

# Generalization of the Entropic Information Theory.

## Abstract

The generalization of the entropic information approach can explain various processes as various aspects of the entangled informational content of the visible universe. Some basic theoretical concepts about the quantum information approach such as the definition of the concept of quantum information, non-locality, the process of quantum entanglement or some considerations about the measurement problem have been explained in the conceptual approach to entropic information. Entropic information theory as an informational approach is mathematically based on the mass of the bit of information, the mass of the bit of information which was injected into Louis de Broglie's hidden thermodynamics providing new entropy formulations. After that, we have introduced the Hawking temperature into these formulas, leading to new formulations to calculate the entropy of black holes independently of the law of area. Being able to express the fine-grained gravitational entropy of a black hole using the rules of gravity, we can speak, at this level, of quantum gravity as emerging through the fundamentality of entangled quantum information. In addition, we calculated the informational content of the observable universe using the entropic information formula to obtain results remarkably close to some previous estimates to account for all the dark matter missing in the visible Universe. After what, we calculated the amount of energy associated with this informational content using Landauer's principle, to obtain a result that we can relate to dark energy estimates. Moreover, some deep considerations based on the perspectives of entropic information theory have been explored. This new complete mathematical framework of entropic information theory capable of explaining various processes being several aspects of one and the same, entangled information, it is the theory framework of the entangled informational universe.

**KEYWORDS:** information, dark matter, dark energy, entropy, quantum gravity, Landauer's principle, entropic information theory

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## 1. Introduction

To begin with, we will examine some of the fundamental definitions and claims regarding the quantum information approach and explain them using entropic information theory to shed new light on these basic concepts. Indeed, after defining the notion of quantum information, entanglement, non-locality as well as the consideration of the measurement problem in the context of the generalization of the entropic information approach, we can see that this approach makes it possible to open new perspectives, indeed it provides new formulations of entropy from the theory of entropic information, as well as a new formulation of the entropy of black holes. In addition, new formula for calculating the number of bits of the entire current observable universe and formula about the energy currently associated with it have been used, leading to important new considerations on entropy, quantum gravity, dark matter or dark energy, respectively, presenting a global explanatory mathematical framework of the life and the universe.

### 1.1 Definition of The Information notion within the entropic information theory framework.

Some attempts to define the concept of information have been attempted, for example, Lucien Hardy's attempt at the informational properties of a system, where "the informational properties of a system should depend only on the number of distinct messages that can be encoded on that system" [1], this can be related to the notion of bit of information, which, in information theory, is the minimum amount of information conveyed by a message. But we can say about quantum information which is the basic entity of study in quantum information theory, and which can be manipulated using quantum information processing techniques; that following John Preskill's approach that quantum information is information about the state of a quantum system, that information is something physical that is encoded in the state of a quantum system [2]. Indeed, Gleason's theorem says that "a quantum state is completely determined by knowing only the answers to all possible yes/no questions", a yes/no question is presumably a self-adjoint operator with two distinct eigenvalues. But information can only exist in a region that also contains energy. In addition, this energy has properties by which we can describe it. It has degrees of freedom. These degrees of freedom can be considered the minimum number of coordinates required to specify a configuration. "Each spin can be taken to represent a bit, and each spin flip corresponds to a bit flip. Almost any interaction between degrees of freedom is sufficient to perform universal quantum logic on these degrees of freedom" [3],[4]. This leads entropic information theory to define the notion of information in a more precise and relevant way; indeed, it can be said by following the theory of entropic information that "information is a change of quantum state due to the modification of a degree of freedom in the quantum system considered".

### 1.2 Definition of the Entanglement notion within the entropic information theory framework.

In quantum physics, certain states, called entangled states, show certain statistical correlations between measurements on particles that cannot be explained by classical theory. An entangled system is defined as a system whose quantum state cannot be factored as a product of the states of its local constituents. That is, they are not individual particles but an inseparable whole. In quantum entanglement, one constituent cannot be described in

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detail without taking into account the other or others. The entanglement process is carried out when two particles are linked together, regardless of the separation from one to the other. In quantum mechanics, even if these entangled particles are not physically connected; they are always able to instantly share information with each other. In other words, in quantum mechanics, entanglement occurs when one or more properties of two particles or molecules such as spin, polarization, or momentum are shared at the quantum level; this connection persists even if the particles move away from each other.

Quantum information can be measured using Von Neumann entropy. Indeed, entropy is considered a measure of entanglement. Entropy provides a tool that can be used to quantify entanglement. The entropy of a mixed state can be thought of as a measure of quantum entanglement. Von Neumann entropy is a measure of the statistical uncertainty represented by a quantum state. Von Neumann entropy connects to all other forms on entropy and glimpses the real origin of the second law of thermodynamics. The growth of entanglement leads to both the second law of thermodynamics and the emergence of the classical world from quantum. In addition, it also defines the arrow of time that itself points in the direction of increasing entropy and multiplying entanglement.

Following the theory of entropic information and its definition of information as a change in quantum state due to the modification of a degree of freedom in the quantum system under consideration; quantum entanglement can be defined as the physical phenomenon that occurs when a group of particles is generated, interacts or shares a spatial proximity such that one or more degrees of freedom are shared at the quantum level between each particle in the group; a particle that cannot be described independently of the quantum state of others because they share degrees of freedom, even when the particles are separated by a great distance.

### 1.3 Definition of the Nonlocality within the entropic information theory framework.

Quantum nonlocality is sometimes understood as being equivalent to entanglement. However, this is not the case. Quantum entanglement can be defined only within the formalism of quantum mechanics, i.e., it is a model-dependent property. In contrast, nonlocality refers to the impossibility of a description of observed statistics in terms of a local hidden variable model, so it is independent of the physical model used to describe the experiment. Non-locality means that measuring the properties of a quantum particle in one location can instantaneously affect the properties of another, even if the two particles are in different locations. A local hidden variable theory (LHV) of quantum mechanics assumes that all natural processes are local: information and correlations are propagated at most at the speed of light. In physics, hidden-variable theories are proposals to provide explanations of quantum mechanical phenomena through the introduction of unobservable hypothetical entities. However, quantum mechanics does not assume the existence of local hidden variables associated with individual particles. Quantum nonlocality does not allow for faster-than-light communication,[5] and hence is compatible with special relativity and its universal speed limit of objects. In the words of Lewis Carroll Epstein: "You can't go faster than the speed of light because you can't go slower than the speed of light. You are always going the speed of light through spacetime. If you use some of your speed to go through space, then there is less speed through time". Indeed, a semi-classical perspective can explain the notion of entanglement; indeed, entanglement of photons can be explained in terms of the relativistic properties of space-time as defined by Einstein as well as by quantum mechanics. Yet, regarding photon and theory of special relativity, all photons moving at the speed of light, the separation between those two points would be zero from the perspective of those photons. Those entangled photons which share one or more degrees of freedom at quantum level between each photon cannot be described independently of the quantum state of the

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others because they share degrees of freedom. We can say thus that quantum theory is local in the strict sense defined by special relativity and, as such, the term "quantum nonlocality" is sometimes considered a misnomer.

#### 1.4 Definition of the Measurement Problem within the entropic information theory framework.

What complicates this somewhat is that the major interpretations of quantum mechanics claim that a "measurement" causes the decoherence. This is because the purpose of interpretations of quantum mechanics is to link the world of quantum mechanics to a hypothetical world governed by classical mechanics. This connection is made through vague terms like "measure" and "collapse." The correlation predicted by quantum mechanics is due to the quantum entanglement of the pair, with the idea that their state is determined only at the point where a measurement is made on one or the other. This idea is entirely consistent with Heisenberg's uncertainty principle, one of the most fundamental concepts in quantum mechanics [6].

"It's one thing to say that measurement requires information. It's another thing to say that the thing being measured is created by the observer doing the measuring" [7].

Following the approach of entropic information theory, we can say that during a measurement, to be informed by the measurement system, the measurer needs information; take information from the system during the measurement process; but with measurement, the new system to consider is the system: "measurer-the thing to be measured", both parts of the system under consideration share one or more degrees of freedom. The two parts of the quantum system considered are entangled. It is a question of relating to the definition of information given by the approach of entropic information theory, where a change in a quantum state due to the modification of a degree of freedom at the quantum level is the definition of quantum information, and by the proposed definition of entanglement, where, following the approach of entropic information theory, quantum entanglement is the physical phenomenon that occurs when a group of particles is generated, interacts, or shares spatial proximity in such a way that one or more degrees of freedom are shared at the quantum level between each particle in the group, these particles cannot be described independently of the quantum state of the others because they share degrees of freedom.

#### 1.5 Definition of the core of the entropic information theory framework.

Entropic information theory is based on the information bit, where the quantum information bit is defined as a change in a quantum state due to the modification of a degree of freedom at the quantum level. Entropic information theory is based on the degree of freedom of the quantum system under consideration, based on the number of bits needed to specify the actual microscopic configuration among the total number of allowed microstates and thus characterize the macroscopic states of the system under consideration [8]. Indeed, a probability distribution of possible states through a statistical set of all microstates defines a macrostate which is a state that can be observed experimentally. Entropy makes it possible to estimate the amount of information lost when microscopic information is summarized by macroscopic information. "We can say that entropy is information: indeed, in micro-canonical language, entropy is determined by the number of microstates compatible with a given macrostate [9]". Considering the principle of mass-energy-information equivalence already stated by Melvin Vopson [10] proposing that a bit of information is not only physical, as already demonstrated, but that it has a finite and quantifiable mass while it stores

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information. The entropic theory of information sheds light on new global perspectives by introducing the mass of the information bit into some famous equations in physics such as the hidden thermodynamics formula of Louis de Broglie, the classical formulation of entropy, the Bekenstein-Hawking entropy formula, the Bekenstein bound, or even some of Casini's work [11] leading to a new global mathematical framework based on the entropic information theory approach.

## 2 Methodology

We start from the hidden thermodynamics of Louis de Broglie where we inject the mass of bit of information of the principle of equivalence mass-energy-information of Vopson, proposing that a bit of information is not only physical, as already demonstrated, but that it has a finite and quantifiable mass while it stores information, leading to the expression of a new formulation on entropy by calculation based on the degree of freedom, on the number of bits in the system [8]. Several formulations of this entropy based on different relations, Einstein's mass-energy equivalence, The Einstein Planck relation or based on the Avogadro number are formulated. After that, we inject the Hawking temperature into the new general entropy formula leading to a new formulation of the Bekenstein-Hawking entropy formula, a new formulation that can explain how this entropy occurs, based on an evaporation time of the black hole and the degree of freedom of the black hole seen as an entire quantum system. After that, we dive into Casini's work on Von Neumann entropy and the Bekenstein bond, giving the proof that the Bekenstein bound is valid using quantum field theory [12-20], we can talk about the process of emergence of quantum gravity through the fundamentality of entangled quantum information. Then, with the help of the formula of entropic information theory, we answer the question of "What is the informational content of the observable universe?". And based on this approach, we estimated the amount of energy associated with this number of bits representing the informational content of the observable universe. These two estimates are candidates for dark matter and dark energy respectively.

### 2.1 Generalization of the Entropic Information Theory framework to Entropy with entropic formulas

The generalization of the framework of entropic information theory is based on a new concept of entropy based on the introduction of the mass of the bit of information revealing new entropic formulas.

In general, entropy is related to the number of possible microstates according to Boltzmann's principle:

$$S = k \ln (W)$$

Where

S is the entropy of the system,

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$k$  Boltzmann's constant,  
 $W$  is the number of microstates.

$W$  = number of states or microstates, characterized by the position and velocity of all particles so if we consider that the degree of freedom of a system can be viewed as the minimum number of coordinates required to specify a configuration. So YES ...  $W$  reflects the degree of freedom of a system [8].

Boltzmann entropy formula can be derived from Shannon entropy formulae when all states are equally probable

$$S = -k \sum p_i \ln(p_i) = k \sum \frac{\ln(W)}{w} = k \ln(W)$$

So, you have  $w$  microstate equiprobable with probability  $p_i = \frac{1}{w}$

We start this generalization by introducing the mass of bit of information formula into the hidden thermodynamics of Louis De Broglie:

$$\frac{\text{action}}{h} = - \frac{\text{entropy}}{k}$$

Where:

$k$ : Boltzmann's constant  
 $h$ : Planck constant

With  $\text{action} = \text{Energy} \times \text{Time}$   
 and  $\text{Energy} = mc^2$

$$\frac{\text{action}}{h} = \frac{mc^2t}{h} = - \frac{\text{entropy}}{k} = \frac{k \ln(w)}{k}$$

We introduce the mass of bit of information into this relation as being:

$$\text{mass}_{bit} = \frac{k T \ln(2)}{c^2}$$

Where:

$k$ : Boltzmann's constant  
 $T$ : temperature at which the bit of information is stored.  
 $t$ : Time required to change the physical state of the information bit

$c$ : speed of light

$$\frac{\text{action}}{h} = \frac{mc^2 t}{h} = \frac{\frac{k T \ln(2)}{c^2} c^2 t}{h} = - \frac{\text{entropy}}{k} = \frac{k \ln(w)}{k}$$

$$\frac{\text{action}}{h} = \frac{mc^2 t}{h} = \frac{k T \ln(2) t}{h} = - \frac{\text{entropy}}{k} = \ln(W)$$

$$\ln(W) = \frac{k T \ln(2) t}{h}$$

We obtain the validity proof by the Landauer limit as

$$\frac{k T \ln(2) t}{h} = \frac{mc^2 t}{h}$$

$$k T \ln(2) = m c^2$$

Indeed, the Landauer limit being the minimum possible amount of energy required to erase one bit of information, known as the Landauer limit:

As  $E = mc^2$

$$E = k T \ln(2)$$

Landauer's principle can be derived from microscopic considerations [21] as well as derived from the well-established properties of the Shannon-Gibbs-Boltzmann entropy [22]. The principle thus appears to be fundamental and universal in application. Landauer's principle applies both to classical and to quantum information

Moreover, as Entropy:

$$S = k \ln(W)$$

We obtain a new value for the general entropy  $S$  formula based on the hidden thermodynamics of de Broglie with the introduction of mass of bit of information [23]:

$$k \ln (W) = k \frac{k T \ln (2) t}{h}$$

$$S = k^2 \frac{T \ln(2) t}{h}$$

With  $S$  entropy expressed in number of bits of information

regarding the relation Energy =  $kT$ , this new entropy concept [8] can take the form of

$$S = kT \frac{k \ln(2) t}{h}$$

with Einstein mass–energy equivalence, Energy =  $mc^2$ , this is

$$S = mc^2 \frac{k \ln(2) t}{h},$$

with Planck Einstein relation  $E = h \nu$ , this is with  $h = \frac{E}{\nu}$

$$S = h \nu \frac{k \ln(2) t}{\frac{E}{\nu}},$$

$$S = k \ln(2) t \nu$$

With  $kT = \frac{RT}{N_A}$ ,

$$S = \frac{RT}{N_A} \frac{k \ln(2) t}{h}$$

$$\text{With } N_A = \frac{MuA_r(e)c\alpha^2}{2R_\infty h}$$

$$S = \frac{RT}{\frac{MuA_r(e)c\alpha^2}{2R_\infty h}} \frac{k \ln(2) t}{h}$$

$$S = \frac{2R_\infty RT k \ln(2) t}{MuA_r(e)c\alpha^2}$$

as the Boltzmann constant may be used in place of the molar mass constant by working in pure particle count,  $N$ , rather than amount of substance,  $n$ .

where:

$R_\infty$  : Rydberg constant

$R$  molar gas constant

$T$  is temperature at which the bit of information is stored

$k$  Boltzmann constant

$t$  Time required to change the physical state of the information bit

$Mu$  molar mass constant

$A_r(e)$  relative atomic mass of the electron.

$c$  speed of light in a vacuum,

$\alpha$  fine-structure constant,

## 2.2 Generalization of the Entropic Information Theory framework to Quantum Gravity with black hole entropic formulas

The theory of entropic information enters the problem of the black hole by introducing hawking temperature into equations where the mass of the bit of information has been implemented in entropic formulas.

We start from this equation obtained by the introduction of mass bits of information into the hidden thermodynamics of Louis de Broglie

$$\ln(W) = \frac{k T \ln(2) t}{h}$$

wherein we inject the Hawking Temperature represented by this formula:

$$T_H = \frac{1}{k} \frac{\hbar c^3}{8\pi GM}$$

With  $\hbar = \frac{h}{2\pi}$ ,

$$\ln(W) = \frac{k \frac{1}{k} \frac{hc^3}{16\pi^2 GM} \ln(2) t}{h}$$

we obtain after simplification of k and h:

$$\ln(W) = \frac{c^3 \ln(2) t}{16\pi^2 GM}$$

It can also be shown that the Boltzmann entropy is an upper bound to the entropy that a system can have for a fixed number of microstates meaning [24]:

$$S = k \ln(W)$$

With W reflecting the degree of freedom of the considered system

W = number of states or microstates, characterized by the position and velocity of all particles so if we consider that the degree of freedom of a system can be viewed as the minimum number of coordinates required to specify a configuration. So YES ... W reflects the degree of freedom of a system [8]

We obtain the black hole entropy formula, at the Hawking temperature, based on bits of information and on the mass of bit of information,

$$S_{BH} = k \ln(W) = k \frac{c^3 \ln(2) t}{16\pi^2 GM}$$

The universal form of the bound was originally found by Jacob Bekenstein in 1981 as the inequality

$$S \leq \frac{2\pi kRE}{\hbar c}$$

Where:

S is the entropy,  
 k is Boltzmann's constant,  
 R is the radius of a sphere that can enclose the given system,  
 E is the total mass–energy including any rest masses,  
 $\hbar$  is the reduced Planck constant,  
 c is the speed of light.

Note that while gravity plays a significant role in its enforcement, the expression for the bound does not contain the gravitational constant G.

In informational terms, the relation between thermodynamic entropy S and Shannon entropy H is given by

$$S = kH \ln(2)$$

whence

$$H \leq \frac{2\pi RE}{\hbar c \ln(2)}$$

With  $E=mc^2$

$$\frac{c^3 t}{16\pi^2 GM} = \frac{2\pi cRM}{\hbar \ln(2)}$$

$$H \leq \frac{c^3 t}{16\pi^2 GM}$$

In the frame of Black hole evaporating by Hawking radiation at the Hawking temperature with a time required to change the physical state of all the bit of information of the system equal to the time required by the black hole to evaporate ( $t_{evap}$ ).

$$S_{BH} = k \ln(W) = k \frac{c^3 \ln(2) t_{evap}}{16\pi^2 GM}$$

With W reflecting the degree of freedom of a system.

We clearly see the relation between the degree of freedom and the process of evaporation of the black hole.

Where:

$$t_{evap} = \frac{64\pi^3 M^3 G^2}{\hbar \ln(2) c^4}$$

We have showed that the black holes entropic information formula  $k \frac{c^3 \ln(2) t_{evap}}{16\pi^2 GM}$  is equal to the universal bound,  $\frac{2\pi k R E}{\hbar c}$  and that it is in relation with degree of freedom of black hole seen as a whole quantum system.

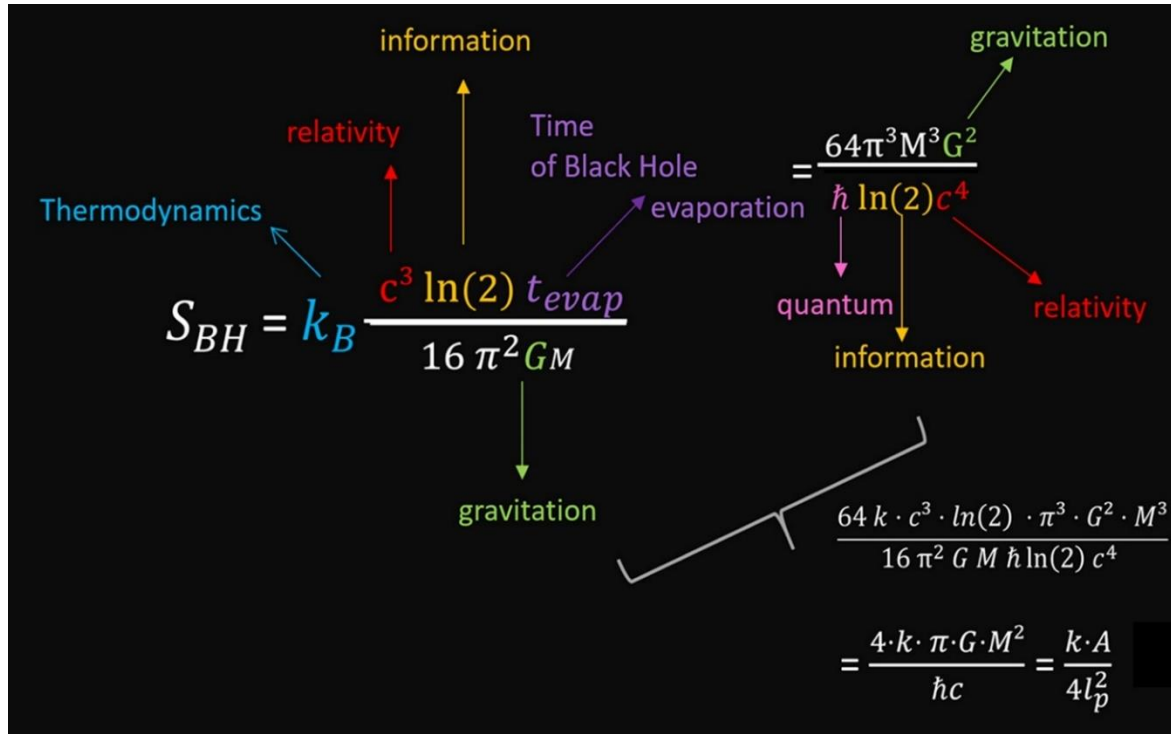


Figure 1 Black holes Entropic Information Formula with the time of evaporation of the black hole ( $t_{evap}$ ) as new formulation of The Bekenstein–Hawking entropy additionally including the information notion; where  $k$ =Boltzmann constant,  $c$ =speed of light,  $\hbar$  = Reduced Planck's constant,  $G$ =Gravitational Constant,  $M$ = mass of the black hole.

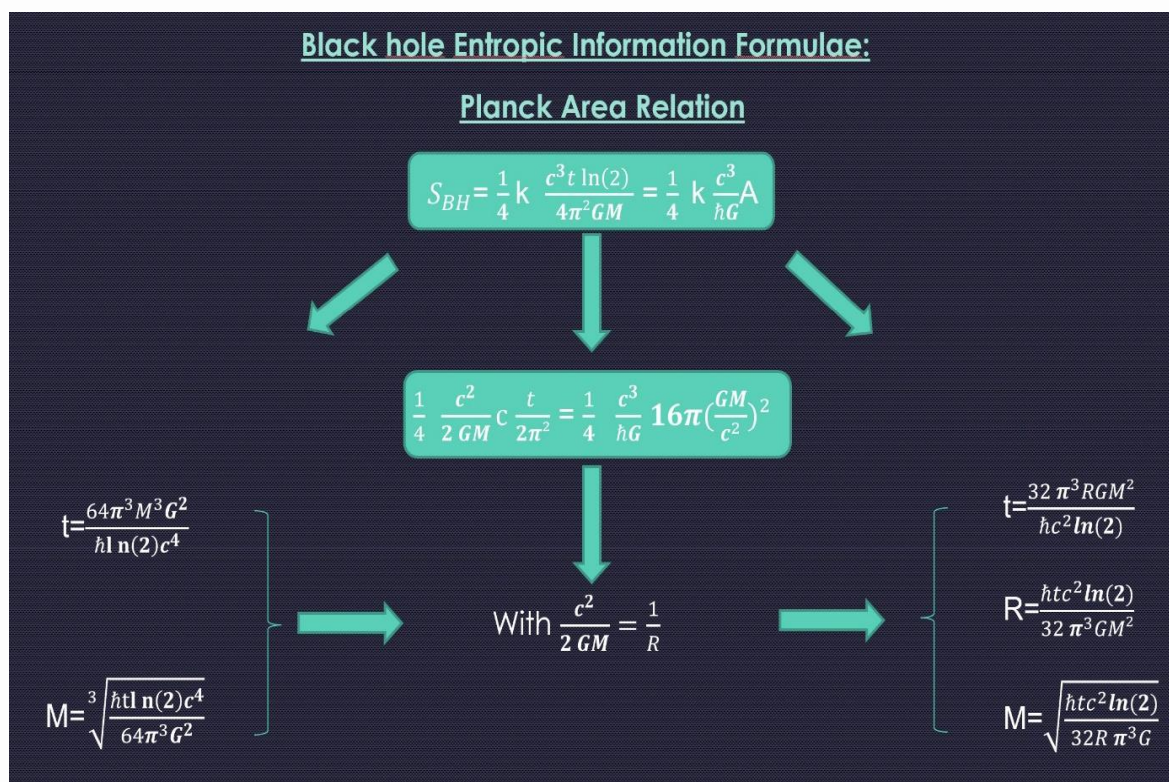


Figure 2 The black holes entropic information formula with the time of evaporation of the black hole ( $t_{\text{evap}}$ ): Planck area relation

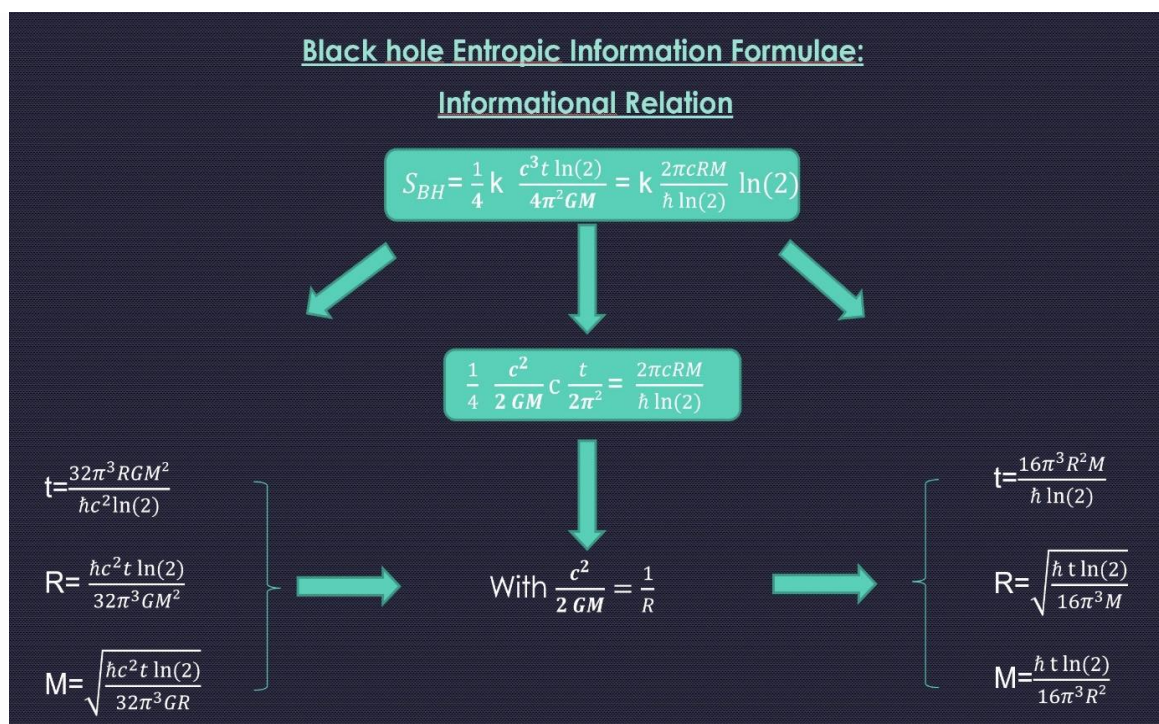


Figure 3 The black holes entropic information formula with the time of evaporation of the black hole ( $t_{\text{evap}}$ ): Informational relation

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Mass of Black Hole in solar mass                :2.7

Classical method Results :
//////////\/////

Masse of the black Hole in kg                  : 5.3703e+30
Schwarzschild radius of black hole in meter   : 7976.022193357674
Entropy of the Black Hole Classical Method Dimensionless : 7.650843355747418e+77
Entropy of the Black Hole Classical Method     : 1.0563129228269317e+55

Entropic Information Results :
//////////\/////

Entropic Information Time Shannon               : 2.3186715453271243e+75
Entropic Information Entropy with time Shannon : 1.0563129228269317e+55

Entropic Information Time Bekenstein          : 2.3186715453271247e+75
Entropic Information Entropy with time Bekenstein : 1.056312922826932e+55

Comparison Results :
//////////\/////

Rapport between Classical Method and Entropic Information with Shannon time : 1.0
Rapport between Classical Method and Entropic Information with bekenstein time : 0.9999999999999998
    
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Figure 4 . GW170817 with a given mass of 2.7 solar mass

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Mass of Black Hole in solar mass                :8.7

Classical method Results :
//////////\/////

Masse of the black Hole in kg                  : 1.73043e+31
Schwarzschild radius of black hole in meter   : 25700.515956374722
Entropy of the Black Hole Classical Method Dimensionless : 7.943653410103184e+78
Entropy of the Black Hole Classical Method     : 1.096739713700555e+56

Entropic Information Results :
//////////\/////

Entropic Information Time Shannon               : 7.757212663783706e+76
Entropic Information Entropy with time Shannon : 1.0967397137005548e+56

Entropic Information Time Bekenstein          : 7.757212663783709e+76
Entropic Information Entropy with time Bekenstein : 1.0967397137005552e+56

Comparison Results :
//////////\/////

Rapport between Classical Method and Entropic Information with Shannon time : 1.0000000000000002
Rapport between Classical Method and Entropic Information with bekenstein time : 0.9999999999999998
    
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Figure 5 Cygnus X1 Black hole with a given mass of 8.7 solar mass

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Mass of Black Hole in solar mass                :6500000000

Classical method Results :
//////////\
Masse of the black Hole in kg                  : 1.2928500000000002e+40
Schwarzschild radius of black hole in meter   : 19201534909935.145
Entropy of the Black Hole Classical Method Dimensionless : 4.434130751444835e+96
Entropy of the Black Hole Classical Method     : 6.12197818785156e+73

Entropic Information Results :
//////////\
Entropic Information Time Shannon              : 3.2351022361198057e+103
Entropic Information Entropy with time Shannon : 6.121978187851557e+73

Entropic Information Time Bekenstein          : 3.2351022361198073e+103
Entropic Information Entropy with time Bekenstein : 6.121978187851561e+73

Comparison Results :
//////////\
Rapport between Classical Method and Entropic Information with Shannon time : 1.0000000000000004
Rapport between Classical Method and Entropic Information with bekenstein time : 0.9999999999999998
    
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Figure 6 . Messier87 Black Hole with a given mass of  $6.5 \times 10^9$  solar mass

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Mass of Black Hole in solar mass                :66000000000

Classical method Results :
//////////\
Masse of the black Hole in kg                  : 1.31274e+41
Schwarzschild radius of black hole in meter   : 194969431393187.6
Entropy of the Black Hole Classical Method Dimensionless : 4.571615042199692e+98
Entropy of the Black Hole Classical Method     : 6.311795736397963e+75

Entropic Information Results :
//////////\
Entropic Information Time Shannon              : 3.3867235411033223e+106
Entropic Information Entropy with time Shannon : 6.311795736397963e+75

Entropic Information Time Bekenstein          : 3.386723541103322e+106
Entropic Information Entropy with time Bekenstein : 6.3117957363979615e+75

Comparison Results :
//////////\
Rapport between Classical Method and Entropic Information with Shannon time : 1.0
Rapport between Classical Method and Entropic Information with bekenstein time : 1.0000000000000002
    
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Figure 7 Black hole TON618 with a given mass of 66 billion solar mass

We can see in figures 4 to 7, the results obtained by the entropic information theory approach in comparison to the Black hole entropy classical method of calculation. Regarding Black hole value of mass taken for the black holes considered all along the scale of size of the black hole, those values are extremal, indeed we begin with GW170817 with a mass of 2.7 solar mass to

go until the mass of the TON618 black hole which is equal to 66 billion of solar mass [23] passing by CYGNUS X1 or by MESSIER87 for the others black holes masses considered. In this way, we have pass in review all the scale sizes of the black holes.

This allows us to have a global vision of the mass of the black holes along the scale. The results obtained are based on the two different relations from the entropic information approach as shown in figure 2 and 3, the informational relation and the Planck relation; the other part of the results is from the classical method of black hole entropy.

As a side note, we must take in account that the black Hole information paradox can be established using the standard Copenhagen method of keeping the system separate from the measuring device. The black hole information paradox is independent of the quantum measurement problem. As such we can discuss the solutions to the information paradox without committing to any particular interpretation of quantum mechanics.

We know, following the work of Casini in 2008 about the Von Neumann entropy and the Bekenstein bound, that the proof of the Bekenstein bound is valid using quantum field theory [12-20].

For example, given a spatial region,  $V$ , Casini defines the entropy on the left-hand side of the Bekenstein bound as

$$S_V = S(\rho_V) - S(\rho_V^0) = -(\text{tr}(\rho_V \log \rho_V)) + \text{tr}(\rho_V^0 \log \rho_V^0)$$

Casini defines the right-hand side of the Bekenstein bound as the difference between the expectation value of the modular Hamiltonian in the excited state and the vacuum state,

The ingenious proposal of Casini is to replace  $2 \pi R E$ , by:

$$K_v = \text{tr}(K\rho_V) - \text{tr}(K\rho_V^0)$$

With these definitions, the bound reads

$$S_V \leq K_V$$

which can be rearranged to give:

$$\text{tr}(\rho_V \log \rho_V) - \text{tr}(\rho_V \log \rho_V^0) \geq 0$$

This is simply the statement of positivity of quantum relative entropy, which proves the Bekenstein bound.

In Casini's work, on the right-hand side of the Bekenstein bound, a difficult point is to give a rigorous interpretation of the quantity  $2 \pi R E$ , where  $R$  is a characteristic length scale of the system and  $E$  is a characteristic energy.

This product has the same units as the generator of a Lorentz boost, and the natural analog of a boost in this situation is the modular Hamiltonian of the vacuum state

$$K = -\log (\rho_v^0)$$

As black holes entropic information formula is equal to Bekenstein universal bound.

As a side note, it can also be shown that the Boltzmann entropy is an upper bound to the entropy that a system can have for a fixed number of microstates

Where  $S(\rho_v)$  is the Von Neumann entropy of the reduced density matrix  $\rho_v$  associated with  $V$ ,  $V$  in the excited state  $\rho$ , and  $S(\rho_v^0)$  is the corresponding Von Neumann entropy for the vacuum state  $\rho^0$

Finally, we obtain:

$$\begin{aligned} K_v &= \text{tr}(K\rho_v) - \text{tr}(K\rho_v^0) = \\ &= \frac{kc^3 \ln(2) t_{evap}}{16\pi^2 GM} = \\ S_v &= S(\rho_v|\rho_v^0) = S(\rho_v) - S(\rho_v^0) = \\ &= -(\text{tr}(\rho_v \log \rho_v)) + \text{tr}(\rho_v^0 \log \rho_v^0) = \\ &= \frac{2\pi kRE}{\hbar c} \end{aligned}$$

Naïve definitions of entropy and energy density in quantum field theory suffer from ultraviolet divergences. In the case of the Bekenstein bond, ultraviolet divergences can be avoided by

taking the differences between the quantities calculated in the excited state and the same quantities calculated in the empty state.[25]

### 2.3 Generalization of the Entropic Information Theory framework to Dark Matter with the informational content of the universe

How much bits of information are there in the content of the observable universe?"

Going back as far as the late 1970s, since, in several studies, this question has been addressed and several answers has been given. Per example, using the Bekenstein–Hawking formula for the black-hole entropy [26], [27] the information content of the universe has been calculated by Davies [28]

$$I \approx \frac{2\pi G M_u^2}{hc} = 10^{120} \text{ Bits}$$

where

G is the gravitational constant,  
 $M_u$  is the mass of the universe enclosed within its horizon,  
 h is Planck's constant,  
 c is the speed of light.

The Wheeler's approach which estimated the number of bits in the present universe at  $T = 2.735 \text{ K}$  from entropy considerations, resulting in  $8 \times 10^{88}$  bits content [29]. Or Lloyd which took a similar approach and estimated the total information capacity of the universe as [30]

$$I = \frac{S}{k_B \ln(2)} \approx (\rho c^5 t^4)^{3/4} \approx 10^{90} \text{ Bits}$$

where:

S is the total entropy of the matter dominated universe,  
 $k_B$  is the Boltzmann constant,  
 $\rho$  is the matter density of the universe,  
 t is the age of the universe at the present time,  
 c is the speed of light,  
 h is the reduced Planck constant.

"If the principle of mass-energy-information equivalence is correct and information does have mass, a digital informational universe would contain a lot of it, and perhaps the missing dark matter could be just information," Vopson said.

Vopson even went so far as to speculate that the dark matter that holds galaxies together could also be composed of information. He said that since for more than 60 years we have been trying unsuccessfully to understand what dark matter is, it could very well be information.

It is well accepted that the matter distribution in the Universe is ~5% ordinary baryonic matter, ~27% dark matter and ~68% dark energy [31].

The estimation of the bits of information contained in the observable universe can be calculated via the entropic information approach giving an accurate calculation of the bit content from the new formulation of entropy, entropy based on the mass of the information bit given as follows:

$$S = k^2 \frac{T \ln(2) t}{h}$$

with Einstein mass–energy equivalence, Energy= mc<sup>2</sup>, this is

$$S = mc^2 \frac{k \ln(2) t}{h},$$

We use the mass of the universe and the age of this one for the calculations as

$$mass_{univ} = 2.78 \times 10^{54} \text{ kg}$$

$$t_{univ} = 4.361170766 \times 10^{17} \text{ seconds}$$

$$2.78 \times 10^{54} \times 299792458^2 \times 1.380649 \times 10^{-23} \times \ln(2) \times 4.361170766 \times 10^{17} \div (6.6267015 \times 10^{-34}) =$$

$$1.5736228e+99$$

Figure 8 Estimation of the number of bits of information in the observable universe based on entropic information entropy new alternative formulation :  $S = mc^2 \frac{k \ln(2) t}{h}$  with  $mass_{univ} = 2.78 \times 10^{54} \text{ kg}$  and  $t_{univ} = 4.361170766 \times 10^{17} \text{ seconds}$

N.B: Here, with the entropic information theory approach to the notion of entropy, the relationship between thermodynamic entropy S and Shannon entropy H given by  $S = kH \ln(2)$ . is not used to convert the result into informational terms because the result of the entropic information theory approach to entropy is expressed in informational terms, indeed It is the mass bit of the information that has been implemented to the initial equation considered, so the results of the entropy are expressed in number of bits. Indeed, as seen above, the theory of entropic information is based on the number of bits of the system, the number of bits necessary to specify the real microscopic configuration among the total

number of microstates allowed and thus characterize the macroscopic states of the system considered.

This number estimated by calculation from entropic information theory is remarkably close to an estimate of the bit-of-information content of the Universe with  $10^{94}$  bits that would be sufficient to account for all the dark matter missing in the visible Universe following Vopson using the reasoning developed following [32]

Taking the estimated mass of our Milky Way galaxy as  $\sim 7 \times 10^{11} M_{\odot}$  solar masses [33], and using the mass of the sun  $M_{\odot} \sim 2 \times 10^{30}$  Kg, then the estimated dark matter mass in our galaxy is  $M_{\text{Dark\_Matter}} \sim 3.78 \times 10^{12} M_{\odot} = 7.56 \times 10^{42}$  Kg. Assuming that all the missing dark matter is made up of bits of information, then the entire Milky Way galaxy has Nbits (Milky Way) =  $M_{\text{Dark\_Matter}} / m_{\text{bit}}(T=2.73\text{K}) = 2.59 \times 10^{82}$  bits. The estimated number of galaxies in the visible Universe is  $\sim 2 \times 10^{12}$  [34], so the estimated total number of bits of information in the visible Universe is  $\sim 52 \times 10^{93}$  bits. Remarkably, this number is reasonably close to another estimate of the Universe information bit content of  $\sim 10^{87}$  given by Gough in 2008 using Landauer's principle via a different approach [35].

Vopson estimated that around  $5,2 \times 10^{94}$  bits would be enough to account for all the missing dark matter in the observable universe [32] [36],[37].

The estimated bit content of the observable universe from entropic information is  $10^{99}$  bits. Note that  $10^{99}$  bits is significantly less than the theoretical maximum information content of the universe of  $10^{123}$  bits provided by applying the holographic principle [26],[38] to the universe. In the maximum information scenario, corresponding to the universe being a single black hole,  $10^{123}$  elementary squares of Planck length are needed to cover the surface of the current known universe

## 2.4 Generalization of the Entropic Information Theory framework to Dark Energy with the energy of informational content of the universe

From [35], we can read that Landauer's principle was originally proposed to describe the energy dissipation when information is overwritten in computer systems and subsequently used to predict the future limits to shrinking computer circuit size [39-46]. Moreover, Landauer showed that any erasure of information is necessarily accompanied by heat dissipation [47]. A corresponding minimum  $k_B T \ln(2)$  of heat energy has to be dissipated into the surrounding environment to increase the environment's thermodynamic entropy in compensation, and in accord with the second law of thermodynamics. The total amount of information is conserved as the surrounding environment effectively contains the erased information, although clearly no longer in a form that the computer can use. More generally, Landauer's principle applies to all systems in nature so that any system, temperature T, in which information is 'erased' by some physical process will output  $k_B T \ln(2)$  of heat energy per bit 'erased' with a corresponding increase in the information of the environment surrounding that system. Information is therefore directly bound up with the fundamental physics of nature. This strong interdependence between nature and information is emphasized by astrophysicist John Wheeler's slogan "it from bit" and computer scientist Rolf Landauer's maxim "information is physical". Landauer's principle is fully compatible with the laws of thermodynamics [22], [46], [48], [49]. Landauer's principle can be derived from

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microscopic considerations [21] as well as derived from the well-established properties of the Shannon-Gibbs-Boltzmann entropy [22]. The principle thus appears to be fundamental and universal in application.

Landauer's principle provides an equivalent energy for each bit of information or bit of entropy. Landauer showed that information is physical since the erasure of a bit of information in a temperature system,  $T$  results in the release of a minimum  $k_B T \ln(2)$  of energy into the system environment [22][50]. Landauer's principle has now been experimentally verified for classical bits and quantum qubits [38], [51-52].

This important physical prediction that links information theory and thermodynamics was experimentally verified for the first time in 2012 [53].

However, information entropy is equivalent to thermodynamic entropy when the same degrees of freedom are considered. The information entropy of the physical world is thus the number of bits needed to account for all possible microscopic states. Then each bit of information is equivalent to  $\Delta S = k_B T \ln(2)$  of thermodynamic entropy leading to Landauer's principle that  $\Delta S T = k_B T \ln(2)$  of heat is dissipated when a computer logic bit is erased [35].

Landauer's principle clearly identifies temperature as the only parameter connecting information to energy.

The equivalent energy of the total information, given by  $N k_B T \ln(2)$ , is proportional to both the total number of bits and the temperature, and therefore proportional to the volume of the universe as in the volume model, and proportional to the surface of the universe in the holographic model.

The equivalent Landauer energy of these elementary bits would be defined in a form and value identical to the characteristic energy of the cosmological constant.

We obtain the equivalent Landauer energy of a fundamental bit of information in a universe at temperature,  $T_{univ}$ ,  $\rho_{tot}$  is the total density of matter in our universe (baryon + dark).

$$k_B T_{univ} \ln(2) = \sqrt[4]{\frac{15 \rho_{tot} \hbar^3 c^5}{\pi^2}} \ln(2)$$

This Landauer bit energy is defined identically to the characteristic energy of the cosmological constant, the cosmological constant is closely associated with the concept of dark energy. The right-hand side of this equation is identical to equation 17.14 of [54] for the characteristic energy of the cosmological constant—with the sole addition of  $\ln(2)$  to convert between entropy units—between natural information units, nats, and bits. Information bit energy might then explain the low milli-eV (0.003 eV) characteristic energy of  $\Lambda$ , which Peebles [54] considered to be 'too low to be associated with any relevant particle physics.[55]

The dark energy density in the universe is about  $7 \times 10^{-30} \text{g/cm}^3$  on average according to Wikipedia. This is uniform throughout the Hubble volume of the entire universe i.e. the volume of the universe with which we are in causal contact. The Hubble volume is  $10^{31} \text{ly}^3$  i.e. cubic light years. This gives  $8.46732 \times 10^{84} \text{cm}^3$  as the volume of the universe. Using the mass-energy equivalence, you find that the total dark energy content in the entire universe is around  $10^{69} \text{Joules}$ .

If we use the value obtained by the informational entropy for the estimation of the number of bits corresponding to all the mass content of the whole observable universe

$$2.78 \times 10^{54} \times 299792458^2 \times 1.380649 \times 10^{-23} \times \ln(2) \times 4.361170766 \times 10^{17} \div (6.6267015 \times 10^{-34}) =$$

$$1.5736228e+99$$

Figure 9 Estimation of the number of bits of information of the observable universe based on entropic information entropy  
 new alternative formulation :  $S = mc^2 \frac{k \ln(2)t}{h}$  with  $mass_{univ} = 2.78 \times 10^{54} \text{ kg}$  and  $t_{univ} = 4.361170766 \times 10^{17} \text{ seconds}$

With the formula of total information equivalent energy, given by the Landauer's principle:

$$N_{Bits} k T_{univ} \ln(2)$$

$$T_{univ} = 2.725 \text{ K}$$

$$1.5736228 \times 10^{99} \times 1.380649 \times 10^{-23} \times 2.725 \times \ln(2) =$$

$$3.5092427e+76$$

Figure 10 Estimation of the energy associated with the number of bits of information of the observable universe based on the Landauer's principle  $N_{Bits} k T_{univ} \ln(2)$  with temperature of universe,  $T_{univ} = 2.725 \text{ K}$

The entropic information theory can provide a quantitative account for dark energy, accounting to the present energy value,  $\sim 10^{70}$  Joules. The entropic information theory provides an estimation of  $3.5 \times 10^{76}$  Joules remarkably close to this estimation.

### 3. Results and Discussion

By way of introduction, we have reviewed some fundamental claims about quantum information theory from the perspective of the entropic information theory approach based on the mass of the information bit. Indeed, after having defined the notion of quantum information, quantum entanglement, non-locality as well as the consideration of the measurement problem within the approach of the generalization of entropic information, this generalization lifts the veil on certain new formulations of notions of entropy based on the degree of freedom of the system considered. This new approach to entropic information is built on the bit of information considered as the change of the quantum state due to the modification of a degree of freedom of the system considered. This new informational approach taking into account the mass of the bit of information reveals new formulas of entropy based on different mathematical and physical perspectives; leading to the expression of new formulas of this particular entropy based on different relations, Einstein's mass-energy equivalence, the Einstein Planck relation or based on the Avogadro number.

These perspectives, as part of this generalization of entropic information theory, can lead us to in-depth reflection. Indeed, we can see a new perspective of defining life in the approach of entropic information theory. Based on the mass of the information bit and entropic considerations, we can see life following entropic information theory as the storage of information and the possibility of updating it through a self-learning process to perpetuate this form of information, because the structures are ordered by storing information, structure is information building. We can think of entropic structures as ordered as follows: non-living entropic structure, living entropic structure, "thinking entropic structure", "knowing entropic structure" (conscious one). With the fact that without information, the conscious has nothing to know. The conscious is considered as a specific processing of information based on "conscious individuality", individuality based on memory and memory on the storage of information. More specifically on the human brain, it calculates but not only, in fact, some human brain processes (such as intuition or creation seen as a not computable brain processes from what emerge an new concept out of logical algorithm) are outside algorithmic logics so that consciousness or the universe cannot be coded.

After all these distant considerations based on entropic considerations, following the continuation of the new formulations of the new different forms of entropy in the approach of entropic information theory, we have shown that the entropic information formula of the black hole taking into account the mass of bit of information at Hawking temperature reveals a new formulation of bekenstein-Hawking entropy.

With the help of Casini's work on the entropy of von Neumann and Bekenstein related. And since the new formulation of the entropic information black hole formula is also equal to the Bekenstein bound, this leads to new considerations about quantum gravity. Indeed, the entropic information formula of black holes calculates the entropy of Hawking radiation as the entangled information of the black hole initially considered, this up to the quantum level of the system, the degrees of freedom describing the black hole, and this regardless of the law of the Bekenstein-Hawking entropy area, providing a sufficient microscopic description of how this entropy occurs, showing that the evaporation process of black holes conforms to the principle of unitarity. In addition, this approach avoids ultraviolet divergences. These perspectives should be understood as the fine-grained entropy formulas discovered by Ryu and Takayanagi. In fact, the entropy of black holes turns out to be a special case of the Ryu-Takayanagi conjecture. The Ryu-Takayanagi formula is a general formula for fine-grained entropy of quantum systems coupled with gravity. This focused on the process of emerging quantum gravity through the fundamentality of entangled quantum information [25].

After that, we looked at an ancient but still topical question about the informational content of the observable universe: "How much information content is in the observable universe? We were able to answer this question using the entropic information formula regarding entropy and the mass of the information bit. We obtain an estimate remarkably close to the dark matter of the universe with regard to the approach of entropic information theory with a result obtained equal to  $10^{99}$  bits to be related to Melvin Vopson recent previous estimate based on a result of  $10^{94}$  bits of information, which would be sufficient, following Vopson, to account for all the dark matter missing in the visible Universe [32], [36], [37]. The result of the entropic information theory approach can also be related to Lloyd's approach which estimated the total information capacity of the universe at  $10^{90}$  bits of information [30]. In addition, we calculated the energy associated with this number of bits of informational content of the observable universe; to obtain again a result remarkably close to the actual estimate of dark energy being  $10^{70}$  joules; indeed, the entropic information theory approach gives an estimate of  $10^{76}$  joules for estimating all the dark energy of the entire observable universe. A more detailed perspective on the estimation of dark energy, leads to see the result obtained in the

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form of the energy characteristic of the cosmological constant which is closely associated with the concept of dark energy.

#### 4. Conclusions

A generalization of the entropic information approach has been presented as a complete new explanatory mathematical framework capable of explaining various processes finding their explanation through the fundamentality of entangled quantum information. The Entropic information theory is based on the mass of the bit of the information of the system considered. This is to be putted in relation with the definition of quantum information which, from the point of view of entropic information, is a change of quantum state due to the modification of a degree of freedom in the quantum system considered. This definition of the notion of quantum information being itself consistent with the definition of entanglement proposed, in fact, according to entropic information, quantum entanglement can be defined as the physical phenomenon that occurs when a group of particles is generated, interacts, or shares spatial proximity in such a way that one or more degrees of freedom are shared at the quantum level between each particle of the group; a particle that cannot be described independently of the quantum state of others because they share degrees of freedom. We must be aware that a semi-classical perspective can explain the notion of entanglement; regarding entangled photon and theory of special relativity, all photons moving at the speed of light, the separation between those two points would be zero from the perspective of those photons. Quantum theory is thus local in the strict sense defined by special relativity. These quantum perspectives are themselves consistent with the entropic information approach of the measurement problem, in essence that during a measurement, to be informed by the measurement system, the measurer needs information; take information from the system during the measurement process; but with measurement, the new system to consider is the system: "measurer-the thing to be measured", the two parts of the system considered share one or more degrees of freedom of the considered quantum system.

Entropic information theory as an informational approach is mathematically based on the mass of bits of information, this was injected into the hidden thermodynamics of Louis de Broglie revealing a new entropic relationship based on the degree of freedom, on the number of bits of the system considered. That lets appears some new entropy formulations based on different mathematical and physical perspectives; leading to the expression of new formulations of this particular entropy based on different relations; Einstein mass–energy equivalence, Planck Einstein relation or based on the Avogadro number. Based on those perspectives some far deep considerations have been explored. Indeed, based on the mass of the bit of information and entropic considerations, we can define life following the entropic information theory as the storage of the information and the possibility to updated it by a process of self-learning to perpetuate that form of information; as the structures are ordered by storage of information. Mathematically, at this point, we have been capable to express some several new entropy formulations wherein we have introduced the Hawking temperature formula which express itself by all the constant in modern physics bringing together: relativity, gravitation, quantum physics and thermodynamics, leading to new expressions of black holes entropy. The entropic Information Theory approach give a mathematical interpretation of the microstates considered when calculating the entropy of black holes. Moreover, the new formulation of the entropic information approach, based on the information bit, shows that the evaporation process of black holes is consistent with the principle of unitarity while avoiding ultraviolet divergences and expressing the fine-grained gravitational entropy of a black hole using the rules of gravity. This allows a semi-classical gravitational approach to expressing the fine-grained gravitational entropy of the black hole.

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Moreover, the black hole entropy information formula solves the black hole information paradox problem by explaining the information processes involved in black hole entropy down to the quantum level independently of the area law by providing a sufficient microscopic description of how this entropy arise. Indeed, the entropic information formula of black holes calculates the entropy of Hawking radiation as the entangled information of the black hole initially considered, this up to the quantum level of the system, the degrees of freedom describing the black hole, and this regardless of the law of the Bekenstein-Hawking entropy area, providing a sufficient microscopic description of how this entropy occurs. At this level we can relate the entropic information of black holes with the Ryu-Takayanagi formula which is a general formula for the fine-grained entropy of quantum systems coupled to gravity. As The black hole entropy horizon law which turns out to be a special case of the Ryu-Takayanagi conjecture, we can put emphasis on the process of emergence of quantum gravity through the fundamentality of entangled quantum information [25].

After that, we looked at an old but still topical question about the informational content of the observable universe and calculated the energy associated with this number of bits of informational content in the visible universe. These calculations obtained respectively accordingly,  $10^{99}$  bits of informational content and for the energy associated with this informational content a result of  $10^{76}$  joules. These estimates are consistent with the previous estimate such as Melvin Vopson's estimate based on a result of  $10^{94}$  bits of information, which would be sufficient, following Vopson, to account for all the dark matter missing in the visible Universe [32] [36] [37]. The result of the entropic information theory approach can also be related to Lloyd's approach which estimated the total information capacity of the universe at  $10^{90}$  bits of information [30]. The entropic information theory approach gives an estimate of  $10^{76}$  joules for estimating all the dark energy of the entire observable universe, again a result remarkably close to the actual estimate of dark energy being  $10^{70}$  joules. A more detailed perspective on the estimation of dark energy, leads to see the result obtained in the form of the energy characteristic of the cosmological constant which is closely associated with the concept of dark energy. We are here, in the presence of a particular mathematical theoretical framework capable of explaining in the approach of entropic information theory based on the mass of the bit of information various processes as being several aspects of the entangled informational content of the universe.

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