A note on the new method of computing the inverse of 3×3 square matrices and its applications

Abstract

This paper presents a simple and easy way of finding the inverse of 3×3 square matrices and some applications of matrices in the real world and theirs solutions. Many challenging problems can be addressed surprisingly rapidly by applying the correct strategy. This work presents such strategies that grant a more complete way of tackling complicated problems .

Keywords: Matrix, inverse, 3×3, applications

1 Introduction

In the 18th and 19th centuries, matrices and determinants were discovered and improved. Initially, they were concerned with the transformation of geometric objects and the solution of linear equation systems. Historically, the determinant was prioritized over the matrix. Matrices are the initial consideration in modern Linear Algebra.

Numerous issues can be resolved using matrices, such as rigid body equilibrium, a system of linear equations, graph theory, game theory, the Leontief economics model, forest management, computer graphics and computed tomography, genetics, cryptography, and electrical networks, to name a few. For displaying and discussing problems that arise in real-world circumstances, matrices are a useful tool. Matrix calculations are used in electrical circuit design, quantum physics, optics, battery power output estimation, and resistor-based electrical energy conversion into other useful forms of energy.

Matrices are important in projecting three-dimensional pictures onto a two-dimensional screen, which gives the illusion of motion. In Economics, matrices are used to calculate gross domestic products, which aids in the efficient calculation of goods production. Matrices are the building blocks of robot movement. Robots' movements are pre-programmed. Geodesy is the branch of applied mathematics concerned with measuring or determining the shape of the earth or with precisely locating points on the earth's surface. Gauss developed elimination around 1800 and used it to solve least square problems in celestial computations and later in computations to measure the earth and its surface.

Despite the fact that Gauss' name is connected with this technique for removing variables from a system of linear equations, there has been previous research on the issue. The method of solving a system of three equations in three unknowns by "Guassian" elimination has been discovered in Chinese writings dating back several centuries. For a long time, Gaussian elimination was thought to be a part of the evolution of geodgesy rather than mathematics. In a manual on geodesy produced by Wihelm Jordan, Gaussian-Jordan elimination had its first appearance in print. Many people mistakenly believe that Camille Jordan, a great mathematician, is the Jordan in the "Gauss-Jordan elimination.

Some of the current strategies for determining the inverse of a matrix, such as Cramer's rule, Gauss elimination method, Gauss-Jordan method, and Crout's method, were presented by.[5] Knowing the definition of a matrix, how to add and multiply matrices, how to use matrix notation to solve systems of linear equations, and how to evaluate a determinant are all included in the phrase "minimal mathematical background." The majority of this information is covered in.[6]Article[7] used a general formula of finding the inverse of a matrix. All these techniques are considered to be cumbersome and times wasting. Matrices and their inverses are used by programmers for coding or encrypting a message.

For communication, a message is created as a binary integer sequence that uses code theory to be decoded. Consequently, those equations are resolved using matrices. Matrix-based word problems with practical applications are common in the fields of pure and applied mathematics. In order to handle problems affecting real-life, we aim to apply a different approach in this work to obtain the inverse of matrices. In,[4] the power of Excel was exploited to tackle matrices-based word problems and exercises.[4]

We present the alternative method of finding the inverse of 3×3 matrix and using that method to solve a real word problems and exercises involving matrices. We think is method is easy and faster.

1.1 matrix inverse

When working with an ordinary number, if we state $ab = 1$, we can then get $a = \frac{1}{b}$ if $b \neq 0$. It is expressed as $b = a^{-1}$ or $aa^{-1} = a^{-1}a = 1$. The division operation in ordinary numbers is analogous to the matrix inverse. Assume that matrix multiplication produces an identity matrix $AB = I$ as the matrix product. If such a matrix B exists, then it is a unique matrix and we can write $B = A^{-1}$ or $AA^{-1} = I = A^{-1}A$. Only a square matrix can have an inverse matrix (that is a matrix having the same number of rows and columns). Matrix inverse does not always exist, though. Therefore, if a square matrix does not have an inverse, we refer to it as singular; otherwise, we refer to it as nonregular. There are numerous approaches to compute matrix inverse in linear algebra because it is such a crucial operation.

Using Cramers rule, which makes use of the determinant of the original matrix, is the most straightforward method for finding the matrix inverse for a small matrix (order 2 or 3). Remember that if and only if the determinant is zero, a square matrix is unique (i.e., has no inverse). Scaling the adjoint of the original matrix with the determinant allows one to get the matrix inverse.
The formulae for inverse matrix is given by

$$A^{-1} = \frac{1}{detA} AdjA \tag{1}$$

Let consider Gauss-Jordon Method to find the inverse of the matrix

A= $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

$A = IA$

$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ A

Reducing the matrix A to reduced row echelon form

$\begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ A

$R_3 - 4R_1, R_3 - 2R_1$

$\begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -15 \\ 0 & -1 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -4 \\ 1 & 0 & -2 \end{bmatrix}$ A

$\frac{-1}{5} R_2$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 3\\0 & -1 & -4\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\0 & \frac{-1}{5} & \frac{-4}{5}\\1 & 0 & -2\end{bmatrix} A$$

$R_3 + R_2$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 3\\0 & 0 & -1\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\0 & \frac{-1}{5} & \frac{-4}{5}\\1 & \frac{-1}{5} & \frac{-6}{5}\end{bmatrix} A$$

$(-1)R_3$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 3\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\0 & \frac{1}{5} & \frac{4}{5}\\-1 & \frac{1}{5} & \frac{6}{5}\end{bmatrix} A$$

$R_2 - 3R_3, R_1 - 4R_3$

$$\begin{bmatrix}1 & 2 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}4 & \frac{-4}{5} & \frac{-19}{5}\\3 & \frac{-4}{5} & \frac{-14}{5}\\-1 & \frac{1}{5} & \frac{6}{5}\end{bmatrix} A$$

$R_1 - 2R_2$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-2 & \frac{4}{5} & \frac{9}{5}\\3 & \frac{-4}{5} & \frac{-14}{5}\\-1 & \frac{1}{5} & \frac{6}{5}\end{bmatrix} A$$

$I = A^{-1}A$

$$A^{-1} = \begin{bmatrix}-2 & \frac{4}{5} & \frac{9}{5}\\3 & \frac{-4}{5} & \frac{-14}{5}\\-1 & \frac{1}{5} & \frac{6}{5}\end{bmatrix}, A^{-1} = \frac{1}{5}\begin{bmatrix}-10 & 4 & 9\\15 & -4 & -14\\-5 & 1 & 6\end{bmatrix}$$

Let consider another method of finding inverse of a matrix

$$A = \begin{bmatrix}2 & 3 & 4\\4 & 3 & 1\\1 & 2 & 4\end{bmatrix}$$

We use the normal procedure of finding the determinant of matrix.

$detA = -5$.

We compute the co-factors using plus and minus

Cofactor of $2 = +\begin{bmatrix}3 & 1\\2 & 4\end{bmatrix}$

$= 10$

Cofactor of $3 = -\begin{bmatrix}4 & 1\\1 & 4\end{bmatrix}$

$= -15$

Cofactor of $4 = +\begin{bmatrix}4 & 3\\1 & 2\end{bmatrix}$

$= 5$

Cofactor of $4 = -\begin{bmatrix}3 & 4\\2 & 4\end{bmatrix}$

$= -4$

Cofactor of $3 = +\begin{bmatrix}2 & 4\\1 & 4\end{bmatrix}$

$= 4$

Cofactor of $1 = -\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$

$= -1$

Cofactor of $1 = +\begin{bmatrix}3 & 4\\3 & 1\end{bmatrix}$

$= -9$

Cofactor of $2 = -\begin{bmatrix} 2 & 4 \\ 4 & 1 \end{bmatrix}$
$= 14$
Cofactor of $4 = +\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$
$= -6$
$A = \begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & -6 \end{bmatrix}$
$A^T = \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$
$Adj\,A = \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$
From equations1
$A^{-1} = \frac{-1}{5} \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

We present an altervative approach of finding inverse of 3×3.
Let consider the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The order of the matrix is 3×3. We tried to find the inverse of this matrix using different approach. For the determinant, we use the usual processes of finding the determinant. Our emphasis will be how to find the cofactors. Here we are not going to consider plus or minus to find the minors.We apply the trick from.[1] We copy the first to two columns from the left and place them in the right. We then copy the first two rows (that is row1, and row 2) and place it at the bottom of rows 3 to obtain the order 5×5.

$$\begin{pmatrix} \text{a\_11} & \text{a\_12} & \text{a\_13} & \text{a\_11} & \text{a\_12} \\ \text{a\_21} & \text{a\_22} & \text{a\_23} & \text{a\_21} & \text{a\_22} \\ \text{a\_31} & \text{a\_32} & \text{a\_33} & \text{a\_31} & \text{a\_32} \\ \text{a\_11} & \text{a\_12} & \text{a\_13} & \text{a\_11} & \text{a\_12} \\ \text{a\_21} & \text{a\_22} & \text{a\_23} & \text{a\_21} & \text{a\_22} \end{pmatrix}$$

The first row and the first column is cancelled out to obtain the matrix below

$$\begin{bmatrix} a_{22} & a_{23} & a_{21} & a_{22} \\ a_{32} & a_{33} & a_{31} & a_{32} \\ a_{12} & a_{13} & a_{11} & a_{12} \\ a_{22} & a_{23} & a_{21} & a_{22} \end{bmatrix}$$

$$M = \begin{pmatrix} \text{a22} & \text{a23} \\ \text{a32} & \text{a33} \end{pmatrix}$$

$$\begin{bmatrix} a_{22}a_{23} - a_{32}a_{23} & a_{32}a_{13} - a_{33}a_{12} & a_{12}a_{23} - a_{13}a_{22} \\ a_{31}a_{23} - a_{33}a_{21} & a_{33}a_{11} - a_{31}a_{13} & a_{13}a_{21} - a_{11}a_{23} \\ a_{21}a_{32} - a_{22}a_{31} & a_{31}a_{12} - a_{32}a_{11} & a_{22}a_{11} - a_{12}a_{21} \end{bmatrix}$$

Consider the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

We use the normal procedure of finding the determinant of matrix.
$detA = -5$.
Adjoint of the matrix

$$\begin{pmatrix} 2 & 3 & 4 & 2 & 3 \\ 4 & 3 & 1 & 4 & 3 \\ 1 & 2 & 4 & 1 & 2 \\ 2 & 3 & 4 & 2 & 3 \\ 4 & 3 & 1 & 4 & 3 \end{pmatrix}$$

$$A = \begin{bmatrix} 3 & 1 & 4 & 3 \\ 2 & 4 & 1 & 2 \\ 3 & 4 & 2 & 3 \\ 3 & 1 & 4 & 3 \end{bmatrix}$$

We take two columns each and find the determinant but we write it in rows form

To obtain the adjoint $A = \begin{bmatrix} (3\times 4)-(2\times 1) & (2\times 4)-(3\times 4) & (3\times 1)-(3\times 4) \\ (1\times 1)-(4\times 4) & (2\times 4)-(1\times 4) & (4\times 4)-(1\times 2) \\ (2\times 4)-(1\times 3) & (1\times 3)-(2\times 2) & (2\times 3)-(3\times 4) \end{bmatrix}$

$$Adj A = \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$$

From equations1

$$A^{-1} = \frac{-1}{5} \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$$

Let consider another example from[2]

Example 1.1 $A = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix}$

Det A$=2(0-24)-3(0-6)+5(16-1)=45$
We apply the trick from[1] to find the adjoin matrix of A.

$$\begin{pmatrix} 2 & 3 & 5 & 2 & 3 \\ 4 & 1 & 6 & 4 & 1 \\ 1 & 4 & 0 & 1 & 4 \\ 2 & 3 & 5 & 2 & 3 \\ 4 & 1 & 6 & 4 & 1 \end{pmatrix}$$

We now cancel the first row and first column to obtain the matrix of the form $\begin{pmatrix} 1 & 6 & 4 & 1 \\ 4 & 0 & 1 & 4 \\ 3 & 5 & 2 & 3 \\ 1 & 6 & 4 & 1 \end{pmatrix}$,

$$Adj A = \begin{pmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{pmatrix}$$

$$A^{-1} = \frac{1}{45} \begin{pmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{pmatrix}$$

2 Application of matrices to the real life

2.1 In Agriculture

Let consider the barnyard problem from[3] ,but this time envision a menagerie of chickens, 1-horned Sheep, and 2-horned goats to get a feel for linear systems and how to solve them. You make a fast count and discover 12 heads, 38 feet, and 10 horns. Can you guess out how many of each animal there are? To proceed, give each animal a variable (c for chickens, s for sheep, and g for goats) and write an equation for each property. The numbers in front of each variable, or coefficients, represent the quantity of that attribute held by each animal.

$$c + s + g = 12\ heads$$
$$2c + 4s + 4g = 38\ feet$$
$$0c + s + 2g = 10\ horns$$

To solve this problem, we are going to use the method from[1] We represent this in augmented forms:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} c \\ s \\ g \end{pmatrix} = \begin{pmatrix} 12 \\ 38 \\ 10 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

$$DetA = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = 1$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 2 & 4 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 2 & 4 \end{pmatrix}$$

We cancelled the first row and the first column to get this matrix

$$A = \begin{pmatrix} 2 & 2 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 2 \end{pmatrix}$$

$$AdjA = \begin{pmatrix} 2 & -1 & 0 \\ -2 & 2 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

By using the formula1

$$A^{-1} = \begin{pmatrix} 2 & -1 & 0 \\ -2 & 2 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

Applying equation **??**,

$x = \begin{pmatrix} 2 & -1 & 0 \\ -2 & 2 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 12 \\ 19 \\ 10 \end{pmatrix}$, *where* x= $\begin{pmatrix} c \\ s \\ g \end{pmatrix}$

$$x = \begin{pmatrix} 2(12) - 19 + 0 \\ -2(12) + 2(19) - 10 \\ 12 - 19 + 10 \end{pmatrix}$$

x= $\begin{pmatrix} 5 \\ 4 \\ 5 \end{pmatrix}$

2.2 In Finance

After his father's passing, Stephen received his father's possessions. Stephen divided his \$12,000 inheritance into three distinct investment vehicles: money markets earning a 3% annual interest, municipal bonds paying a 4% annual interest, and mutual funds providing a 7% annual interest. Stephen spent more money on mutual funds than he did on municipal bonds (\$4000 more). In one year, \$670 in interest was generated. Our goal is to determine the amount he invested in each sort of fund.

solution
Let x be amount invested in money-market funds
Let y be amount invested in Municipal bonds funds
Let Z be amount invested in Mutual funds

$$\begin{cases} x + y + z = \$12000 \\ z = y + \$4000 \\ 0.03x + 0.04y + 0.07z = \$670 \end{cases}$$

We multiply the last equation by 100

$$\begin{cases} x + y + z = \$12000 \\ -y + z = \$4000 \\ 3x + 4y + 7z = \$67000 \end{cases}$$

We now write the system of equations in the augmented form

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 3 & 4 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12000 \\ 4000 \\ 67000 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 3 & 4 & 7 \end{pmatrix}$$

$detA$=-5

$$\begin{pmatrix} \not{1} & \not{1} & \not{1} & \not{1} & \not{1} \\ 0 & -1 & 1 & 0 & -1 \\ 3 & 4 & 7 & 3 & 4 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 0 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 1 & 0 & -1 \\ 4 & 7 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 0 & -1 \end{pmatrix}$$

$$AdjA = \begin{pmatrix} -11 & -3 & 2 & \\ 3 & 4 & -1 & 4 \\ 3 & -1 & -1 & \end{pmatrix}$$

By using the formulae 1:

$x = \frac{1}{-5} \begin{pmatrix} -11 & -3 & 2 \\ 3 & 4 & -1 \\ 3 & -1 & -1 \end{pmatrix} \begin{pmatrix} 12000 \\ 4000 \\ 67000 \end{pmatrix}$

where $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$,

$v = \begin{pmatrix} \$2000 \\ \$3000 \\ \$7000 \end{pmatrix}$

3 Conclusion

Alternative methods of determining the inverse of a 3×3 matrix have been shown in this paper, along with examples of how to use them to address matrices-related problems in the real world. Although the methods discussed in this work are not rigorous solutions to find inverse a matrix, such techniques can be an essential utility for students, particularly in exam settings where time is a major constraint in solving problems and obtaining good scores.

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