{bk}{}^0 - A_b^{ik}A_{ak}{}^0\\
&= i\big(\partial_a A_b^i - \partial_b A_a^i + A_a^{ik}A_{bk} - A_b^{ik}A_{ak}\big)\\
&= i\big(\partial_a A_b^i - \partial_b A_a^i + e_{ijk}A_a^j A_b^k\big) = i\mathcal{F}^i{}_{ab}
\end{aligned}$$

Now, to replace the second term of the corresponding action Nn^b by $\mathcal{O}^b - n^b$, we need,

$$\mathcal{L}_{\mathcal{O}}A_b^i = \mathcal{O}^a\partial_a A_b^i + A_a^i\partial_b\mathcal{O}^a$$

And,

$$\mathcal{D}_b\big(\mathcal{O}^a A_a^i\big) = \partial_b\big(\mathcal{O}^a A_a^i\big) + e_{ijk}A_b^j\big(\mathcal{O}^a A_a^k\big)$$

To obtain,

$$\begin{aligned}
\mathcal{L}_{\mathcal{O}}A_b^i - \mathcal{D}_b\big(\mathcal{O}^a A_a^i\big) \qquad &= \mathcal{O}^a\big(\partial_a A_b^i - \partial_b A_a^i + e_{ijk}A_a^j A_b^k\big)\\
&= \mathcal{O}^a\mathcal{F}_{ab}^i
\end{aligned}$$

The resultant action shows,

$$S = \int d^4x \left(-i\frac{1}{2}\left(\frac{N}{\sqrt{\mathcal{T}}}\right)\tilde{E}_I^a \tilde{E}_J^b e^{IJ}{}_M \mathcal{F}_{ab}{}^{M0} - 2(\mathcal{O}^a - N^a)\tilde{E}_I^b \mathcal{F}_{ab}{}^{I0}\right)$$
$$= \int d^4x \left(-2i\tilde{E}_I^b \mathcal{L}_{\mathcal{O}} A_b^i + 2i\tilde{E}_I^b \mathcal{D}_b\left(\mathcal{O}^a A_a^i\right) + 2iN^a \tilde{E}_I^b \mathcal{F}_{ab}{}^i - \left(\frac{N}{\sqrt{\mathcal{T}}}\right) e_{ijk} \tilde{E}_I^a \tilde{E}_J^b \mathcal{F}_{ab}{}^k\right)$$

Where swapping of the dummy variable a and b occurs in the second term of the first line, which integrating by parts gives,

$$\int d^4x\, \tilde{E}_i^b \mathcal{D}_b\left(\mathcal{O}^a A_a^i\right) = d\mathcal{O} d^3x \tilde{E}_i^b \left(\partial_b\left(\mathcal{O}^a A_a^i\right) + e_{ijk} A_b^j \left(\mathcal{O}^a A_a^k\right)\right)$$
$$= -\int d\mathcal{O} d^3x\, \mathcal{O}^a A_a^i \left(\partial_b \tilde{E}_i^b + e_{ijk} A_b^j \tilde{E}_k^b\right)$$
$$= -\int d^4x\, \mathcal{O}^a A_a^i \mathcal{D}_b \tilde{E}_i^b$$

Where the boundary term has been thrown away and the covariant form of the vector density $\tilde{\mathcal{G}}_i^b$ implicates the relation as,

$$\mathcal{D}_b \tilde{\mathcal{G}}_i^b = \partial_b \tilde{\mathcal{G}}_i^b + e_{ijk} A_b^j \tilde{\mathcal{G}}_i^b$$

Yields, the final form of the action as,

$$S = \int d^4x \left(-2i\tilde{E}_i^b \mathcal{L}_{\mathcal{O}} A_b^i - 2i\left(\mathcal{O}^a A_a^i\right)\mathcal{D}_b \tilde{E}_i^b + 2iN^a \tilde{E}_i^b \mathcal{F}_{ab}{}^i + \left(\frac{N}{\sqrt{\mathcal{T}}}\right) e_{ijk} \tilde{E}_i^a \tilde{E}_j^b \mathcal{F}_{ab}{}^k\right)$$

Here in the calculation and for the sake of understanding, we have taken $\mathcal{T}$ as the momentum in the appendix section, where there has been portraying from the equation an equally term in the form of "$p\dot{\mathcal{T}}$" which is representating the quantity $\tilde{E}_i^a$ as a conjugate momentum to A_a^i, another thing worth mentioning is that, for the calculation purpose we have changed the time variable t by $\mathcal{O}$ thus the dt has been modified to $d\mathcal{O}$.

Here, it is plausible to write the relation,

$$\left\{A_a^i(x), \tilde{E}_j^b(y)\right\} = \frac{i}{2}\delta_b^a \delta_j^i \delta^3(x,y)$$

Giving us the action variation of the non-dynamical quantities $\mathcal{O}^a A_a^i$ which is the temporal component of the four-connection, the lapse function N and shift function N^b giving the constraints,

$$\mathcal{D}_a \tilde{E}_i^a = 0$$

$$\mathcal{F}_{ab}^i \tilde{E}_i^b = 0$$

$$e_{ijk} \tilde{E}_i^a \tilde{E}_j^b \mathcal{F}_{ab}^k = 0$$

Varying with the lapse function N and the factor $e_{ijk} \tilde{E}_i^a \tilde{E}_j^b \mathcal{F}_{ab}^k \big/ \sqrt{\mathcal{T}}$ gives the polynomial connection of the fundamental variables, representating the term or rescaling the term A_a^i given as,

$$A_a^i = \frac{1}{2} e_{jk}^i A_a^{jk}$$
$$= \frac{1}{2} e_{jk}^i \left(\varpi_a^{jk} - i\frac{1}{2}\left(e^{jk}{}_{m0}\varpi_a^{m0} + e^{jk}{}_{0m}\varpi_a^{0m}\right)\right)$$
$$= \Gamma_a^i - i\varpi_a^{0i}$$

And,

$$E_{ci}\varpi_a^{0i} = -\mathcal{T}_a^b E_{ci}\varpi_b^{i0}$$
$$= -\mathcal{T}_a^b E_{ci}\varepsilon^{di}\nabla_b \varepsilon_d^0$$
$$= \mathcal{T}_a^b \mathcal{T}_c^d \nabla_b n_d$$
$$= K_{ac}$$

Where it has been used,

$$e_d^0 = \mathcal{V}^{0I} g_{dc} \varepsilon_I^c$$
$$= -g_{dc}\varepsilon_0^c$$
$$= -nd$$

Therefore, $\varpi_a^{0i} = K_a^i$ which gives us the required result,

$$\boldsymbol{A_a^i = \Gamma_a^i - iK_a^i}$$

This is called the chiral spin connection. This connection is essential in order to retain the connectivity between the emergent granular structures of the space-time through diffeomorphisms in the Hilbert space.

Appendix B

An interesting concept that has been arises from LQG is the holonomy where there lies a difference after the parallel transport is completed from the initial and final positions by a spinor in a closed loop. This can be represented by,

$$\hbar_\omega[A]$$

For a closed loop, with the initial and final connecting points be the same $(\beta \to \alpha)$, under Gauss law, these loops transforms as in order of,

$$\begin{aligned}(\hbar'_\omega)_{\beta\beta} &= U^{-1}_{\beta\gamma}(x)(\hbar_\omega)_{\gamma\rho}U_{\rho\beta}(x)\\ &= \left[U_{\beta\beta}U^{-1}_{\beta\gamma}(x)\right](\hbar_\omega)_{\gamma\rho}\\ &= \delta_{\gamma\rho}(\hbar_\omega)_{\rho\gamma}\\ &= (\hbar_\omega)_{\gamma\gamma}\end{aligned}$$

The trace could be written in the form of a Wilson loop as $\ell_\omega[A]$, which provides a holonomy of an explicit form given by,

$$\hbar_\omega[A] = \mathcal{W}_\ell\, exp\left\{-\int_{\gamma_1}^{\gamma_2} ds\, \dot{\omega}^\beta A^i_a\left(\omega(s)\right)T_i\right\}$$

Where ω is the curve satisfying holonomy, s, the affine valued parameter along the curve and $\mathcal{W}_\ell$ denotes the path of the loop, while T_i is the matrices obeying the $SU(2)$ algebra as,

$$\left[T^i, T^j\right] = 2i\epsilon^{ijk}T_k$$

The Wilson loop satisfies the quantum Hamiltonian constraint by,

$$\widetilde{H}^\dagger \ell_\omega[A] = -\epsilon_{ijk}\hat{\mathcal{F}}^k{}_{ab}\frac{\delta}{\delta A^i_a}\frac{\delta}{\delta A^j_a}\ell_\omega[A]$$

Where in $\hat{\mathcal{F}}^k{}_{ab}$, has an anti-symmetric indices a and b where the tangent vector $\dot{\omega}^a$ of the loop ω satisfies the equation of the vector potential $\hat{\mathcal{F}}^i{}_{ab}\dot{\omega}^a\ \dot{\omega}^b$.

Appendix C

Here we will proceed with [38], for a large negative energy, the Schrӧdinger equation $E = \mathcal{K}^2 = -\sigma^2$, the equation takes the implicit form,

$$V(r)\psi(r) = \psi''(r) - \left(\sigma^2 + \frac{\ell(\ell+1)}{r^2}\right)\psi(r)$$

The power series expansion could be done taking the potential $V(r)$ satisfying the relation,

$$v = -\frac{a}{x^2} - \frac{a_0}{x} - a_1 - a_2x - \ldots$$

Where at the origin $v \propto \frac{1}{r^2}$ which can otherwise be regarded as a superposition of the Yukawa potential, having the mass spectrum $m(\mu)^2$ behaving as $\frac{1}{\mu}$ for large μ. As the term $\frac{a}{x^2}$ has the dependence on the centrifugal barrier term x, then the transformation could be achieved by,

$$\sqrt{\left(\ell + \frac{1}{2}\right)^2 - a} - \frac{1}{2} \rightsquigarrow \ell_0$$

Then, the recursion formula will be given by,

$$[(v+1+s)(v+s) - \ell(\ell+1) + a]_{C_{v+1}} - (v+s)_{C_v} + \sum_{n=0}^{v} a_n C_{v-n} = 0$$

Giving the value $v = -1$ and to satisfy the above equation, only if s takes the value of the form,

$$s = \frac{1}{2} \pm \sqrt{\left(\ell + \frac{1}{2}\right)^2 - a}$$

Taking the positive value, choosing $s = \frac{1}{2} + \sqrt{\left(\ell + \frac{1}{2}\right)^2 - a}$ the resultant equation suffices to,

$$(v+1)\left(v + 1 + 2\left[\sqrt{\left(\ell + \frac{1}{2}\right)^2 - a}\right]\right)C_{v+1} = \left(v + \frac{1}{2}\left[\sqrt{\left(\ell + \frac{1}{2}\right)^2 - a}\right]\right)C_v - \sum_{n=0}^{v} a_n C_{v-n}$$

And by the substitution of $\sqrt{\left(\ell + \frac{1}{2}\right)^2 - a} - \frac{1}{2} \rightsquigarrow \ell_0$ reduces to the form,

$$(v+1)(v+2\ell+2)_{C_{v+1}} = (v+\ell+1)_{C_v} - \sum_{n=0}^{v} a_n C_{v-n}$$

The meromorphic form of C_v cutting the line along the ℓ plane, connecting two fixed branch points,

$$\ell = -\frac{1}{2} - \sqrt{a}$$
$$\ell = -\frac{1}{2} + \sqrt{a}$$

the cut has been chosen in this form, such that $\sqrt{\left(\ell+\frac{1}{2}\right)^2 - a}$ when moves along the ℓ changes its sign from a large positive value to a large negative value where the cut if on the real axis $a > 0$ which is attractive at small distances and on the vertical line Re $\ell = -\frac{1}{2}$ if the value is $a < 0$ which is repulsive at small distances, the Regge pole in this case at $a = 0$ located at $\ell = \ell_0$ and for $\ell \neq \ell_0$, the pole can be found at,

$$\ell = -\frac{1}{2} + \sqrt{\left(\ell_0 + \frac{1}{2}\right)^2 - a}$$

If the value of $\ell_0 > -\frac{1}{2}$ and is real, then the Regge pole will shift from the ℓ_0 plane to the right as $a > 0$ which is attractive at small distances or shift to the left as $a < 0$ which is repulsive at small distances, in accordance that, more energy potentials will accommodate at the attractive range in the case of the higher momenta as oppose to the repulsive case. However, if $\ell_0 < -\frac{1}{2}$ then the Regge poles will move to the left or right either in form $a > 0$ or $a < 0$. This is because the cut along the horizontal axis pushes out the complex ℓ_0 plane along the positive and negative directions while the vertical cut along ℓ_0 behaves in the opposite. Thus in the limit, $\sigma \nrightarrow \infty$ the ℓ_0 takes the values of $-1, -2 \ldots$ and the Regge pole is found at $a > -\frac{1}{4}$. However, if the behaviour of a is such that, it decreases beyond the value of $-\frac{1}{4}$ the ℓ_0 plane will cut out from the real axis taking the value as -1. As long as $\sigma = \infty$ the Regge pole decreases but cannot overtake the branch point. As the energy value E increases from $-\infty$ $(-\sigma^2)$, the Regge pole sitting on the real axis shifts to the right as $a_0 > 0$ (potential varying attractively as $\frac{1}{r}$) and as $a_0 < 0$ (potential varying repulsively as $\frac{1}{r}$). And finally as E becomes (or approaches closely) to $+\infty$, the Regge poles comeback to the initial position having $\ell = -\frac{1}{2} + \sqrt{\left(\ell_0 + \frac{1}{2}\right)^2 - a}$ takes through complex values.

Now, from the potentials as expressed earlier as $v = -\frac{a}{x^2} - \frac{a_0}{x} - a_1 - a_2 x - \ldots$, if σ approaches infinity, then Regge poles tend to take negative integers where the poles could be of $\ell = -b - 1$ $(b = 0,1,2,\ldots)$ is related to the term σ^{2b-1} with the superposition of the potentials r^n the Born scattering could be found at,

$$\int d^3x\, e^{i\Delta k \cdot x_\sigma 2b-1} \propto t^{-b-1}$$

$$\int d^3x\, e^{i\Delta k \cdot x_\sigma 2b} = 0$$

This relation compels us to believe that, for the even powered term $\sigma^{2b}(b \geq 0)$ will influence the formation of Regge poles at *finite energy limits*. If we consider the coordinate origin $x = y = z = 0$ then, there exists a singularity for the form σ^{2b-1}, however, as b increases from the coordinate origin, then the singularity becomes smoother and the Regge poles seem to move away from the complex ℓ plane at the origin.